

# End-shortening and electric field in edge plasmas with application to field-reversed configurations

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The shortening of open field lines where they intersect external boundaries strongly modifies the transverse electric field all along the field lines. The modified electric field is found by an extension of the familiar Boltzmann relation for the electric potential. This leads to a prediction of the electric drift. Flow generation by electrical shorting is applied here to three aspects of elongated field-reversed configurations: plasma rotation rate; the particle-loss spin-up mechanism; and the sustainability of the rotating magnetic field current drive method. © 2002 American Institute of Physics. [DOI: 10.1063/1.1494823]

## I. INTRODUCTION

Open magnetic field lines make electrical contact with external boundaries. If this contact produces a shorting effect at the boundary, then the transverse electric field is strongly modified along the entire length of the field line. Finding the self-consistent electric field produced by shorting and the resulting electric drift is the main focus of this paper. The electric potential is determined by an extension of the familiar Boltzmann relation for potentials along a field line. The analysis leads to several surprising results. The shorting does not force the radial electric field in the plasma to zero. Indeed the resulting electric field is actually outward. Consequently the electric drift augments the diamagnetic drift of the ion fluid.

When open magnetic field lines intersect external boundaries they make thermal as well as electrical contact. Because of the high electron mobility, this contact raises the specter of large electron energy losses along the field lines. However, parallel thermal loss does not appear to be a significant problem based on previous work, which will be revisited briefly.

These results are applied to three aspects of elongated field-reversed configurations (FRC). (1) The rotation rate in the edge layer is found to be surprisingly fast. This compared with experimental observations suggests that there is strong shear in the rotational flow. (2) The inclusion of the electric drift allows the particle-loss spin-up mechanism to be revisited. The electric drift is strong enough that this mechanism actually predicts spin-up in the wrong direction from what is observed. (3) The rotating magnetic field (RMF) current drive method is sustainable only if the ion spin-up problem can be solved. End-shortening on field lines opened up by the RMF allows the ion rotation to be controlled so that current drive can be sustained.

The outline of this paper is as follows. Section II addresses the basic physics of end-shortening, including its relation to fluid drifts, the self-consistent electrostatic potential, and energy loss along field lines. Sections III–V applies these results with three important FRC issues: Sec. III considers the rotation rate in the edge of an FRC and its impli-

cations on rotational stability; Sec. IV revisits the particle-loss spin-up mechanism in the light of the electric fields predicted here; and Sec. V considers the effect of shorting on the sustainability of rotating magnetic field (RMF) current drive in FRCs. Section VI is a summary and discussion.

## II. BASIC END-SHORTING PHYSICS

### A. Electric potential-fluid drift relationship

If the steady equation of motion for a fluid species is crossed with the magnetic field vector then an equation follows relating the fluid drifts to the electric potential. Ignoring inertial effects this gives

$$\mathbf{u}_{\perp\alpha} = \frac{c}{B^2} \left( \mathbf{B} \times \nabla \phi + \frac{1}{q_\alpha n} \mathbf{B} \times \nabla p_\alpha \right). \quad (1)$$

Here  $q_\alpha$ ,  $p_\alpha$ ,  $n$ , and  $\mathbf{u}_{\perp\alpha}$  are the charge, pressure, density, and drift perpendicular to the magnetic field  $\mathbf{B}$ , respectively, of fluid species  $\alpha (= i, e)$ . Equation (1) exploits the fact that in steady state, Faraday's law implies the existence of an electric potential  $\phi$  such that the electric field is  $\mathbf{E} = -\nabla \phi$ . Adopt the straight field line approximation appropriate to the mid-plane region of an elongated FRC,  $\mathbf{B} = B\hat{\mathbf{z}}$ ,  $\nabla = \hat{\mathbf{r}}d/dr$ ,  $\mathbf{u}_{\perp\alpha} = u_{\perp\alpha}\hat{\boldsymbol{\theta}}$ ; then Eq. (1) becomes

$$u_{D\alpha} = V_E + V_{d\alpha} = \frac{c}{B} \frac{d\phi}{dr} + \frac{c}{q_\alpha n B} \frac{dp_\alpha}{dr}, \quad (2)$$

