Educational Systems, Growth and Income Distribution: A Quantitative Study

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Abstract

This paper studies how human capital connects economic growth with income distribution under imperfect credit markets. We construct a model where households can choose between private and public schools. The government provides public education and public support for private education through voucher programs. We find that in a mixed educational system, the government can control economic performance by changing the structure of the educational system. If the government wishes to reduce income inequality, a policy to increase the enrollment rate in public schools should be adopted. However, having a high enrollment rate in private schools with the liberalization of credit markets can keep the growth rate high.

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1. Introduction

The relationship between economic development and income inequality has always been an important issue in macroeconomics. One of the most famous hypotheses related to this issue is Kuznets' hypothesis: income inequality first increases and then decreases with the development of the economy[1]. While Kuznets' hypothesis emphasizes the effect of growth on income distribution, the opposite causal link between growth and inequality is also possible because income distribution can affect the allocation of capital and therefore has an impact on growth.

Although most of the growth literature focuses on the importance of technological change to explain the linkages between growth and inequality, we argue that human capital is also a key factor and study how human capital connects growth with income distribution. In particular, we analyze how the educational system affects the accumulation of human capital and how this in turn affects growth and income distribution in the long run. Both causal effects work in our model. The impact of growth on income distribution will depend on who will benefit from economic development. If rich agents benefit from economic growth and invest more in their human capital, income inequality will increase. On the other hand, income distribution will also have an impact on growth through the role of human capital and financial development. Due to the imperfections of credit markets, investments in human capital are subject to borrowing constraints. Hence, some poor agents will underinvest in education if they are credit-rationed.

[1] Empirical support for this inverted U-shape between the level of income and inequality for the United States and Great Britain can be found in Williamson and Lindert (1980) and Williamson (1985). For surveys of the cross-sectional literature on Kuznets' hypothesis, see Fields (1980), Bigsten (1987) and Kanbur (1999).

What distinguishes this paper from the literature that uses human capital to study growth and inequality is that we allow for the coexistence of private and public education regimes. Public education is provided by the government while the amount of investment in private education is decided by households. Table 1 presents data on private school enrollments as a percentage of secondary school enrollments for both developed and developing countries for 1985. None of the countries in the data has only private or public schools. Instead, all countries have both types of schools. In order to mimic the educational system in the real world, we allow households to choose between public and private schools. A modified endogenous growth model is used and two main features are introduced into our model. First, financial development is included in the model to determine the tightness of credit constraints. This distorts agents' decisions regarding investments in education. Second, some resources from the public sector can be "transported" to the private sector[2] through voucher programs and this allows the government to control the size of the private/public sector[3].

We find that income inequality is lower under a public education regime than under a private education regime. This is because, under a public education regime, everyone has the same investment in education. For an economy with a mixed educational system, income inequality decreases as the size of the public sector increases and vice versa.

In this paper, we are able to quantify the effects of financial development on income inequality through the role of human capital. Our second finding is that income inequality will increase during the process of financial development. The reason for this is that rich

[2] We refer to private schools as the "private sector" and to public schools as the "public sector".
[3] What we mean by "the size of the private sector" is the enrollment in private schools as a percentage of school enrollments. Also, "the size of the public sector" refers to the enrollment in public schools as a percentage of school enrollments. Because the summation of these two is one, the size of the private sector increases while the size of the public sector decreases.

or able households will benefit from the liberalization of credit markets and they will invest more in education.

Our model also allows us to quantify the impacts of different government policies regarding tax rates, financial developments and the scales of voucher programs. A third finding is that the government is able to control economic performance (high economic growth or low income inequality) by using combinations of policies. If the government wishes to reduce income inequality, it should adopt policies that increase the size of the public sector. However, fast growth takes place if the enrollment rate in private schools is high and credit markets are liberalized.

Glomm and Ravikumar (1992), Glomm (1997) as well as Zhang (1996) analyze long-run economic performance with a private education regime and a public education regime. Because analytical solutions only exist for a pure public or private education regime, without a quantitative study, they are not able to examine the impact of a mixed educational system on the evolution of growth and inequality. Papers in the literature that study school choices with peer effects include Fernandez and Rogerson (1996), Caucutt (2000), Epple and Romano (1998) and Snipes (1998). Fernandez and Rogerson (1996) only set up static models while intergenerational settings are used in Epple and Romano (1998) and Snipes (1998). Our work differs from those papers in terms of the way in which we present a dynamic model with a mixed educational system to explore the relationship between growth and income distribution.

Mckinnon (1973), Shaw (1973) and others (Kapur (1976), Galbis (1977), Fry (1978, 1995), Mathieson (1980)) study the importance of financial development in the process of economic growth. While the economy grows, the deregulation of the financial sector usually takes place. Financial development can help an economy grow by allocating financial resources among the best uses[4]. Although the positive correlation between financial development and growth is well known, the relationship between financial development and income inequality is not clear. The quantitative study performed in this paper shows that the correlation between financial development and inequality is also positive. Under a private education regime, inequality (which is measured by the Gini coefficient) will increase by 11.7% when an economy experiences a financial reform by transitioning from an imperfect credit market with exogenous constraints to a perfect market.

We construct an overlapping generations model where agents differ in their innate abilities and in terms of parental human capital. Both innate abilities and parental human capital affect households' choices of investments in education and each of them will affect the accumulation of human capital. Our work in relation to a private education regime with market imperfections is closely related to that of Galor and Zeira (1993). Their theoretical work shows that, in the presence of credit market imperfections and indivisibilites in human capital investment, the initial distribution of wealth affects aggregate output and investment both in the short run and in the long run. In this paper, we first study an economy with perfect credit markets. This means that households can borrow as much as they want to invest in education and we also assume that no one will default. Another extreme case is that none of the agents can borrow. This might happen when there is no enforcement of punishment for defaulting. Without any enforcement of punishment, it is optimal for households to default. Because there is no punishment to

[4] Empirical studies conducted by Demetriades and Hussein (1996) and Luintel and Khan (1999) find that there is a bi-directional causality between financial development and economic growth.

prevent borrowers from defaulting, no one is willing to lend. We refer to these things as exogenous borrowing constraints. Unlike Galor and Zeira (1993), borrowing constraints do not only ration poor agents, but also those able individuals. Agents who are credit-rationed will underinvest in education. Our study indicates that, with exogenous credit constraints, income inequality will be transmitted from one generation to another. However, exogenous credit constraints are misleading because people with higher future income are more likely to repay their loans and it should also be easier for them to borrow. Following Lochner and Monge (2002), we assume that agents will lose a fraction of their income if they default[5]. Given the punishment for defaulting, an endogenous credit constraint is derived as a fraction of an agent's future income[6]. Far fewer agents are credit rationed with endogenous credit constraints than with exogenous credit constraints. By allowing an economy to be transformed from one with exogenous credit constraints to one with perfect markets through a stage of endogenous credit constraints, we find that loosening credit constraints improves economic growth but also increases income inequality.

By switching from a private education regime to a public education regime, it is possible to avoid the problem of underinvestment in education when financial markets are imperfect. Under a public education regime, tax revenue is collected from wage income tax and is used for public education. With the coexistence of public and private schools, we allow the government to provide public support for private education through

[5] Lochner and Monge (2000) study the life-cycle behavior of consumption, labor supply and human capital accumulation in an economy where credit constraints arise endogenously.
[6] Kehoe and Levine (1993) study the case where households are indifferent between repaying loans and defaulting.

voucher programs[7]. The role of voucher programs is like that of a subsidy from the government to households who choose private schools. We find that the structure of the educational system is important for determining the growth and income inequality both in the short run and the long run.

The model is calibrated by using 1980 U.S. data. The initial income distribution is calibrated to match the median households' income and the Gini coefficient in 1980. The tax rate, borrowing constraints and the scale of voucher programs (these are instruments controlled by the government) are set to match the ratio of public spending on education to gross national product, the ratio of consumer credit to net national product and the public school enrollment rate, respectively. Changing the values of these three instruments allows us to analyze the impact of fiscal policies.

Section 2 presents the model of a private education regime. We first analyze a private education regime with perfect credit markets. We then impose an exogenous borrowing constraint: every young agent can borrow up to a certain limit regardless of his learning ability and parental human capital. Endogenous borrowing constraints are introduced into the model at the end of this section. The definition of general equilibrium is also given in section 2. The model of a public education regime is presented in section 3. A mixed educational system is analyzed in section 4. In order to study the impact of the structure of the educational system on economic growth and income distribution, we simulate equilibrium in section 5. The calibration of the parameters and the results of the simulation are given in section 5. Section 6 studies the impact of government policies. The conclusion is given in section 7.

[7] An alternative way of providing public support for a private education regime is to subsidize private schools.

2. **The Model of a Private Education Regime**

We consider an infinite-horizon, discrete-time overlapping generations model where agents live for three periods. Each period is approximately 20 years, corresponding to youth, middle age (adulthood) and old age. Every adult gives birth to a young person. We refer to young agents as children, to adults as parents, and to old people as retirees. There is no population growth and we normalize population size to one. Agents have the same utility function over their lifecycle. The utility function is

$$\ln(c_t) + \beta \ln(c_{t+1}) + \beta^2 \ln(c_{t+2}) , \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, and c_t, c_{t+1}, c_{t+2} represent the consumption of the young, the middle aged and the old for the cohort born at time $t > 0$.

Each individual is endowed with one unit of time. Young agents devote all of their time to studying, middle-aged agents use all of their time for work, and old agents enjoy leisure. Adults use a fixed amount of time to work at home and the rest of their time to work for earnings. We assume that the output produced at home equals each adult's human capital (h_t). Hence, the home production function is

$$Y_t^{\text{home}} = h_t .$$

Adults give their home products to their children as endowments. Each young agent, after he/she is born, receives parental endowment and his/her innate ability is revealed. Both h_t and the innate/learning ability (z_t) of each young agent are public knowledge. Given these endowments and innate abilities, the young individuals in cohort t choose how much they want to spend on consumption (c_t) and education (q_t), and how much they want to save (s_t). When s_t is negative, it refers to debt. The budget constraint for a young agent is

$$c_t + q_t + s_t = h_t \,. \tag{2}$$

Human capital is accumulated according to the constant-returns-to-scale learning technology:

$$h_{t+1} = z_t (q_t)^{\gamma} (h_t)^{\delta} (H_t)^{1-\gamma-\delta} , \qquad \gamma ,\ \delta \in (0,1). \tag{3}$$

where H_t is the average human capital for the cohort. We assume that the distribution of learning ability (g_z) is log-normal with mean μ_z and variance σ_z^2. We also assume that the initial distribution of human capital (g_{h1}) is log-normal with mean μ_{h1} and variance σ_{h1}^2.

$$z_t \sim LN(\mu_z, \sigma_z^2),$$

$$h_1 \sim LN(\mu_{h1}, \sigma_{h1}^2).$$

We assume that the accumulation of human capital does not affect the realization of innate ability. The human capital accumulation function in eq (3) depends on learning ability, educational investment, parental human capital and average human capital. One externality arises, because community quality, H_t, is a function of the human capital of all of the residents in this economy.

