

# Inductive Flux Build-Up of RMF Formed FRCs

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## Abstract

Slow formation of high beta plasmas, such as FRCs, is difficult due to the need to build up plasma pressure rapidly enough to balance the dominant poloidal field pressure. Slow formation was attempted in the Coaxial Slow Source (CSS) device where 4-turn inner and outer coils were used to explore slower formation options, but it was not possible to operate at low enough densities to avoid radiative collapse.<sup>1</sup> FRCs have also been formed slowly at relatively low pressures and magnetic fields using Rotating Magnetic Fields (RMF), but it has also been difficult to overcome radiation barriers and achieve high temperatures due to relatively low power inputs.<sup>2</sup> We have performed calculations that show FRCs could be formed using RMF, and then augmented in flux and energy using the inductive input from a CSS type internal coil. With correct tailoring of the coil current profiles, the FRC could then be translated off the central coil. This methodology should be capable of producing hot, high flux FRCs using slow, low voltage technology.

<sup>1</sup> Z.A. Pietrzyk, et. al., Nuclear Fusion **27**, 1478 (1987).

<sup>2</sup> H.Y. Guo, et. al., Phys. Plasmas **9**, 185 (2002).

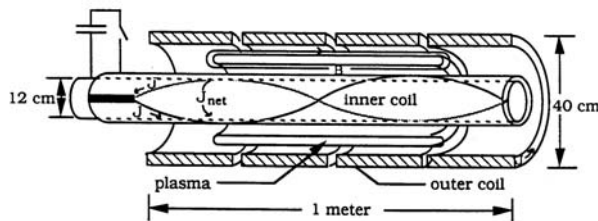
## Motivation and Goal

- Combine the benefits of RMF formation with inductive current drive using a central core to form a high flux FRC suitable for Tangential Neutral Beam Injection (TNBI).
- Past CSS experiments did not successfully burn through the radiation barrier. These experiments were not starting with a sufficiently low density plasma. This problem can be overcome when the initial plasma is created with RMF.
- RMF can be used to form a low density FRC. Flux can then be transferred from the inner coil to the FRC; in the process increasing the FRC flux and resistively heating it.
- This combination of techniques could be ideal for forming high flux FRCs suitable for studying stability limits, and the effects of TNBI.

W.F. Pierce, R.J. Maqueda, R.D. Brooks, R. Farengo, "Initial results from parallel coil operation of the coaxial slow source Field Reversed Configuration device", Nucl. Fusion **33**, 117 (1993)

Z.A. Pietrzyk, G.C. Vlases, R.D. Brooks, K.D. Hahn, and R. Raman, "Initial results from the coaxial slow source FRC device", Nucl. Fusion **27**, 1478 (1987)

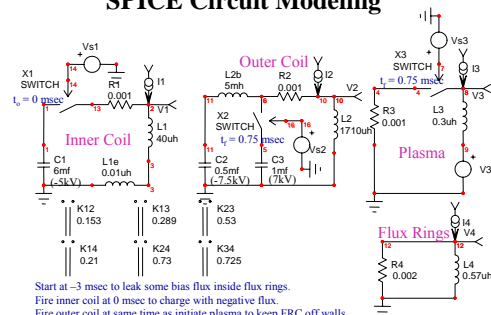
## The Coaxial Slow Source (CSS) Concept



### Two Modes of Operation:

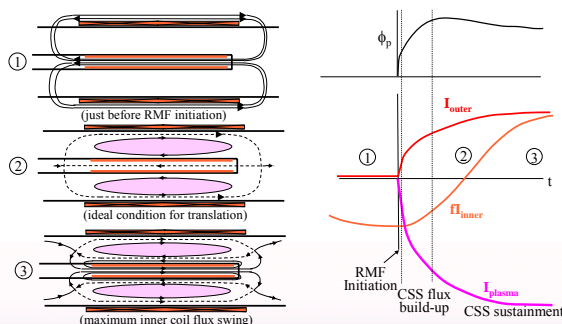
- Fast, high voltage:  $t_{1/4} \ll \tau_{L/R}$  of FRC.
  - All inner coil flux will be transferred to FRC.
- Slow, multi-turn coils:  $t_{1/4} \gg \tau_{L/R}$  of FRC.
  - FRC current will be equal to  $(V/N)_{coil}/R_{FRC}$ .

### SPICE Circuit Modeling

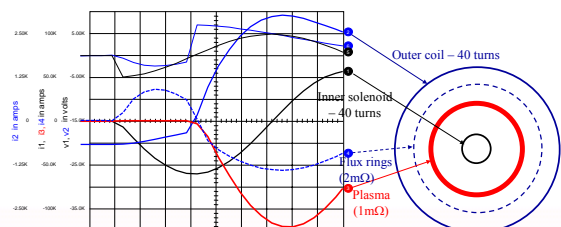


Start at -3 msec to leak some bias flux inside flux rings.  
Fire inner coil at 0 msec to charge with negative flux.  
Fire outer coil at same time as initiate plasma to keep FRC off walls.