where  $V_E$  is the electric drift and  $V_{d\alpha}$  is the diamagnetic drift. It is useful to express the latter in terms of familiar parameters. The relative gradients of the temperature and density are given by the familiar  $\eta$  parameter

$$\eta_\alpha \equiv d \ln T_\alpha / d \ln n. \quad (3)$$

Other parameters introduced are the local density length scale,  $L_n = [(-1/n)dn/dr]^{-1}$ ; the ion skin depth,  $l_i \equiv (m_i c^2 / 4\pi e^2 n)^{1/2}$ ; the species beta (ratio of thermodynamic pressure to local magnetic pressure),  $\beta_\alpha = 8\pi n k T_\alpha / B^2$ ; and the local Alfvén speed  $V_A = B / (4\pi m_i n)^{1/2}$ . Then the diamagnetic drift is

$$V_{d\alpha} = -(1 + \eta_\alpha) \beta_\alpha \frac{V_A}{2} \frac{l_i}{L_n}. \quad (4)$$

If the electric potential is allowed to float, then Eq. (2) can be regarded as an equation for  $\phi$  (or its gradient). Suppose the pressure gradient is given, and suppose the ions rotate at a prespecified rate (e.g., initially nonrotating ion fluid), then the potential is determined by Eq. (2). Here the arrow of cause-effect proceeds from the pressure gradient and the transverse drift to the resulting potential. This is the nonshorted case. However, if end-shortening occurs, the electric potential is determined by the shorting process. The resulting potential and the pressure gradient (again given) determine the transverse ion drift by Eq. (2). Thus, with end-shortening, the arrow of cause-effect reverses, proceeding from the pressure gradient and the shorted potential to the resulting fluid drift.

End-shortening and its effect on plasma rotation has long been known in the context of open-field line plasmas such as  $\theta$  pinches.<sup>1,2</sup> It also occurs in the open-field-line “edge layer” of an FRC.<sup>3</sup> If a shorting path is introduced where a magnetic field line intercepts a chamber wall, then the cross-field structure of the electric potential may be modified significantly. In the simplest picture, the transverse electric field at the boundary is simply shorted out. This then launches a torsional Alfvén wave which travels from the boundary along the field lines. The wave puts a twist on adjacent field lines; this imparts a torque to the fluid species and changes their drift speed (see Fig. 2 of Ref. 3). The torque is exerted on the plasma by the boundary itself to which the field lines are “line-tied.” The energy for the resulting drift comes not from the twist but from the release of electrostatic potential energy as the plasma spins up.<sup>3</sup> In short order the Alfvén waves damp out producing a steady state in which the electric potential alongside the FRC is consistent with the shorted potential at the end boundary. In steady state the field lines are again untwisted, unless there remains a residual torque from some other source, e.g., a torque on the ions caused by friction with an injected neutral beam. (The usefulness of a residual torque on the ions will be taken up later in one of the applications, Sec. V.)

End-shortening may not always happen. It depends on the end boundary. If the boundary material is a conductor then shorting probably occurs. If it is a dielectric then shorting may not occur. In the analysis that follows, it is assumed that the boundary is such that shorting does take place, i.e., the transverse electric field at the boundary is forced to zero.

## B. Self-consistent electric potential on a shorted field line

The electric potential in the quasisteady state is determined by ambipolarity, the establishment of a self-consistent electric potential so that electrons and ions maintain quasineutrality everywhere except in a thin sheath at the boundary. This potential is set up almost instantly because of the high electron mobility: an electron almost instantly detects and responds when the field line on which it is located contacts a boundary. This happens even if there is flow along

the field line. It also happens even if the field line thrashes about due to dynamical motion or some other cause. (An example of a “thrashing” field line will be taken up in Sec. V.)

The self-consistent electric potential is found using the parallel (to  $\mathbf{B}$ ) component of the electron equation of motion. This leads to the familiar Boltzmann relation. However, more can be said. End-shortening at the boundary causes the potential at the boundary to be constant. This “boundary condition” together with the Boltzmann relation allows the transverse structure of the potential far from the boundary to be determined. From the potential structure, the transverse electric field and the electric drift will follow.