Assume that w_t and R_t are the real wage rate per unit of human capital and real interest rate in period t, respectively. Middle-aged agents can earn $w_{t+1}h_{t+1}$. Given the real interest rate and the real wage rate, an adult decides how much he wants to consume and save for old age. The budget constraint for an adult is thus

$$c_{t+1} + s_{t+1} = w_{t+1}h_{t+1} + R_{t+1}s_t \,. \tag{4}$$

Old agents consume what they save. Hence, the budget constraint for an old person is

$$c_{t+2} = R_{t+2}s_{t+1} \tag{5}$$

2.1. Perfect Credit Markets

We first consider the case of perfect credit markets. Under a private education regime, young agents decide how much they want to invest in education. If we assume that households can commit to repay all debts (and hence there is no upper limit on borrowing), their optimal choice of investment in education is

$$q_t = \left(\frac{w_{t+1}\gamma z_t}{R_{t+1}} h_t^{\delta} H_t^{1-\gamma-\delta}\right)^{\frac{1}{1-\gamma}} . \tag{6}$$

Eq (6) tells us that investment increases as learning ability/parental human capital increases and decreases as the real interest rate increases. Hence, people who are smarter or have bigger endowments will invest more in education.

The optimal choice of consumption in middle and old age satisfies the standard Euler equations:

$$\frac{1}{c_{t+t}} = \frac{\beta R_{t+1+i}}{c_{t+1+i}} \,, \text{ i=0,1.} \tag{7}$$

The human capital accumulation function becomes

$$h_{t+1} = [z_t (\frac{w_{t+1}\gamma}{R_{t+1}})^{\gamma} h_t^{\delta} H_t^{1-\gamma-\delta}]^{\frac{1}{1-\gamma}} . \tag{8}$$

Since young agents with high ability or high parental human capital invest more in education if they can borrow freely, their adult human capital will be higher. Eq (8) also shows that, if the real interest rate is high, young individuals will borrow less to invest in education and the adult human capital will be low. This is because a high real interest rate means a high opportunity cost of borrowing.

The intertemporal budget constraint is

$$c_t + c_{t+1}/R_{t+1} + c_{t+2}/R_{t+1}R_{t+2} = h_t - q_t + w_{t+1}h_{t+1}/R_{t+1}\,. \tag{9}$$

This implies that the optimal saving and consumption for a young agent are:

$$s_t = \frac{1}{1+\beta+\beta^2}[\beta(1+\beta)h_t - (\frac{w_{t+1}\gamma z_t}{R_{t+1}} h_t^{\delta} H_t^{1-\gamma-\delta})^{\frac{1}{1-\gamma}} \frac{1+\gamma\beta(1+\beta)}{\gamma}] \tag{10}$$

$$c_t = \frac{1}{1+\beta+\beta^2}[h_t - q_t + (\frac{w_{t+1}z_t}{R_{t+1}}\gamma^{\gamma} h_t^{\delta} H_t^{1-\gamma-\delta})^{\frac{1}{1-\gamma}}]. \tag{11}$$

Let us now define g_{t+1} as the growth rate of average human capital from period t to period $t+1$ ($g_{t+1} = \frac{H_{t+1}}{H_t}$), $\bar{z}$ as the average of learning ability and $b_t = \frac{h_t}{H_t}$ as relative human capital.

Proposition 1. If the interest rates are constant over time ($R_t = R, \forall t$ [8]), then under a private education regime with perfect markets, there is a balanced growth path (b_t =1, $z_t = \bar{z}$ $\forall t$) with a growth rate (g^*) equal to $[\bar{z}(\frac{w\gamma}{R})^{\gamma}]^{\frac{1}{1-\gamma}}$.

Proof: See Appendix 2.

2.2. Exogenous Credit Rationing

[8] This implies that the real wage rate is also constant, $w_t = w, \forall t$. We will explain this in section 2.4.

Suppose that agents are subject to borrowing constraints. Without the commitment of paying back loans, it is optimal for borrowers to default. Therefore, savings for young agents are restricted to being non-negative ($s_t \geq 0$)[9] because no one is willing to lend. We refer to the zero borrowing as the exogenous credit constraint.

From eq (10), for a given z_t, saving is non-negative under a perfect market if

$$h_t \geq [(\frac{1+\gamma\beta(1+\beta)}{\gamma\beta(1+\beta)})^{1-\gamma} \frac{w_{t+1}\gamma z_t}{R_{t+1}}]^{\frac{1}{1-\gamma-\delta}} H_t \,. \tag{12}$$

Eq (12) means that for a certain learning ability, poor agents will want to borrow. Hence, the exogenous credit constraint is binding on poor young agents, i.e. those with a low stock of inherited parental human capital.

Given h_t, saving is non-negative under the perfect market if

$$z_t \leq (\frac{\gamma\beta(1+\beta)}{1+\gamma\beta(1+\beta)})^{1-\gamma} (\frac{R_{t+1}}{w_{t+1}\gamma})(\frac{h_t}{H_t})^{1-\gamma-\delta} \,. \tag{13}$$

Eq (13) says that for a certain level of parental human capital, agents with high innate ability will want to borrow. The constraint is binding on the able young agents.

To summarize, young agents might face problems with credit limits if they have low parental human capital or high learning abilities or both. Everyone in middle age chooses to save to provide for consumption in old age.

For young individuals who are credit-rationed, their savings are zero. Their investments in education are

$$q_t = \frac{\beta\gamma(1+\beta)}{1+\beta\gamma(1+\beta)} h_t \,. \tag{14}$$

[9] We can also assume that young individuals can borrow up to a certain fixed level. However, the result will be similar to what we get when we set that level to zero.

Eq (14) shows that the expenditure on education of a constrained agent is a fraction of the parental human capital. Such an investment is independent of learning ability, the real interest rate and average human capital. The law of motion of human capital for constrained agents is

$$h_{t+1} = z_t [\frac{\beta\gamma(1+\beta)}{1+\beta\gamma(1+\beta)}]^{\gamma} h_t^{\gamma+\delta} H_t^{1-\gamma-\delta} . \tag{15}$$

2.3. Endogenous Credit Constraints

Although the assumption of exogenous credit rationing makes it easier to analyze human capital accumulation with imperfect credit markets, it does not seem very reasonable to make this assumption. Jappelli and Pagano (1994), for example, study households' savings in an overlapping generations model in which liquidity constraints are a fraction of discounted lifetime income. Without introducing human capital into their model, their finding that the relationship between credit constraints and growth is positive is quite different from much of the literature that states capital market imperfections tend to deter growth. In this section, we analyze the situation where borrowing constraints differ across individuals. There are two types of young agent: those who are constrained and those who are not. If young agents choose to borrow, they can choose to repay their loans or default in their middle age. All adults and old agents are unconstrained because adults need to save for their old age and old people only consume their savings.

In order to make credit constraints arise endogenously, we follow the assumptions made by Lochner and Monge-Naranjo (2000) who study the effects of endogenous credit constraints on the accumulation of human capital. We assume that, if agents default in their middle age, they will lose a fraction $\phi \in (0, 1)$ of their income[10]. There are two main differences between their work and ours: first, we study the impacts of educational system and allow for the coexistence of private and public schools while they only consider a private education regime. Second, the relationship between growth and inequality is not the concern of their paper while it is the object of this paper.

Given the income and saving/debt decision made in his/her youth, an adult chooses to save and consume

$$s_{t+1} = \frac{\beta(w_{t+1}h_{t+1} + R_{t+1}s_t)}{1+\beta} \tag{16}$$

$$c_{t+1} = \frac{w_{t+1}h_{t+1} + R_{t+1}s_t}{1+\beta}\,. \tag{17}$$

Recall that adults are not credit rationed because they need to save for old age. Using eq (17) and eq (7), the value function for an adult who does not default is

$$V_{adult}(h_{t+1}, s_t) = (1+\beta)[\log(w_{t+1}h_{t+1} + R_{t+1}s_t) - \log(1+\beta)] + \beta\log(R_{t+2}\beta)\,. \tag{18}$$

If an adult needs to repay a huge debt, he might choose to default. If he defaults (s_t =0), a fraction of his income will be gone. The value function for an adult who defaults is

$$V^d_{adult}(h_{t+1}, s_t) = (1+\beta)[\log(1-\phi)w_{t+1}h_{t+1} - \log(1+\beta)] + \beta\log(R_{t+2}\beta)\,. \tag{19}$$

Eq (19) is independent of R_{t+1}. The size of the credit limit is determined by letting borrowers be indifferent between repaying and defaulting. So the value function of paying back debt is at least as big as the value function of defaulting. Creditors set the

[10] Using a three period OLG model, an alternative punishment for defaulting that we can use is that for adults who default, they can only obtain the lower rate of return of their savings. However, the results will not be different from those in this paper. Lochner and Monge (2002) use both punishments for defaulting.

upper boundary of debt up to this level so that no default will take place. By setting $V_{adult}(h_{t+1}, s_t) \geq V^{d}_{adult}(h_{t+1}, s_t)$, the credit limit is

$$-s_t \leq \rho w_{t+1} h_{t+1} \qquad \text{where} \qquad \rho = \frac{\phi}{R_{t+1}}. \tag{20}$$

Eq (20) gives the maximal debt that a young individual can carry on to the next period. It is a fraction of discounted future income. Note that ρ is an increasing function of ϕ. When ϕ increases, the cost of defaulting also increases and people are less willing to default. Hence, lenders are willing to lend more.

For unconstrained young agents, the optimal choices of expenditures on education, saving and consumption are the same as those under perfect markets.

The value function for an unconstrained young agent is

$$V_{youth}(h_t, z_t) = [1 + \beta \log \beta R_{t+1} + \beta^2 \log(\beta^2 R_{t+1} R_{t+2})] + (1 + \beta + \beta^2) \log c_t .$$

Poor or able young agents are constrained. The maximal debt they are allowed to have is $\rho w h_{t+1}$. Thus, a young agent's problem becomes

$$\max_{q_t, s_{t+1}} . \log(h_t + \rho w_{t+1} h_{t+1} - q_t) + \beta \log((1 - R_{t+1}\rho) w_{t+1} h_{t+1} - s_{t+1}) + \beta^2 \log(R_{t+2} s_{t+1}) .$$

There are no analytical solutions for q_t and s_{t+1}. The first-order conditions are

$$s_{t+1} = \frac{\beta}{1+\beta}(1 - R_{t+1}\rho) w_{t+1} h_{t+1} \tag{21a}$$

$$\gamma \rho w_{t+1} h_{t+1} - q_t = \beta\gamma(1+\beta)(q_t - h_t - \rho w_{t+1} h_{t+1}) . \tag{21b}$$

2.4. Equilibrium

For a closed economy, the average human capital in this economy is

$$H_{t+1} = \iint h_{t+1} g_{h,z} dhdz . \qquad (22)$$

Aggregate capital is determined by market clearing conditions in the capital market:

$$K_{t+1} = \iint (s_t^{t+1} + s_{t+1}^{t+1}) g_{h,z} dhdz . \qquad (23)$$

Here s_i^{t+1} (i = t, t+1) represents saving in time t+1 while the agent was born at time i and $g_{h,z}$ is the joint distribution of human capital and learning ability.

