A Network Model of Market Prices and Trading Volume

Andrei A. Kirilenko*

Preliminary and Incomplete

Abstract

This paper presents a network model of market prices and trading volume. In the model individual agents are represented by nodes. If any two nodes trade with each other within a unit of calendar time, they are considered connected by an edge. The edges define a network topology. I assume that the price difference between any two nodes is a random variable drawn from a normal distribution with a mean proportional to the number of edges between them and a constant variance. I define trading volume as the number of new edges added to the network per unit of calendar time. I show that depending on the underlying pattern of interaction, different levels of volume lead to the same changes in prices. I also show that while the agents do not directly observe the global pattern of interaction, they are able to consistently estimate it. I argue that an appropriate empirical price-volume model is a system of simultaneous equations in which the variance of prices is a function of volume and volume is a function of price variability and characteristics of the network topology.

*Please do not quote or circulate without an explicit permission from the author. The views expressed in the paper are my own and do not necessarily constitute an official position of the International Monetary Fund. Contact: International Capital Markets Department, International Monetary Fund, Washington, DC 20431. Phone: (202) 623-5642, Fax: (202) 623-5692, E-mail: akirilenko@imf.org.

1. Introduction

"Most people would agree that a fundamental property of complex systems is that they are composed of a large number of components or "agents", interacting in some way such that their collective behavior is not a simple combination of their individual behaviors. Over the years, a large body of research has been directed at understanding both the behavior of individual agents within complex systems and the nature of the interactions between them. As we are just beginning to realize, however, there is a third aspect to these systems which may be even more important and which has so far received little attention, and that is the pattern of interaction between agents, i.e., which agents interact with which others." (Newman, 2001).

Research on financial markets has been following the general trend of research on complex systems. Most studies focus on the behavior of individual agents in the marketplace (e.g., rational expectations), ways in which agents interact among themselves (e.g., market microstructure), and the properties of their collective outcomes (e.g., market efficiency). There are only a few studies that explicitly take into account the pattern of interaction among agents.[1]

The pattern of interaction forms a network with a particular structure or topology. Consider an intuitive way to think about network topologies of financial markets. Suppose that information is dispersed among a large number of economic agents. Markets facilitate information exchange through trading: direct connections between agents. Trading volume records the number of new connections (trades) per unit of calendar time, e.g., one hour. Depending on the underlying topology (pattern of connections), the same number of new connections can make one network much more connected than another. Intuitively, information spreads much faster in a network in which individuals are highly connected to each other than in one where they are far apart. The closer the connection, the more information is impounded into market prices over the same period of calendar time. As more dispersed information is pulled into market prices, they can become more volatile.

This intuition suggests two research questions. Can the pattern of interaction among agents drive a relationship between trading volume and market prices? Conversely, for given trading volume, market prices, and local information (direct trades), can the agents consistently estimate the pattern of interaction for the whole network?

I present a network model of market prices and trading volume. In the model individual agents are represented by nodes. Any two nodes are considered connected by an edge if they trade with each other within a given calendar time interval. The edges define a network topology.

I assume that (in each unit of calendar time) there exists a cross-sectional distribution of prices over the nodes. The price difference between any two nodes is a random variable drawn

[1]See, Kirman (1997) for a survey.

from a normal distribution with mean proportional to the normalized distance between the two nodes and constant variance. Specifically, if any two nodes are connected by a single edge (e.g., they have traded during a given time), then the mean price difference is equal to zero. If any two nodes are connected through chains of edges, then the mean is equal to a constant (scaling factor) times the smallest number of edges between them (distance) save one (normalizing factor).[2]

For the network as a whole, the price difference between any two nodes is a normal random variable with mean equal to the normalized average distance (the smallest number of edges between any two nodes averaged over all pairs of nodes) and constant variance. If the network is fully connected (each node has traded with all other nodes), then the average price difference between any two nodes is equal to zero. If the network is completely disjoint (no trading took place within a unit of calendar time), then the price difference between any two nodes becomes infinite as the number of nodes goes to infinity.

For example, if in the last hour, Joe traded with Mary, Mary traded with Bill, and Bill traded with Alice, then (at the end of the hour) the price difference between Joe and Mary, Mary and Bill, and Bill and Alice is a Normal random variable with zero mean and constant variance. The mean of the price difference between Joe and Bill is μ (the scaling factor) and between Joe and Alice is 2μ. The mean of the price difference for the whole network is $\frac{2\mu}{3}$.[3] Note that even if all transaction prices are observed, without knowing the pattern of trading, the distribution of prices over the nodes cannot be described. For instance, if exactly the same trade took place between Joe and Bill rather than Joe and Mary, the mean of the price difference for the network is $\frac{\mu}{2}$ rather than $\frac{2\mu}{3}$.

In addition to the cross-sectional dispersion of prices, I assume that new edges are added to the network over time. The nodes know the number of edges added to the network during a given calendar time, but do not know the location of the new edges on the network unless a new edge is local to a particular node (i.e., its own trade).

The price distribution changes in accordance with the evolution of the network topology or "network time", irrespective of the calendar time scale. However, observations of financial prices and quantities are typically made in calendar time. As a result, the distribution of prices in calendar time may appear non-stationary, even though it is stationary in network time.

In order to permit the difference between network time and calendar time, I use the methodology based on the work of Bochner (1960), Mandelbrot and Taylor (1967), and Clark (1973), who propose to apply a "time change", following which stationary applies.

[2]For completeness, I assume that if any two nodes are disjoint, then the mean is equal to a constant times the total number of nodes minus two (maximum number of edges connecting any two nodes minus the normalizing factor).

[3]There are 6 possible chains of edges between 4 nodes: (J,M), (J,B), (J,A), (M,B), (M,A), and (B,A). The means of price differences are 2μ for (J,A), μ for (J,B) and (M,A), and zero for (J,M), (M,B) and (B,A). Thus, the mean of the price difference for the network is $\frac{2\mu+\mu+\mu+0}{6} = \frac{2\mu}{3}$.

Agents do not directly observe network time. However, they conjecture that depending on the underlying topology, adding the same number of new edges can make the average distance of one network much smaller compared to another. Conversely, depending on the network topology, different number of edges needs to be added in order to achieve the same evolution of the price process.

I assume that agents know the (global) trading volume per unit of calendar time. I define trading volume as the number of new edges added to the network per unit of calendar time. By construction, trading volume reflects the intensity of network formation. Consequently, agents can use trading volume to convert calendar time into network time.

