Endogenous Price Stickiness, Trend Inflation, and the New Keynesian Phillips Curve

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Abstract

For standard calibration, this paper shows that the optimal price, in a model with Calvo form of price stickiness and strategic complementarities, is only defined for annualised trend inflation rates of under 5.5%. This critical inflation rate is below the average inflation rate over recent decades. Furthermore, over the range for which the optimal price is defined, the slope of the New Keynesian Phillips curve generated by this model is *decreasing* in trend inflation. That contradicts the stylised fact that Phillips curves are flatter in low-inflation environments. Substituting endogenous price stickiness for the Calvo form of time-dependent pricing can help avoid these implications.

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1 Introduction

Recent literature on optimisation-based sticky price models for monetary policy analysis uses the Calvo (1983) price-setting assumption to introduce nominal price inertia.[1] Under the Calvo price-setting assumption, firms are assumed to receive an exogenous probabilistic signal every period as to when they can adjust their price. This adjustment probability is constant and independent across firms and time. The assumption of an exogenous adjustment signal delivers much analytical tractability in dynamic general equilibrium models, as shown by Yun (1996), and is considered reasonable if the inflation environment is one of zero steady-state inflation (see Woodford (2002)). However, in the presence of positive trend inflation, firms with fixed nominal prices experience erosion in their relative prices and are thus likely to reset their nominal prices more frequently. In the Calvo model, however, firms do not choose when to change prices. As trend inflation rises this assumption becomes increasingly unrealistic. Consequently, it is unclear what the upper bound of trend inflation is, below which the Calvo price-setting assumption is a good approximation. The answer to this question is important for applying the Calvo framework to examine issues concerning a move to a low-inflation environment witnessed in the United Kingdom and several other industrialised countries after the early 1990s.[2] Furthermore, the 'New Keynesian Phillips curve' (NKPC) that characterises structural inflation dynamics around zero trend inflation is also based on this form of price-setting, and have come under serious empirical scrutiny (see, for example, Galí and Gertler (1999) and Sbordone (2002)). It is, therefore, also unclear what the consequences of ignoring positive trend inflation are for the structure of the NKPC.

In this paper, we extend Woodford's (2002) exposition to an economy with positive trend inflation to study the interaction between trend inflation and Calvo price-setting. The particular macroeconomic environment we consider is characterised by 'real rigidities', or equivalently, 'strategic complementarity' in firms' pricing decisions. A large body of literature has emphasised the importance of this feature for quantitatively important effects of monetary policy on output and inflation dynamics. See, for example, Ball and Romer (1990), Kimball (1995), Christiano et

[1] A few early examples of this approach are Yun (1996), Woodford (1996), King and Wolman (1996), Goodfriend and King (1997), and Rotemberg and Woodford (1997). Two alternatives to Calvo price-setting are the staggered and overlapping contracts of fixed length (Taylor (1980)) and exogenous costs of adjusting prices (Rotemberg (1982)).

[2] The Calvo price-setting assumption is also commonly used in optimising models to examine monetary policy rules and stabilisation policies (see, for example, Rotemberg and Woodford (1997), Clarida et al. (1999), Clarida et al. (2002), Woodford (2002)).

al. (2001), Dotsey and King (2001), Neiss and Pappa (2002), and Woodford (2002). The notion of strategic complementarity is related to the cost and revenue conditions facing the firms. It explains why a firm's relative price might be insensitive to the quantity that it supplies. This phenomenon could arise on the cost side, for example, in the presence of firm-specific factor inputs, input-output linkages, or variable factor utilisation – factors that make marginal cost less procyclical (see, for example, Basu (1995), Burnside and Eichenbaum (1996)). On the revenue side this may be due to procyclical variation in elasticity of demand, customer markets, collusive pricing strategies among firms, or search costs – factors that make desired mark-ups countercyclical (see, for example, Woglom (1982), Ball and Romer (1990), Rotemberg and Woodford (1992), Galí (1994), Warner and Barsky (1995), Klemperer (1995), Kimball (1995)). For our analysis it is the presence of strategic complementarity that is important and not its precise source. We follow the benchmark model of Woodford (2002), and consider the presence of specific-factor markets as a particular source of strategic complementarity. Using this framework we investigate the consequences of positive trend inflation for NKPC based on the Calvo price-setting model. Our paper builds on the earlier work of King and Wolman (1996) and Ascari (2000) who also consider positive trend inflation in the Calvo model. King and Wolman (1996) discuss the influence of trend inflation on the average mark-up in the steady state in the Calvo model. Ascari (2000) considers the influence of trend inflation on both the steady-state output, and the transitional dynamics and examines welfare costs of disinflation. The key difference between these two papers and ours is that the former ignore strategic complementarity (alternatively, assume 'strategic substitutability in pricing decisions' due to the presence of common factor markets). We show that the interaction of trend inflation and strategic complementarity has important bearing on the results reported in this paper.

We note two implications: first, the optimal relative price for a firm in the steady state is not defined (is infinite) for trend inflation rates above 5.5%. In other words, a firm could maximise profits by not producing at all whenever trend inflation is beyond the upper bound. We compute the upper bound using standard parameter values in the literature. Moreover, the stronger the strategic complementarity or real rigidity, the lower is the upper bound. Surprisingly, the low single-digit upper bound for trend inflation is *below* the average inflation rates over the past few decades for several industrialised countries including the United Kingdom, and in particular, over the 1970s and 1980s. Therefore, our finding suggests that Calvo pricing is a restrictive description of firms' pricing behaviour even for moderate inflation levels.[3]

Second, in the output gap-inflation space, the slope of the NKPC increases as trend inflation falls from the upper bound towards zero. That is, a 1% rise in demand pressure, *ceteris paribus*, has a larger effect on inflation in a low-inflation environment than in a high-inflation environment. This implication sits oddly to the stylised fact from the traditional Phillips curve literature and the conventional wisdom that Phillips curves are flatter at low inflation levels.[4]

The intuition for the results in this paper is as follows: in the presence of positive trend inflation, the exogenous price-setting behaviour implied by the Calvo structure makes firms more concerned about the future erosion of their mark-ups (and hence losses in profits). In other words, their effective discount factor rises towards unity as trend inflation increases (ie they care more about the future) and consequently their current mark-up is relatively less important. The constraint that discount factors cannot exceed unity places the upper bound on the trend inflation rate for which the model can be solved. Because the current mark-up is less important, the current output gap has a smaller effect on inflation in the NKPC. Hence, the slope of the NKPC decreases with trend inflation.

Following Romer (1990), we consider an environment in which firms' price-setting behaviour is influenced by the trend inflation rate. That is, firms adjust prices more frequently at higher trend inflation rates.[5] The trend inflation rate is determined by the monetary policy regime. We show that if the Calvo price signal is *endogenous* in this sense, the model's steady state is defined for trend inflation rates higher than the sample averages of industrialised countries. Furthermore, if the elasticity of firms' pricing response with respect to trend inflation is sufficiently high, the slope of the NKPC is flatter at low inflation levels.

The paper is organised as follows: in Section 2 we present the model. In Section 3 we examine

[3]Typically the Calvo model is perceived as questionable only for double-digit steady-state inflation rates (see, for example, Calvo et al. (2001)).

[4]There is no consensus on why Phillips curves are flatter in a low-inflation environment. For example, Beaudry and Doyle (2000) attribute it to the improvement in information gathering and its use by the central bankers while Ball et al. (1988) link it to changes in the frequency of price adjustments.

[5]The inverse relationship between the Calvo time-dependent frequency of price adjustment and trend inflation is formalised in Romer (1990). Dotsey et al. (1999) present a dynamic general equilibrium model with state-dependent (endogenous) pricing. Burstein (2002) presents a state-dependent pricing model where firms choose a sequence of prices instead of choosing a single price. To address the implausibility of the fixed frequency of price adjustment in double-digit inflation environments, Calvo et al. (2001) allow firms to choose their price level and a firm-specific inflation rate.

the consequences of positive trend inflation for the steady state of the model. In Section 4 we derive NKPC specification under positive trend inflation and strategic complementarities. In Section 5 we consider endogenous price stickiness. Section 6 concludes.