The parallel electron equation of motion assuming massless electrons is

$$0 = -\nabla_{\parallel} p_e + en \nabla_{\parallel} \phi. \quad (5)$$

Unless the electrons are highly collisional, rapid parallel thermal conduction assures a flat electron temperature along the field line, i.e.,  $\nabla_{\parallel} T_e \approx 0$ , except of course in the thin sheath at the boundary. Then with  $p_e = nkT_e$ , Eq. (5) becomes an “exact derivative:”

$$\nabla_{\parallel} [-(kT_e/e) \ln n + \phi] = 0 \quad (6)$$

which integrates at once to

$$\phi = (kT_e/e) \ln n + \phi_0(\psi). \quad (7)$$

The integration “constant”  $\phi_0$  is actually a function of the magnetic surface (with marker  $\psi$ ); it can be regarded as the potential at the end boundary. In the shorted case  $\phi_0 = \text{const}$ . Equation (7) is the familiar Boltzmann relation used in mirror plasmas. It will be applied in a new way here to infer how the potential varies from field line to field line.

## C. Fluid drifts in end-shortened plasmas

The radial electric field with shorting can be found from Eq. (7) simply by differentiating with respect to  $r$ . With Eq. (3) the radial electric field is

$$E_r = (1 + \eta_e) \frac{kT_e}{e} \frac{1}{L_n}. \quad (8)$$

Note two surprising results. (1) The radial electric field is not zero unless there is no density gradient ( $1/L_n \rightarrow 0$ ) and no temperature gradient ( $\eta_e/L_n \rightarrow 0$ ). This is contrary to the conventional wisdom that end-shortening makes the radial electric field go to zero. (2) The radial electric field is actually outward. This contrasts sharply with the inward field in static equilibria (nonrotating ions). Clearly, shorting causes a marked change in the radial electric field.

The electric drift resulting from the “shorted” field Eq. (8) is

$$V_E = -(1 + \eta_e) \beta_e \frac{V_A}{2} \frac{l_i}{L_n}. \quad (9)$$

This form is remarkably similar to that for the ion diamagnetic drift, Eq. (4) with  $\alpha = i$ :

$$V_{\text{di}} = -(1 + \eta_i) \beta_i \frac{V_A}{2} \frac{l_i}{L_n}. \quad (10)$$

The total ion fluid drift is the sum of these two. Note that when shorting occurs, the electric drift actually augments the drift in the minus- $\theta$  direction.

#### D. Energy loss along open magnetic field lines

When open magnetic field lines intersect external boundaries they make thermal as well as electrical contact. Because of the high electron mobility, this contact raises the specter of large electron energy losses along the field lines. The question of electron energy loss along open field lines was taken up in Ref. 4. If the ion and electron temperatures and the particle confinement time in the bulk plasma are measured, then a very simple theoretical model can be constructed to infer the electron energy loss  $W_e$  from the plasma. From this,  $W_e/kT_e$ , the electron energy lost per particle lost, can be inferred from experiment. A number of FRC and  $\theta$ -pinch experiments were analyzed.

The experimental results can easily be compared with a simple theoretical model. If the energy loss is conductive, then the ratio  $W_e/kT_e$  should be proportional to  $\lambda_e/L_{\text{conn}}$ , where  $\lambda_e$  is the electron–electron mean-free-path, and  $L_{\text{conn}}$  is the connection length to the boundary. If the energy loss is convective, then the  $W_e/kT_e$  should be a constant (5/2). The sum of the two loss rates shows that convective loss dominates for  $\lambda_e/L_{\text{conn}} < 0.01$  (highly collisional), and the conductive loss dominates for  $\lambda_e/L_{\text{conn}} > 0.01$  (weakly collisional).