Using aggregate physical capital (K) and average human capital (H) as inputs, aggregate domestic output is produced by a standard neoclassical production function:

$$Q_t = AF(K_t, H_t) \qquad \text{where } A \text{ is the total factor productivity}$$

Assuming that the production function has constant returns to scale, then

$$Q_t = A * H_t * F(K_t / H_t) = A * H_t * f(k_t), \quad \text{where } k_t = K_t / H_t .$$

The market clearing prices in every period satisfy

$$R_t = A\frac{\partial F}{\partial K} \text{ - depreciation rate} = A \ f'(k_t) - \text{ depreciation rate,}$$

$$w_t = A\frac{\partial F}{\partial H} = A(f(k_t) - k_t f'(k_t)) .$$

Definition 1. Equilibrium under a Private Education Regime in a Closed Economy

Given the initial distribution of human capital g_{h1}, the distribution of learning ability g_z, preferences, a home production function, human capital accumulation technology, production technology and financial regulations, an equilibrium consists of aggregate capital stocks { H_t , K_t }, sequences of prices { w_t , R_t }, the distribution of human capital g_{ht} and individual decisions { q_t , s_t , c_t , c_{t+1} , c_{t+2} } such that :

1. Given { w_t , R_t }, the { q_t , s_t , c_t , c_{t+1} , c_{t+2} } solves the households' problems (maximizing the utility function subject to eqs (2), (3), (4) and (5)).
2. { w_t , R_t } clear markets.
3. Eqs (22) and (23) hold.
4. Given g_{ht} and g_z, the distribution of human capital at $t+1$, g_{ht+1}, is determined by

$$h_{t+1} = z_t (q_t)^{\gamma} (h_t)^{\delta} (H_t)^{1-\gamma-\delta} .$$

For a small open economy, if we assume that there is no depreciation of physical capital and the world interest rate is constant ($R_t = R \ \ \forall t$), the equilibrium value k^* of physical capital per unit of human capital is determined by the factor-price equation of R and is constant. From the factor-price equation of w, the wage rate is also constant over time ($w_t = w \ \ \forall t$).

Definition 2. Equilibrium under a Private Education Regime in a Small Open Economy Given the initial distribution of human capital g_{h1}, the distribution of learning ability g_z, constant interest rate R and constant real wage rate per unit of human capital w, preferences, a home production function, human capital accumulation technology, production technology and financial regulations, an equilibrium consists of average capital stocks { H_t , K_t }, the distribution of human capital g_{ht} and individual decisions { q_t , s_t , c_t , c_{t+1} , c_{t+2} } such that :

1. Given $\{w, R\}$, the $\{q_t, s_t, c_t, c_{t+1}, c_{t+2}\}$ solves the households' problems (maximizing the utility function subject to eqs (2), (3), (4) and (5)).
2. Factor price equations hold.
3. Eq (22) holds and $K_t = (k^*)H_t$.
4. Given g_{ht} and g_z, the distribution of human capital at $t+1$, g_{ht+1}, is determined by $h_{t+1} = z_t (q_t)^{\gamma} (h_t)^{\delta} (H_t)^{1-\gamma-\delta}$.

In the remainder of this paper, we focus our research on a small open economy and assume that the world interest rate is constant.

3. A Public Education Regime

Our discussion so far has only focused on a private education regime. Under a private education regime, young people decide how much to spend on education. However, because of borrowing constraints, some of them are credit-rationed and underinvest in education. By introducing a public education regime, we can overcome the problem of underinvestment in education due to credit constraints. Under a public education regime, adults need to pay tax on wage income. We assume that the tax rate is constant ($\tau_t = \tau$) over time and is determined by the government. Young agents do not need to decide how much they want to spend on education since quality schooling is provided by the government. We rule out the case of government debt by assuming that the government always maintains a balanced budget. Hence the tax revenue equals the school expenditure. So the school expenditure is

$$q_{ut} = \tau w_t H_t . \tag{24}$$

The human capital accumulation under a public education regime becomes

$$h_{t+1} = z_t (\tau w)^\gamma h_t^\delta H_t^{1-\delta} . \tag{25}$$

Definition 3. Equilibrium under a Public Education Regime in a Small Open Economy Given the initial distribution of human capital, the distribution of learning ability, constant interest rate R and constant real wage rate per unit of human capital w, preferences, a home production function, a constant tax rate, human capital accumulation technology, production technology and financial regulations, an equilibrium is average capital stocks $\{H_t, K_t\}$, a distribution of human capital g_{ht} and individual decisions $\{s_t, c_t, c_{t+1}, c_{t+2}\}$ such that :

1. Given $\{w, R\}$, $\{s_t, c_t, c_{t+1}, c_{t+2}\}$ solves the households' problem.
2. $q_{ut} = \tau w_t H_t$.
3. The factor price equations hold.
4. eq (22) holds and $K_t = (k^*) H_t$.
5. Given g_{ht} and g_z, the distribution of human capital at $t+1$, g_{ht+1}, is determined by $h_{t+1} = z_t (q_{ut})^\gamma (h_t)^\delta (H_t)^{1-\gamma-\delta}$.

Proposition 2. In a small open economy with a public education regime, a balanced growth path (b_t =1, $z_t = \bar{z}$ $\forall t$) exists with a growth rate (g^*) equal to $\bar{z}(\tau w)^\gamma$.

Proof. See Appendix 2.

The tax on the future income makes young agents save and consume more than is the case without any income tax. However, adults will save less due to the loss of tax on wage income. Proposition 2 tells us that, under a public education regime, the government plays an important role in economic development. The government can control economic growth by using different tax policies. Because we assume that all tax revenue is used for public schools, a high tax rate will increase human capital accumulation and this will in turn increase the economic growth rate. Although a high tax rate can generate a high growth rate, it may not be preferred by households[11]. Along the balanced growth path, the growth rate is higher under a public education regime than under a private education regime with perfect credit markets if the tax rate is greater than $[\frac{\bar{z}\gamma w^r}{R}]^{\frac{1}{1-\gamma}}$.

Because of the tax rate, the endogenous credit constraint becomes

$$-S_t \leq \rho' w_{t+1} h_{t+1} \qquad \text{where } \rho' = (1-\tau)\rho . \qquad (26)$$

Since adults need to pay a fraction of their income in the form of tax, the amount that they can commit to repay decreases as the tax rate increases. Not many households are credit-rationed under a public education regime because endogenous credit constraints are not very tight constraints and young agents do not need to pay for their education under a public education system.

[11] However, if we include the voting system in our model, the preferred tax rate is chosen by solving the maximization problem: $\max_{\tau_t} \log(h_t - s_t) + \beta \log\{(1-\tau_t) w[z_t (\tau_t w)^\gamma h_t^\delta H_t^{1-\delta}] + Rs_t - s_{t+1}\}$.

The first-order condition gives rise to a constant optimal tax rate, $\tau_t = \tau = \frac{\gamma}{1-\gamma}$.

4. A Mixed Educational System

As we show in Table 1, most countries have both private and public schools. In this section, a mixed educational system of private and public schools will be analyzed. Besides providing public education, the government can also subsidize private education by giving tuition reimbursements (vouchers) to the households who choose to attend private schools[12]. Epple and Rmano (1996) analyze the competition between private and public schools in terms of a voucher program and peer effects. They introduce the voucher into the model as a fixed lump-sum subsidy. Their results show that tuition vouchers increase the relative size of the private sector and the extent of student sorting, and benefit high-ability students relative to low-ability students. West (1997) claims that most voucher programs in the world are fractions of school expenditure in public schools. Hence, we assume that the tuition voucher is a fraction (v) of the expenditure on public education. However, the way we treat the voucher is as if it were a lump-sum income subsidy as in Epple and Rmano (1996). If the amount of the voucher is larger than the tuition for the private school, then households can use vouchers for consumption or saving[13]. The government runs a balanced budget and in period t, the government budget constraint is:

$$\tau w_t H_t = p_{ut} q_{ut} + (1 - p_{ut}) v q_{ut},$$

where p_{ut} is the percentage of young agents going to public schools.

[12] Without voucher programs, only households whose ideal education investments are higher than those in public education will choose private schools. Hence, private schools will provide higher school qualities than public schools. But with voucher programs, in order to qualify for vouchers, some households whose desired education investments are lower than public education expenditure will choose private schools.
[13] We make this assumption to simplify the model.

The budget constraints for young agents and adults are

$$c_t + q_t + s_t = h_t + vq_{ut} \tag{27}$$

$$c_{t+1} + s_{t+1} = (1-\tau)w_{t+1}h_{t+1} + R_{t+1}s_t \,. \tag{28}$$

For unconstrained people, because the tax on future income decreases the “return” on investment in education, the optimal choice of school spending decreases as the tax rate increases. The tuition voucher is simply a lump-sum subsidy and so it does not affect an agent’s choice of school spending. However, the saving of young agents increase and fewer young people are constrained because of vouchers.

For a constrained agent, the upper limit on what he/she can borrow is the amount that he/she commits to repay. Eq (26) says that this amount is equal to $\rho' wh_{t+1}$. Therefore, the budget constraints for young agents and adults, respectively, are

$$c_t + q_t = h_t + vq_{ut} + \rho' w_{t+1}h_{t+1} \,, \tag{29}$$

$$c_{t+1} + s_{t+1} = [(1-\tau) - \rho' R_{t+1}]w_{t+1}h_{t+1} \,. \tag{30}$$

5. Simulation

Having constructed the model, our intention to study its long-run implications for growth and inequality. One way to do this is to simulate the equilibrium. By simulating the model, we can quantify the impacts of credit constraints on growth and income inequality through the accumulation of human capital. Before performing a simulation, we need to calibrate the parameter values.

5.1. Calibration

We start by calibrating the human capital accumulation function. The parameters γ and δ are income elasticities with respect to expenditure on education and parental income, respectively. The result of an empirical study by Johnson and Stafford (1973) gives income elasticity of school expenditure (γ) of 0.198. The number used by Fernandez and Rogerson (1997) based on Card and Kreuger's estimates (1992) is 0.2. These two numbers are close and so we set γ=0.2. We then follow Caucutt (2000) and pick δ=0.4. This corresponds to an intergenerational correlation of income of roughly the same number and is consistent with the results of Solon (1992) and Zimmerman (1992).

Because one period consists of 20 years, the discount factor is $\beta = (0.98)^{20}$. To make things easier, we consider a small open economy. As we discussed before, the real wage rate per unit of human capital is constant if the world real interest rate is constant over time. We assume that the real interest rate is 3.5% per year. Thus, $R = (1.035)^{20}$. The wage rate is set as 3.101[14].

We need to calibrate two distributions: a log-normal distribution of innate ability and a log-normal distribution of initial human capital. We use the model under a private education regime with perfect credit markets as our baseline model and calibrate these two distributions under the environment of the baseline model. The average growth rate of output per capita for the U.S. from 1960 to 1998 is about 2%. Choosing a mean of innate ability equal to 1.733 approximates the growth rate of income as being equal to $(1.02)^{20}$ along the balanced growth path in the baseline model. We calibrate the variance

[14] We assume that the capital share in the final good sector equals 1/3 and that there is no depreciation of capital, and we also assign total factor productivity as being equal to 5 as the value used by Lochner and Monge N. (2000). This implies that the equilibrium physical to human capital ratio (k^*) is 0.668. According to the factor-price equation, the wage rate is 3.101.

of log(z) so that the variance of log human capital is roughly constant under perfect markets.