2 Model

To analyse the consequences of positive trend inflation for the NKPC, we adopt the framework of Woodford (2002). This emphasises the importance of strategic complementarities in pricing decisions to generate quantitatively important inflation and output dynamics. We use standard functional forms for preferences and technology and assume sector-specific labour markets as the source of strategic complementarities.[6]

The representative household maximises the expected discounted sum of future expected utility, $E_t[U]$,

$$E_t[U] \equiv E_t \sum_0^{\infty} \beta^j \left[\frac{C_{t+j}^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \int_0^1 \frac{H_{t+j}(i)^{1+\phi}}{1+\phi} di \right] \tag{2.1}$$

where C_t is the Dixit-Stiglitz constant-elasticity-of-substitution aggregate and $H_t(i)$ denotes the firm-specific labour input. The parameter β is the subjective discount factor, $\sigma > 0$ is the intertemporal elasticity of substitution, and ϕ is the inverse of the labour-supply elasticity with respect to real wages. The trade-off between consumption and leisure gives the optimality condition

$$\frac{W_t(i)}{P_t} = \frac{H_t(i)^{\phi}}{C_t^{-\sigma^{-1}}} \tag{2.2}$$

On the supply side, each firm, i, operates in a monopolistically competitive market and faces the following demand curve

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \tag{2.3}$$

where $Y_t(i)$ and Y_t are firm i and aggregate demands, respectively. $P_t(i)$ and P_t are firm i and aggregate price levels, respectively. It produces output using a technology

$$Y_t(i) = H_t(i)^a, \quad 0 < a \leq 1 \tag{2.4}$$

where $H_t(i)$ is the firm-specific labour input in period t and a is the labour share of income.[7] From 2.4, we get $H_t(i) = Y_t(i)^{\frac{1}{a}}$. Total real cost for firm i, $TC_t(i)$, in period t is $TC_t(i) = \frac{W_t(i)}{P_t} H_t(i) \equiv$

[6]See Kimball (1995), Romer (2001), and Woodford (2002) for other ways of introducing strategic complementarities.

[7]Without loss of generality, we assume no productivity shocks.

$\frac{W_t(i)}{P_t} Y_t(i)^{\frac{1}{a}}$. Therefore, the marginal cost, $MC_t(i)$, is

$$MC_t(i) = \frac{1}{a} \frac{W_t(i)}{P_t} Y_t(i)^{\frac{1}{a}-1} \tag{2.5}$$

Using 2.2, 2.3, and the aggregate constraint $C_t = Y_t$, we obtain the expression for real marginal cost

$$MC_t(i) = \frac{1}{a} Y_t(i)^{\omega} Y_t^{\sigma^{-1}} \equiv \frac{1}{a} \Big(\frac{P_t(i)}{P_t}\Big)^{-\omega\theta} Y_t^{\omega+\sigma^{-1}}, \quad \omega = \frac{\phi}{a} + \frac{1}{a} - 1 \tag{2.6}$$

The term ω represents the elasticity of marginal cost with respect to firm's own output. The presence of specific-factor market makes the marginal cost depend on firms' own relative price and the aggregate output.

2.1 Strategic complementarity in firms' pricing decisions

Strategic complementarity (or real rigidity) is independent of nominal rigidity, and depends on the cost and revenue conditions in the economy. It could arise from different sources.[8] The specific-factor market assumption is one way of introducing it. To illustrate this point formally, consider a firm's optimal desired relative price, in the absence of price stickiness, $\frac{P_t^*(i)}{P_t}$, which is a mark-up over its marginal cost

$$\frac{P_t^*(i)}{P_t} = \frac{\theta}{\theta - 1} MC_t(i) \tag{2.7}$$

Substituting 2.6, we get

$$P_t^*(i) = P_t \left(\frac{\theta}{(\theta-1)a} \right)^{\frac{1}{1+\omega\theta}} Y_t^{\frac{\omega+\sigma^{-1}}{1+\omega\theta}} \equiv P_t \Omega(Y_t) \tag{2.8}$$

Given an exogenous nominal demand, Y_t^{nom}, and the identity $Y_t \equiv \frac{Y_t^{nom}}{P_t}$, we can write 2.8 as

$$P_t^*(i) = P_t \Omega \left(\frac{Y_t^{nom}}{P_t} \right) \tag{2.9}$$

Strategic complementarity exists if $\frac{\partial P_t^*(i)}{\partial P_t} > 0$. Taking this partial derivative and rearranging gives,

$$\frac{\partial P_t^*(i)}{\partial P_t} = \Omega(Y_t) \left[1 - \zeta\right], \quad \zeta \equiv \frac{Y_t \Omega'(Y_t)}{\Omega(Y_t)} \tag{2.10}$$

Therefore, $\frac{\partial P_t^*(i)}{\partial P_t} > 0$ if and only if $\zeta < 1$. Under the specific-factor market assumption, the condition for the presence of strategic complementarity is satisfied since $\zeta \equiv \frac{\omega+\sigma^{-1}}{1+\omega\theta} < 1$ for standard calibration of the parameters. Correspondingly, if $\zeta > 1$, $\frac{\partial P_t^*(i)}{\partial P_t} < 0$ and strategic substitutability

[8] See Romer (2001) and Woodford (2002) for a detailed discussion.

in pricing decisions prevails. King and Wolman (1996) and Ascari (2000) consider this latter case in their analysis of trend inflation in the Calvo model. In that case firms are assumed to hire factors in a common economy-wide market and factor prices are always equalised. So, the marginal cost of a firm depends only on aggregate output, and is the same across all firms. This corresponds to the case where $\zeta = \omega + \sigma^{-1} > 1$ for standard calibration of the parameters.

It is, however, the interaction of strategic complementarity with nominal price stickiness that is of particular importance for inflation dynamics. This interaction further slows the adjustment of prices, even more than if nominal price stickiness alone were present, and consequently the effect on output and inflation is drawn out. The intuition is that firms that do choose their prices, adjust them by less when other firms' prices are sticky.

2.2 Optimal pricing decision

Under the Calvo (1983) pricing structure each firm faces an exogenous probability, $(1 - \alpha)$, of adjusting its price in each period. A firm chooses its price $P_t(i)$ to maximise current and discounted future (real) profits:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left[\left(\frac{P_t(i)}{P_{t+j}} \right)^{1-\theta} Y_{t+j} - \frac{W_{t+j}(i)}{P_{t+j}} \left(\left(\frac{P_t(i)}{P_{t+j}} \right)^{-\theta} Y_{t+j} \right)^{\frac{1}{a}} \right] \tag{2.11}$$

where $Q_{t,t+j} = \beta^j \frac{C_t^{-\sigma}}{C_{t+j}^{-\sigma}}$ is the real stochastic discount factor. We define $P_t^*/P_t \equiv X_t$, $P_t/P_{t+j} = 1/\prod_{i=1}^{j} \Pi_{t+i}$, where Π_t is the gross inflation rate. Using 2.2, 2.3, 2.5, the first-order condition from 2.11, and imposing $Y_t = C_t$ we derive a closed-form expression for the optimal relative price as

$$X_t = \left[\frac{\theta}{\theta - 1} \frac{1}{a} \frac{E_t \sum\limits_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left(\left(\frac{1}{\prod_{i=1}^{j} \Pi_{t+i}} \right)^{-(\theta+\omega\theta)} Y_{t+j}^{1+\omega+\sigma^{-1}} \right)}{E_t \sum\limits_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left(\frac{1}{\prod_{i=1}^{j} \Pi_{t+i}} \right)^{1-\theta} Y_{t+j}} \right]^{\frac{1}{1+\omega\theta}} \tag{2.12}$$

The optimal relative price depends on current and future demand, aggregate inflation rates, and discount factors. The effect of specific-factor markets is reflected by the term $\omega\theta$. Under the common factor-market assumption, a firm's marginal cost no longer depends on its own relative price but only on aggregate variables. Relative factor prices get equalised instantaneously and consequently the $\omega\theta$ term no longer appears in 2.6. The expression for optimal relative prices in that case is identical to that in King and Wolman (1996) and Ascari (2000).

