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Confinement & Current Drive Measurements for Steady-State FRCs

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Detailed measurements in the TCS Rotating Magnetic Field (RMF) driven FRC device display a highly non-uniform resistivity profile, highly peaked near the separatrix where the ratio of electron drift velocity v_{de} to ion sound speed v_s is large. The RMF parameters determine the plasma density. The plasma temperatures are governed by power balance, and higher temperatures result in higher diamagnetic currents, mostly inside the magnetic field null, and higher magnetic fields, with surprisingly little increase in absorbed power. The results are well modeled by a 'Chodura' type resistivity scaling with electron collision frequency scaling as $\nu_{ch} \sim \omega_{pi}(1 - \exp[-v_{de}/v_s])$.

KEY WORDS: FRC; Rotating Magnetic Field current drive; anomalous resistivity.

INTRODUCTION

FRCs have the highly desirable and unique features for a fusion confinement scheme of very high beta, a natural unconstrained divertor, and simple singly-connected cylindrical geometry. The recent adaptation of Rotating Magnetic Field (RMF) current drive pioneered in Australian rotamak experiments [1] has provided a means of sustaining FRCs in steady-state in the reactor relevant density regime of 10^{19}–10^{20} m^{-3} [2]. The FRC currents are diamagnetic, and simply a consequence of the plasma pressure, but plasma resistivity leads to a negative azimuthal electric field and poloidal flux decay. In non-sustained FRCs this flux decay sets the ultimate FRC lifetime.

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RMF CURRENT DRIVE

RMF produces an azimuthal force on the electrons which counteracts the resistive friction with ions. The force is distributed throughout the FRC by various mechanisms such as radial flow ($F_\theta = -j_{er}B_z$), and an analysis based on torque is most appropriate for calculation of E_θ at the FRC field null and the rate of change of FRC poloidal flux.

$$\frac{d\phi_p}{dt} = 2\pi R E_\theta(R) \simeq \frac{2}{n_e e r_s^2 \ell_s}\left(T_{RMF} - T_\eta\right) \quad (1)$$

$r_s = \sqrt{2}R$ and ℓ_s are the FRC separatrix radius and length. $T_{RMF} = 2\pi(B_\omega^2/\mu_o) r_s \delta^* \ell_s$ is the RMF torque and $T_\eta = 0.5\pi e^2 \langle n_e^2 \omega_e \eta_\perp \rangle r_s^4 \ell_s$ is the resistive torque. B_ω is the RMF magnitude in vacuum (with frequency ω), δ^* is its penetration distance into the FRC, ω_e is the electron rotation rate (assuming the ions are stationary), and $\eta_\perp$ is the cross-field plasma resistivity. The average electron rotation rate needed to reverse the external field B_e is $\langle \omega_e \rangle = 4(B_e/\mu_o)/\langle n_e \rangle e r_s^2$. The RMF will self consistently increase its penetration into the

FRC and raise both B_e and $\langle\omega_e\rangle$ until T_η equals T_{RMF}, resulting in $\delta^*/r_s \sim \langle\omega_e\rangle/\omega$.

For a prolate FRC confined inside a flux conserver of radius r_c with initial bias field B_o, an increase in poloidal flux increases $x_s \equiv r_s/r_c$ and $B_e = B_o/(1 - x_s^2)$. Torque balance yields an equilibrium density of

$$n_e^* = \frac{2B_\omega}{\left(\mu_o \eta_\perp e^2 \omega r_s^2\right)^{1/2}} \sqrt{\frac{\delta^*/r_s}{\langle\omega_e\rangle/\omega}}, \qquad (2)$$

and once this value of density is reached both RMF penetration and flux build-up will cease. The attainable external magnetic field $B_e = (2\mu_o n_m k T_t)^{1/2}$, where n_m is the peak plasma density and $T_t = T_e + T_i$ is the sum of electron plus ion temperature. The above result is fairly insensitive to temperature and, as long as $\langle\omega_e\rangle \sim <0.5\omega$ (where the previous RMF torque expression is valid), increases in temperature (with n_e approximately constant) will result in automatic increases in B_e and $\langle\omega_e\rangle$. Further penetration of the RMF may also occur, depending on details of the resistivity profile. If $\langle\omega_e\rangle$ should approach about ω/2, the RMF frequency should be increased to preserve the highly desirable partially penetrated RMF condition, which both helps maintain closed field lines during anti-symmetric RMF application [3], and provides strong stabilization against interchange instabilities [4]. The external bias flux should also be adjusted to keep $x_s \sim 0.8$ to produce average beta values commensurate with the more moderate $\langle\beta\rangle$ resulting from the low separatrix densities obtained with RMF drive ($\langle\beta\rangle = 1 - x_s^2/2$ for a prolate FRC confined in a cylindrical flux conserver).