In network time, the mean price dispersion is either constant or decreases in the number of nodes. Thus, different number of new edges needs to be added to different networks, in order to make the price process evolve in the same way. Thus, a pattern of interaction among agents is a latent variable that drives a relationship between trading volume and market prices.

This results supports suppositions made in earlier studies.[4] For example, Clark (1973) posits that trading volume is a proxy for "operational time" according to which information flows into the market.

I show that for a given price process, the average intensity of network formation (trading volume) is not necessarily constant as assumed by earlier studies, but can also be decreasing in the number of nodes, depending on the network topology. In other words, depending on the underlying pattern of interaction among agents, different levels of volume (the addition of new edges in calendar time) lead to the same changes in prices.

I also show that while the agents do not directly observe the pattern of interaction, they are able to consistently estimate it if they know prices, volume and network locations of other agents. I argue that for a broad class of network topologies, agents can use information about network locations of other agents to form consistent estimators of the unknown parameters underlying the global network topology. Intuitively, some agents not only trade on their own, but intermediate transactions between other counterparties. The knowledge of their clients' trades and the identities of counterparties is the sort of necessary information that, in addition to prices and volume, can help to form a consistent opinion about the global trading pattern.

Finally, I provide support for earlier studies that empirically model the variance of prices as a power function of volume (e.g. Clark (1973)). However, unlike the earlier studies I argue that the exponential in the power function should be modelled not as a constant, but as a function of the (latent) network topology among agents. Thus, the correct empirical model is a system of simultaneous equations (rather than one equation), in which the variance of prices is a power function of volume and volume is a function of price variability and

[4] See Ane and Geman (2000) and Howison and Lamper (2000) for recent studies on stochastic subordination.

characteristics of the network topology.

The rest of the paper is organized as follows. Section 2 positions this paper in relation to the existing literature on trading volume. Section 3 gives a brief description of network topologies. Section 4 presents the model. Section 5 describes how to go from local information to global network topology. Section 6 evaluates the robustness of results with respect to the underlying assumptions. Section 7 concludes.

2. Literature

The network model presents a formal framework to understand the relationship between prices and volume. According to standard rational expectations models with supply uncertainty, trading orders have both informational (or "signal") and "noise" components.[5] If the signals are independently and identically distributed or dependencies among them cannot be learned from volume, then volume contains only "noise" and all valuable information is in the prices. In other words, although prices and volume and determined simultaneously, innovations to changes in prices and volume are generated by independent stochastic processes.

However, if signals of the informed agents are dependent and the degree of signal correlation and/or precision could be forecasted from volume, then prices and trading volume together are more informative than prices alone. Specifically, a more precise estimator of the variance-covariance matrix of individual private signals can be constructed from prices and volume together rather than from prices alone.

A number of empirical studies support positive empirical relationship between trading volume and price changes. Clark (1973) models conditional volatility of price changes as a power function of the form $\sigma^2_{\Delta P} \mid v = Cv^b$, where ΔP is per period (daily) differences in prices, v is per period (daily) trading volume and C and b are positive constants. Consequent studies by Epps and Epps (1976) and Tauchen and Pitts (1983) support a nonlinear positive empirical relationship between trading volume and price changes.

A survey by Karpoff (1987) reports that trading volume is positively related to volatility[6] Although when the number of trades and measures of quoted liquidity are included into the estimation, trading volume has no additional explanatory power. Jiang and Kryzanowski (1998) report that the number of trades is associated with the rate of information flow and Jones, Kaul and Lipson (1994) find that volatility is primarily determined by number

[5] Private signals of informed traders can either be independently and identically distributed conditional on the true value of the asset (Diamond and Verrechia (1981), Brown and Jennings (1989)) or correlated (Foster and Viswanathan (1996)). Signals can either have the same precision (Grundy and McNichols (1989)) or different precisions across agents (Blume, Easley and O'Hara (1994)). The signals can also be both correlated and have different precisions (Dupont (1997)).

[6] For another survey, see Gallant, Rossi, and Tauchen (1992).

of trades rather than trading volume. Bessembinder, Chan, and Seguin (1996) find that trading volume rises as a function of divergence in opinions.

These results can be better understood in the context of network topology. Even when it is not explicitly specified, the pattern of interaction (network topology) is always imbedded in a model. Standard rational expectations models assume that trades of independent agents form a regular lattice: either they trade all at once (a star topology) or sequentially (a segment or ring topology). For example, Kyle (1985) assumes a star topology, while Glosten (1989) assumes a segment topology. In a batch market, the common agent (market maker) uses information contained in the order flow to forecast the parameters (variance-covariance matrix) of the aggregate distribution of signals (Foster and Viswanathan (1996), Dupont (1997)). In a sequential setting, each consecutive trader uses preceding and contemporaneous price-volume sequences to learn about the dispersion of beliefs (e.g., the main diagonal of the variance-covariance matrix) and utilize this information to form expectations (Blume, Easley, O'Hara (1994)).

Under these assumptions, because information is incorporated into prices through a regular lattice, volume does not have any information about the trading process and, unless it is assumed to be correlated with the dispersion of beliefs, lacks value.

Moreover, according to the Central Limit Theorem for theoretical markets with independent traders, the distribution of returns (log changes in price over discrete time intervals) converges to a Gaussian. Empirically, however, the tails of the distribution of returns decay according to the power law rather than exponentially (as in the Gaussian). Usefully, for a general class of network topologies, neither the Central Limit Theorem nor informational redundancy of volume need to be true.

Therefore, explicitly modeling the pattern of interaction can help explain some empirical properties of prices and volume that are hard to reconcile with predictions coming from earlier theoretical models.

3. Network Topologies

This section gives a brief description of network topologies. A network topology is defined by the connection configuration and does not depend on the specifics of interconnections or message characteristics.

3.1. Definitions

Two connected nodes are called neighbors. The degree (or clustering coefficient) of a node is defined to be the number of its neighbors. If the degree is equal to all but one nodes (i.e. itself), then the agent is connected to all other agents.

Sometimes, messages must go through several nodes in order to reach their final destination. The average distance of a network (also known as characteristic path length or the number of degrees of separation) is the smallest number of edges between any two nodes averaged over all pairs of nodes. The longer the average distance, the more edges it takes to go from any node to any other.

