

# Improved Stability and Confinement in a High- $\beta$ Relaxed Plasma State

H. Y. Guo,<sup>1,2</sup> A. L. Hoffman,<sup>1</sup> K. E. Miller,<sup>1</sup> and L. C. Steinhauer<sup>1</sup>

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Relaxation in low- $\beta$  plasmas has long been recognized and usually been described by the Taylor relaxation principle. Recently, relaxation has also appeared in an extremely high- $\beta$  (over 85%) plasma formed by highly super Alfvénic translation of a spheromak-like compact toroid (CT) in the translation, confinement and sustainment (TCS) experiment. Strong flux conversion from toroidal to poloidal occurs during the highly dynamic relaxation process while the magnetic helicity is approximately preserved, leading to a novel relaxed state that exhibits a spherical-torus- (ST-) like field-reversed configuration (FRC). Modeling using the newly developed nearby-fluids theory shows that a broad core of the FRC-ST resembles a two-fluid minimum energy state. This FRC-ST state exhibits a significant improvement in confinement and remarkable stability. The latter is explained by a simple stability model, taking into account magnetic shear and centrifugal effects.

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**KEY WORDS:** Relaxation; high  $\beta$ ; two-fluid minimum energy state; field-reversed configuration; stability.

## INTRODUCTION

Experimental relaxed plasma states were first identified by Taylor 30 years ago [1]. He conjectured that a plasma may self-organize into a state of minimum magnetic energy subject to preserved magnetic helicity. However, the Taylor theory predicts force-free plasma states with  $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ , where  $\lambda$  is a global constant. These plasmas have intrinsically  $\beta = 0$ . Recognizing that realistic plasmas have finite  $\beta$ , several more general relaxation principles have been advanced [2,3]. One of the more promising is the *modern relaxation principle* [2] based on a two-fluid plasma model where the total magnetofluid energy is minimized while preserving both electron (magnetic) and ion helicities.

The state of minimum energy represents an important subset of flowing two-fluid equilibria [4]. The possibility of relaxation into a minimum energy state (MES) is very intriguing because a MES plasma object would be stable to all two-fluid instabilities and would thus have very low transport. In axisymmetric two-fluid equilibria there are two distinct sets of characteristic surfaces, the magnetic (and massless electron) surfaces  $\psi(r,z) = \text{const}$ , and the ion surfaces  $Y(r,z) = \psi + \varepsilon r u_\theta = \text{const}$ . This differs from standard MHD which has only one set of surfaces  $\psi = \text{const}$ . The parameter characterizing two-fluid effects is  $\varepsilon = \ell_i/L$ , the ratio of the ion skin depth  $\ell_i = c/\omega_{pi}$  to the characteristic length in the system  $L$ ; here  $\omega_{pi} = \sqrt{ne^2/m_i \varepsilon_0}$  is the ion plasma frequency. Note that the magnetic and massless-electron flow surfaces coincide, while the ion flow surfaces differ from the electron (magnetic) surfaces due to the toroidal rotation ( $u_\theta$ ) and two-fluid effects ( $\varepsilon$ ). In a two-fluid MES the ion and electron stream functions, which characterize the poloidal flows, are linear

<sup>1</sup> Redmond Plasma Physics Laboratory, University of Washington, Seattle, Washington, USA.

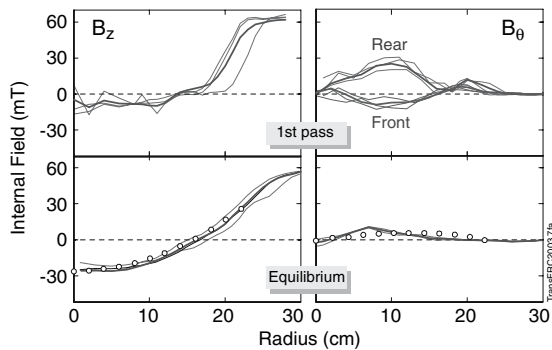
<sup>2</sup> To whom correspondence should be addressed: E-mail: hguo@aa.washington.edu.

functions of their respective surface variables. This is analogous to constant  $\lambda$  in a Taylor state.

This paper presents the first evidence of relaxation in an extremely high- $\beta$  plasma state, as indicated by the flux conversion and helicity preservation, as well as the confinement and stability properties of such a relaxed high- $\beta$  plasma state.