The comparison of the theory with experiment gives what is at first blush a surprising result. In the experiments  $W_e/kT_e \sim 4-8$ , independent of the collisionality (within error bars) for all experimental examples. This holds over the quite broad collisionality range in the experiments,  $10^{-3} < \lambda_e/L_{\text{conn}} < 2$ . Thus the thermal-conduction-dominated regime predicted by theory,  $W_e/kT_e \propto \lambda_e/L_{\text{conn}}$ , *never* appears. The likely reason is that the low-collisionality regime where thermal conductivity would be strong is precisely the condition where Spitzer conductivity breaks down. “Fick’s law” formulas for the thermal flux ( $\mathbf{q}_e = -\kappa \nabla T_e$ ) are only valid if  $\lambda_e$  is shorter than  $L_T$ , the temperature gradient length scale. In *parallel* thermal conduction, the bulk of the energy is transported by superthermal electrons, thus the requirement is more stringent:  $\lambda_e$  must be *much less* than  $L_T$  for Fick’s law to be valid. Thus the collisionality range  $\lambda_e/L_{\text{conn}} > 0.01$  where Spitzer conductivity  $\lambda_e/L_{\text{conn}} > 0.01$  (if valid) would dominate is precisely the regime where it breaks down.

Note that there is an alternative explanation for why  $W_e/kT_e$  exceeds the simple convective value 5/2. Ignoring recycled particles would lead to an underestimate of the particles loss rate along open field lines. This would increase the inferred value of  $W_e/kT_e$ . On the other hand, the lifetime of typical FRC plasmas (<hundreds of  $\mu\text{s}$ ) is short enough that significant recycling may not have occurred. Further, the experimentally inferred values  $W_e/kT_e \sim 4-8$  are consistent

TABLE I. Rotation parameter for experiments.

Device	Log No.	$B_{\text{ex}}$ (kG)	$T_i+T_e$ (eV)	$T_e$ (eV)	$r_c$ (cm)	$r_s$ (cm)	$w_n$	$S_*$	$K$	$\alpha$
FRX-B	1	6.5	310	100	12.5	5.4	3.5	9.8	0.53	3.2
FRX-C	2	8	800	175	25	9	4.1	12.5	0.44	4.6
FRX-C	1	7	250	100	25	10	6.1	21.7	0.49	4.7
TRX-1	2	8	450	150*	12.5	6.3	2	11.6	0.62	5.3
TRX-1	3	7.5	350	140*	12.5	7	2	13.7	0.69	5.6
TRX-1	1	9	300	130*	12.5	4.2	2.2	10.7	0.41	8.6
TRX-2	9	10	400	140*	12	4.7	3.1	11.5	0.48	5.0

with the theoretical model which include conductive and ambipolar convective energy loss effect without recycling.<sup>4</sup>

The observation of the ratio  $W_e/kT_e$  is 4–8 rather than 5/2 reflects the influence of the electrical potential, Eq. (7). Because the plasma runs “positive” relative to the boundary, electrons are confined electrostatically. The only electrons that can escape the electrostatic barrier at the sheath are superthermal electrons with energy of 4 to  $8kT_e$ .

Apparently then the energy loss is convective on open field line plasmas in theta-pinches and FRCs. Convective-dominated energy loss is well known in mirror plasmas.

However, this conclusion is not universal. In the open field lines of a tokamak divertor, refluxing plasma from the divertor plate causes a back flow that keeps the pressure roughly constant along the field lines. Also, the long connection length to the divertor plate makes it much easier to satisfy the condition,  $\lambda_e \ll L_T$ , for thermal conduction to be valid.

#### III. ROTATION RATE IN FRCs

Plasma rotation causes the rotational instability. The theoretical threshold for the rotational instability is  $\alpha \equiv \Omega_i/\Omega_{\text{di}} \approx 1-2$ , where  $\Omega_i$  is the ion fluid rotation and  $\Omega_{\text{di}}$  is the ion diamagnetic drift frequency. In theoretical treatments the ion rotation is usually assumed to be rigid and the diamagnetic drift frequency is an average over the plasma cross section,  $\langle \Omega_{\text{di}} \rangle$ . For the commonly used rigid-rotor profile  $\Omega_{\text{di}}$  is uniform.