3 The steady state under positive trend inflation

In this section we illustrate the impact of positive trend inflation on the steady state of the model. We also discuss how it leads to difficulties from both theoretical and empirical perspectives. From 2.12, the optimal relative price in steady state is

$$X = \left[\frac{\theta}{\theta-1}\frac{Y^{\omega+\sigma^{-1}}}{a}\frac{\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta+\omega\theta})^j}{\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta-1})^j}\right]^{\frac{1}{1+\omega\theta}} = \left[\frac{\theta}{\theta-1}\frac{(1-\alpha\beta\pi^{\theta-1})}{(1-\alpha\beta\pi^{\theta+\omega\theta})}\right]^{\frac{1}{1+\omega\theta}}\left(\frac{1}{a}\right)^{\frac{1}{1+\omega\theta}} Y^{\frac{\omega+\sigma^{-1}}{1+\omega\theta}} \tag{3.1}$$

For the optimal relative price to be defined (ie less than infinity) in steady state, the effective discount factor in 3.1, $\alpha\beta\pi^{\theta+\omega\theta}$, must be less than one otherwise the term $\sum_{j=0}^{\infty}\alpha\beta\pi^{\theta+\omega\theta}$ will not be finite (and the second equality in 3.1 will not hold). That is, the steady state is defined only when $\pi < \bar{\pi}$, where $\bar{\pi}$ (the upper bound) is such that $\alpha\beta\bar{\pi}^{\theta+\omega\theta} = 1$. The optimal relative price approaches infinity as trend inflation approaches the upper bound. Given a concave profit function, a firm could maximise profits by not producing at all whenever trend inflation is beyond the upper bound.

We use calibrated values from Woodford (2002) for $\alpha = 0.75$, $\theta = 10$, $\beta = 0.99$, $\sigma = 1$, $\omega = 1.25$ to compute $\bar{\pi}$. These are standard parameter values in the literature and they imply that firms adjust prices, on average, every four quarters and charge a mark-up of approximately 11%; the discount factor is consistent with the evidence on real interest rates and consumers have log utility. The value $\omega \equiv \frac{\phi}{a} + \frac{1}{a} - 1 = 1.25$ is consistent with the degree of diminishing returns to labour in the production function and the degree of marginal disutility of work.

The upper bound on the trend inflation rate (in annualised terms) is

$$\bar{\pi}^{annual} = \left(\left(\left(\frac{1}{\alpha\beta}\right)^{\frac{1}{\theta+\omega\theta}}\right)^4 - 1\right) * 100 \tag{3.2}$$

Table 1 presents $\bar{\pi}^{annual}$ when prices are sticky, on average, for three, four, and five quarters as implied by values α of 0.66, 0.75, and 0.8, respectively (baseline case in bold).[9]

[9]Note that a similar upper bound can be derived for steady-state wage inflation in the Erceg et al. (2000) model. That model also uses the Calvo probabilistic signal to introduce nominal wage rigidities in the model.

Table 1: Maximum trend inflation (annualised): $\bar{\pi}^{annual}$

$\sigma = 1$, $\beta = 0.99$		Strategic complementarity	Strategic substitutability
α	θ		
0.66	10	7.86	18.56
0.75	**10**	**5.44**	**12.65**
0.80	10	4.23	9.78

Table 2: Average inflation rates (annualised) based on the GDP deflator

Country	1960s	1970s	1980s	1990s	1961-2000
UK	3.84	**12.92**	**6.92**	3.16	**6.12**
US	2.68	**6.73**	4.60	2.27	4.07
Canada	3.38	**8.30**	**5.64**	1.91	4.78
Euro area	-	**9.64**	**6.67**	2.75	**6.21***

*Average over 1970:01 to 2000:04.

In the baseline case, under strategic complementarity, this critical inflation rate is under 5.5%. In contrast, the upper bound under strategic substitutability is slightly over 12.5% (as in Ascari (2000)). So, the presence of strategic complementarity sharply reduces the upper bound on trend inflation for which the optimal relative price is defined. The stronger the strategic complementarity, reflected by the absolute magnitude of the term $\omega\theta$ in 3.2, the lower is the upper bound. Table 2 shows that the upper bound for our calibration is below the average inflation rates for several countries, in particular, the 1970s and 1980s. Only for the 1990s period is the average inflation rate below the upper bound. These findings suggest that Calvo pricing is a restrictive description of firms' pricing behaviour even for moderate inflation levels.

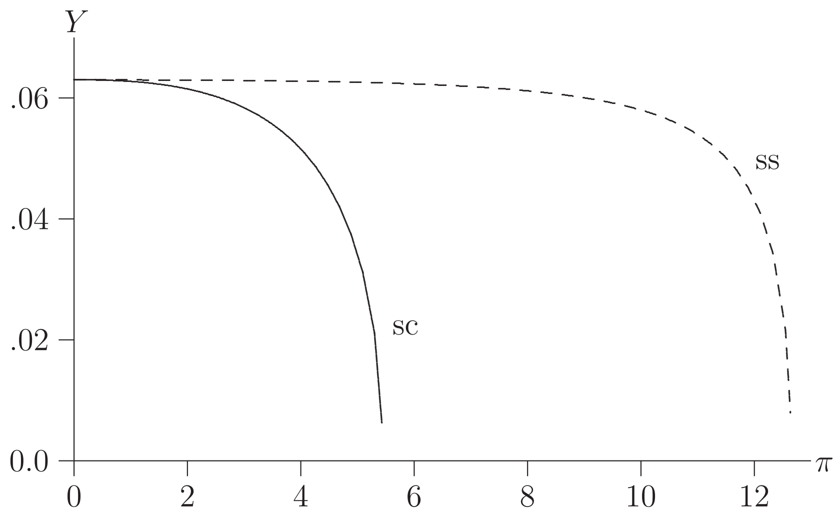
Note that we have assumed that there is no real output growth in the model. If we do account for real growth, γ, as in King and Wolman (1996), then our conclusion is further strengthened. The presence of real growth raises the effective discount factor to $\alpha\beta\pi^{\theta+\omega\theta}\gamma^{\theta+\omega\theta-\sigma^{-1}}$, and consequently lowers the upper bounds in Table 1 even further.

Using the aggregate price level the steady-state optimal relative price is

$$X = \left[\frac{1-\alpha}{1-\alpha\pi^{\theta-1}}\right]^{\frac{1}{\theta-1}} \tag{3.3}$$

This term reflects the price adjustment gap that captures the erosion of relative prices chosen by firms in the past due to positive trend inflation (see King and Wolman (1996)). The term $\alpha\pi^{\theta-1}$ in the price adjustment gap also implies an upper bound on the trend inflation rate. For

Figure 1: Steady-state output and trend inflation
sc = strategic complementarity, ss = strategic substitutability



standard calibration, however, it is the effective discount factor, $\alpha\beta\pi^{\theta+\omega\theta}$ that determines the upper bound since it reaches unity faster than the term $\alpha\pi^{\theta-1}$ in the price adjustment gap. Even under common-factor markets, the effective discount factor, $\alpha\beta\pi^{\theta}$, approaches unity faster than the term $\alpha\pi^{\theta-1}$.

