Comparing Linear and Nonlinear Solution Methods for Dynamic Equilibrium Economies*

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EXTENDED ABSTRACT

This paper addresses the following question: how different are the computational answers provided by linear and non-linear solution methods for dynamic equilibrium economies?

Over the last ten years a number of nonlinear solution methods, as several variants of projection (see Judd (1992) and McGrattan (1996)) and asymptotic procedures (Judd and Guu (1997)), have been proposed as alternatives to more traditional (and relatively simpler) linear approaches and to the proven, but expensive, value function iteration. However little is known about the relative performance of the different possibilities since the last systematic comparison of solution procedures (Taylor and Uhlig (1990)) is by now 13 years old and it did not review any of the newest approaches. This paper tries to fill part of this gap in the literature.

To do so we use the canonical stochastic neoclassical growth model with leisure choice, the workhorse of modern dynamic macroeconomics. We compute and simulate the model using three linear methods (a Linear Quadratic approximation to the utility function and two undetermined coefficients methods, one based on the linearization of the first order conditions of the problem and another one based on their loglinearization) and three nonlinear methods (a finite element method with Galerkin weighting, a pseudospectral procedure with Chebyshev polynomials and a perturbation approach). In addition, for comparison purposes and given the amount of knowledge about its theoretical properties (see Santos and Vigo (1998)), we also solve the model using value function iteration with a multigrid scheme.

In our simulations we keep a fixed set of stochastic shocks common for all methods. That allows us to observe the dynamic responses of the economy to the same driving process under different solution methods. We also perform Euler Equations tests (Den Haan and Marcet (1994) and Judd and Guu (1997)) and compare the results with the bounds in Santos (2000).

We document substantial differences in algorithmic complexity, time performance and accuracy of the solution and how our findings depend on parameter values, specially the degree of risk aversion. Two main results deserved to be highlighted. First, perturbation methods deliver an interesting compromise between accuracy, speed and programming burden but they suffer from the need of computing analytical derivatives, a difficult task to implement in low level but efficient languages as C++ or Fortran 95. Also, since their validity is local, they perform relatively poorly far away from the steady state in comparison with other nonlinear methods. However we show how low order perturbations (of 4th or 5th order) provide a clear advantage over all the linear methods for a trivial marginal cost. Second the finite elements approach is extremely accurate over a long range of the state space, which can be of importance in applications like estimation procedures where this accuracy is required to obtain unbiased estimates (Fernández-Villaverde and Rubio (2002)). However it suffers from being probably the most complicated method to implement in practice. Our results should serve as an encouragement of a wider use of high order perturbation approaches, to suggest the reliance on finite element methods for problem that demand high accuracy and to support the phasing out of pure linearizations.

We finish pointing out to future lines of research and we offer a preview of some preliminary results from the application of newer nonlinear methods as the Adaptative Finite Element method (Verfürt (1996)) and the Weighted extended B-splines finite element approach (Höllig (2003)) from the companion paper Fernández-Villaverde and Rubio (2003).