## FIRST EVIDENCE OF RELAXATION IN HIGH- $\beta$ PLASMAS

Relaxation in low- $\beta$  plasma states, such as spheromaks and reversed field pinches (RFP), has long been recognized and has usually been described by the Taylor relaxation principle. Recently, strong evidence for relaxation in an extremely high- $\beta$  (over 85%) plasma state has been obtained from the translation, confinement and sustainment (TCS) experiment [5]. This high- $\beta$  plasma state was produced by highly super Alfvénic translation of a spheromak-like compact toroid (CT). Detailed internal field measurements have demonstrated, for the first time, that the transition from the early highly dynamical state to the final equilibrium approximately preserves the magnetic helicity, while converting toroidal to poloidal flux. Flux conversion is a marker of a turbulent relaxation. Flux conversion from toroidal flux to poloidal is evident from the radial profiles shown in Figure 1 for the initial translated CT and the final equilibrium phase, measured at the midplane of the confinement chamber. The initial plasmoid has very little poloidal flux, but has strong net toroidal flux, with a slight reversal in toroidal field at one end. After extremely violent



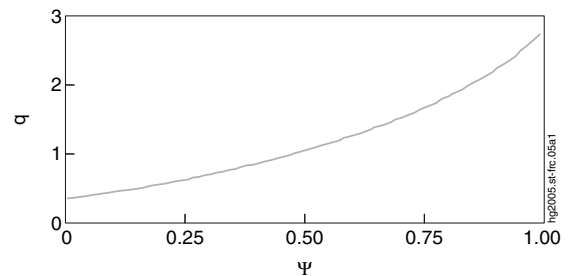
**Fig. 1.** Radial profiles of axial and toroidal fields  $B_z$  and  $B_\theta$  for first pass and equilibrium FRC. The thick solid lines represent the averaged profiles for many shots and time slices. Several individual profiles are also shown to illustrate possible variability. The circles shown during the equilibrium phase are for a Hill's vortex-like interpretive model.

reflections at the end mirrors of the confinement chamber, the disorganized plasmoid settles into a *preferred* near-FRC state with a substantial increase in the poloidal flux. Most of the oppositely directed toroidal fluxes are annihilated with only a reduced unidirectional toroidal flux remaining. Significant toroidal rotation has been observed, but flow measurements are incomplete for determining whether the ion helicity is conserved.

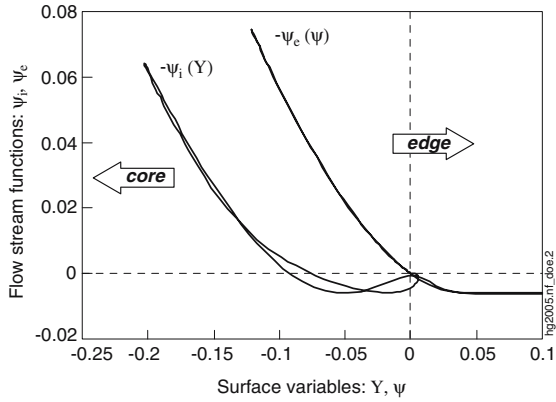
## A NOVEL RELAXED PLASMA STATE: FRC-ST

The final *relaxed* state is quiescent with high  $\beta$ , i.e. it is clearly *not* a Taylor state. Such a high- $\beta$  relaxed state can only arise as the result of a more general regulating principle, which controls the plasma evolution during the relaxation process. Compared to the other relaxed states such as spheromaks and RFPs, this new relaxed state has a much smaller and highly localized toroidal field. However, when combined with the high elongation ( $E \sim 4$ ), it results in a safety factor,  $q$ , exceeding unity over much of the configuration. Moreover, the highly localized toroidal field leads to a strong magnetic shear near the edge, as shown in Figure 2. Note that the  $q$ -profile is similar to that produced in a spherical-torus (ST), but is obtained here without a center column. Hence, this newly discovered high- $\beta$  relaxed plasma state has been named “FRC-ST”.

A recently developed nearby-fluids equilibrium model for flowing two-fluid plasma equilibria has been used to interpret the field and flow structure observed in the FRC-ST [7]. The most remarkable outcome is the discovery that a broad core of this *relaxed* high- $\beta$  plasma object is very close to a MES. As aforementioned, in a two-fluid MES state, the electron stream function  $\psi_e$  and the ion stream function  $\psi_i$  have a *linear* dependence on the surface functions, i.e.,  $d\psi_e/d\psi = M_e = \text{const}$  and  $d\psi_i/d\psi = M_i = \text{const}$ .



**Fig. 2.**  $q$  versus normalized flux  $\Psi = 1 - \psi/\psi_{\text{max}}$ ,  $\Psi = 0$  at the magnetic axis, and  $\Psi = 1$  at the edge.