As trend inflation approaches the upper bound, firms' profit maximising output levels fall, and consequently, the aggregate steady-state output falls. To illustrate this impact, we combine 3.1 with 3.3 to get the steady-state output level

$$Y^{SC} = \left(\frac{a}{\mu}\right)^{\frac{1}{\omega+\sigma^{-1}}} \left(\frac{1-\alpha}{1-\alpha\pi^{\theta-1}}\right)^{\frac{1+\omega\theta}{(\omega+\sigma^{-1})(\theta-1)}} \left(\frac{1-\alpha\beta\pi^{\theta+\omega\theta}}{1-\alpha\beta\pi^{\theta-1}}\right)^{\frac{1}{\omega+\sigma^{-1}}}, \qquad \mu = \frac{\theta}{\theta-1} \tag{3.4}$$

under strategic complementarity.[10] The corresponding expression for the strategic substitutability case is

$$Y^{SS} = \left(\frac{a}{\mu}\right)^{\frac{1}{\omega+\sigma^{-1}}} \left(\frac{1-\alpha}{1-\alpha\pi^{\theta-1}}\right)^{\frac{1}{(\omega+\sigma^{-1})(\theta-1)}} \left(\frac{1-\alpha\beta\pi^{\theta}}{1-\alpha\beta\pi^{\theta-1}}\right)^{\frac{1}{\omega+\sigma^{-1}}} \tag{3.5}$$

For standard calibration the steady-state output level is decreasing in trend inflation in both cases. However, as shown in Chart 1, the output level decreases faster under strategic complementarity than under strategic substitutability. This occurs since the average mark-up in the economy rises faster with rising inflation in the former case.

The problem of existence of the steady state in the Calvo model would not arise in the staggered

[10] See Appendix B.

contracting model of Taylor (1980). The optimal relative price in that model is always defined since firms have a finite horizon and the sums in 3.1 always exist. The focus of our paper, however, is on the implications of trend inflation for the Calvo model which is the underlying structural framework for the NKPC.[11]

4 The NKPC under positive trend inflation

In this section we derive an NKPC specification under positive trend and strategic complementarities. We assume that the steady-state trend inflation is below the upper bound derived in Section 3.

We log-linearise 2.12 around the steady-state values $Y_t = Y$, $X_t = X$, $Q_{t,t+j} = \beta^j$, and $\Pi_t = \Pi \equiv \pi$ (lower-case variables indicate log-deviations from steady-state levels) to get

$$x_t = \frac{(1-\alpha\beta\pi^{\theta+\omega\theta})}{1+\omega\theta} E_t \sum_{j=0}^{\infty} (\alpha\beta\pi^{\theta+\omega\theta})^j \left[q_{t,t+j} + (\theta+\omega\theta)\sum_{i=1}^{j}\pi_{t+i} + (1+\omega+\sigma^{-1})y_{t+j} \right]$$
$$- \frac{1-\alpha\beta\pi^{\theta-1}}{1+\omega\theta} E_t \sum_{j=0}^{\infty} (\alpha\beta\pi^{\theta-1})^j \left[q_{t,t+j} - (1-\theta)\sum_{i=1}^{j}\pi_{t+i} + y_{t+j} \right] \quad (4.1)$$

Log-linearising the aggregate price level in the model, $P_t = \left[(1-\alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}$, gives

$$x_t = \frac{\alpha\pi^{\theta-1}}{1-\alpha\pi^{\theta-1}}\pi_t \quad (4.2)$$

Using 4.1 and 4.2, we derive the NKPC under trend inflation and strategic complementarity[12]

$$\pi_t = \left(\beta \left[(1-\alpha\pi^{\theta-1}) \left((\frac{\theta+\omega\theta}{1+\omega\theta})\pi^{1+\omega\theta} - \frac{\theta-1}{1+\omega\theta} \right) + \alpha\pi^{\theta+\omega\theta} \right] \right) E_t\pi_{t+1}$$
$$+ \left(\frac{(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta+\omega\theta})}{\alpha\pi^{\theta-1}} \left(\frac{\omega+\sigma^{-1}}{1+\omega\theta} \right) + \frac{\beta(1-\alpha\pi^{\theta-1})(1-\pi^{1+\omega\theta})}{1+\omega\theta} \right) y_t$$
$$+ (\pi^{1+\omega\theta}-1)\frac{(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta-1})}{1+\omega\theta} E_t \sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta-1})^j \left[(\theta-1)\sum_{i=1}^{j}\pi_{t+1+i} + y_{t+1+j} \right] \quad (4.3)$$

The presence of trend inflation alters the structure of the NKPC in two ways.[13] First, the coefficient on one-period ahead expected inflation is a function of structural parameters now while it is the

[11] The Phillips curve specification under the Taylor model is different from the NKPC and typically assumes that all firms change their prices every two periods.

[12] Note that unlike the NKPC under zero trend inflation, stochastic variation in discount factor is not eliminated in the log-linearisation. For simplicity, we ignore this variation in our derivation of the NKPC. See Appendix A for the details of the derivations.

[13] Although we have assumed trend inflation to be exogenous (and constant) here, it is ultimately determined by the monetary policy regime.

steady-state discount factor in the absence of trend inflation. Second, there is an additional forward-looking structure.[14] Note that under zero trend inflation (ie, $\pi = 1$), 4.3 reduces to the standard NKPC

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \left(\frac{\omega + \sigma^{-1}}{1+\omega\theta} \right) y_t \tag{4.4}$$

Under strategic substitutability, 4.3 changes to

$$\begin{aligned} \pi_t = & \left(\beta \left[\left(1 - \alpha\pi^{\theta-1}\right) \left(\theta(\pi - 1) - 1\right)\right) + \alpha\pi^{\theta} \right] \right) E_t \pi_{t+1} \\ & + \left(\frac{(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta})}{\alpha\pi^{\theta-1}} \left(\omega + \sigma^{-1}\right) + \beta(1-\alpha\pi^{\theta-1})(1-\pi) \right) y_t \\ & + (\pi-1)(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta-1}) E_t \sum_{j=0}^{\infty} (\alpha\beta\pi^{\theta-1})^j \left[(\theta-1) \sum_{i=1}^{j} \pi_{t+1+i} + y_{t+1+j} \right] \end{aligned} \tag{4.5}$$

This formulation is similar to that in Ascari (2000).

4.1 Implication for the slope of the NKPC

In the output gap-inflation space, the slope of the NKPC in 4.3, S^{SC}, is

$$S^{SC} = \left[\frac{(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta+\omega\theta})}{\alpha\pi^{\theta-1}} \left(\frac{\omega+\sigma^{-1}}{1+\omega\theta} \right) + \frac{\beta(1-\alpha\pi^{\theta-1})(1-\pi^{1+\omega\theta})}{1+\omega\theta} \right] \tag{4.6}$$

with the expectation terms as shift-factors. Similarly, the slope of the NKPC in 4.5 is

$$S^{SS} = \left[\frac{(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta})}{\alpha\pi^{\theta-1}} (\omega+\sigma^{-1}) + \beta(1-\alpha\pi^{\theta-1})(1-\pi) \right] \tag{4.7}$$

Table 3 shows that a higher trend inflation is associated with a flatter NKPC under both strategic complementarity and substitutability. As expected, the NKPC under strategic complementarity is flatter relative to that under strategic substitutability, other things being equal. This implication, as mentioned in the introduction, is at odds with the stylised fact that Phillips curves are flatter in low-inflation environments.

[14] Using simulated data from a dynamic general equilibrium model, Bakhshi et al. (2003b), (i) examine the quantitative importance of the additional forward-looking structure that emerges in the presence of trend inflation, and (ii) estimate the underlying structural parameters of the NKPC when trend inflation is taken into account.

Table 3: Slope of NKPC and trend inflation

$\alpha = 0.75$, $\sigma = 1$, $\beta = 0.99$, $\theta = 10$	Strategic complementarity		Strategic substitutability	
π (annual)	S^{SC}	D	S^{SS}	D
0	0.0143	4	0.1931	4
2	0.0067	4	0.1350	4
4	0.0008	4	0.0881	4
6	–	–	0.0516	4
8	–	–	0.0252	4
10	–	–	0.0081	4
12	–	–	0.0001	4

$D = \frac{1}{1-\alpha}$ is the average duration of price stickiness in quarters.