The mathematical theory of networks is known as graph theory. A graph is a pair (N, E), where N is a finite set and E is a binary relation on N. Elements u, v of N are called nodes (or vertices) and elements of E are called edges (or bonds). An edge is a pair (u, v) with $(u, v) \in E$. In a directed graph, edges are ordered pairs, connecting a source node to a target node. In an undirected graph, edges are unordered pairs and connect the two nodes in both directions. Hence, a directed edge (u, v) can be different from a directed edge (v, u), while an undirected edge (u, v) is the same as (v, u).

3.2. Main types of network topologies

There are three general types of network topologies: regular lattice, random graph, and disordered or "small-world" topology. Regular lattice is a regular network. There is no randomness in the connection structure. Regular lattices have high average distances and high clustering (degrees). Specifically, their average distances are proportional to the number of agents and, thus, go to infinity when the number of agents increase. For example, an exogenous sequential arrival of agents forms a regular lattice, a segment or a ring. Each arriving agent is connected only with her immediate predecessor.

A random graph is constructed by taking a set of n nodes and randomly introducing each of the possible $\frac{n(n-1)}{2}$ edges among them with equal probability $0 \leq p \leq 1$.[7] If $p = 0$, all nodes remain disconnected and if $p = 1$, all nodes are connected.[8] Random graphs have small average distances and low clustering. Random graphs also have a property of a rapid formation of a global connected component when the number of added edges exceeds half the number of nodes (a phase transition). Random graphs have been used to describe herd behavior in financial markets: large changes in prices accompanied by low volume levels.[9]

Disordered or "small-world" networks—called so by analogy with a "small-world" phenomenon, popularly known as the 'six degrees of separation'—are neither completely regular, nor purely random. A small world network is constructed by "rewiring" a regular lattice either (i) by taking one end of an existing edge and assigning it to a new node chosen uniformly at random (so some existing edges are removed from the lattice) or (ii) by adding new shortcuts between pairs of nodes chosen uniformly at random (without removing any of the existing edges from the regular lattice).[10] In other words, disordered networks are

[7] Random graphs were introduced in Erdös and Rényi (1960).

[8] An edge connecting a node to itself is allowed.

[9] See Cont and Bouchaud (2000), Eguiluz and Zimmermann (2000), and D'Hulst and Rogers (2000).

[10] Under the second type of "rewiring", a message could be simultanously transmitted to more agents than

constructed by superimposing a “random” subgraph over a “regular” subgraph. Watts and Strogatz (1998) show that small-world networks exhibit properties of both regular lattices (high clustering) and random graphs (small average distances). In this paper, I will use a small-world framework to model the pattern of interaction in financial markets.

Barthelemy and Amaral (1999) give the following intuitive descriptions of a regular lattice and random graph network topologies: “The regular network is similar to the streets of Manhattan: Walking along 5th Avenue from Washington Square Park on 4th Street to Central Park on 59th Street, we have to go past 55 blocks. ... The random network is similar to a strange subway system that would directly connect different parts of Manhattan and enable us to go from Washington Square Park to Central Park in just one stop.” A disordered network could be thought of as using both the ordered grid and the strange subway to go between any two points in Manhattan.

3.3. Boundary conditions

Boundary conditions specify what happens at the ends of a network. There are two main types of boundary conditions: free and periodic. If a part of the network repeats itself after a specified number of nodes, then boundary conditions are periodic. For example, a network with a finite number of nodes, in which each node is connected to only one other node forms a simple ring. This network (infinitely) repeats itself after every node. With periodic boundary conditions, each node is always connected to the same number of neighbors irrespective of it’s location within the network. Periodic boundary conditions represent an approximation to an infinite system in that the connection structure does not change with the addition of more nodes.

By contrast, if a network has ends at which the nodes have fewer neighbors than in the interior of the network, then the boundary conditions are free. For example, if one node is removed from a simple ring defined above, then the node that lost the link is not connected to any other node. This network can be visualized a finite line segment. It has ends where the number of connections is smaller than that in the interior. Free boundary conditions approximate a finite system with the interior and exterior characteristics, e.g., a drop of liquid on a solid surface or an isolated cluster of agents. Naturally, a multi-dimensional network can be periodic in some dimensions and free in others. For example, a network can have periodic boundary conditions across space and free boundary conditions across time.

The way in which the boundary conditions are constructed affects the speed of message diffusion. For example, if every node is connected to six other nodes rather than just one, then a message sent by one node will be received by six neighbors, passed on to six more distinct neighbors (some neighbors will receive the message from multiple nodes), and diffuse faster than a one-to-one transmission.

specified by the regular lattice.

3.4. Nodes and edges

Typically, nodes represent distinct agents and edges specify interaction and communication among them. The objective is to describe the behavior of a group of economic agents rather than viewing them in isolation. Connections among groups of agents are assumed to be direct.

There are two main ways of defining an edge in the literature.[11] According to one way, an edge represents a match/trade between an agent willing to buy and an agent willing to sell an asset.[12] This is the approach I use in this paper.

Alternatively, an edge between two traders means that they have common (coordinated) information/beliefs. This definition has been used previously both for regular lattices (Ellison (1993), Blume (1993), Kosfeld (1998), Bala and Goyal (2000)) and random graphs (Cont and Bouchaud (2000), Eguiluz and Zimmermann (2000), and D'Hulst and Rogers (2000)).

4. Model

The following network model is based on a small-world model of Kleinberg (1999).

4.1. Setup

Consider a two-dimensional regular integer lattice, (N, E) with N nodes, E edges and free boundary conditions. Individual agents are represented by nodes. Each of the N nodes, $(i; j)$, is identified as a point on a grid. Any two nodes are considered connected by an edge if they trade with each other within a given calendar time interval.

Define the distance between two nodes $(i; j)$ and $(k; l)$ to be the number edges separating them, $d((i; j); (k; l)) = |k - i| + |l - j|$. For a constant $\delta \geq 1$, the node $(i; j)$ has a directed edge to every other node within distance δ. These are it's local δ-neighbors. In addition, for constants $M \geq 0$ and $0 \leq \beta \leq 2$, node $(i; j)$ has long-range contacts (β-acquittances) with M other nodes constructed by randomly adding edges between $(i; j)$ and $(k; l)$ with probability

$$P(d, \beta) = \frac{[d((i; j); (k; l))]^{-\beta}}{\sum_{(k;l)} [d((i; j); (k; l))]^{-\beta}}. \tag{1}$$

For fixed M, this specification describes a general family of parametric network models

[11] Kirman (1997) and Brock and Durlauf (2000) provide extensive surveys of the use of networks and graph theory in economic modelling.