We calibrate μ_{h1} and σ_{h1} to match the median of U.S. household incomes (\$17,710)[15] and the Gini coefficient (35.2) in 1980 under the baseline model. Due to the structure of the model, the households' income is $(1+w)h_t$. This gives the median human capital as being equal to \$4318.46. Because we assume that the initial human capital stock is log-normally distributed, the median of human capital is exp(μ_{h1}). Accordingly, μ_{h1} is set to 8.3707[16] and σ_{h1} is set to 0.645. We also use two different numbers for the variance of $\log h_1$ (0.645 and 1) to study the impact of initial inequality on economic growth and income inequality within an economy.

Jappelli and Pagano (1994) report that in 1980, consumer credit was 16.1% of net national product in the U.S. Hence, we choose ϕ to be 28.5% to match this data to allow agents to borrow up to 16.1% of their future income.

Using the parameter values we have just calibrated, the annual growth rate along the balanced growth path under a public education regime with average innate ability will be less than 2% if the tax rate is less than 14.7%. In 1980, the ratio of public spending on education to gross national product was 6.7%[17]. Accordingly, τ is set as 8.86%. This implies that the growth rate equals 1.5% along the balanced path under a public education regime.

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[15] Data source: U.S. Census Bureau.

[16] If the initial human capital is log-normally distributed, the average human capital $H_1 = \exp(\mu_{h1} + \frac{\sigma_{h1}^2}{2})$ = 5317.1887.

[17] Source: UNESCO, United Nations.

The last parameter we need to calibrate is the scale of the voucher. In his survey, West (1997) shows that the scales of most voucher programs are between 30% and 100%. We choose a number that matches the enrollment rate in public schools in the U.S of 88%. This makes *v* equal to 63%. Later, we will try different scales of voucher programs to study the impact of voucher programs on economic performance. Table 2 gives a summary of the calibration of parameters.

5.2. Results

The simulation results are given in Table 3. Figure 1 shows that the transitions in the annual growth rate and the Gini coefficient for the youth under perfect markets over 10 periods with homogeneous innate ability. The annual growth rate converges to the growth rate along the balanced path (2%) and the Gini coefficient decreases to zero. Using eq (8),

the growth of human capital is

$$\frac{h_{t+1}}{h_t} = [z_t (\frac{w\gamma}{R})^{\gamma} (\frac{H_t}{h_t})^{1-\gamma-\delta}]^{\frac{1}{1-\gamma}} .$$

With homogeneous innate ability, income inequality decreases over time because the growth of human capital is a decreasing function of human capital (this means that agents with high human capital will accumulate their human capital slowly and vice versa).

Figure 2 shows regions for agents being constrained or unconstrained in the case of a private education regime with exogenous credit constraints and heterogeneous innate ability. Using the parameter values calibrated in the previous section, about 93% of young agents are credit-constrained in the first period. We find that the pattern of the growth rate under perfect markets with heterogeneous innate ability is quite different from the one with homogeneous innate ability. With homogeneous innate ability, the annual growth rate converges to 2%. However, with heterogeneous innate ability, the average annual growth rate over 10 periods is 1.9645%. This is close to the average U.S. growth rate, but the variation in innate ability lowers the average growth rate. The explanation for this is given in Appendix 4. With exogenous credit constraints, the average annual growth rate decreases dramatically by 53.6% to 0.8829%. This is not surprising because, with exogenous credit constraints, a great number of young agents underinvest in education as we can see from Table 3. The average ratio of school expenditure to the income per capita under perfect markets is 2.59 times the ratio with exogenous borrowing constraints. The Gini coefficient is lower with exogenous borrowing constraints than with perfect markets. With exogenous borrowing constraints, the Gini coefficient is lower because more middle income or able young individuals are constrained and cannot get their desired school investments. The average saving rate for youth is –0.5765 with perfect markets and 0.0121 with exogenous borrowing constraints. Saving rates for the youth with exogenous borrowing constraints are positive because exogenous borrowing constraints rule out negative savings.

In the case where there are endogenous borrowing constraints, only 64.2% of young individuals are credit constrained in the first period. Figure 3 presents the histogram of education investments for the first period[18]. The simulation results are also given in Table 3. Because endogenous credit constraints are not as strict as exogenous credit constraints, the average saving rate for the youth, which equals –0.3899, is much lower than is the case with exogenous credit constraints, but it is higher than is the case with perfect credit

[18] We assume that there are 2000 agents for each cohort.

markets. The ratio of school expenditure to per capita GDP is 0.2679. The average ratio of school expenditure to per capita GDP is lower than is the case with perfect markets, and so some young agents underinvest their education. The average growth rate (1.6881%) is less than is the case with perfect market. The average of the Gini coefficient is 33.9372%.

Figure 4 shows the time paths of the annual growth rate and the Gini coefficient for the youth under a public education regime with homogeneous innate ability. Based on our calibration of parameter values, the annual growth rate with homogeneous innate ability converges to 1.5% and the Gini coefficient goes to zero. With heterogeneous innate ability, the simulation results show that none of the individuals in the economy is credit-constrained. The explanation of different patterns of the growth rate with homogeneous/heterogeneous innate ability under a public education regime is given in Appendix 4. The annual growth rate under a public education regime is lower than that with perfect markets because of the low tax rate. Although poor agents can benefit from public education, some agents who prefer to invest more in education now are forced to accept the public education. For example, an agent with high ability and a huge parental endowment will want to invest much in education and accumulate his human capital more rapidly, but now he can only enjoy the same investment in education as other agents under a public education regime. Notice that, under a public education regime, the average Gini coefficient drops to 27.139%. Hence, a public education regime can be used to decrease the income inequality between households. The Gini coefficient is smaller under a public education regime than under a private education regime because, under a public education regime, everyone is forced to have the same investment in education.

This is consistent with the theoretical results derived by Glomm and Ravikumar (1992). However, the reasons are different. Because they do not introduce imperfections in credit markets into their model, the higher income inequality in a private education system is generated because the time devoted to human capital accumulation in a private education economy is higher than in a public education economy (p.827). Glomm (1997) also obtains the same results by using a two-period overlapping generations model where parents make schooling decisions for their children.

When both public and private schools exist in the economy, about 88% of agents choose public schools. Agents might be endogenously credit-constrained if they choose private schools. Figure 5 shows which type of agents will choose private/public schools. As we can see, most rich or able agents will choose private schools, while most poor or unable agents will choose public schools. Because Figure 5 does not cover all types of agents due to the limited ranges of parental human capital and innate ability shown in the figure, Figure 6 presents the histogram of education investments for young agents with different parental human capital in the first period. It shows that some people with low parental human capital or innate ability will choose private schools in order to qualify for vouchers. Their investments in education are lower than the expenditure on public education. The average annual growth rate (1.6067%) lies between the average growth rate under a private education regime with endogenous credit constraints and the average growth rate under a public education regime. The Gini coefficient is 31.1492% and lies between that under a private education regime with endogenous credit constraints and the one under a public education regime. We also compare the long-run economic impacts for two different initial distributions of human capital. Two different standard deviations

(0.645 and 1) of $\log h_1$ are considered. Under a mixed educational system, the initial income inequality will be transmitted from one generation to another.

6. Policies

This section analyzes the impacts of fiscal policies. Our main concern is how these policies alter the transitions of growth and income inequality. Three instruments that the government can use to affect the economic performance are the scales of the voucher programs, the regulation of financial markets and the tax rate. The simulation results for the policies are given in Table 4. Figures 8, 9 and 10 show the time path of the average level of human capital and the Gini coefficient over 10 periods. Note that the average income is proportional to the average human capital. Each dot on the graph indicates the level of human capital and income inequality in each period.

Figure 7 shows the dynamic transitions of the Gini coefficient during the process of development for three different voucher programs v (20%, 63% and 90%). A high v means large subsidies for agents who choose private education. More rich and able young agents will choose private schools as the scales of the voucher programs increase. Hence, the Gini coefficient will increase as v goes up. In addition, as the subsidy becomes larger, the expenditure on public education will go down because of the balanced budget. Poor young agents who go to public schools will accumulate their human capital more slowly while rich or able agents accumulate their human capital more rapidly. Hence, the large scales of the voucher programs will deter an economy to grow if the former dominates the latter.

Figure 8 illustrates economic performance for three different situations of financial development (ϕ =10%, 28.51% and 56.7%). Because the upper limit of debt increases while ϕ increases, fewer young agents are constrained for larger ϕ (in our model, no one is credit constrained when ϕ equals 56.7%). If fewer young individuals are constrained, more agents will choose to go to private schools. Hence, the Gini coefficient increases as ϕ increases. Since the liberalization of financial markets can help the economy grow, a large ϕ will generate a high growth rate.

The last instrument that we analyze is the tax rate. The transitions of inequality and the level of human capital for three different tax rates (8.86%, 15% and 30%) are given in Figure 9. Because higher tax rates imply more expenditure on public education and also more subsidies for agents who choose private schools, the enrollment rates in public schools will increase if the former dominates the latter. In our model, enrollment rate in public schools increases first when the tax rate increases and decreases after a turning point. The average enrollment rate in public schools is 93.57% when the tax rate equals 15% and only 37.65% when the tax rate equals 30%. Hence, the Gini coefficient first decreases then increases. Thus, the growth rate increases as the tax rate increases because of more investment in education.

7. Conclusion

This paper indicates that the structure of the educational system is important in explaining the relationship between growth and inequality. Human capital is an engine for growth, but not many economists pay attention to the composition of educational systems. We develop a dynamic model where households are allowed to make choices between private and public schools. The existence of imperfect credit markets prohibits some agents from choosing private schools. However, through voucher programs, the government can provide public support for private education. By changing the scales of the voucher programs, the government is able to control the education sector.

The findings in this paper suggest ways in which the impacts of policies can be considered. Our simulation results show that low scales of voucher programs can increase the public school enrollment rates and this in turn will raise the economic growth rate if the tax rate is high enough and financial markets are imperfect. Financial reforms without any other interventions will increase both the growth rate and inequality. If the government intends to use financial reforms to help the poor, a policy to increase the size of the public sector (e.g., decreasing the amounts of vouchers) should also be used. Our results provide a cautionary note to those developing countries that are experiencing financial reforms. A high tax rate can contribute to growth, but it may increase or decrease income inequality.