### Flux Build-Up & Sustainment of RMF Generated FRC



### SPICE CSS Calculation for 1 mΩ FRC Driven by 40-turn, 5 kV solenoid



Initial bias ( $I_0$ ) applied so that there is ~zero field inside flux rings at time of plasma initiation. 125 V/turn generates ~125 kA in 1 mΩ FRC. Applied power ~15 MW. Equilibrium field ~75 mT and equilibrium flux ~20 mWb

# Numerical Simulations of Coaxial Slow Source Flux Build-up

Flex2d Code: Dan Barnes, 1980's vintage

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{u} = 0$$

$$Mn \left[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = \frac{\mathbf{J} \times \mathbf{B}}{c} - \nabla P - \nabla \cdot \Pi$$

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - \eta c^2 \left( \frac{\mathbf{J}}{c} \right) - \frac{c}{en} \left[ \left( \frac{\mathbf{J}}{c} \times \mathbf{B} \right) - \nabla P_e \right]$$

$$\frac{\partial S}{\partial t} + \nabla \cdot S\mathbf{u} = \frac{\gamma-1}{n^{\gamma-1}} [\eta \mathbf{J}^2 + \nabla \cdot (k_{\perp} \nabla T) - \Pi : \nabla \mathbf{u} - R]$$

$$S = n^{2-\gamma} T$$

$$P = nk_B T$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B}$$

$$\Pi = -\frac{\mu}{Mn} \nabla \mathbf{u}$$

## Resistivity

-  $\eta = \max(\eta_{const}, \eta_{chod})$ : Normally  $\eta_{const} = 100 \mu\Omega\text{-m}$ .

- Chodura resistivity

$$\eta_{chod} = \frac{m_e v_{chod}}{ne^2}$$

$$v_{chod} = C_c \omega_{pi} \left( 1 - e^{-\frac{v_e}{fv_e}} \right)$$

-  $C_c = 0.1, f = 3$

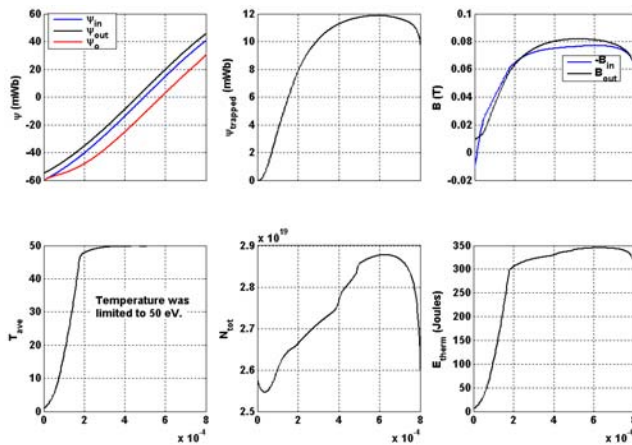
## Parameters

- $R_{ci} = 8 \text{ cm}$
- $R_{wi} = 10 \text{ cm}$
- $R_{wo} = 40 \text{ cm}$
- $R_{co} = 42 \text{ cm}$
- $Z_c = 150 \text{ cm}$
- Circuit applies approximately 125 Volt-turns on the inner and outer coils.