[12] See, for example, Ioannides (1990).

characterized by parameters δ and β.[13] For $\beta = 0$, the distribution over long-range contacts is uniform (as in the Watts and Strogatz (1998) original "small-world" model) and the choice of long-range contacts is independent of a node's location on the grid. As β increases, the probability of connecting to a node within a smaller distance becomes higher than that for a higher distance, and the long-range contacts become more localized.

Specifically, by construction, for this probability distribution, for any two long-range contacts i and j, the ratio $\frac{P(j,\beta)}{P(i,\beta)} = \frac{i}{j}^{\beta} \leq \frac{N}{\beta}$, where $P(j,\beta)$ is the probability of connecting to a long-range acquittance located at a distance j. In other words, the further is j from the original node compared to i, the less are its chances of becoming a long-range contact. When $\beta = 2$, the probability distribution (1) is inverse-square power. For this value of β, for any two long-range contacts i and j, $\frac{P(j,2)}{P(i,2)} = \frac{i}{j}^{2}$, proportional to the dimension of the grid.

For $\delta = 1$, each node has four local contacts (trades), one in each direction on the grid. As δ increases, the number of local neighbors increases. For $\delta = N$, there exists a node for which any other node on the grid is a neighbor. By construction, the structure of local neighbors is deterministic and the same for every node. However, the long-range acquaintances are determined stochastically and their position could differ from node to node. Consequently, I assume that every node has information about the set of local δ-neighbors for all nodes (the ordering of the lattice), but knows with certainty only its own long-range contacts. About the long-range contacts of other nodes, each node knows that they have M long-range contacts constructed by randomly adding (directed) edges with probability drawn from a power distribution with parameter β.

At the beginning, I assume that both β and M are known constants. Later, I relax this assumption and assume that while M is known, only the functional form of the parametric process (power distribution) is known to the agents, but the value of the parameter β is not. However, in either case, each node has complete local information, but does not know the global network structure.

4.2. Prices and average distances

I assume that for all pairs of nodes, $(i;j)$, $(k;l)$, the price difference between any two connected nodes, $\Delta P_{ij,kl}$, is a random variable drawn from a normal distribution with mean equal to $\mu\,(min\,(d((i;j);(k;l))) - 1)$ and variance equal to σ_N^2, where $\mu > 0$ and $\sigma_N^2 > 0$ are constants. I also assume that the price difference between any two unconnected nodes is a random variable drawn from a normal distribution with mean equal to $\mu\,(N-2)$ and variance equal to σ_N^2.[14]

[13] Accoring to Kleinberg, "this model has a simple "geographic" interpretation: individuals live on a grid and know their neighbors for some number of steps in all directions; they also have some number of acquaintances distributed more broadly across the grid."

[14] This assumption is without the loss of generality. $(N-2)$ is the difference between the maximum number

Intuitively, if any two nodes are connected by a single edge (e.g., they have traded during a given calendar time), then the mean price difference is equal to zero. If any two nodes are connected through chains of edges, then the mean is proportional to the distance between them. If any two nodes are not connected (they have not traded within a given time), then the mean is proportional to the maximum number of edges between them.

Note that the cross-sectional mean price difference does not have to be equal to zero. Intuitively, during the same calendar period of time, different pairs of agents could trade the same asset at different prices. Arbitrage (the same agent trading with many different agents) makes the network more connected and, thus, forces the mean price difference to zero. However, I do not assume cross-sectional arbitrage, a priori.

Denote by P_{ij}, the price corresponding to a randomly drawn edge (trade) originating at some node $(i; j)$. By construction, the price difference between this node and any other node is a random variable drawn from a Normal distribution with mean equal to the normalized average distance (the smallest number of edges between any two nodes averaged over all pairs of nodes) and constant variance. For example, if the network is fully connected (each node has traded with all other nodes), then the average price difference between any two nodes is equal to zero. If the network is completely disjoint (no trading took place within a unit of calendar time), then the price difference between any two nodes is proportional to the total number of nodes.

The average distance of a network depends on the values of δ, M, and β. For given $\delta \geq 1$, $M \geq 0$, and $0 \leq \beta \leq 2$, each node can calculate how many edges there are on average between any two nodes. Consider the following lemmas.[15]

Lemma 1 For $\delta \geq 1$, $0 \leq \beta \leq 2$, when $M = 0$, the average distance between any two nodes is proportional to $\frac{N}{4\delta(\delta+1)}$, linear in the number of nodes.

When $M = 0$, the lattice is regular, i.e. no node has random (long-range) edges irrespective of the value of β. For a two-dimensional grid, the number of local neighbors is equal to $2\delta(\delta + 1)$ as each node is connected to all nodes within the distance δ along two dimensions. Each "step" in any of the four directions connects with $\delta(\delta + 1)$ new nonoverlaping (local) contacts. Consequently, the average distance between any two nodes increases as $\frac{N}{4\delta(\delta+1)}$, and thus grows linearly with the number of nodes. For example, when $\delta = 1$ each node has $2 \times 1 \times (1 + 1) = 4$ local contacts, one in each direction on the grid. Each step in any direction connects with 2 new nonoverlaping contacts and the average distance between any two nodes grows at the rate $\frac{N}{8}$. When $\delta = 2$, each node has 12 neighbors, each step connects with 6 new nonoverlaping neighbors, and the average distance between any two nodes increases at the rate $\frac{N}{24}$. For more general types of regular lattices, the average distance grows as $\frac{N}{\delta C(\delta)}$, where $C(\delta) > 0$ is a constant independent of N.

of edges connecting any two nodes, $(N - 1)$ and one, the normalizing factor.

[15] The results in lemmas 1 and 2 are standard in random graph theory.

Lemma 2 For $\delta \geq 1$ and $M > 0$, when $\beta = 0$, with high probability the distance between any two nodes is bounded by $\frac{lnN}{lnM}$, exponentially smaller than the total number of nodes.

The value of $\beta = 0$ corresponds to a uniform distribution over long-range contacts. Thus, each "step" connects with $M > 0$ new long-range contacts. With k steps, with high probability, the number of edges increases with the number of "steps" as $N \sim M^k$. This implies that the average distance increases proportionally to $\frac{lnN}{lnM}$, exponentially smaller than the total number of nodes. Note that when $M = N$, i.e. each node has long-range contacts with all other nodes (including itself), the average distance between any two nodes is equal to 1.

Lemma 3 For $\delta = 1$ and $M > 0$, when $0 < \beta < 2$, the average distance between any two nodes is bounded by a polynomial in N. Moreover, when $\beta = 2$, the average distance is bounded by $\alpha_1 ln^2(N)$, where α_1 is a constant independent of N.