Intuitively, optimal pricing behaviour under exogenous price stickiness becomes more forward-looking under trend inflation relative to the case with no trend inflation. Current marginal costs (and hence the current output gap) matter relatively less for setting the optimal price under positive trend inflation compared with the case with no trend inflation. Price-setting firms are more concerned about the erosion of future mark-ups in the positive trend inflation case.[15] This is reflected in the rise of the effective discounting of profits from $\alpha\beta$ under no trend inflation to $\alpha\beta\pi^{\theta}$ under positive trend inflation and strategic substitutability (common-factor markets), and to $\alpha\beta\pi^{\theta+\omega\theta}$ under trend inflation and strategic complementarity (specific-factor markets). That is, $\alpha\beta < \alpha\beta\pi^{\theta} < \alpha\beta\pi^{\theta+\omega\theta}$. Strategic complementarity amplifies the effect of nominal price stickiness in the future periods. At the aggregate level, because the current mark-up is less important, the current output gap has a smaller effect on inflation in the NKPC.

The relatively small effect of current output gap (or marginal cost) on inflation in the presence of trend inflation is due to the interaction between exogenous price stickiness and forward-looking price-setting behaviour. Therefore, the implication that the slope of the Phillips curve falls with a rise in inflation would also be present in other models of exogenous price stickiness, in particular, the Taylor model.[16]

[15] This point is related to the exogenous price stickiness, and hence, would also be present in other models in this class (for example, Taylor price-setting).

[16] Although the duration of price stickiness is exogenous in both Calvo and Taylor models, the infinite horizon structure of the Calvo model implies that relative price distortions, and hence, output distortions, rise with trend inflation in the latter model. Kiley (2002) shows that the welfare cost of these distortions in the Calvo model is substantially higher relative to the Taylor model with same average duration of price stickiness. Khan and Rudolf (2003), however, show that under endogenous Calvo pricing (as considered in this paper) the relative output distortions are small. The reason is that under endogenous price stickiness,

From an empirical perspective, the modified NKPC in 4.3 suggests that, in contrast to the existing estimates in the literature, the structural estimates should take into account the effect of positive trend inflation. In empirical implementations, one could use average inflation as a proxy for trend inflation. The difficulty, however, is that sample averages are higher than what the model theoretically admits. This aspect makes the estimation of the modified NKPC over periods of high average inflation difficult.[17]

In the next section we consider an extension of the Calvo model in which the frequency of price adjustment is endogenous. We show how it helps avoid the implications discussed above.

5 Endogenous price stickiness

In this section we show that both implications of the Calvo model in the previous section are avoided if the degree of nominal rigidity is endogenous. Following Romer (1990), we postulate that the frequency of price adjustment depends inversely on the trend inflation rate.[18] That is, $\alpha \equiv \alpha(\pi)$. This functional form is assumed to satisfy the following properties:

$$\alpha(1) = \bar{\alpha} \tag{5.1}$$

$$\frac{\partial \alpha(\pi)}{\partial \pi} \equiv \alpha_\pi(\pi) < 0 \tag{5.2}$$

$$\lim_{\pi \to \bar{\pi}} \alpha(\pi) = 0 \tag{5.3}$$

Equation (5.1) says that in the absence of trend inflation ($\pi = 1$), price stickiness still exists due to fixed 'menu' costs. Equation (5.2) says that when trend inflation rises, the probability that a firm keeps its price unchanged falls (or the corresponding average duration of price stickiness, $\frac{1}{1-\alpha(\pi)}$, falls). Finally, (5.3) indicates that as trend inflation reaches a very high level, the probability of not adjusting prices tends to zero and pricing decisions tend to full flexibility.

5.1 Consequence for the existence of steady state

The effective discount factor with endogenous price stickiness is $\alpha(\pi)\beta\pi^{\theta+\omega\theta}$. Given (5.2), it is no longer necessarily monotonic in trend inflation. Thus, allowing firms to vary the timing of their pricing decisions supports the existence of the steady-state optimal price for a larger range of trend

the number of firms in the tail of the relative price distribution falls sharply, therefore, relative output distortions are small.

[17]See Bakhshi et al. (2003b).

[18]See also Kiley (2000) and Devereux and Yetman (2002).

inflation rates (potentially as large a range as the sample averages in Table 2). Consequently, the steady-state output, shown in Chart 1, does not decline rapidly with a rise in trend inflation.

5.2 Consequence for the slope of the NKPC

Proposition: *For a given elasticity of demand, θ, the slope of the NKPC increases with trend inflation if the elasticity of price stickiness with respect to trend inflation, ($\epsilon_{\alpha\pi}$), is sufficiently high.* That is,

$$\epsilon_{\alpha\pi} > 1-\theta+\frac{(1-T)\beta\pi^{1+\omega\theta}(1+\omega+\sigma^{-1})}{\left(\frac{\omega+\sigma^{-1}}{1+\omega\theta}\right)\left(\beta T\pi^{1+\omega\theta}-\frac{1}{T}\right)+\frac{\beta\pi^{1+\omega\theta}-1}{T(1+\omega\theta)}},\quad T=\alpha(\pi)\pi^{\theta-1} \tag{5.4}$$

Proof: See Appendix C.

The intuition for this result is as above: endogenous price stickiness offsets the rise in the effective discount factor when trend inflation rises. Even though firms are concerned about the erosion of their mark-ups in the presence of trend inflation, the fraction of these firms itself declines. The latter occurs since endogenous price stickiness allows firms to choose when to change their price. The relative responsiveness of the two forces determines the relationship between trend inflation and the slope of the short-run NKPC.[19]

5.3 A quantitative example

We consider a simple functional form for the Calvo price signal to illustrate the consequences of endogenous price stickiness. Let

$$\alpha(\pi)=\frac{\bar{\alpha}}{\pi^b} \tag{5.5}$$

This functional form satisfies (5.1) to (5.3). The elasticity of price stickiness with respect to trend inflation, $\epsilon_{\alpha\pi}\equiv b$, is constant. We use the same calibration as in Section 3. The parameter $\bar{\alpha}=0.75$ indicates that in the absence of trend inflation firms adjust their prices, on average, every four quarters.

In Tables 4 and 5 we evaluate the effect of endogenous price stickiness on the slope of the NKPC under trend inflation. Under zero trend inflation endogenous price stickiness as given in 5.5 implies that the slope of the NKPC is 0.014 (for the calibration in Section 3). This value is independent

[19] In the presence of endogenous price stickiness the structure of NKPC will necessarily change. To examine that Bakhshi et al. (2003a) derive a state-dependent Phillips curve using the Dotsey et al. (1999) state-dependent pricing model. In that paper, the requirement that the slope of the Phillips curve should rise with trend inflation is used to calibrate the adjustment cost distribution.

of the elasticity parameter b. In the presence of positive trend inflation $\alpha(\pi)$ is decreasing in π for $b \geq 1$. Table 4 illustrates the necessary elasticity of price stickiness, b, for which the slope of the NKPC is invariant to trend inflation. In the third column we show the elasticity of price stickiness that ensures a constant slope of $\bar{S} = 0.014$. The results in Table 4 indicate that the higher the trend inflation, the stronger the endogenous response required to replicate the zero trend inflation slope of the NKPC. That is, as trend inflation rises, b must rise in 5.5, and hence the average duration of price stickiness (column 4), must fall to ensure that the slope of the NKPC remains invariant to trend inflation. Furthermore, with strategic complementarity, a stronger endogenous response of pricing decisions to trend inflation is required relative to strategic substitutability to keep the slope $\bar{S}$ constant.