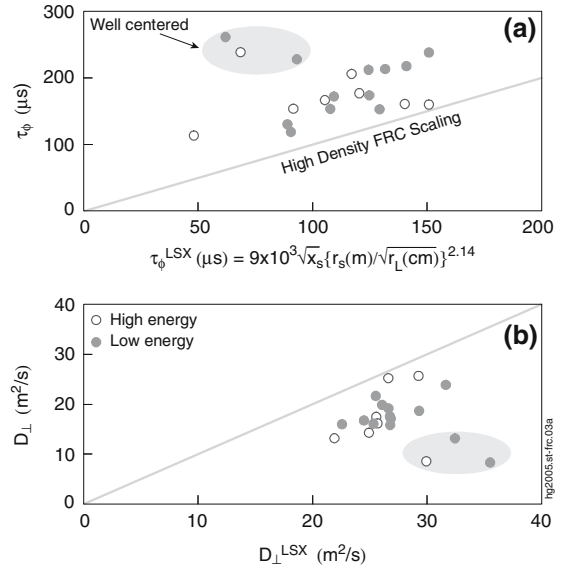
**Fig. 3.** Surface functions for electron and ion stream functions  $\psi_e$  ( $\psi$ ) and  $\psi_i$  ( $Y$ ), calculated using a dimensionless variable scheme.

$dY = M_i$ . Figure 3 shows the inferred electron and ion stream functions from the TCS data. Both the electron and ion stream functions appear to be linear in a large core region, indicating that a broad core of the plasma conforms to a minimum energy state. This likely results in the suppression of instabilities in the core, thus promoting good stability and confinement.

### IMPROVED CONFINEMENT AND STABILITY

Indeed, the FRC-ST configuration exhibits significantly reduced transport, with up to 4 times improvement in magnetic flux confinement times [6], compared to the previous scaling for the conventional  $\theta$ -pinch formed FRCs [8],  $\tau_\phi^{LSX} (\mu s) = 9 \times 10^3 x_s^{0.5} \{r_s(m)/\sqrt{r_L(cm)}\}^{2.14}$ , where,  $x_s \equiv r_s/r_c$  is the ratio of separatrix radius to flux conserver radius and  $r_L$  is the ion Larmor radius. This is shown in Figure 4(a), assuming  $T_i = T_e$ . The magnetic flux decays on a resistive diffusion time scale. For a rigid rotor profile and a uniform plasma resistivity, the flux lifetime  $\tau_\phi \approx (\mu_0/\eta)r_s^2/16$ . Figure 4(b) shows the corresponding values of  $D_\perp = \eta/\mu_0$  plotted against the predictions from the conventional  $\theta$ -pinch FRC scaling. The resistivity and, thus, the cross-field transport are significantly improved.

Furthermore, the FRC-ST exhibits remarkable stabilizing properties for the normally lifetime terminating  $n = 2$  centrifugally driven interchange mode present in the conventional  $\theta$ -pinch formed FRCs. It has usually been stabilized by applying static multipole fields to  $\theta$ -pinch formed FRCs. Recently, it has been demonstrated that rotating magnetic fields also provide strong stabilizing effects [9]. To examine the stability property of the FRC-ST, we have developed



**Fig. 4.** (a) Measured magnetic flux lifetimes for the FRC-STs versus the conventional  $\theta$ -pinch FRC scaling. (b)  $D_\perp$  derived from the flux life times.

a simple model by extending the energy principle approach by Bellan [10]. The added feature is the centrifugal effect due to the plasma rotation, represented by an extra term, i.e.,  $\Omega^2 r \nabla \rho$ , in the expression for  $\delta W$ , where  $\Omega$  is the plasma rotation rate, and  $\rho$  the mass density. We have obtained the following stability criterion for the rotational modes [6]:  $(\frac{dq}{d\Psi})^2 > \frac{3R^2 Z_0 \Omega^2}{n^2 V_A^2} \left| \frac{\nabla \rho}{\rho_0} \right| \sim \frac{4}{n^2}$  for typical FRC-STs, where  $\Psi = 1 - \psi/\psi_{max}$ ,  $Z_0$  is the axial distance to the midplane,  $V_A = B_z/\sqrt{\mu_0 \rho_0}$  is the Alfvén speed based on the edge poloidal field. From the inferred  $q$  profile shown in Figure 2,  $(dq/d\Psi)^2 \sim 5$ , which is sufficient to stabilize the  $n = 2$  rotational instability.

### SUMMARY & CONCLUSIONS

Relaxation in an extremely high- $\beta$  plasma has been observed in the TCS translation experiment, producing a novel ST-like FRC state. It is clearly not a Taylor state, strongly suggesting that a more general principle regulates the relaxation. Modeling using the newly developed nearby-fluids theory shows that a broad core of the FRC-ST is very close to a two-fluid minimum energy state. This relaxed plasma state exhibits significantly reduced transport with up to 4 times improvement in confinement. The FRC-ST state also exhibits remarkable stability to the generally ubiquitous centrifugally driven  $n = 2$  inter-

change mode in  $\theta$ -pinch formed FRCs; this is attributed to the strong magnetic shear.

### ACKNOWLEDGMENTS

The authors acknowledge fruitful discussions with R.D. Milroy, Z.A. Pietrzyk and G.C. Vlases, and support from the rest of the RPPL team. This work was funded by the U.S. Department of Energy.

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