It is useful to estimate the value of the rotational parameter  $\alpha$  based on the rotation rates with end shorting. Then in the edge region  $\Omega_i = (V_E + V_{\text{di}})/r_s$  where  $r_s$  is the separatrix radius and  $V_E$ ,  $V_{\text{di}}$  are the two components of the drift velocity, Eqs. (9) and (10). For this average diamagnetic frequency for a rigid rotor profile is used:<sup>5</sup>

$$\Omega_{\text{di}} = -8K(\beta_i)_{\text{ex}} \frac{(V_A)_{\text{ex}}}{2} \frac{l_i}{a^2}. \quad (11)$$

Here the subscript “ex” denotes a quantity evaluated using the external (vacuum) magnetic field. The values of  $\beta$  used in Eqs. (9) and (10) are based on the local field. The conversion between these two (using pressure balance  $p + B^2/8\pi = B_{\text{ex}}^2/8\pi$ ) is  $\beta = \beta_{\text{ex}}/(1 - \beta_{\text{ex}})$ ;  $\beta_i = \beta T_i/(T_e + T_i)$ ;  $\beta_e = \beta T_e/(T_e + T_i)$ . The density length scale in the edge layer is sometimes expressed as a multiple ( $w_n$ ) of the ion gyro-radius,  $L_n = w_n(\rho_i)_{\text{ex}}$ .

Table I shows seven FRC examples where the normal-

ized density size in the edge,  $w_n$ , has been inferred from measurements.<sup>6</sup> Also shown are other relevant parameters taken from the FRC confinement data base.<sup>7</sup> The electron temperatures with an asterisk are estimated. The density is derived assuming pressure balance,  $n_{\max} = 8\pi B_{\text{ex}}^2 / (T_e + T_i)$ . The rigid rotor shape parameter is determined using the average- $\beta$  relation:  $K \approx (3/2)r_s/r_c$  (small  $K$  expansion;  $r_c$  = coil radius). The  $S_* = r_s/l_i$  shown is the familiar radial size parameter. In computing the rotational parameter, it is assumed that  $\eta_e = 2/3$  and  $\eta_i = 1/3$ .

Most examples show  $\alpha \sim 4-6$ . This edge value is considerably higher than the threshold level for the rotational instability. It is also considerably higher than the measures values of  $\alpha \sim 1-2$  the most reliable of which give  $\alpha \sim 1-1.2$ .<sup>2</sup> If the edge value here is valid then the implication is considerable flow shear, that is the edge is rotating much faster ( $\alpha \sim 5$ ) average than the average rotation of the interior ( $\alpha \sim 1-2$ ).

#### IV. PARTICLE-LOSS SPIN-UP MECHANISM IN FRCs

One of the proposed mechanisms to explain the spin-up of FRCs is the particle-loss mechanism.<sup>8</sup> The end-loss of ions from the edge of an FRC will produce a rocket-like effect on the rotation of the plasma if the typical "lost" ion has an angular momentum that differs from the average angular momentum of ions in the bulk FRC. For example, an ion with a rotational drift in the plus- $\theta$  direction is suddenly lost out the ends, its disappearance causes the remaining plasma to recoil in the minus- $\theta$  direction. It was noted that the grad- $B$  drift of ions near the plasma edge is in the plus- $\theta$  direction so that the resulting spin-up is in the minus- $\theta$  direction, as observed in experiments. However this mechanism as invoked previously ignores the effect of electric drift on the ions. The calculation of the radial electric field in Sec. II allows the electric drift to be properly included. The result is surprising.

The ion fluid drift is the sum of the electric and ion diamagnetic drifts. The ion particle drift is the sum of the electric and the grad- $B$  drifts. The grad- $B$  drift,  $\mathbf{V}_{\nabla B} = V_{\nabla B} \hat{\theta}$ , in an elongated configuration is

$$V_{\nabla B} = \frac{v_{thi}^2}{2\omega_{ci}} \frac{1}{B} \frac{dB}{dr}. \quad (12)$$

Observe that this is in the plus- $\theta$  direction. Using radial force balance,  $p + B^2/8\pi = \text{const}$ , and variables used earlier this is

$$V_{\nabla B} = \frac{\beta_i}{4} [(1 + \eta_e)\beta_e + (1 + \eta_i)\beta_i] \frac{V_A}{2} \frac{l_i}{L_n}. \quad (13)$$

Note the appearance of terms similar to the ion diamagnetic drift Eq. (10). The total ion particle drift is the sum of the electric drift Eq. (9) and the grad- $B$  drift Eq. (13):

$$V_{\perp i} = \left[ \left( \frac{\beta_i}{4} - 1 \right) (1 + \eta_e) \frac{T_e}{T_i} + \frac{\beta_i}{4} (1 + \eta_i) \right] \frac{\beta_i V_A}{2} \frac{l_i}{L_n}. \quad (14)$$