This paper also offers another approach to thinking about the educational system. For a developed country with low income inequality, a mixed educational system with a higher private school enrollment rate may be a better choice than an educational system with a lower private school enrollment rate because financial markets are already liberalized and a high tax rate is politically unfeasible[19]. Table 5 gives the average growth rates (1985-1994) and the Gini coefficients[20] for 26 developed countries. We use 90% of

[19] This is because in democratic countries, tax rates are determined by the voting. However, we do not include the voting system in this paper. See Perotti (1993) for a paper that considers voting.
[20] According to Deininger and Squire (1996), there is no significant difference between Gini coefficients defined on the basis of net or gross income and between household-based or individual-based estimates. However, the difference is significant between income-based and expenditure-based estimates. Following their empirical results, we add 6.6 to the expenditure-based coefficients.

the public secondary school enrollment rate as our criterion. If a country has a public secondary school enrollment rate higher than 90% in 1985, we include that country in a group of countries that have a larger public sector. On the other hand, if a country has a public secondary school enrollment rate of less than 90% in 1985, we include that country in a group of countries that have a larger private sector[21]. The average growth rate for the countries with a larger private sector is 3.355% (13 countries), while the average growth rate for the countries with a larger public sector is 2.95% (10 countries). Income inequality is higher for those countries that correspond to a larger private sector than for those countries that correspond to a larger public sector[22]. For a developing country, our model predicts that a large public sector is preferred to a large private sector due to the underdevelopment of financial markets. Table 6 gives the average growth rates (1985-1994) and the Gini coefficients for 46 developing countries. The average growth rate for the countries with a larger public sector is 3.86% (14 countries), while the average growth rate for the countries with a larger private sector is 3.58% (31 countries). As with developed countries, income inequality is higher for those countries which correspond to a larger private sector than for those countries which correspond to a larger public sector[23]. Although the differences in the growth rates and the Gini coefficients between the two groups are not very significant, these findings provide some support for the predictions of our model. If we focus on the data for low income countries, we find that most low income countries do not have high enrollment rates in public secondary

[21] We use 90% of the public school enrollment rate as our criterion in order to have more equal sizes for each group.

[22] The average Gini coefficient for the countries with a larger private sector is 34.43% (9 countries), while the average Gini coefficient for the countries with a larger public sector is 32.57% (7 countries).

[23] The average Gini coefficient for the countries with a larger private sector is 50.12% (20 countries), while the average Gini coefficient for the countries with a larger public sector is 46.41% (10 countries).

schools. Table 6 also shows that these countries have high income inequality and not a very high growth rate as predicted by our model for a developing country with a larger private sector. Given these results, governments should think carefully about the long-run consequences of educational policies.

This paper also provides a theoretical explanation of Kuznets' curve found in time series and cross-sectional regressions. During the early stage of economic development, the liberalization of credit markets will increase income inequality. By increasing the size of the public sector, the government can reduce income inequality. This will generate an inverted U-shape between levels of income and inequalities in a time series regression. In a cross-section regression, we show that the high Gini coefficients of some developing countries might be a consequence of low public school enrollment rates and imperfections in credit markets.

Appendix 1.

Procedures to Derive the Endogenous Credit Limit

At the beginning of the adult period, each adult is given his own h_{t+1} and s_t. He then needs to decide how much he wants to consume and save in this period. Using eq (7), we can solve for

$$s_{t+1} = \frac{\beta(w_{t+1}h_{t+1} + R_{t+1}S_t)}{1+\beta} \tag{16}$$

$$c_{t+1} = \frac{w_{t+1}h_{t+1} + R_{t+1}S_t}{1+\beta}. \tag{17}$$

Given h_{t+1} and S_t, the value function for an adult who does not default is

$$\begin{aligned} V_{adult}(h_{t+1}, S_t) &= \log(c_{t+1}) + \beta\log(c_{t+2}) \\ &= \log(c_{t+1}) + \beta\log(\beta R_{t+1}c_{t+1}) \\ &= (1+\beta)\,[\log(w_{t+1}h_{t+1} + R_{t+1}S_t) - \log(1+\beta)] + \beta\log(R_{t+2}\,\beta) \end{aligned} \tag{18}$$

If the adult defaults, he loses the portion ϕ of his income. The value function for an adult who defaults is

$$V^d_{adult}(h_{t+1}, S_t) = (1+\beta)[\log(1-\phi)\,w_{t+1}h_{t+1} - \log(1+\beta)] + \beta\log(R_{t+2}\,\beta) \tag{19}$$

Therefore, there is no defaulting if $V_{adult}(h_{t+1}, S_t) \geq V^d_{adult}(h_{t+1}, S_t)$. That is

$$\frac{w_{t+1}h_{t+1} + R_{t+s}S_t}{(1-\phi)w_{t+1}h_{t+1}} > 1$$

This implies

$$-S_t \leq \rho w_{t+1}h_{t+1} \qquad \text{where} \quad \rho = \frac{\phi}{R_{t+1}} \tag{20}$$

For young agents who are not constrained, their optimal choices of saving and consumption are the same as their choices under the perfect market (eq (10) and eq (11)). If an agent is constrained, he can only borrow $\rho w_{t+1} h_{t+1}$. The budget constraints for the young and adult periods are

$$c_t + \mathrm{q}_t = \mathrm{h}_t + \rho w_{t+1} h_{t+1} \tag{A1.1}$$

$$c_{t+1} + \mathrm{s}_{t+1} = (1\text{-}\, \rho R_{t+1})\; w_{t+1} h_{t+1} \tag{A1.2}$$

The young agent then chooses S_{t+1} and q_t to maximize his utility. The first-order conditions for S_{t+1} and q_t are

$$\frac{1}{c_t}\left(\frac{w_{t+1}h_{t+1}}{q_t} - 1\right) + \frac{\beta\gamma}{c_{t+1}}(1 - R_{t+1})\frac{w_{t+1}h_{t+1}}{q_t} = 0$$

$$\frac{1}{c_{t+1}} = \frac{\beta R_{t+2}}{c_{t+2}}$$

After a few steps, we can rewrite the first-order conditions as

$$S_{t+1} = \frac{\beta}{1+\beta}(1 - R_{t+1}\rho) w_{t+1} h_{t+1} \tag{21a}$$

$$\gamma\rho w_{t+1} h_{t+1} - q_t = \beta\gamma(1+\beta)(q_t - h_t - \rho w_{t+1} h_{t+1}) \tag{21b}$$

Appendix 2.

Proof of proposition 1.

From eq (8), we can get

$$\frac{h_{t+1}}{H_{t+1}} = \frac{[z_t(\frac{w\gamma}{R})^{\gamma} h_t^{\delta} H_t^{-\delta}]^{\frac{1}{1-\gamma}}}{H_{t+1}/H_t} = \frac{[z_t(\frac{w\gamma}{R})^{\gamma}]^{\frac{1}{1-\gamma}} (\frac{h_t}{H_t})^{\frac{\delta}{1-\gamma}}}{g_{t+1}}.$$

So $$b_{t+1} = \frac{[z_t(\frac{w\gamma}{R})^{\gamma}]^{\frac{1}{1-\gamma}} b_t^{\frac{\delta}{1-\gamma}}}{g_{t+1}}. \tag{A1}$$

Along the balanced growth path, b_t =1and $z_t = \bar{z}$ for all t. From eq (A1), we can solve for growth rate g_{t+1} = g* = $[\bar{z}(\frac{w\gamma}{R})^{\gamma}]^{\frac{1}{1-\gamma}}$. Q.E.D.

Proof of proposition 2.

By using eq (25), the human capital accumulation function can be written as

$$h_{t+1} = z_t(\tau w H_t)^{\gamma} h_t^{\delta} H_t^{1-\gamma-\delta} = z_t(\tau w)^{\gamma} h_t^{\delta} H_t^{1-\delta}$$

So,

$$b_{t+1} = \frac{h_{t+1}}{H_{t+1}} = \frac{z_t(\tau w)^{\gamma}(h_t/H_t)^{\delta}}{H_{t+1}/H_t} = \frac{z_t(\tau w)^{\gamma}(h_t/H_t)^{\delta}}{g_{t+1}}. \tag{A2}$$

Along the balanced growth path, b_t =1and $z_t = \bar{z}$ for all t. From eq (A2), we can solve for growth rate g_{t+1} = g* = $\bar{z}(\tau w)^{\gamma}$. Q.E.D.

Appendix 3.

Computation of the equilibrium when public and private schools coexist[24]

1. First, we assign the scale of vouchers.
2. Then we guess the amount of public education expenditure (e.g. Initially, we assign expenditure of public education equal to 15% of the average income.).
3. Assume p_u is the fraction of individuals who choose public schools. Given the policy of tax rate τ and voucher v, we can solve p_u by balancing government budget. The tax revenue is

τwH = expenditure on education provided by government + the amount of vouchers

$= p_u q_u + (1 - p_u) v q_u$.

Then

$$p_u = \frac{\tau wH - vq_u}{(1-v)q_u}.$$

4. Households optimize their utilities by choosing the school regime. Notice that young agents might encounter the endogenous credit constraint if they want to borrow.
5. Computing the actual fraction (p_u') of agents who choose to go to public schools.
   (1) If $p_u = p_u'$, then go to step 6.
   (2) If $p_u > p_u'$, then we increase q_u and repeat steps 3 to 5.
   (3) If $p_u < p_u'$, then we decrease q_u and repeat steps 3 to 5.
6. Adjusting the scale of vouchers by using the enrollment rate of public schools.
   (1) If the enrollment rate of public schools = 88%, then stop.

[24] In this appendix, I do not write down the index of time to make the context easier to read.

(2) If the enrollment rate of public schools > 88%, then we increase v and repeat steps 2 to 6.

(3) If the enrollment rate of public schools < 88%, then we decrease v and repeat steps 2 to 6.

Appendix 4.

Simulation Results of Growth Rate with Homogeneous/Heterogeneous Innate Ability

Case 1. With homogeneous innate ability:

(A) Under a Private Education Regime with Perfect Credit Markets

This is our baseline model. The annual growth rate will converge to the growth rate along the balanced growth path. That is, annual growth rate will converge to 2%. Income inequality converges to zero because human capital accumulation technology is a decreasing function of human capital.

(B) Under a Public Education Regime

Using parameter vales we calibrated in section 5, annual growth rate will converge to 1.5% while Gini coefficient will converge to zero.

