A politico-economic equilibrium of unemployment insurance with precautionary savings and liquidity constraint

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Abstract

In this paper, I build a recursive politico-economic equilibrium of unemployment benefits. Agents can save, dissave, but face a strict borrowing constraint. The true recursive formulation of this problem would require to let the whole distribution of agents be part of the state space. To overcome this difficulty, I use a limited set of statistics of the distribution, in the spirit of Krusell and Smith (1998). I then study the nature of the equilibrium by focusing on how agents' expectations and savings shape the equilibrium. Quantitatively, the recursive equilibrium is quite different from that of an economy where agents are myopic and do not internalize the impact of their current choice on their future ones. Rather high levels of benefits are sustainable in the recursive equilibrium, since agents know that a downward deviation of the replacement rate today will generate a downward deviation tomorrow as well. In the absence of savings, the equilibrium would also substantially differ. Quantitatively, rather high levels of benefits are sustained for plausible voting frequencies.

1 Introduction

In this paper, I develop a model where unemployment benefits are the outcome of a politico-economic equilibrium. Agents can accumulate assets, but

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cannot borrow. Whenever a vote occurs, each agent chooses her most preferred replacement rate, and the median voter determines the vote outcome. The equilibrium is recursive: agents choose today the replacement rate, given their expectations of future votes. Their expectations are rational in that they correctly anticipate the outcomes of future votes.

Wright's (1986) seminal work built the first politico-economic equilibrium of unemployment insurance, but ignored savings. However, in the presence of income fluctuations, risk-adverse agents tend to accumulate assets as a precautionary device. Precluding savings is therefore a major assumption which may not be neutral as to the various conclusions one can derive. Although it has been established that the income risk only marginally affects the saving rate (Aiyagari, 1994), this does not mean that precautionary savings are systematically unimportant. Pallage and Zimmermann (1999) build a politico-economic equilibrium with precautionary savings, liquidity constaint and moral hazard. The authors study how differences in the skill composition of the labor-force affect the political outcome. In a recent paper, Hassler and Mora (1999) build a politico-economic model of unemployment insurance in the presence of precautionary savings. They show that the assumption regarding savings is crucial. Since savings act like a substitute for the public insurance system, agents' choices are quite different from those in an economy without savings.

In this (draft, at the moment) paper, I build a politico-economic equilibrium of unemployment insurance with precautionary savings and liquidity constraint. Apart from the political determination, the setup is quite simple: agents face exogenous employment shocks, can save or dissave, but cannot borrow. In particular, the unemployment rate is unique and does not depend on the level of benefits. It is quite possible that the disincentive effect of unemployment insurance would affect agents' choices and the political equilibrium. However, this issue is ignored here, in order to focus on a basic setup.

Unlike Hassler and Mora (1999), the utility function yields a constant relative risk aversion. The political process is the following: at each period, and with a given probability, a vote occurs, and the outcome is the choice of the median voter. The presence of a borrowing constraint and a CRRA utility function implies that employed differing in asset holdings have different choices over benefits. The true recursive formulation of the equilibrium would then require to add the whole distribution of agents in the state space, and to describe all possible transitions of this distribution. Since this cannot be implemented numerically, in the spirit of Krusell and Smith (1998), I use a limited number of statistics of the distribution to compute the equilibrium.

The nature of the equilibrium is explored, by underlining how agents' expectations affect the equilibrium. The comparison of the recursive equilibrium with the equilibrium of an economy where agents are myopic with respect to the outcome of future votes reveals that high levels of benefits are sustainable in the recursive economy, while they are not in the myopic one. Expectations play a crucial role in the determination of agents' most preferred replacement rate. The intuition behind this result is the following. In a recursive equilibrium, agents internalize the effect of their current choice on future outcome. When the median voter (employed) decides upon her most preferred replacement rate, she realizes that reducing slightly the replacement rate will tend to increase savings during the on-going voting cycle. The increase in savings will push down the outcome of the next vote, since the median voter will be better protected against an unemployment shock. Therefore, although the current median voter knows that benefits chosen today will apply only for a limited length of time, during which she is more or less likely to remain employed, a downward deviation will reduce benefits not only in the short, but also in the median run. Over this longer horizon, the median voter considers benefits as an insurance, since the probability to begin an unemployment spell is much higher. In a word, the median agent is willing to pay generous benefits in the short run, to ensure that this generous level will be sustained afterwards.

I then explore how the possibility to save affects the equilibrium. To this purpose, I compare the model results with those of an economy without savings. It appears that the divergence between the two economy can be very significant. The higher the voting frequency, the higher the temptation to deviate toward lower replacement rates. Agents then decide to save more, so that the median voter gets richer. Her decisions will diverge more from those of the no-savings economy. Indeed, the temptation to deviate is quite different in these two setups. When savings are precluded, no agent can afford a single unemployment spell. When agents can save, they can partly protect themselves against the next unemployment shock.