Consider the example:  $\eta_e = 2/3$  and  $\eta_i = 1/3$  again;  $\beta_{\text{ex}} = 0.6$  at the separatrix; and  $T_e/T_i = 1/2$ . Then the rotational drift of ions that are lost out the ends is

$$V_{\perp i} = \left[ -\frac{7}{24} \right] \frac{\beta_i V_A}{2} \frac{l_i}{L_n}, \quad (15)$$

where the quantity in square brackets is the same as that in Eq. (14). Thus the electric drift is strong enough that it more than cancels the grad- $B$  drift. Thus the net rotational drift of the ions is in the minus- $\theta$  direction. The disappearance of such an ion by end-loss would impart a momentum to the plasma in the plus- $\theta$  direction. This is exactly the opposite from what is observed in FRC spin-up. This result is not sensitive to the assumptions. If the two temperature profiles are flat  $\eta_e = \eta_i = 0$ , then the factor in Eq. (14) is  $[\dots] = -1/8$ , which is still negative.

Thus, the particle loss spin-up mechanism appears *not* to explain FRC spin-up; indeed it would lead to spin-up in the opposite direction from what is observed.

#### V. SUSTAINABILITY OF RMF CURRENT DRIVE IN FRCs

Another application of the shorted-field theory is to the rotating magnetic field (RMF) current drive technique. RMF is an attractive method for driving the bulk electron current in FRCs. However, its effectiveness as a current drive method depends on whether the ions can be kept from spinning up. If ions are allowed to free-wheel, they will simply begin to spin-up as a result of friction from the rotating electrons. Then the cause-effect sequence (discussed in Sec. II A) proceeds from cause (pressure gradient plus ion rotation, as set by electron friction) to effect (electric potential). The result is that the electrons rotation to increase by the same amount as the ions. In so doing the electron rotation frequency  $\Omega_e$  catches up to the RMF frequency,  $\omega_{\text{RMF}}$ . As this happens the RMF torque on the electrons  $\propto \omega_{\text{RMF}} - \Omega_e$  decays to zero. Thus if the ions are free-wheeling the RMF current drive simply turns off.<sup>9,10</sup>

Sustainable RMF current drive requires that additional torque be applied to the ions. Two inherent mechanisms that impose a torque on the ions have been suggested: (1) ion drag against a neutral background; and (2) ion drag against the edge layer by shear viscosity.<sup>10</sup> In addition, active measures may also apply a torque to the ions: (1) a particle source, as in a refueled FRC, is equivalent to a ion momentum drag and hence a torque;<sup>11,12</sup> (2) neutral beam injection;<sup>11</sup> and (3) a second component of the RMF tuned so as to drag the ion fluid.<sup>13</sup> All of these mechanisms have significant liabilities: they are either too weak to sustain the RMF, ineffective in fusion-relevant energetic plasmas, or require the introduction of difficult technologies.

Another inherent mechanism that can apply a torque to the ions is end-shortening on magnetic field lines opened up by the RMF. Ordinarily the separatrix surface separates the regions of open and closed magnetic surfaces. RMF, however, introduces a transverse field into the interior of the plasma and opens up the magnetic field lines. This "opening" is necessary because it allows the current drive to work, i.e., it provides a channel for torque to be transferred from external RMF antennas to electrons in the FRC interior. The opening also causes two other effects: it makes (1) electrical and (2)

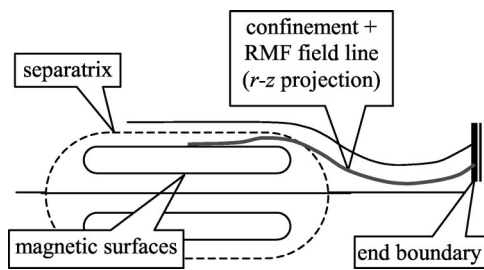


FIG. 1. Field-line opening by RMF.

thermal contact with the wall. The former provides a means for applying a torque to the ions, and is examined here. The latter, thermal contact with the wall, should not cause excessive energy loss, as discussed earlier in Sec. II D.