Case 2. With heterogeneous innate ability

(A) Under a Private Education Regime with Perfect Credit Markets

From eq (8), the human capital accumulation function is

$$h_{t+1} = [z_t (\frac{w\gamma}{R})^{\gamma} h_t^{\delta} H_t^{1-\gamma-\delta}]^{\frac{1}{1-\gamma}}.$$

Hence,

$$\log h_{t+1} = \frac{1}{1-\gamma}[\log z_t + \gamma \log(\frac{w\gamma}{R}) + \delta \log h_t + (1-\gamma-\delta)\log H_t].$$

Because both $\log z_t$ and $\log h_t$ are normally distributed, $\log h_{t+1}$ is also normally distributed with mean μ_{ht+1} and variance σ_{ht+1}^2. Note that the average human capital is period t equals $\exp(\mu_{ht} + \frac{\sigma_{ht}^2}{2})$. Then we can derive the mean of $\log h_{t+1}$ in period t+1 as

$$\mu_{ht+1} = E(\log h_{t+1})$$

$$= \frac{1}{1-\gamma}[E(\log z_t) + \gamma \log(\frac{w\gamma}{R}) + \delta E(\log h_t) + (1-\gamma-\delta)\log H_t]$$

$$= \frac{1}{1-\gamma}[\mu_z + \gamma \log(\frac{w\gamma}{R}) + \delta\mu_{ht} + (1-\gamma-\delta)(\mu_{ht} + \frac{\sigma_{ht}^2}{2})]$$

$$= \frac{1}{1-\gamma}[\mu_z + \gamma \log(\frac{w\gamma}{R}) + (1-\gamma)\mu_{ht} + (1-\gamma-\delta)\frac{\sigma_{ht}^2}{2}].$$

If σ_{ht}^2 is constant, using parameter values calibrated in section 5, the difference between μ_{ht+1} and μ_{ht} is

$$\mu_{ht+1} - \mu_{ht} = \frac{1}{1-\gamma}[\mu_z + \gamma \log(\frac{w\gamma}{R}) + (1-\gamma-\delta)\frac{\sigma_{ht}^2}{2}] = 0.3751 > 0.$$

Because μ_{ht} increases over time and the difference between μ_{ht+1} and μ_{ht} is constant, the growth rate of logarithm of human capital decreases over time. Because the average human capital equals $\exp(\mu_{ht} + \frac{\sigma_{ht}^2}{2})$, growth rate of human capital is

$$\frac{H_{t+1} - H_t}{H_t} = \frac{\exp(\mu_{ht+1} + \frac{\sigma_{ht+1}^2}{2}) - \exp(\mu_{ht} + \frac{\sigma_{ht}^2}{2})}{\exp(\mu_{ht} + \frac{\sigma_{ht}^2}{2})} = \exp[(\mu_{ht+1} - \mu_{ht}) + \frac{1}{2}(\sigma_{ht+1}^2 - \sigma_{ht}^2)]\text{-1}$$

$$= \exp[\mu_z + \gamma \log(\frac{w\gamma}{R}) + \frac{1}{2}(\sigma_{ht+1}^2 - (\gamma+\delta)\sigma_{ht}^2)]\text{-1}.$$

If σ_{ht}^2 is constant and equals $(0.645)^2$, growth rate of human capital over one period (20 years) is 45.5%. That is, annual growth rate is 1.89% (this number is slightly less than 2%). However, due to the limited number of agents in our simulation (the number of agents we use in the simulation is 2000), σ_{ht}^2 slightly varies around $(0.645)^2$=0.416 and annual growth rate of average human capital varies around 1.9% because of variations of variance of logarithm of human capital.

(B) Under a Public Education Regime

From eq(25), the human capital accumulation function is $h_{t+1} = z_t (\tau w)^{\gamma} h_t^{\delta} H_t^{1-\delta}$. Following steps we do in part (A), we can derive the μ_{ht+1} as

$$\mu_{ht+1} = \mu_z + \gamma \log(\tau w) + \delta\mu_{ht} + (1-\delta)(\mu_{ht} + \frac{\sigma_{ht}^2}{2}) .$$

If σ_{ht}^2 is constant, by using parameter values calibrated in section 5, the difference between μ_{ht+1} and μ_{ht} is

$$\mu_{ht+1} - \mu_{ht} = \mu_z + \gamma \log(\tau w) + (1-\delta)\frac{\sigma_{ht}^2}{2} = 0.31653 > 0 .$$

Because μ_{ht} increases over time and the difference between μ_{ht+1} and μ_{ht} is constant, the growth rate of logarithm of human capital decreases over time. Growth rate of human capital becomes

$$\frac{H_{t+1} - H_t}{H_t} = \exp[(\mu_{ht+1} - \mu_{ht}) + \frac{1}{2}(\sigma_{h+1t}^2 - \sigma_{ht}^2)] - 1$$
$$= \exp[\mu_z + \gamma \log(\tau w) + \frac{\sigma_{h+1t}^2}{2} - \delta \frac{\sigma_{ht}^2}{2}] - 1$$

If σ_{ht}^{2} is constant and equals 0.416, growth rate of human capital over one period (20 year) is 37.2%. This implies annual growth rate is 1.6% (this number is slightly larger than 1.5%). Again, due to the limited number of agents, σ_{ht}^{2} varies around 0.416 over time and annual growth rate varies around 1.3%.

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Table 1: Enrollment in private schools as a percentage of total enrollment in secondary schools in 1985[25].

Country	Private school enrollment rate (%)
High Income Countries(Non OECD)	
Australia	28.7795
Belgium	64.877
Canada	6.5367
Denmark	13.7139
France	21.6396
Greece	3.3341
Ireland	64.0131
Italy	6.3332
Japan	13.0121
Luxembourg	8.087
Netherlands	72.057
New Zealand	4.5214
Norway	2.891
Portugal	8.5942
Spain	34.7616
Switzerland	5.8584
UK	8.4712
US	9.9055
High Income Countries (Non OECD)	
Bahamas	23.627
Cyprus	13.0283
Kuwait	13.756
Malta	31.1517
New Caledonia	38.121
Qatar	10.8256
Singapore	27.7426
Reunion	6.5349
United Arab Emirates	15.3827

[25] Source: UNESCO, United Nations (except Ireland and U.S.).
Ireland: OECD education database, OECD organization.
U.S.: U.S. National Center for Education Statistics: Digest of Education Statistics, annual.

Table 1: Enrollment in private schools as a percentage of total enrollment in secondary schools in 1985 (Continued).

Country	Gini (%), 1985-1994	Growth rate(%), 1985-1994
Upper Middle Income Countries		
Argentina*	NA	2.6143
Bahrain	NA	3.3812
Bostwana*	60.81	7.6893
Brazil*	58.015	2.766
Chile*	57.185	6.8313
Gabon*	NA	0.262
Korea*	34.09	8.6708
Mauritius*	44.765	6.435
Mexico	54.1567	2.5745
Oman	NA	5.502
Panama	56.47	2.8986
Saudi Arabia	NA	2.6312
St. Kitts and Nevis	NA	5.8362
St. Lucia	NA	6.6341
Turkey	44.09	4.2235
Uruguay	NA	4.0008
Venezuela*	47.697	2.7225
Lower Middle Income Countries		
Colombia*	51.26	4.3823
Costa Rica	44.035	4.4759
Fiji*	NA	2.4
Jamaica	47.598	2.4821
Morocco	45.8	4.1615
Paraguay*	NA	3.4911
Peru	50.415	1.9252
Philippines*	45.603	2.2771
St. Vincent and the Grenadines*	NA	5.052
Sri Lanka	41.7	3.8703
Swaziland*	NA	6.6172
Syrian Arab Republic	NA	4.552
Thailand	48.775	9.0347
Tonga*	NA	2.318
Tunisia	48.22	3.7757

Table 1: Enrollment in private schools as a percentage of total enrollment in secondary schools in 1985 (Continued).

Country	Private school enrollment rate (%)
Low Income Countries	
Bangladesh	93.034
Burkina Faso	48.4858
Burundi	13.2623
Cameroon	48.6366
Cote d'Ivoire	28.9022
Haiti	84.2507
Indonesia	49.7396
Mali	8.7706
Nicaragua	19.7232
Niger	10.7273
Senegal	29.4094
Tanzania	49.0686
Zambia	6.3299
Zimbabwe	66.9242

Table 2 Parameter Values

Sources	Parameters	Number
Discount Factor	β	$(0.98)^{10} = 0.668$
Real Interest Rate	R	$(1.035)^{20} = 1.99$
Wage Rate	w	3.101
Defaulting	ϕ	0.2851
Human Capital Accumulation Fn	γ	0.2
	δ	0.4
Distribution of $\log h_1$	μ_{h1}	8.3707
	$sd_{\log(h_1)}$	$\{\mathbf{0.645}, 1\}$
Distribution of Innate Ability	$\bar{z}$	1.7332
Tax Rate	τ	8.86%
Scale of Vouchers	v	63%

Table 3. Simulation Results[26]

	g (%)	Gini (%)	q/(GDP p.c.)	s/(GDP p.c.)
Pri. Edu				
Perfect Mkt				
z_t : homogeneous	1.9645	3.5746	0.3365	0.2835
z_t : heterogeneous	1.9030	35.1724	0.3336	-0.5765
Exog. B.C.	0.8829	31.4882	0.1290	0.0121
End. B.C.	1.6881	33.9372	0.2679	-0.3889
Pub. Edu.				
z_t : homogeneous	1.4385	2.3887	0.2010	-0.2691
z_t : heterogeneous	1.3017	27.1390	0.1964	-0.2530
Pub. Edu. + Pri. Edu.				
$sd_{\log(h)}$ =0.645	1.6067	31.1492	0.2514	-0.3669
$sd_{\log(h)}$ =1.00	1.6694	47.7139	0.2667	-0.0834

[26] Note that g, Gini, q/(GDP p.c.) and s/(GDP p.c.) are the average of growth rates (%), Gini coefficients (%), ratios of average school expenditure to per capita GDP and ratios of average saving for youth to per capita GDP over 10 periods

Table 4: Simulation Results of Different Policies

Private + Public	g (%)	Gini(%)	q/(p.c.GDP)	Public school e.r. (%)
v=61%; tau=8.86%; phi=28.51%	1.6067	31.1492	0.2514	88.04
v=20%	1.4938	29.6602	0.2300	95.36
v=90%	1.6938	33.3290	0.2652	60.74
phi=10%	1.3799	27.9362	0.2100	97.17
phi=56.7%	1.7258	32.6544	0.2846	83.74
tau=15%	1.8715	28.6595	0.3373	93.57
tau=30%	1.9714	38.5402	0.4100	37.65

Table 5: Growth rate and Gini coefficient for developed countries[27].

Country	Gini (%),1985-1994	Growth rate(%),1985-1994
High Income Countries (OECD)		
Australia*[28]	39.31	3.0879
Belgium*	26.59	2.0805
Canada	30.3	2.4508
Denmark*	33.175	1.8724
France*	34.91	2.1596
Ireland*	34.6	4.5167
Italy	33.52	2.0334
Japan*	36.17	3.34
Luxembourg	27.13	5.8916
Netherlands*	29.36	2.7222
New Zealand	36.92	1.4863
Norway	32.6	2.8325
Spain*	32.06	2.9066
Switzerland	NA	1.6718
UK	29.77	2.374
US	37.72	2.4976
High Income Countries(Non OECD)		
Bahamas*	43.708	1.282
Bahrain*	NA	3.3812
Cyprus	NA	5.6262
Kuwait*	NA	9.3959
Malta*	NA	5.1954
New Caledonia*	NA	NA
Qatar	NA	NA
Reunion	NA	NA
Saudi Arabia	NA	2.6312
United Arab Emirates*	NA	1.6742

[27] Source: Growth rate: World Development Data, World Bank.
Gini Coefficient: Deininger and Squire, World Bank.

[28] A country with a star means its private school enrollment rate is larger than 10% and is defined as having a bigger private sector. A country without a star means its private school enrollment rate is less than 10% and is defined as having a bigger public sector.

Table 6: Growth rate and Gini coefficient for developing countries.