Kleinberg (1999) provides a formal proof. The intuition is as follows. Under the assumed probability distribution of long-range contacts (1), the placement of long-range links is correlated with the lattice distance. For $\beta = 2$, a node has the same correlation with each of it's long-range contacts. For $\beta < 2$, the correlation with its long-range contacts at greater distances is higher than with those at closer distances. Given this property, a variation of breadth-first search algorithm (described as a breadth-closest search) is able to find a known target within a square-logarithmic number of steps.

A breadth-first search algorithm on a directed graph is defined as follows. Start at node u and proceed to build up a series of layers. Layer 1 consists of all nodes to which u points with a directed edge. Layer i consists of all nodes to which there is a directed edge from some node in layer $i-1$, but are not in any earlier layer. By construction, layers are disjoint. The smallest distance from u to v is the index of the layer that v belongs to. Since the lattice under study has a connected ordered subgraph, such a layer always exists.

For $\beta = 2$, each long-range is equally likely to belong to any of the layers, while for $\beta < 2$, the probability of belonging to layers at greater distances is higher than for those at lower distances. Consequently, for $\beta = 2$, the search is equally likely to move closer to a target at any layer, while for $\beta < 2$, the search is less effective at closer distances to the source.

Consider the following variation of a breadth-first search algorithm, which could be described as the breadth-closest algorithm. After constructing the layers, forward the message to the layer that is as close as possible to the target. Kleinberg (1999) shows that such a decentralized algorithm is able to find a known target within number of steps that is smaller than the number of nodes on the grid. In particular, the algorithm can reach from any source to any target within the number of steps that is proportional to (i) a power function in the number of nodes with the exponent less than or equal to $\frac{2}{3}$ for $\beta < 2$ or (ii) a square-logarithmic function in the number of nodes for $\beta = 2$.

4.3. Volume and network time

In addition to the cross-sectional dispersion of prices, I assume that over time new edges are added to the network. I also assume that unless a new edge is local to a particular node (its own trade), the nodes do not observe where the new edges have been added to the network. However, the nodes do know how many edges have been added to the network during a given calendar time.

Intuitively, the pattern of trading evolves as new trades are being added.[16] The agents only observe the total number of trades that occurred between calendar time observations. However, during the same length of calendar time (e.g., one hour), the pattern of trading among agents may change anywhere between a lot and very little.

By assumption, the distribution of prices changes with the evolution of the network topology (the pattern of trading). Intuitively, the pattern of trading affects the transmission of valuable information through the network. In a highly connected network, information travels very quickly between any two nodes. Consequently, the distribution of price differences among agents is centered very tightly at zero. In a mostly disjoint network, information between nodes travels slowly. Correspondingly, the distribution of price differences has a possibly nonzero mean and a high variance.

During one hour, the network topology may change from moderately connected to highly connected or become almost disjoint (very few trades). During the same hour, the price distribution changes in accordance with the evolution of the network topology or “network time”, irrespective of the calendar time scale. However, observations of financial prices and quantities are typically made in calendar time. As a result, the distribution of prices in calendar time may appear non-stationary, even though it is stationary in network time.

In order to permit the difference between network time and calendar time, I use the methodology based on the work of Bochner (1960), Mandelbrot and Taylor (1967), and Clark (1973), who propose to apply a “time change”, following which stationary applies.[17]

I suppose that the intensity of network formation is controlled by a strictly increasing, continuous, adapted network-time process, $T(t)$ with $T(0) = 0$. $T(t)$ denotes the amount of market time that passed after t units of calendar time. For example, when $T(t) = t$, network time is equivalent to calendar time. The agents do not directly observe $T(t)$.

Denote by $P_{ij}(t)$ the price corresponding to a randomly drawn edge (trade) originating at some node $(i; j)$ at calendar time t. From here on, I suppress the subscript ij with the understanding that the location of the node $(i; j)$ at which the edge (trade) originates is

[16] In this paper, the network topology evolves as new edges are being added to a constant number of nodes. There is an expanding literature on growing networks that evolve as both new nodes and new edges are being added. See, for example, Jin et al (2001) and Callaway et al (2001).

[17] For recent studies invloving this technique, see Ghysels and Jasiak (1994), Ghysels, Gourerioux and Jasiak (1995), Conley, Hansen, Luttmer and Scheinkman (1997), Russel and Engle (1998), Duffie and Glynn (2001), and Geman, Madan and Yor (2001).

known only to itself and the node to which the edge connects. All other nodes view the location as random. In other words, only the counterparties specific to the trade know that the transaction price, $P(t)$, observed by other agents corresponds to their trade.

Denote by $\Delta P(t+1) = P(t+1) - P(t)$, the evolution of the price process in calendar time $t \geq 0$. By construction, $\Delta P(t+1)$ is a random variable drawn from a Normal distribution with mean zero and variance σ_N^2. While for each $t \geq 0$, the cross-sectional mean of $P(t)$ could be nonzero depending on the network topology, the increments of the price process in calendar time are zero due to arbitrage.

Denote by $\Delta T(t+1, t) = T(t+1) - T(t)$, $t = 0, 1, ...$, the interval of network time that passed during the change in calendar time from t to $t+1$.

I assume that agents know the (global) trading volume per unit of calendar time. I define trading volume as the number of new edges added to the network per unit of calendar time. By construction, trading volume reflects the intensity of network formation. Consequently, the agents can use trading volume to convert calendar time into network time.

5. Network topology, prices, and volume

In this section I address two questions. Can the pattern of interaction among agents drive a relationship between trading volume and market prices? And, for given trading volume, market prices, and local information (direct trades), can the agents consistently estimate the pattern of interaction for the whole network?

5.1. From network topology to prices and volume

Clark (1973) proves the following result (Theorem 4 and Corollary 4.2): Suppose that $P(t)$ is a process with stationary independent increments and $T(t)$ is another process with possibly nonstationary and correlated increments. Let the increments of $P(t)$ be drawn from the distribution with mean zero and finite variance σ_N^2 and the increments of $T(t)$ be drawn from a positive distribution with mean λ independent of the increments of $P(t)$. Then the subordinated stochastic process $P(T(t))$ has a stationary uncorrelated increments with mean zero and variance $\lambda\sigma_N^2$.

The intuition is as follows. Suppose that on average λ new edges are added to a network. Depending on the network topology, the cross-sectional characteristics of the network change. The average distance becomes much smaller in a random graph than in a regular lattice.