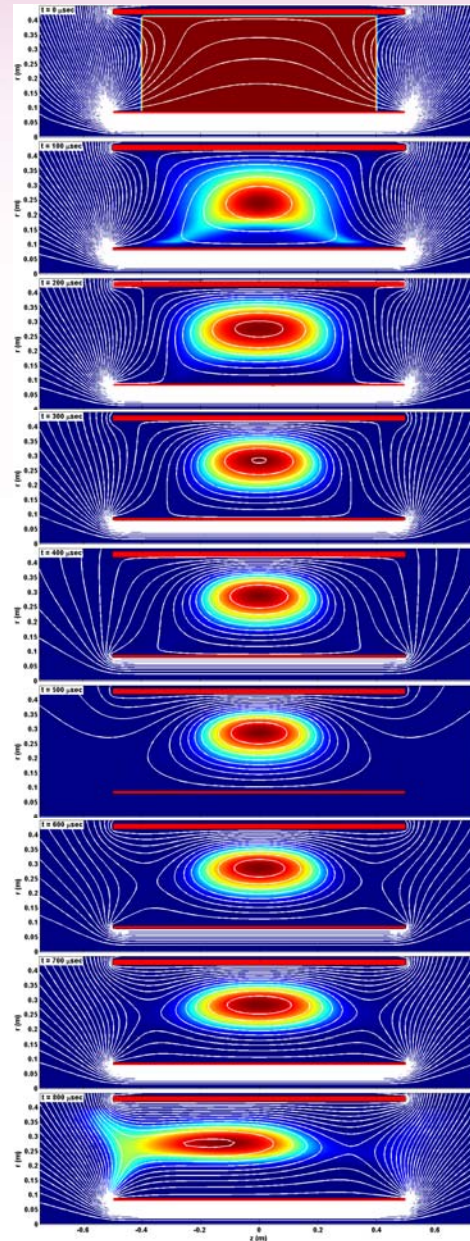
## Initial Conditions

- $n_0 = 7 \times 10^{19} \text{ m}^{-3}$
- $T_e = T_i = 1 \text{ eV}$

## Time History of Key Variables



## Flux and pressure contours



## Equilibrium Flux Level

For a rigid-rotor profile:  $\tau_{\phi} = \frac{r_s^2}{16D} = \frac{r_s^2}{13\eta(\mu\Omega - m)}$

$\phi = -\frac{\varphi}{\tau_{\phi}} = V$  where  $V$  is the voltage applied to the coil.

Equilibrium flux is:

$$\phi = \dot{\varphi} \tau_{\phi} = V \tau_{\phi} = \frac{V r_s^2}{13\eta(\mu\Omega - m)}$$

For this calculation:  $r_s = .35 \text{ m}$ ,  $V = 125 \text{ Volts}$ , and  $\eta = 100 \mu\Omega\text{-m} \rightarrow \phi = 11.8 \text{ mWb}$ .



## Analysis of Resistive Heating a High $\beta$ Plasma

### Ohmic Heating

It is difficult to heat a high- $\beta$  plasma with Ohmic power alone.

For a rigid-rotor profile, the absorbed (Ohmic) power is:

$$P_{abs} = \frac{8\pi \eta B_e^2}{\mu^2 \langle \beta \rangle} \left[ 1 - \frac{1}{3} \tanh^2(K) \right] \quad \langle \beta \rangle = \frac{\tanh(K)}{K}$$

For  $K = 1$ , this is:

$$P_{abs} \text{ (MW/m)} = 16.9\eta(\mu\Omega - m)B_e^2(T)$$

The line-density can be expressed as:

$$N = \frac{\pi r_s^2 \langle \beta \rangle B_e^2}{2\mu k_B T_r}$$

Thus, the power absorbed per particle (eV/particle/sec) is:

$$\frac{P_{abs}}{Nk_B} = \frac{16}{\mu} \frac{\eta T_r}{\langle \beta \rangle^2 r_s^2} \left[ 1 - \frac{1}{3} \tanh^2(K) \right]$$

For  $K = 1$ , this is:

$$\frac{P_{abs}}{Nk_B} = 1.77 \times 10^7 \frac{\eta T_r}{r_s^2} \text{ (eV/particle/sec)}$$

Expressing  $\eta(\mu\Omega - m)$ , this is:

$$\frac{P_{abs}}{Nk_B} = 18 \frac{\eta(\mu\Omega - m)T_r}{r_s^2} \text{ (eV/particle/sec)}$$

The heating rate per particle only depends on  $\eta$ ,  $T_r$ , and  $r_s$ .  
The heating rate is not increased by increasing B through a larger voltage on the coils!

For  $r_s \sim 0.3$  m

$$\frac{P_{abs}}{Nk_B} = 197\eta(\mu\Omega - m)T_r \text{ (eV/particle/sec)}$$

### Radiation

Until burn through, the power lost to impurity radiation can be crudely approximated as:

$$P_r = 10^{-31} f n^2 \text{ W/m}^3$$

where  $f$  is the impurity fraction and  $n$  is the plasma density.

The heating rate exceeds the impurity losses, providing the impurity fraction is below the threshold:

$$f < 2.84 \frac{\eta(\Omega - m)T_r \text{ (eV)}}{n(10^{19})r_s^2 \text{ (m)}}$$

For  $\eta = 100 \mu\Omega - m$ ,  $T_r = 20$  eV, and  $n = 1.7 \times 10^{19}$ ;  $f < 3.7\%$ .

Since the heating rate per particle does not depend on density, but the radiation rate per particle is proportional to density, it is important that the magnetic field rises slow enough to allow the plasma time to burn through impurities, before the density rises too far.

Numerical calculations with Chodura resistivity show this analysis to be somewhat pessimistic, because the heating rate does in fact increase with increased voltage on the coils.

### Summary

- RMF current drive, combined with inductive flux build-up using a central core could lead to a powerful technique for generating high flux hot FRCs suitable for Tangential Neutral Beam Injection (TNBI).
- This method utilizes low voltage technology that can scale to large sizes.
- RMF will allow us to overcome difficulties of previous CSS experiments:
  - Start with a pre-formed, warm, low-density plasma.
  - Act to stabilize the FRC during the transition from the collisional to collisionless regime.
- The CSS technique has the potential of inducing a very large amount of trapped flux.
- Central core has an added benefit will allow an exploration of the effects of an added  $B_\theta$ .