Country	Gini (%), 1985-1994	Growth rate(%), 1985-1994
Middle Income Countries		
Argentina*	NA	2.6143
Botswana*	60.81	7.6893
Brazil*	58.015	2.766
Cameroon*	NA	-1.5482
Chile*	57.185	6.8313
Colombia*	51.26	4.3823
Costa Rica	44.035	4.4759
Cote d'Ivoire*	45.7825	1.2789
Fiji*	NA	2.4
Greece	41.79	1.8553
Indonesia*	38.863	6.981
Jamaica	47.598	2.4821
Korea*	34.09	8.6708
Mauritius*	44.765	6.435
Mexico	54.1567	2.5745
Morocco	45.8	4.1615
Nicaragua	56.92	-1.6101
Oman	NA	5.502
Panama	56.47	2.8986
Paraguay*	NA	3.4911
Peru	50.415	1.9252
Philippines*	45.603	2.2771
Portugal	36.195	3.0951
Singapore*	40	7.479
St. Kitts and Nevis	NA	5.8362
St. Lucia	NA	6.6341
St. Vincent and the Grenadines*	NA	5.052
Sri Lanka	41.7	3.8703
Swaziland*	NA	6.6172
Syrian Arab Republic	NA	4.552
Thailand	48.775	9.0347
Tonga*	NA	2.318
Tunisia	48.22	3.7757
Turkey	44.09	4.2235
Uruguay	NA	4.0008
Zimbabwe*	63.43	2.683
Venezuela*	47.697	2.7225

Table 6: Growth rate and Gini coefficient for developing countries (continued).

Country	Gini (%), 1985-1994	Growth rate(%), 1985-1994
Low Income Countries		
Bangladesh*	NA	4.0435
Burkina Faso*	NA	3.4146
Burundi*	NA	2.6478
Haiti*	NA	-2.9963
Mali	60.6	3.0196
Niger*	42.7	1.5482
Senegal*	60.72	2.2758
Tanzania*	44.7	NA
Zambia	54.055	0.5475

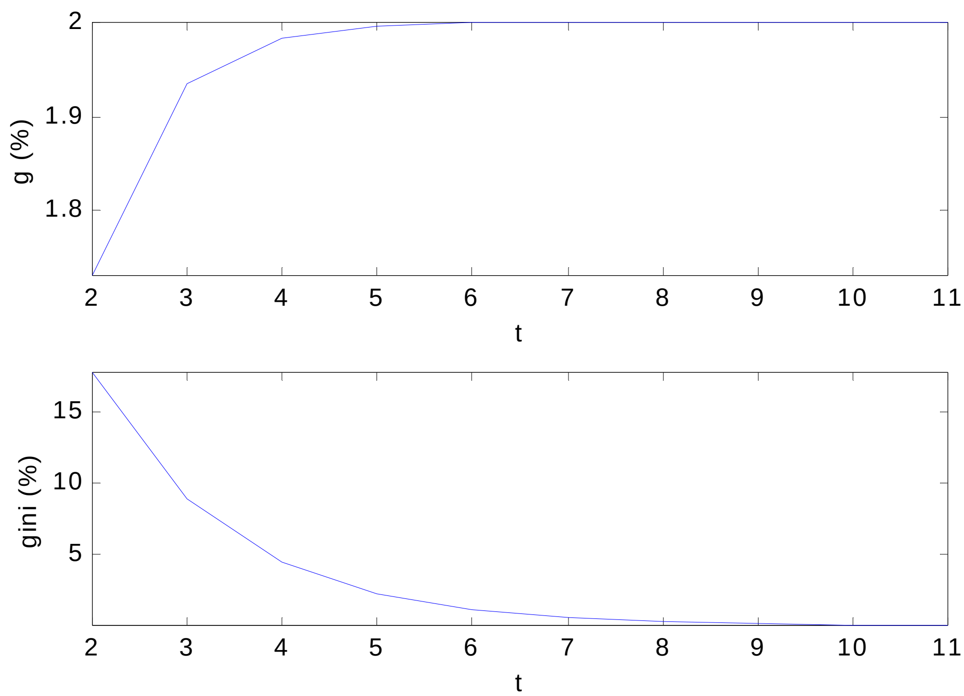

Figure 1. Public Education with Perfect Credit Markets and Homogenous Innate Ability

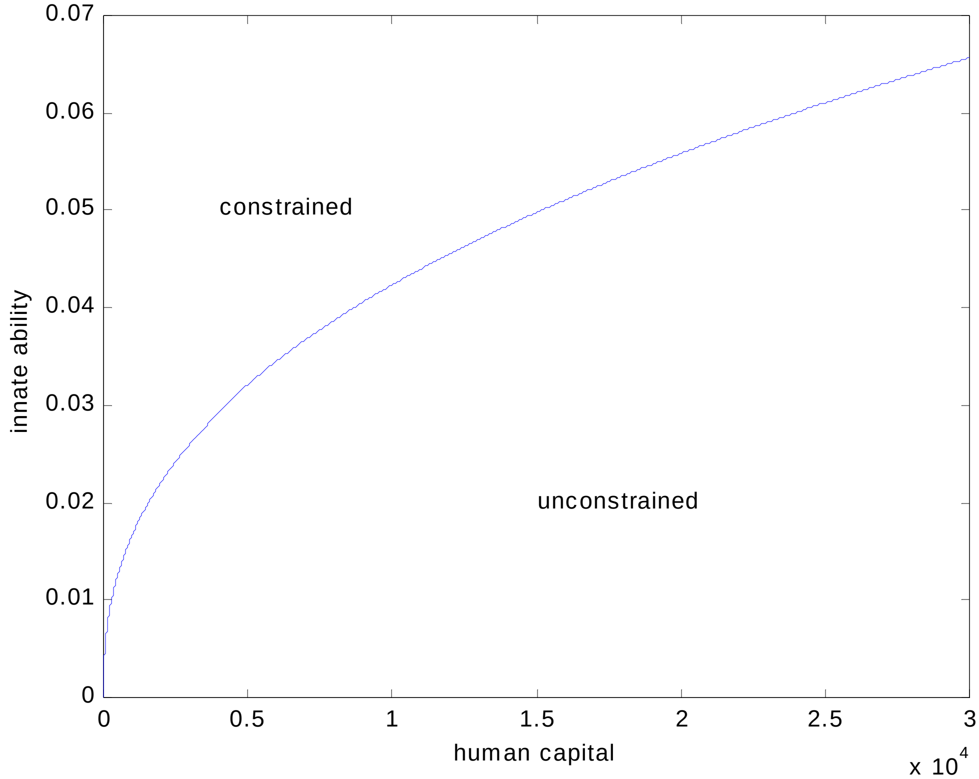

Figure 2. Regions for Agents Being Constrained or Unconstrained with Exogenous Borrowing Credit Constraints

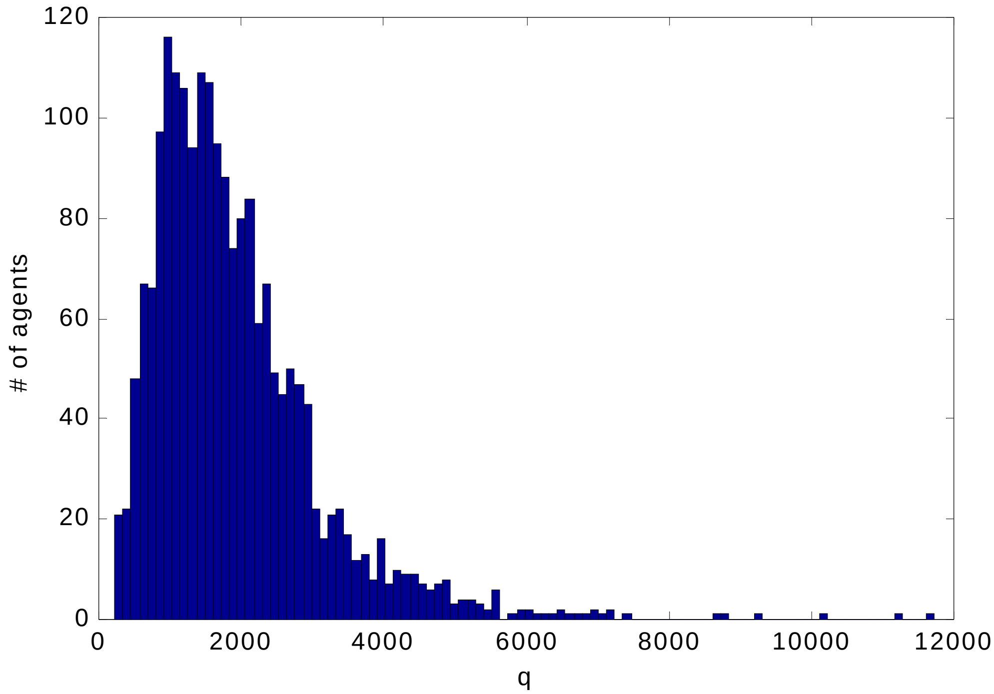

Figure 3. Histogram of Investment in Education under A Private Education Regime with Endogenous Credit Constraints