In Table 5, we present the case for which the slope is always increasing in trend inflation. For that to occur, the elasticity of price stickiness with respect to trend inflation should be sufficiently high. For example, if annualised trend inflation is 4% the frequency of price adjustment should rise from every four quarters on average to every two and a half quarters. In the case of strategic complementarity, the elasticity necessary to ensure an upward-sloping NKPC is higher than in the case of strategic substitutability. The reason is that in the presence of positive trend inflation, firms are relatively more forward-looking under strategic complementarity than under strategic substitutability. Therefore, emphasis on current marginal cost declines faster under the former case, and hence, price stickiness needs to be relatively more elastic to offset this effect.

Table 4: Trend inflation, endogenous price stickiness, and constant slope

$\bar{\alpha} = 0.75$, $\sigma = 1$, $\beta = 0.99$, $\theta = 10$	Strategic complementarity			Strategic substitutability		
Annual inflation (%)	$\bar{S}^{SC}$	b	D^*	$\bar{S}^{SS}$	b	D^*
0	0.014	-	4.00	0.193	-	4.00
2	0.014	18.49	3.17	0.193	9.611	3.51
4	0.014	19.60	2.63	0.193	9.619	3.15
6	0.014	20.45	2.26	0.193	9.627	2.87
8	0.014	21.07	2.00	0.193	9.634	2.65
10	0.014	21.51	1.82	0.193	9.641	2.47
12	0.014	21.84	1.68	0.193	9.647	2.33

*in quarters.

Table 5: **Trend inflation, endogenous price stickiness, and increasing slope**

$\bar{\alpha} = 0.75$, $\sigma = 1$, $\beta = 0.99$, $\theta = 10$	Strategic complementarity			Strategic substitutability		
Annual inflation (%)	S^{SC}	b	D^*	S^{SS}	b	D^*
0	0.014	-	4.00	0.193	-	4.00
2	0.018	25	2.96	0.230	15	3.29
4	0.020	25	2.42	0.270	15	2.83
6	0.025	25	2.09	0.312	15	2.51
8	0.029	25	1.86	0.357	15	2.28
10	0.034	25	1.70	0.404	15	2.10
12	0.038	25	1.59	0.453	15	1.96

*in quarters.

5.4 Indexation to inflation

To offset influences of positive trend inflation on Calvo price-setting, one may assume some form of indexation of prices to trend inflation. This assumption would prevent erosion of firms' relative prices between price adjustments. This possibility, however, is less appealing relative to endogenous price stickiness, if the underlying rationale behind sticky prices is that there are fixed adjustment costs associated with price changes. In that case, indexation itself should be costly.

There are at least two additional technical reasons why indexation of price contracts is used in the literature. First, Yun (1996) assumes indexation of prices to a constant trend inflation rate. This formulation removes the long-run output-inflation trade-off in the Calvo model. However, as Calvo et al. (2001) note, this form of indexation is problematic because it implies that during transitions across steady states, all firms change their indexation rule immediately. Second, Christiano et al. (2001), Smets and Wouters (2002), and Batini et al. (2002), all use a backward-looking indexation rule where firms that do not choose optimal prices, automatically adjust them by last period's inflation rate. This feature introduces lags of inflation in the NKPC and helps account for the observed inertia of inflation. There is, however, little consensus on the precise form of indexation of price contracts. In particular, whether indexation should be to inflation in the previous period or to average inflation over a recent past.[20] Furthermore, there is some empirical evidence which supports the prediction of the endogenous sticky price model that high-inflation economies should

[20]Furthermore, it is unclear whether indexation itself should be backward-looking or forward-looking. Minford and Peel (2002) compare backward-looking indexation versus rational indexation (forward-looking) and argue in favour of the latter on theoretical grounds.

have steeper Phillips curves (see, for example, Ball et al. (1988)).[21] Based on these theoretical and empirical considerations we favour endogenous price stickiness as a natural extension of the Calvo model.

6 Conclusions

The Calvo price-setting assumption is extensively used in the literature to introduce nominal inertia in models for monetary policy analysis in a tractable manner. These models examine inflation dynamics, monetary policy rules, and welfare effects of stabilisation policies. Much of the literature, however, ignores trend inflation. We have studied the interaction of Calvo price-setting and trend inflation in a macroeconomic environment characterised by strategic complementarities. In particular, for standard calibration, the optimal relative price in the steady state is only defined for trend inflation rates below the sample averages for several industrialised countries. Furthermore, over this range, the slope of the short-run NKPC is decreasing in trend inflation. That is opposite to the stylised fact that Phillips curves are steeper at high-inflation levels. These findings present theoretical and empirical difficulties for the sticky-price model with exogenous price stickiness. In particular, it shows that Calvo pricing becomes a restrictive description of firms' pricing behaviour even at moderate inflation levels. We show that both implications are avoided in an intuitively appealing extension of the Calvo model where price-setting behaviour is influenced by the trend inflation — determined by the monetary policy regime. In other words, the Calvo price adjustment signal is an endogenous feature of the economy.

The structure of NKPC is modified substantially in the presence of positive trend inflation and strategic complementarity. Both theoretical literature on monetary policy rules and optimal stabilisation policies (for closed and open economies), and empirical literature on the estimates of the NKPC have ignored this modification. Our findings suggest that (a) it is important to investigate whether the results and conclusions from this earlier literature change under the modified NKPC, and (b) it is important to examine inflation dynamics under more realistic endogenous (or state-dependent) price-setting behaviour. We plan to do future work in this direction.

[21] Yates and Chapple (1996) show that this prediction is robust to different time periods. Khan (2000) shows that the prediction is robust to the exclusion of episodes of hyperinflation.

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Appendix A: Derivation of the NKPC under positive trend inflation and strategic complementarity

Log-linearization of the aggregate price level: Given that all firms face Calvo-type price rigidity and all adjusting firms choose the same optimal nominal price $P_t^*(i) = P_t^*$, the Dixit-Stiglitz aggregator describing the aggregate price level in discrete time space is

$$P_t = \left[1 - \alpha) \sum_{j=0}^{\infty} \alpha^j (P_{t-j}^*)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{A.1}$$

Consider an inflationary steady state where nominal prices are growing at the rate $\pi > 1$. We can rewrite the aggregate price level (A.1) such that all of its elements are constant along the inflationary steady state. That is,

$$1 = (1-\alpha) \sum_{j=0}^{\infty} \alpha^j \left(\frac{P_{t-j}^*}{P_t}\right)^{1-\theta} = (1-\alpha) \sum_{j=0}^{\infty} \alpha^j \left(\frac{P_{t-j}^*}{P_{t-j}} \frac{P_{t-j}}{P_t}\right)^{1-\theta} \tag{A.2}$$

Replacing P_{t-j}^*/P_{t-j} by X_{t-j} and P_{t-j}/P_t by $1/\prod_{i=0}^{j-1} \Pi_{t-j}$, we obtain

$$1 = (1-\alpha) \sum_{j=0}^{\infty} \alpha^j \left(\frac{X_{t-j}^{1-\theta}}{\prod_{i=0}^{j-1} \Pi_{t-i}^{1-\theta}}\right) \tag{A.3}$$

Next, we log-linearise (A.3) around the constant steady state values $\Pi = \pi$ and $X^{(1-\theta)} = \frac{1}{(1-\alpha)\sum_{j=0}^{\infty} \alpha^j \pi^{j(\theta-1)}}$ to get

$$0 = (1-\alpha) \sum_{j=0}^{\infty} \left[(1-\theta)\alpha^j \frac{X^{1-\theta}}{\pi^{j(1-\theta)}} x_{t-j} - (1-\theta)\alpha^j \frac{X^{1-\theta}}{\pi^{j(1-\theta)}} \pi_{t-j}\right] \tag{A.4}$$