ANALYSIS

Internal magnetic field profiles have been analyzed at two times for many experiments. At an early time the temperature is higher, before being limited by impurity ingestion and radiation barriers. At a later time, quasi-steady temperatures persist for as long as the RMF is applied. The density stays fairly constant, decreasing approximately proportional to any reduction in B_ω due to use of a capacitor bank powered RMF supply. Density, magnetic field, and $\alpha = \omega_e/\omega$ profiles are shown at early and late times for a typical discharge in Figure 1. The density profiles are approximately rigid rotor (RR—uniform ω_e), but slightly broader near the field null due to an

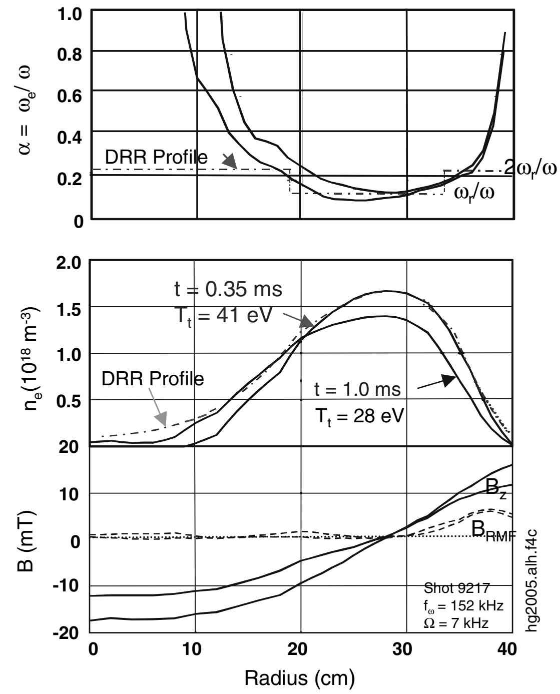

Fig. 1. Internal FRC profiles for RMF driven FRCs obtained from internal $B_z - B_\theta$ probes and a cross-tube interferometer. $B_\omega \sim 4$ mT and $\omega = 0.95 \times 10^6$ rad/s.

increased ω_e near the separatrix, which is the feature that allows the RMF to penetrate into a highly conducting plasma column. We utilize a simple double rigid rotor (DRR) model, with twice the electron rotation rate near the edge as in the center (as indicated in Figure 1) to produce a good approximation to the experimental density and magnetic field profiles.

At the typical $\langle\omega_e\rangle/\omega$ values of ~ 0.18 seen in Figure 1, the RMF penetration does not change much with plasma temperature. This is due to an edge resistivity that is very much higher than the central plasma resistivity. The higher x_s, B_e, and overall higher azimuthal current obtained at the higher temperature are due to an increased electron diamagnetic rotation rate, mostly inside the field null, which does not strongly affect the RMF current drive process.

Many features of detailed experimental measurements provide proof of the assertion that the resistivity profile is highly peaked near the FRC edge [5,6]. Using the DRR model we thus assign an inner resistivity η_i to the bulk of the plasma column, and use $\eta_\perp = \eta_e$ near the edge. The DRR analytic model, and the analytic expression for the RMF screening currents, give the following results for the resistive torque and the power absorption due to both the azimuthal diamagnetic currents and the axial screening currents. Calling $\omega_e = \omega_r$ in the center, and $\omega_e = 2\omega_r$ near the edge,

$$T_\eta = 0.29\pi(en_m)^2\omega_r r_s^4[\eta_i + 0.29\eta_e]\ell_s = 1.07\pi n_m\left(\frac{B_e}{\mu_o}\right)r_s^2[\eta_i + 0.29\eta_e]\ell_s. \quad (3)$$

$$P_{abs\theta} = \int \eta_\perp j_\theta^2 = 0.29\left(en_m\omega_r r_s^2\right)^2[\eta_i + 0.58\eta_e]\ell_s = 4\pi\left(\frac{B_e}{\mu_o}\right)^2[\eta_i + 0.58\eta_e]\ell_s \quad (4)$$

$$P_{absz} = \int \eta_\perp j_z^2 \simeq 2\pi\left(\frac{r_s}{\delta^*}\right)\left(\frac{B_\omega}{\mu_o}\right)^2\eta_e\ell_s. \quad (5)$$

Experimental data in TCS experiments ($r_s \sim 40$ cm) are best modeled in numerical calculations by using a resistivity profile $\eta_\perp(r) = \eta_i + \eta_e[1 + \exp((r_s - r)/\delta_\eta)]^{-1}$ with $\eta_e \sim 25\eta_i$ and $\delta_\eta \sim 1$ cm [6]. If we use the less extreme DRR analytical model, with $\eta_e = 10\eta_i$ over the outer 5 cm, and plot the measured peak electron density n_m against a relationship based on equation (3), as done in Figure 2 for operation at various RMF frequencies, we arrive at a resistivity scaling $\eta_i = 15n_m^{-1/2}(10^{20}\text{m}^{-3})\mu\Omega$-m. This is similar to the density dependence inferred from high density theta-pinch formed FRC decay rates [7].