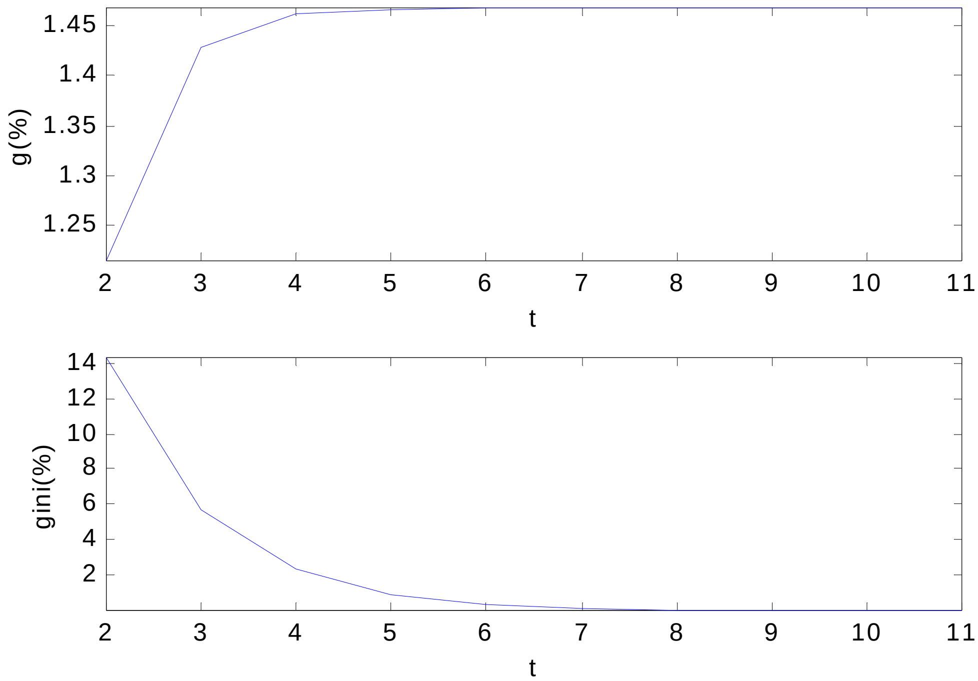

Figure 4. Public Education with Homogenous Innate Ability Shock

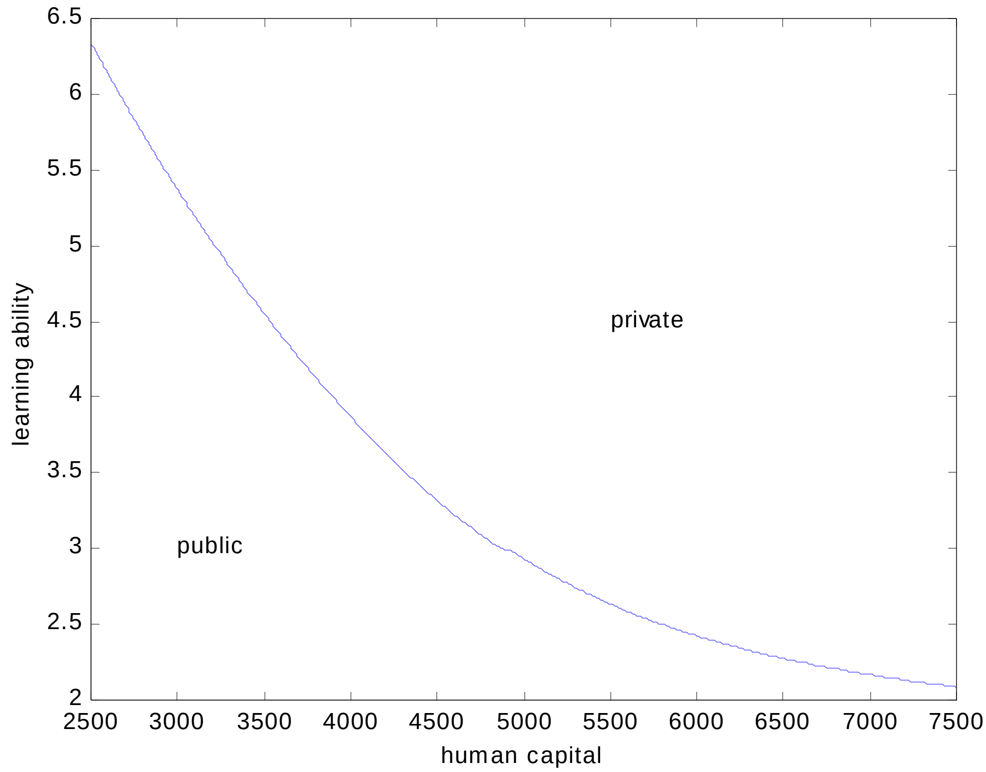

Figure 5. Regions of Choosing Private or Public Schools with Endogenous Credit Constraints

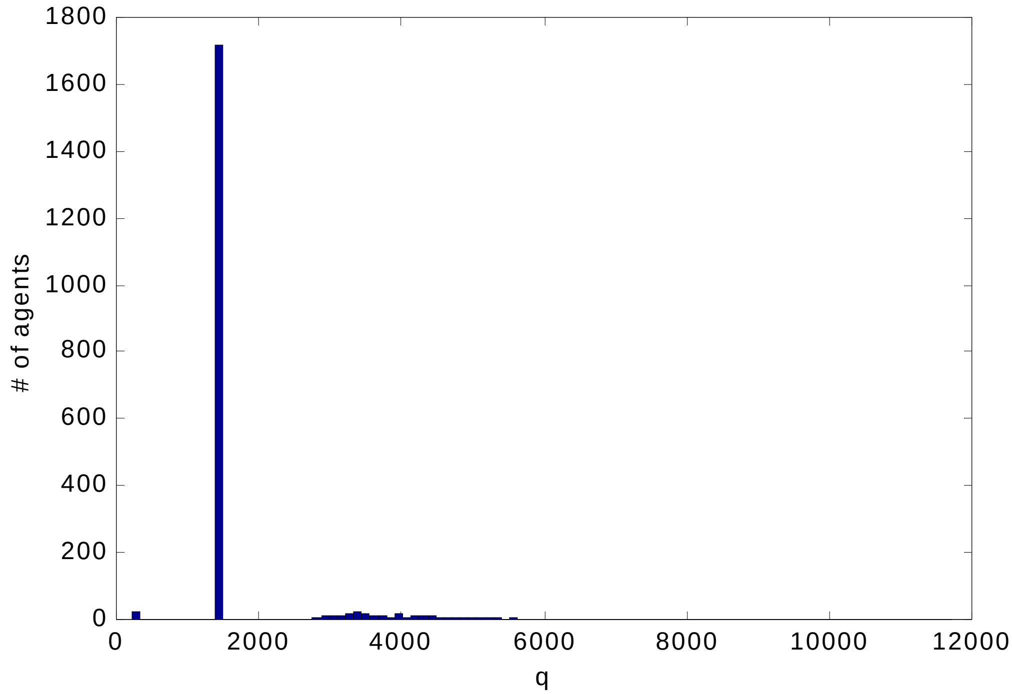

Figure 6. Histogram of Investment in Education under A Mixed Educational System

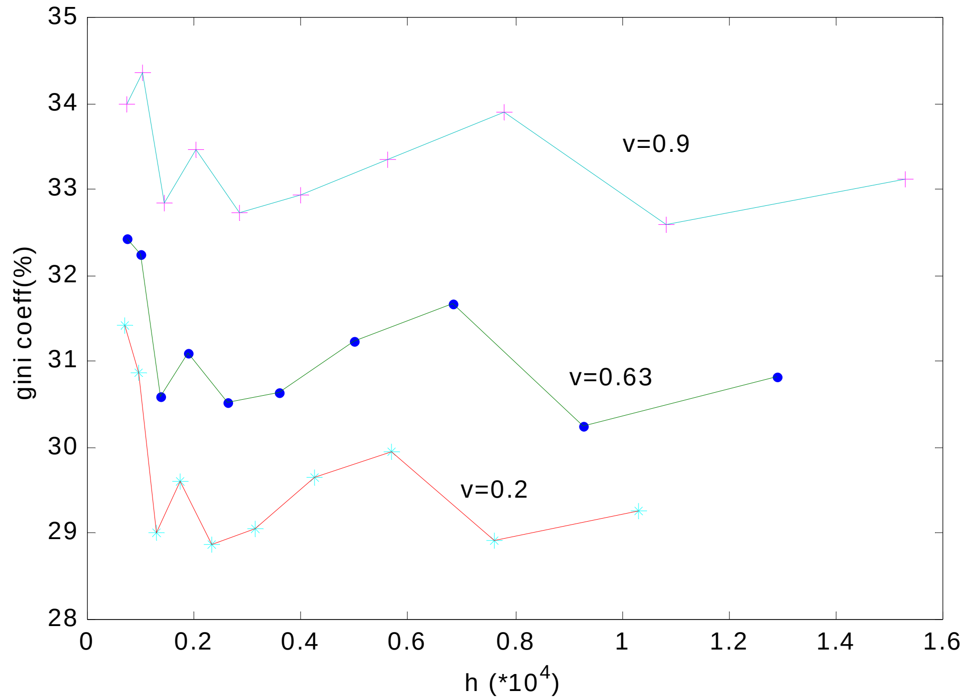

Figure 7. Transitions of Growth and Income Inequality with Different Voucher Programs

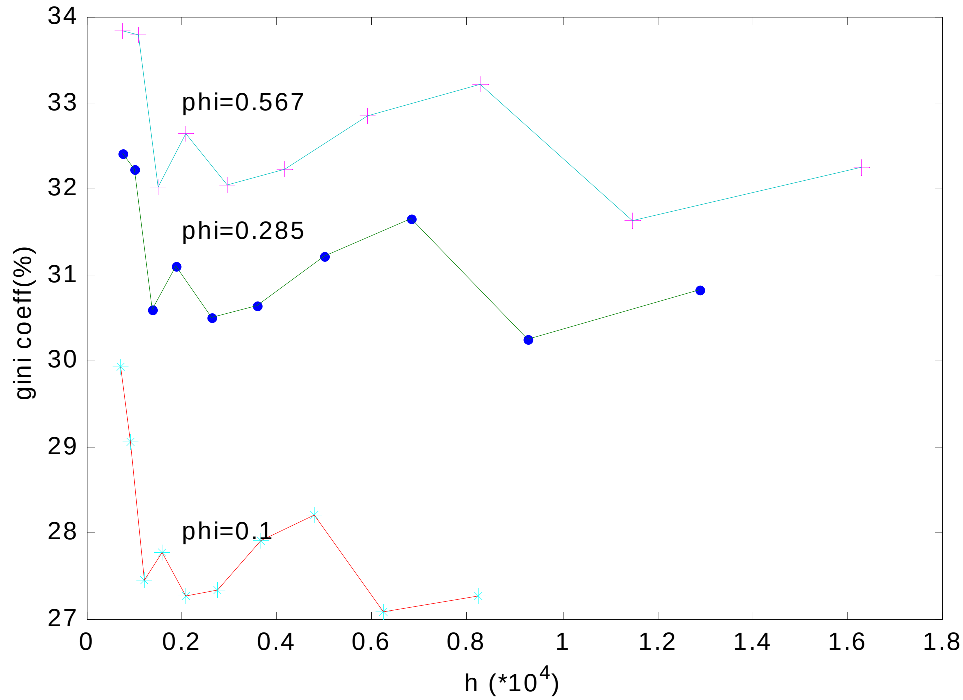

Figure 8. Transitions of Growth and Income Inequality with Different Financial Regulations

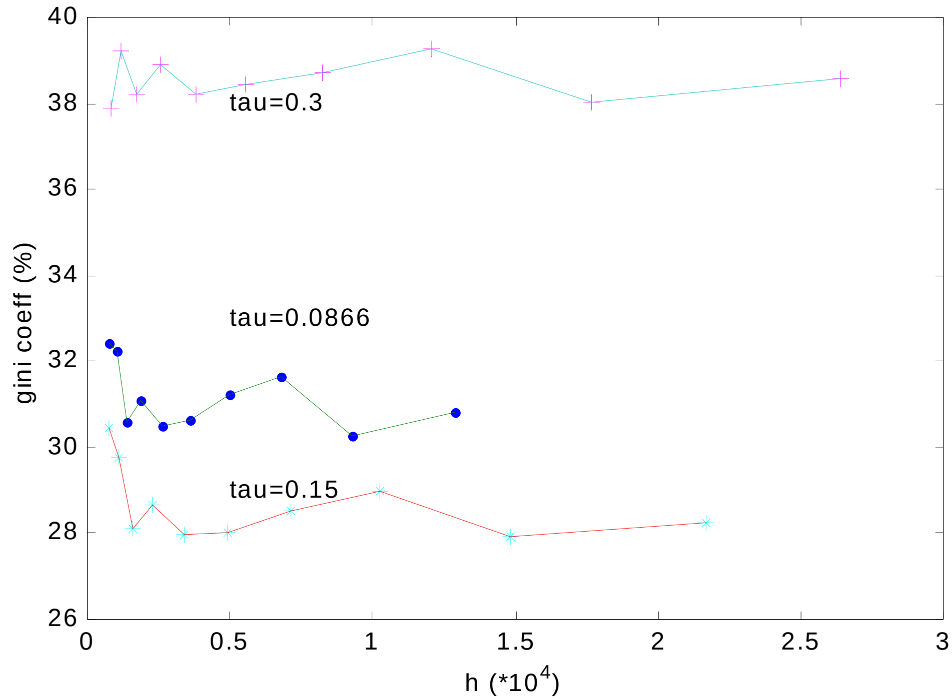

Figure 9. Transitions of Growth and Income Inequality with Different Tax Rates