The lower-case letters denote the percentage deviations of the variables from their steady-state values. Note that since $x_{t-j} = p_{t-j}^* - p_{t-j}$ and $\pi_{t-j} = p_t - p_{t-j}$, we can eliminate the p_{t-j}-terms. Using these two relations in (A.4), the log-linearized aggregate price level equation is

$$p_t = (1 - \alpha\pi^{(\theta-1)}) \sum_{j=0}^{\infty} \alpha^j \pi^{j(\theta-1)} p_{t-j}^* \tag{A.5}$$

and in terms of the optimal relative price of a currently adjusting firm, it is

$$x_t = \frac{\alpha\pi^{\theta-1}}{1 - \alpha\pi^{(\theta-1)}} \pi_t \tag{A.6}$$

Log-linearisation of the optimal nominal price set by adjusting firms: We start from the first-order condition of (2.11) that determines the optimal nominal price

$$0 = E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left((1-\theta)\Big(\frac{P_t^*(i)}{P_{t+j}}\Big)^{-\theta} \frac{Y_{t+j}}{P_{t+j}} + \frac{\theta}{a} \frac{W_{t+j}(i)}{P_{t+j}} \left(\Big(\frac{P_t^*(i)}{P_{t+j}}\Big)^{-\theta} Y_{t+j}\right)^{\frac{1}{a}-1} \Big(\frac{P_t^*(i)}{P_{t+j}}\Big)^{-\theta-1} \frac{Y_{t+j}}{P_{t+j}}\right) \tag{A.7}$$

Multipying (A.7) by $P_t^*(i)$, and dividing by $(1-\theta)$, and rearranging leads to

$$E_t\sum_{j=0}^{\infty}\alpha^j Q_{t,t+j}\Big(\frac{P_t^*(i)}{P_{t+j}}\Big)^{1-\theta}Y_{t+j}=\frac{\theta}{a(\theta-1)}E_t\sum_{j=0}^{\infty}\alpha^j Q_{t,t+j}\frac{W_{t+j}(i)}{P_{t+j}}\left(\Big(\frac{P_t^*(i)}{P_{t+j}}\Big)^{-\theta}Y_{t+j}\right)^{\frac{1}{a}-1}\Big(\frac{P_t^*(i)}{P_{t+j}}\Big)^{-\theta}Y_{t+j} \tag{A.8}$$

Next, we use (2.5) and (2.6) to express the individual real marginal costs, $MC_t(i)=\frac{1}{a}\frac{W_t(i)}{P_t}Y_t(i)^{\frac{1}{a}-1}$ in terms of aggregate output and the firm's relative price, $\frac{1}{a}\Big(\frac{P_t^*(i)}{P_t}\Big)^{-\omega\theta}Y_t^{\omega+\sigma^{-1}}$, to get

$$E_t\sum_{j=0}^{\infty}\alpha^j Q_{t,t+j}\Big(\frac{P_t^*(i)}{P_{t+j}}\Big)^{1-\theta}Y_{t+j}=\frac{\theta}{a(\theta-1)}E_t\sum_{j=0}^{\infty}\alpha^j Q_{t,t+j}\Big(\frac{P_t^*(i)}{P_{t+j}}\Big)^{-(\theta+\omega\theta)}Y_{t+j}^{1+\omega+\sigma^{-1}} \tag{A.9}$$

Substituting $P_t^*(i)/P_t=X_t$, $P_t/P_{t+j}=1/\prod_{i=1}^{j}\Pi_{t+i}$, in (A.9) yields

$$E_t\sum_{j=0}^{\infty}\alpha^j Q_{t,t+j}\Big(\frac{X_t(i)}{\prod_{i=1}^{j}\Pi_{t+i}}\Big)^{1-\theta}Y_{t+j}=\frac{\theta}{a(\theta-1)}E_t\sum_{j=0}^{\infty}\alpha^j Q_{t,t+j}\left(\Big(\frac{X_t(i)}{\prod_{i=1}^{j}\Pi_{t+i}}\Big)^{-(\theta+\omega\theta)}Y_{t+j}^{1+\omega+\sigma^{-1}}\right) \tag{A.10}$$

We log-linearise (A.10) around the steady state $Y_t=Y$, $\Pi_t=\pi$, $Q_{t,t+j}=\beta^j$ and

$$X^{(1+\omega\theta)}=\frac{\theta}{a(\theta-1)}Y^{\omega+\sigma^{-1}}\frac{1-\alpha\beta\pi^{\theta-1}}{1-\alpha\beta\pi^{\theta+\omega\theta}}, \tag{A.11}$$

to get

$$\begin{aligned}&E_t\sum_{j=0}^{\infty}\left[q_{t+j}+(1-\theta)x_t-(1-\theta)\sum_{i=1}^{j}\pi_{t+i}+y_{t+j}\right](\alpha\beta\pi^{\theta-1})^jX^{1-\theta}Y\\&=\frac{\theta}{a(\theta-1)}E_t\sum_{j=0}^{\infty}\left[q_{t+j}-(\theta+\omega\theta)x_t+(\theta+\omega\theta)\sum_{i=1}^{j}\pi_{t+i}+(1+\omega+\sigma^{-1})y_{t+j}\right](\alpha\beta\pi^{\theta+\omega\theta})^jX^{-(\theta+\omega\theta)}Y^{1+\omega+\sigma^{-1}}\end{aligned} \tag{A.12}$$

Solving (A.12) for x_t, we get

$$\begin{aligned}x_t=&\frac{1-\alpha\beta\pi^{\theta+\omega\theta}}{1+\theta\omega}E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta+\omega\theta})^j\left[q_{t+j}+(\theta+\omega\theta)\sum_{i=1}^{j}\pi_{t+i}+(1+\omega+\sigma^{-1})y_{t+j}\right]\\&-\frac{1-\alpha\beta\pi^{\theta-1}}{1+\theta\omega}E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta-1})^j\left[q_{t+j}+(\theta-1)\sum_{i=1}^{j}\pi_{t+i}+y_{t+j}\right]\end{aligned} \tag{A.13}$$

NKPC under positive inflation and strategic complementarity: Combining the log-linearised pricing equations (A.6) and (A.13) we get

$$\begin{aligned}\pi_t=&\frac{1-\alpha\pi^{\theta-1}}{\alpha\pi^{\theta-1}}\frac{1-\alpha\beta\pi^{\theta+\omega\theta}}{1+\theta\omega}E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta+\omega\theta})^j\left[q_{t+j}+(\theta+\omega\theta)\sum_{i=1}^{j}\pi_{t+i}+(1+\omega+\sigma^{-1})y_{t+j}\right]\\&-\frac{1-\alpha\pi^{\theta-1}}{\alpha\pi^{\theta-1}}\frac{1-\alpha\beta\pi^{\theta-1}}{1+\theta\omega}E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta-1})^j\left[q_{t+j}+(\theta-1)\sum_{i=1}^{j}\pi_{t+i}+y_{t+j}\right]\end{aligned} \tag{A.14}$$

The NKPC expression, (2.15), is obtained after isolating the next period's expected inflation and the current output gap in (A.14). We lead (A.14) one period to get

$$\begin{aligned} E_t\pi_{t+1} = & \frac{1-\alpha\pi^{\theta-1}}{\alpha\pi^{\theta-1}}\frac{1-\alpha\beta\pi^{\theta+\omega\theta}}{1+\theta\omega}E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta+\omega\theta})^j\left[(\theta+\omega\theta)\sum_{i=1}^{j}\pi_{t+1+i}+(1+\omega+\sigma^{-1})y_{t+1+j}\right] \\ & -\frac{1-\alpha\pi^{\theta-1}}{\alpha\pi^{\theta-1}}\frac{1-\alpha\beta\pi^{\theta-1}}{1+\theta\omega}E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta-1})^j\left[(\theta-1)\sum_{i=1}^{j}\pi_{t+1+i}+y_{t+1+j}\right] \end{aligned} \tag{A.15}$$