Conversely, depending on the network topology, different number of edges, λ needs to be added in order to achieve the same evolution of the price process in calendar time. Consider the following proposition.

Proposition 1 The values of $\lambda = \lambda(\delta, \beta, M, N)$ have the following upper bounds:

(i) $\frac{N}{4\delta(\delta+1)}$ for a regular lattice,

(ii) $\frac{lnN}{lnM}$ for a random graph,

(iii) $\alpha_\beta N^{\frac{2-\beta}{3}}$ for a disordered network with $0 \leq \beta < 2$, and

(iv) $\alpha_1 (lnN)^2$ for a disordered network with $\beta = 2$.

The proof follows directly from lemmas 1 - 3. Proposition 1 establishes the relationship between the parameters describing the network topology and the mean of the (unobserved) network time process. In contrast to earlier studies, Proposition 1 shows that for a given price process, the average intensity of network formation (trading volume) is not necessarily constant, but can also be decreasing in the number of nodes. In other words, depending on the underlying pattern of interaction among agents, different levels of volume (the addition of new edges in calendar time) lead to the same changes in prices.

The intuition is as follows. Consider a fully connected network and a fully disjoint network. The average distance in the fully connected network is one and in the fully disjoint network is proportional to the number of nodes, N. In order to make the price process evolve in the same way in the two networks, up to N new edges need to be added to the fully disjoint network, while up to one edge needs to be added to the fully connected network.

5.2. From prices, volume, and locations to network topology

In the previous subsection, I argued that the average intensity of network formation can be decreasing in the number of nodes for topologies other than the regular lattice. However, since the pattern of interaction is not directly observed, can the agents do better than assuming constant average intensity for the driving (network time) process?

What kind of additional information would an agent need in order to understand the global network topology? In this section I argue that while the agents do not directly observe the global pattern of interaction, they are able to consistently estimate it, if they know prices, volume and lattice locations of other agents.

Is it realistic to assume that agents know lattice locations of other agents? In practice, some agents not only trade on their own, but intermediate transactions between other counterparties. The knowledge of their clients' trades and the identities of counterparties is the sort of necessary information that, in addition to prices and volume, can help to form a consistent opinion about the global trading pattern.

If agents know the values of δ, M, and β, they can solve for the values of λ as a function of N. I begin by showing for which parameter values of δ, M, and β, even with the knowledge of lattice locations of other agents, the local geometry of the grid is either completely uninformative or fully informative about the global network topology. Consider the following lemmas.

Lemma 4 *If there exists a node for which any other node on the grid is a neighbor, i.e. $\delta = N$, then the local geometry of the grid provides full information about the global network topology.*

The distance between any source and any target is equal to at most two. The simple message transmission algorithm is as follows: if the target is a neighbor, send the message directly, otherwise, send the message to a central node. Information about message transmission does not add any new information about the global network topology.

Lemma 5 *If no random long-range contacts are introduced into the regular lattice, i.e. $M = 0$, then the local geometry of the grid provides full information about the global network topology.*

If the network topology is a regular lattice, then its global geometry consists of repeated identical local geometries (e.g. each node is connected by fours edges to four other nodes in each direction). Thus, local geometry provides full information about the global topology. No additional information is needed to characterize the network topology.

Lemma 6 *If the long-range contacts are generated by a uniform distribution, i.e. $\beta = 0$, then the local geometry of the grid does not provide any additional information about the global network topology.*

The uniform distribution corresponds to a random drawing of agents that is independent of the underlying grid structure. Thus, although with high probability there exist paths with lengths bounded by lnN, a decentralized algorithm cannot find these paths. Consequently, there does not exist a local information-based algorithm capable of improving upon a sequential grid search. Kleinberg (1999) shows that the smallest number of steps of a decentralized algorithm is bounded by $\alpha_0 N^{\frac{2}{3}}$.

In summary, lemmas 4 - 6 show that for boundary parameter values, i.e. $\delta = N$, $M = 0$, and $\beta = 0$, the knowledge of lattice locations of other agents does not add any new information about the global topology, because the local geometry of the grid is either completely uninformative or fully informative.

It is reasonable to assume that agents know their local and long-range trading interaction, δ and M. However, the value of the global parameter β is typically unknown. Suppose that only the functional form of the general parametric process (power distribution) is known to the agents, but the value of the global parameter β is not. Can an agent combine information about prices, volume, and locations of other traders to form a consistent estimate of the value of β? In other words, can estimates formed on the basis of local probability laws converge to the global parameter characterizing the aggregate probability law?[18]

[18] See Blume and Durlauf (1999) for a discussion of equilibrium concepts for social interaction models.

Proposition 2 There exists a decentralized algorithm such that local estimators of the unknown global parameter β are consistent.

Consider the following algorithm. Since lattice locations of source-target pairs are known, an agent can order these pairs by distance from the smallest to the largest. The agent can then observe how the realized distances behave as the number of (ordered) edges separating the source from the target grows. Specifically, the agent can estimate coefficients in the following linear regression, $lnY = \phi_0 + \phi_1 lnZ$, where Y is the vector of realized distances (number of edges between each source-target pair) and Z is the ordered vector of lattice distances (number of edges between the coordinates of the source-target pairs). The estimated coefficient $\hat{\phi}_1$ is an estimate of β. Namely, $\hat{\beta} = 2 - 3\hat{\phi}_1$, where $\hat{\beta}$ denotes an estimate. If $\hat{\phi}_1 = \frac{2}{3}$, then $\hat{\beta} = 0$. If $\hat{\phi}_1 = 0$, then $\hat{\beta} = 2$. Finally, if $0 < \hat{\phi}_1 < \frac{2}{3}$, then the value of $0 < \hat{\beta} < 2$. From the random graph theory, errors around the upper bound on average distances are $O(1)$. Therefore, $\hat{\beta}$ is a consistent estimator of β. This completes the proof.

Intuitively, average distances found by a local information-based algorithm are bounded by a power function. Using repeated observations of average distances, agents can form an estimate that converges to the exponent of the power function (global property).

Standard tests can be applied to test whether $\hat{\phi}_1$ is significantly different either from zero or from $\frac{2}{3}$. If $\hat{\phi}_1$ is not statistically significantly different from zero, then with high probability there exists a local information-based algorithm that could find a short distance from any source to any target. Similarly, if $\hat{\phi}_1$ is not statistically significantly different from $\frac{2}{3}$, then with high probability the local geometry of the grid does not provide any additional information about the global network topology.