The temperatures listed next to the data points on Figure 2 are the total plasma temperatures. T_t was limited in all TCS experiments by high impurity content and strong radiation barriers. However, based on the available data, better performance (lower effective average resistivity) was always achieved at higher temperatures. This was also seen vividly when the temperatures were raised using antisymmetric RMF [3]. The two calculated data points shown in Figure 2 were performed with identical resistivity profiles (chosen to model the 258 kHz data), but with different temperatures. The higher temperature calculation resulted in a lower plasma density since the resistive torque is proportional to $\eta_\perp n_m j_\theta$ and j_θ is proportional to the external axial field $B_e \propto (n_m T_t)^{1/2}$. The opposite result is seen in the experiments, and our best inference over the limited experimental temperature range is that $\eta_\perp$ is also proportional to $T_t^{-1/2}$.

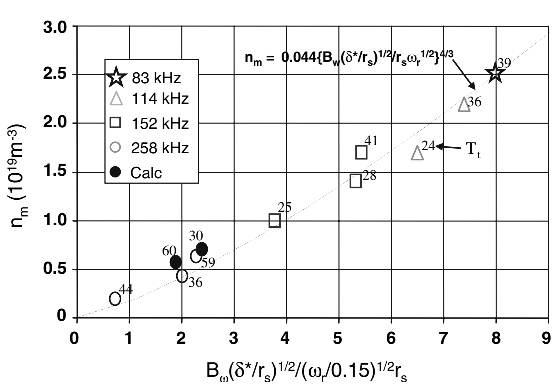

Fig. 2. Experimentally measured peak FRC density versus scaling parameter. The RMF frequencies $f_\omega = \omega/2\pi$ and total temperatures T_t are indicated.

The resistivity profile used in the calculations was chosen based on the formula, first attributed to R. Chodura and used to model θ-pinch implosions, $v_{ch} \sim \omega_{pi}(1 - \exp[-v_{de}/v_s\ ])$ with $\eta_{ch} = m_e v_{ch}/n_e e^2$ [8]. v_{de} is the electron drift speed relative to the ions, and v_s is the ion sound speed. This formulation has provided good modeling of θ-pinch FRC formation and decay in numerical calculations where the measured flux decay rate drops by up to two orders of magnitude after the current profile is broadened (reducing v_{de}) [9]. Its validity is also supported by recent 2-fluid calculations at the UW which show rapid non-linear turn-on of anomalous drift wave turbulence when v_{de} approaches v_s [10]. Near the edge of the FRC $v_{de} \approx \omega r_s$, which is much larger than v_s, and the resistivity is very high. Power absorption measurements also attest to the high edge resistivity, with P_{abs} barely increasing as the temperature and overall current increase. Most of the power absorption occurs in the edge region, partly due to the FRC azimuthal currents there, but also due to the axial RMF j_z screening currents.

For typical TCS operating conditions B_ω was about one-third of B_e. $P_{abs\theta}$ calculated from equation (4), using the resistivities inferred from torque balance, was only about one-half of the total measured absorbed power. The same factor of two increase in absorbed power due to the axial screening currents was seen in the numerical calculations. Based on equation (5), this excess power absorption can be reduced if the ratio of B_e/B_ω is increased. $B_e \propto (n_m T_t)^{1/2}$ and RMF drive produces a density $n_m \propto B_\omega$, so B_e/B_ω scales with $(T_t/n_m)^{1/2}$. Since the temperature in the TCS experiments was pegged to a fairly constant value by a high impurity content and radiation barriers, high values of B_e/B_ω (up to 6) were only achievable by operating at low values of B_ω and low n_m. In those cases P_{absz} became negligible compared to $P_{abs\theta}$. A new facility, TCS/upgrade is nearing completion, with vastly improved vacuum and wall conditioning, which should permit high temperatures to be reached and high values of B_e/B_ω to be achieved at high density.

SUMMARY

The drift wave scaling represented by $v_{ch} \sim \omega_{pi}$ (1 – exp[$- v_{de}/v_s$]) is favorable for scaling to larger and hotter devices. The reduction in effective plasma resistivity with size (lower values of v_{de} for the same overall current and density) has been noted in scaling from the smaller r_s = 20 cm STX experiment to TCS, and the improvement in resistivity with temperature (larger v_s) has been seen in all TCS experiments. The ratios of v_{de}/v_s near the FRC edge in TCS were about 4, but would be reduced to ~ 0.15 in a reactor sized device. If the 'Chodura' scaling continued to hold, it would be highly favorable for RMF as an efficient means of maintaining steady state FRCs.

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