To express the accumulated inflation terms as future inflation terms we make use of the following identities:

$$E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta+\omega\theta})^j(\theta+\omega\theta)\sum_{i=1}^{j}\pi_{t+i}=(\theta+\omega\theta)E_t\sum_{j=1}^{\infty}(\alpha\beta\pi^{\theta+\omega\theta})^j\pi_{t+j} \tag{A.16}$$

$$E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta-1})^j(\theta-1)\sum_{i=1}^{j}\pi_{t+i}=(\theta-1)E_t\sum_{j=1}^{\infty}(\alpha\beta\pi^{\theta-1})^j\pi_{t+j} \tag{A.17}$$

Using the right-hand side of (A.16) and (A.17) together with (A.15) in (A.14), and after simplification we get

$$\begin{aligned} \pi_t = & \left(\beta\left[(1-\alpha\pi^{\theta-1})\left((\frac{\theta+\omega\theta}{1+\omega\theta})\pi^{1+\omega\theta}-\frac{\theta-1}{1+\omega\theta}\right)+\alpha\pi^{\theta+\omega\theta}\right]\right)E_t\pi_{t+1} \\ & +\left(\frac{(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta+\omega\theta})}{\alpha\pi^{\theta-1}}\left(\frac{\omega+\sigma^{-1}}{1+\omega\theta}\right)+\frac{\beta(1-\alpha\pi^{\theta-1})(1-\pi^{1+\omega\theta})}{1+\omega\theta}\right)y_t \\ & +(\pi^{1+\omega\theta}-1)\frac{(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta-1})}{1+\omega\theta}E_t\sum_{j=0}^{\infty}(\alpha\beta\pi^{\theta-1})^j\left[(\theta-1)\sum_{i=1}^{j}\pi_{t+1+i}+y_{t+1+j}\right] \end{aligned} \tag{A.18}$$

Appendix B: Steady state output level and trend inflation

The aggregate price level

$$P_t^{1-\theta} = (1-\alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{B.1}$$

can be rewritten in terms of the optimal relative price set by adjusting firms

$$X_t = \left[\frac{1-\alpha}{1-\alpha\pi_t^{\theta-1}}\right]^{\frac{1}{\theta-1}} \tag{B.2}$$

where $X_t = \frac{P_t^*}{P_t}$ and $\frac{P_t}{P_{t-1}} = \pi_t$. In the steady state where inflation evolves at the constant rate π we can drop the time subscript to get,

$$X = \left[\frac{1-\alpha}{1-\alpha\pi^{\theta-1}}\right]^{\frac{1}{\theta-1}} \tag{B.3}$$

Along the zero inflation steady state where $\pi = 1$, the steady-state relative price is one. In contrast as the level of steady state inflation increases, ie. as π increases, the optimal relative price X rises to account for the higher expected inflation cost occurring between to price adjustments.

Using (3.1) and (B.3) we can eliminate X and solve for the steady-state output level Y under trend inflation to get

$$Y = \left(\frac{a}{\mu}\right)^{\frac{1}{\omega+\sigma^{-1}}} \left(\frac{1-\alpha}{1-\alpha\pi^{\theta-1}}\right)^{\frac{1+\omega\theta}{(\omega+\sigma^{-1})(\theta-1)}} \left(\frac{1-\alpha\beta\pi^{\theta+\omega\theta}}{1-\alpha\beta\pi^{\theta-1}}\right)^{\frac{1}{\omega+\sigma^{-1}}} \tag{B.4}$$

under strategic complementarity and

$$Y = \left(\frac{a}{\mu}\right)^{\frac{1}{\omega+\sigma^{-1}}} \left(\frac{1-\alpha}{1-\alpha\pi^{\theta-1}}\right)^{\frac{1}{(\omega+\sigma^{-1})(\theta-1)}} \left(\frac{1-\alpha\beta\pi^{\theta}}{1-\alpha\beta\pi^{\theta-1}}\right)^{\frac{1}{\omega+\sigma^{-1}}} \tag{B.5}$$

under strategic substitutability, where $\mu = \frac{\theta}{\theta-1}$. For standard calibration the steady-state output level is decreasing in trend inflation in both cases.

Formally, it is easy to check that under strategic complementarity $\frac{\partial Y}{\partial \pi} < 0$ when

$$\left(\frac{\theta+\omega\theta}{\theta-1}\right)\left(\frac{1-\alpha\beta\pi^{\theta-1}}{1-\alpha\beta\pi^{\theta+\omega\theta}}\right)\pi^{\omega\theta-1} > 1. \tag{B.6}$$

For standard calibration, ie. $\alpha < 1$, $\beta < 1$, $\pi > 1$, $\theta > 1$ and $\omega > 1$ (B.6) holds since $\theta+\omega\theta > \theta-1$. The steady-state output level decreases faster under strategic complementarity relative to that under strategic substitutability since the average mark-up in the economy rises faster with rising trend inflation under the former.

The zero trend inflation case: If $\pi = 1$,

$$Y = \left(\frac{a}{\mu}\right)^{\frac{1}{\omega+\sigma^{-1}}} \tag{B.7}$$

That is, the steady-state output is determined only by the real side of the economy. Since, $X = 1$, the steady-state output under strategic complementarity and strategic substitutability is identical.

Appendix C: Proof of the proposition

The slope of the NKPC derived under positive trend inflation and strategic complementarity in Section 3 is

$$S = \left[\frac{(1-\alpha\pi^{\theta-1})(1-\alpha\beta\pi^{\theta+\omega\theta})}{\alpha\pi^{\theta-1}}\left(\frac{\omega+\sigma^{-1}}{1+\omega\theta}\right)+\frac{\beta(1-\alpha\pi^{\theta-1})(1-\pi^{1+\omega\theta})}{1+\omega\theta}\right] \tag{C.1}$$

To simplify the steps, we define $T = \alpha(\pi)\pi^{\theta-1}$.

$$\frac{dT}{d\pi} = \alpha(\pi)\pi^{\theta-2}[\theta-1+\epsilon_{\alpha\pi}], \quad \epsilon_{\alpha\pi} \equiv -\frac{\pi\alpha_\pi}{\alpha(\pi)} \tag{C.2}$$

where $\epsilon_{\alpha\pi}$ is the elasticity of $\alpha(\pi)$ with respect to π.

Rewriting (C.1) as

$$S \equiv \left[\frac{(1-T))(1-\beta T\pi^{1+\omega\theta})}{T}\left(\frac{\omega+\sigma^{-1}}{1+\omega\theta}\right)+\frac{\beta(1-T)(1-\pi^{1+\omega\theta})}{1+\omega\theta}\right] \tag{C.3}$$

For the slope to be increasing in π, the derivative of (C.3) with respect to π should be positive. Taking the derivative and collecting terms we get

$$\begin{aligned}\frac{dS}{d\pi} = &\left(\left((\omega+\sigma^{-1})(\beta\pi^{1+\omega\theta}-\frac{1}{T})+\beta(\pi^{1+\omega\theta}-1)\right)\frac{1}{1+\omega\theta}+(T-1)T^2\left(\frac{\omega+\sigma^{-1}}{1+\omega\theta}\right)\right)\frac{dT}{d\pi} \\ &-\left(\frac{1-T}{1+\omega\theta}\right)\left((\omega+\sigma^{-1})\beta(1+\omega\theta)\pi^{\omega\theta}+(1+\omega\theta)\beta\pi^{\omega\theta}\right) > 0\end{aligned} \tag{C.4}$$

Rearranging (C.4) using (C.2) we get

$$\epsilon_{\alpha\pi} > 1-\theta+\frac{(1-T)\beta\pi^{1+\omega\theta}(1+\omega+\sigma^{-1})}{\left(\frac{\omega+\sigma^{-1}}{1+\omega\theta}\right)\left(\beta T\pi^{1+\omega\theta}-\frac{1}{T}\right)+\frac{\beta\pi^{1+\omega\theta}-1}{T(1+\omega\theta)}} \tag{C.5}$$