6. Robustness

In this section I evaluate the assumptions underlying the results shown in the previous sections.

First, I assume that the network topology is exogenous, i.e. agents have no choice over the design or transformation (e.g., addition of edges) of the topology. This assumption is restrictive. Agents may wish to change the topology. However, Bala and Goyal (2000) show that at the local level, the assumed type of topology (stars) is robust with respect to noncooperative behavior.[19]

[19] Bala and Goyal (2000) study the theory of network formation in a noncooperative framefork: an agent can establish links with others without their permission by simply incurring some costs. They assume that benefits from establising a link can either go to the agent that established the link or to both agents connected by the link. They show that in the limit (one-dimensional) equilibrium networks have one of two types of topologies: ring (or wheel) or star. The type of the limiting topology depends on the degree of required incentive compatibility of agents. If all benefits from establishing a link go to one agent, then each agent

Second, I assume a power family of distributions for the selection of long-range contacts. Power family of distributions is fairly general. It has been empirically observed in a number of social and natural phenomena. Power family of distributions is homogeneous (scale independent), i.e. it holds at all frequencies/magnifications with a different exponent and constant.

Third, I assume that the addition of edges (trades) is costless. This is also a restrictive assumption. It would be more realistic to assume that the degree of a topology has an effect on cost, since the more links a node has the more logic it takes to implement the connections.[20] Although a cost structure as a function of distance could be easily formulated, it would complicate the model, without necessarily adding to its crucial findings. This is a subject of future work.

Fourth, I assume that lattice locations of both the source and the target are known to all nodes. This may seem to be a restrictive assumption. However, Adamic et al (2001) show that for β close to two, the average distance between any two nodes remains bounded by $ln^2(N)$ even if the location of the target is not known. The only information required is that the nodes have some information about their second degree neighbors, i.e. their friends' friends. The algorithm that has this property is specified as follows. Starting with the source node, at each step choose a node with the highest degree (number of local and long-range contacts) until the highest degree is achieved. After visiting the highest degree node, continue by choosing the node with the next highest degree. If that node turns out to have the highest degree continue down the degree sequence until a node with a higher degree is found.

7. Conclusion

This paper presents a network model of market prices and trading volume. In the model individual agents are represented by nodes. If any two nodes trade with each other within a unit of calendar time, they are considered connected by an edge. The edges define the pattern of interaction among agents or a network topology.

I assume that there is cross-sectional distribution of price differences among agents. The price distribution changes in accordance with the evolution of the network topology or "network time." The network time is not directly observable by the nodes. The nodes know the number of edges added to the network during a given calendar time, but do not know the location of the new edges on the network unless a new edge is local to a particular node (its own trade). As a result, the distribution of prices in calendar time may appear

chooses a best response strategy to the other agents' strategies and the limiting topology is a ring. If the benefits are shared between two agents connected by a link, then each pair of agents must choose the best response to the other agents' strategies and the limiting topology is a star.

[20] See, for example, George, Kaul, and Nimalendran (1994) on how transactions costs can affect volume.

non-stationary, even though it is stationary in network time.

Based on the work of Bochner (1960), Mandelbrot and Taylor (1967), and Clark (1973), I apply a "time change", which results in the stationarity of the price process. However, in contrast to earlier studies, I argue that for a given evolution of the price process, the intensity of network formation (trading volume) in calendar time is a function of the network topology. I show that depending on the underlying pattern of interaction among agents, different levels of volume (the addition of new edges in calendar time) lead to the same changes in prices. I also argue that an appropriate empirical price-volume model is a system of simultaneous equations, in which the variance of prices is a function of volume and volume is a function of price variability and characteristics of the network topology.

I show that for a broad class of network topologies, there exists a decentralized efficient mechanism to transmit information dispersed throughout the economy. I argue that while the agents do not directly observe the pattern of interaction, they are able to consistently estimate it, if they know prices, volume and network locations of other agents. Intuitively, some agents not only trade on their own, but also intermediate transactions between other counterparties. The knowledge of their clients' trades and the identities of counterparties is the sort of necessary information that, in addition to prices and volume, can help to form a consistent opinion about the global trading pattern.

Network topology is different from market microstructure. Market microstructure is a set of trading rules. It changes infrequently. Network topology is a set of trading interactions. It changes all the time. Individual traders know market microstructure and their trading decisions. They do not know the global trading pattern.

However, even when it is not explicitly specified, the pattern of interaction (network topology) is always imbedded in a market microstructure model. Standard rational expectations models assume that trades of independent agents form a regular lattice: either they trade all at once (a star topology) or sequentially (a segment or ring topology). For example, Kyle (1985) assumes a star topology, while Glosten (1989) assumes a segment topology. In addition, it is assumed that the agents know the structure of the network topology. Under these assumptions, because information is incorporated into prices through a regular lattice, volume does not have any information about the trading process and, unless it is explicitly assumed to be correlated with the dispersion of beliefs (Blume, Easley, O'Hara (1994)), lacks value.

I argue that when the pattern of interaction is not a regular lattice, but a disordered graph (constructed by superimposing a "random" subgraph over a regular lattice), trading volume contains information about the network topology. Specifically, for a given price process, trading volume is a function of the degree of randomness in the network.

A number of assumptions common in the literature on general complex systems drive the results of this paper. Two of the assumptions merit a specific mention. I assume that the global network topology is exogenous. While Bala and Goyal (2000) show that the type of topology that I assume at the local level (stars) is robust with respect to noncooperative behavior, agents collectively may wish to change the global topology. I also assume a power family of distributions for the selection of random trade pairs. While the power family of distributions is fairly general and has been empirically observed in a number of social and natural phenomena, it parametrically determines the functional form of my results.

Developing a suitable set of modelling assumptions and finding empirical support for the arguments made in this paper are subjects of future work.

References

[1] Adamic, Lada A., Lukose, Rajan M., Puniyani, Amit R., and Bernardo A. Huberman, 2001, Search in power-law networks, mimeo, Stanford University.

[2] Ane, Thierry, and Helyette Geman, 2000, Order flow, transaction clock, and normality of asset returns, Journal of Finance, 2259–2284.

[3] Bala, Venkatesh, and Sanjeev Goyal, 2000, A non-cooperative model of network formation, Econometrica 68, 1181-1229.

[4] Barthelemy, Marc, and Luis A. N. Amaral, 1999, Small-world networks: Evidence for a crossover picture, Physical Review Letters 82, 3180-3183.