Field-line opening lines by RMF is illustrated in Fig. 1. The familiar poloidal field structure of an FRC is shown (thin lines), one of which is the separatrix (dashed line). Also shown as a heavy line is a combined RMF+confinement field line. This is actually the  $r$ - $z$  projection of such a line, since the RMF field experiences considerable azimuthal twist in the plasma interior.

Suppose that the boundary where the combined field line (RMF+confinement field) intersects the wall is a shorting boundary, then the entire line will be shorted as discussed here. The fact that the combined field line shifts its footprint on the wall as the RMF rotates does not change this. As far as the highly mobile electrons are concerned, the combined field line is in quasisteady state. The upshot of the shorting is that the ion fluid drift speed rate is fixed as the sum of the electric and diamagnetic drifts [Eqs. (9) and (10)]. The ions do not spin up so as to cancel the RMF current drive effect. The torque that maintains the ion fluid at the shorted rotation rate is supplied by a persistent twist in the combined field line. Thus the electron drag on the ions, attempting to spin them up, is resisted by the torque transmitted from the RMF antenna along the opened field lines. This allows RMF to be a sustainable current drive method at least in the regions of the FRC where the RMF fields penetrate.

It should be remembered that the parallel electron equation of motion Eq. (5) is valid along the combined field line defined by the instantaneous sum of the confinement field and the RMF field. Thus the RMF force  $(-e/c)(\mathbf{u}_e \times \mathbf{B})$  contributes nothing to the “parallel” equation (5) since  $\mathbf{u}_e \times \mathbf{B}$  is always perpendicular to  $\mathbf{B}$ .

Whether end-shorting occurs depends on the nature of the boundary where the opened field line intersects the wall. If the boundary material is a dielectric then shorting may not occur. If the boundary material is a conductor, then shorting probably occurs. End-shorting can be encouraged in an experiment by placing a conducting limiter in the regions where the combined field lines intersect the wall.

## VI. SUMMARY

The end-shorting of open magnetic field lines was analyzed in Sec. II. The basic physics arises from the parallel electron dynamics by which the familiar Boltzmann relation for the electric potential is derived. Shorting at the bound-

aries ends allows this relation to be used to infer radial electric fields in the plasma. Three surprising results follow. (1) Contrary to the popular rule of thumb, end-shorting does not force the radial electric field zero. (2) The shorted electric field is actually outward, which is opposite to its direction for a static (nonflowing) ion fluid. (3) The self-consistent electric drift actually augments the diamagnetic drift, leading to fairly rapid ion fluid drifts.

Flow generation by electrical shorting is applied here to three aspects of elongated field-reversed configurations (FRC). (1) Rapid rotation in edge layer. The rotation rate in the edge layer is much larger than that in the interior of FRC as inferred in experiments. This suggests that is considerable flow shear with the boundary rotating much faster than the interior. (2) Particle-loss spin-up mechanism. The end-loss of ions with a preferential rotation has been suggested as an explanation of FRC spin-up. Previous estimates of this included the grad- $B$  drift of ions but ignored the electric drift. When shorted electric fields are taken into account, the electric drift is strong enough to more than cancel the spin-up effect of the grad- $B$  drift. Thus this mechanism predicts spin-up in the wrong direction. (3) Sustainable RMF current drive. The RMF as a current drive method is sustainable only if an independent torque can be applied to the ions to keep them from spinning up to follow the electrons. Such a torque is available because of the opening of magnetic field lines by the RMF; if the boundary is such that shorting can occur, then the needed torque is applied by the RMF antenna along the opened field lines.

An important issue for further work is the nature of the contact of the open field line with the boundary, the “footprint” of the field line. This differs from one application to another. If the edge layer is essentially stationary, then the footprint remains at the same point on the boundary. However, in the RMF application, the footprint will move in some cyclic path in response to the rotating field. In this case, the moving footprint implies the presence of a transverse electric field at the boundary. Whether a locally nonshorted electric field can exist if the boundary is “shorting” is of first importance. In the application as developed here in Sec. V, it was assumed that the field could be shorted at the footprint. In fact it is only necessary for it to be partially shorted. It is only necessary that the boundary apply a persistent torque to the field line bundles.

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