[5] Bessembinder, H., K. Chan, and P. J. Seguin, 1996, An empirical examination of information, differences of opinion, and trading activity, Journal of Financial Economics 40, 105-134.

[6] Blume, Lawrence E., 1993, The statistical mechanics of strategic interaction, Games and Economic Behavior 5, 387-424.

[7] Blume, Lawrence E., David Easley, and Maureen O'Hara, 1994, Market statistics and technical analysis: The role of volume, Journal of Finance 49, 153–81.

[8] Blume, Lawrence E., and Steven Durlauf, 1999, Equilibrium Concepts for Social Interaction Models, mimeo.

[9] Bochner, Salomon, 1960, Harmonic Analysis and the Theory of Probability, University of California Press, Berkeley, CA.

[10] Brock, William A., and Steven N. Durlauf, 2000, Interaction-based models, NBER Technical working paper 258.

[11] Brown, D. P. and R. H. Jennings, 1989, On technical analysis, Review of Financial Studies 2, 527-551.

[12] Callaway, Duncan S., Hopcroft, John E., Kleinberg, Jon M., Newman, M. E. J., and Steven H. Strogatz, 2001, Are randomly grown networks really random?, Physical Review E 64, 041902.

[13] Conley, T., Hansen, L., Luttmer, E., and J. Scheinkman, 1997, Short-term interest rates as subordinated diffusions, Review of Financial Studies 10, 525-577.

[14] Cont, Rama, and Jean-Philipe Bouchaud, 2000, Herd behavior and aggregate fluctuations in financial markets, Macroeconomic Dynamics 4, 170-196.

[15] Clark, Peter K., 1973, A subordinated stochastic process model with finite variance for speculative prices, Econometrica 41, 135-155.

[16] D'Hulst, R. and G. I. Rogers, 2000, Exact solution of a model for crowding and information transmission in financial markets, International Journal of Theoretical and Applied Finance 3, 609-616.

[17] Diamond, Douglas W., and Robert E. Verrecchia, 1981, Information aggregation in a noisy rational expectations economy, Journal of Financial Economics 9, 221-235.

[18] Duffie, Darrell, and Peter Glynn, 2001, Estimation of continuous-time Markov processes sampled at random time intervals, mimeo, Stanford University.

[19] Dupont, Dominique, 1997, Trading volume and information distribution in a market-clearing framework, mimeo, Board of Governors of the Federal Reserve System.

[20] Eguiluz Victor M., and Martin G. Zimmermann, 2000, Transmission of information and herd behavior: An application to financial markets, Physical Review Letters 85, 5659-5662.

[21] Ellison, Glenn, 1993, Learning, local interaction, and coordination, Econometrica 61, 1047-1071.

[22] Epps, Thomas W., and Mary L. Epps, 1976, The stochastic dependence of security price changes and transaction volumes: Implications for the mixture-of-distribution hypothesis, Econometrica 44, 305-321.

[23] Erdös, P. and Rényi, A., 1960, On the evolution of random graphs, Publications of the Mathematical Institute of the Hungarian Academy of Sciences 5, 17-61.

[24] Foster, F. D., and S. Viswanathan, 1996, Strategic trading when agents forecast the forecast of others, Journal of Finance 51, 1437-1478.

[25] Gallant, A. Ronald, Rossi, Peter E., and Tauchen, George, 1992, Stock price and volume, Review of Financial Studies 5, 199-242.

[26] Geman, Helyette, Madan, D., and M. Yor, 2001, Time changes for Levy processes, Mathematical Finance 11, 79-96.

[27] George, T. J., Kaul, G., and M. Nimalendran, 1994, Trading volume and transaction costs in specialist markets, Journal of Finance 49, 1489-1505.

[28] Ghysels, E., Gourerioux, C., and J. Jasiak, 1995, Market time and asset price movements: Theory and estimation, Working Paper, CRDE Universite de Montreal and CIRANO.

[29] Ghysels, E., and J. Jasiak, 1994, Stochastic volatility and time deformation: An application of trading volume and leverage effects, Working Paper, CRDE Universite de Montreal and CIRANO.

[30] Glosten, Lawrence R., 1989, Insider trading, liquidity, and the role of the monopolist specialist, Journal of Business 62, 211-235.

[31] Grundy, Bruce and Maureen McNichols, 1989, Trade and the revelation of information through prices and direct disclosure, Review of Economic Studies 2, 495-526.

[32] Howison, Sam, and David Lamper, 2000, Trading volume in models of financial derivatives, mimeo, University of Oxford.

[33] Ioannides, Yannis M., 1990, Trading uncertainty and market form, International Economic Review 31, 619-638.

[34] Jiang, Li, and Lawrence Kryzanowski, 1998, Trading activity, quoted liquidity, and stock volatility, Multinational Finance Journal 3, 199–227.

[35] Jin, Emily M., Girvan, Michelle, and M. E. J. Newman, 2001, The structure of growing social networks, Physical Review E 64, 046132.

[36] Jones, C. M., Kaul, G., and Lipson, M. L., 1994, Transactions, volume, and volatility, Review of Financial Studies 36, 127-154.

[37] Karpoff, Jonathan M., 1987, The relation between price changes and trading volume: A survey, Journal of Financial and Quantitative Analysis 22, 109-126.

[38] Kirman, Alan, 1997, Interaction and markets, ASSET Discussion paper 166.

[39] Kleinberg, Jon, 1999, The small-world phenomenon: An algorithmic perspective, Computer Science Technical report 99-1776, Cornell University.

[40] Kosfeld, Michael, 1998, Stochastic Strategy Adjustment in Coordination Games, mimeo, Tilburg University.

[41] Kyle, Albert S., 1985, Continuous auctions and insider trading, Econometrica 53, 1315-1335.

[42] Mandelbrot B., and H. Taylor, 1967, On the distribution of stock price differences, Operations Research 15, 1057-1062.

[43] Newman, M. E. J., 2001, The structure and function of networks, mimeo, Santa Fe Institute.

[44] Russel, J. and R. Engle, 1998, Econometric analysis of discrete valued, irregularly spaced financial transaction data using a new autoregressive conditional multinomial model, Econometrica 66, 1127-1162.

[45] Tauchen, George E. and Mark Pitts, 1983, The price variability-volume relationship on speculative markets, Econometrica 51, 485-505.

[46] Watts, Duncan J., and Steven H. Strogatz, 1998, Collective dynamics of 'small-world' networks, Nature 393, 440–442.