Quantitatively, the preliminary results tend to show that unlike Hassler and Mora (1999), the equilibrium replacement rate is pretty high for plausible vote periodicities. Their paradox is in fact turned upside down: while the authors find it difficult to reproduce high levels of benefits, I have not (so far) been able to reproduce low levels characterizing the U.S. economy.

The paper is organized as follows. Section 2 presents the model. In section 3, I develop the computational method involved. Section 4 briefly presents the calibration. The model is simulated in section 5, where various implications of the recursive equilibrium are analyzed. Section 6 concludes.

2 The model

The economy consists of a continuum of infinitely-lived agents, represented by the interval $[0;1]$. Agents can be either employed, or unemployed. The transitions between these two states are due to exogenous idiosyncratic shocks following a simple Markov chain, and represented by the matrix $(\pi_{ij})_{i,j=e,u}$ where, for example, π_{eu} is the probability to be next period in state u (unemployed), when current state is e (employed). When they are employed, agents earn a before-tax wage w, exogenous. When they are unemployed, they receice a before-tax replacement income b. Agents can accumulate an asset, denoted a, but cannot borrow : $a \geq 0$. Assets yield an exogenous return r. Each agent maximizes an expected intertemporal utility, assumed to be time-separable, which writes:

$$\max EU = E\left(\sum_{t=0}^{\infty} \beta^t u(c_t)\right)$$

where β, $u(.)$, c_t denote respectively the discount factor, the instantaneous utility and consumption at date t. The instantaneous utility function is of the CRRA type:

$$u(c_t) = \frac{c_t^{1-\lambda}}{1-\lambda}$$

where λ denotes the relative risk aversion. Because of the presence of idiosyncratic income shocks, agents are inclined to accumulate assets on precautionary grounds. Since the income shocks are Markovian, the individual state of an agent consists of her current employment status and her current level of assets.

2.1 The politico-economic equilibrium

When a vote is organized, agents decide upon the level of the replacement rate $\rho = \frac{b}{w}$ which shall prevail until the next vote. Benefits are financed by a proportional tax on income (wages and unemployment benefits), and the budget of the public insurance company is assumed to be balanced at every period. Since, in this simple setup, the transition probabilities are exogenous, so is the unemployment rate u. Therefore, at every period, there is a unique tax level such that the budget is balanced:

$$\tau(\rho).\left((1-u).w + u.b\right) = u.b$$
$$\Leftrightarrow \tau(\rho) = \frac{\rho.u}{1-u+\rho.u}$$

where u denotes the steady state unemployment rate. We assume that a vote occurs, at each period, with a probability μ. That is, the expected length of a spell between two consecutive votes is $\frac{1}{\mu}$. Each agent has his own preferences in terms of unemployment benefits. In particular, we can expect unemployed to favor very high replacement rates, at least if the periodicity of the vote is not too high. Indeed, in the near future, unemployed are more or less likely to remain unemployed, which means that benefits are a pure transfer to them. On the other hand, working agents currently pay for the unemployment benefits. They shall therefore favor smaller replacement rates. They would choose to suppress unemployment benefits if votes happened every period. However, with a longer periodicity, employed know that they are likely to loose their job before the next vote, which is the reason why they favor unemployment insurance up to a certain extent.

To determine the issue of the vote, we check that the condition of the median voter theorem applies. Precisely, the vote bears on a single variable, and preferences are single peaked. Thus, the outcome is the replacement rate chosen by the median voter.

Although this setup is particularly simple, the definition of the recursive politico-economic equilibrium needs take the expectations of future votes into account. Future votes depend on the future state of the median voter. For agents to correctly predict this, the whole current distribution $\Psi(a,i)$ is part of the state space. In addition, since votes do not occur at each period, the current level of benefits is also a state variable. Indeed, the evolution of the distribution depends on this variable until a new vote occurs.

In a setting where agents have rational expectation, they should be able to (i) correctly predict the evolution of the distribution as long as the current level of benefits is maintained and (ii) correctly predict the outcome of potential future votes. That is, agents know the relation between the distribution and the vote outcome, $\rho = \nu(\Psi)$ and the law of motion of the distribution $\Psi' = \phi(\Psi, \rho)$. The recursive formulation of the program of the agent then writes:

$$\begin{aligned} \forall i &= e, u, \forall a, \\ V(a,i,\rho,\Psi) &= \max \left\{ u(c) + \beta \left[\sum_{j=e,u} \pi_{ij} \left((1-\mu) . V(a',j,\rho,\Psi') + \mu . V(a',j,\nu(\Psi'),\Psi') \right) \right] \right\} \\ & s.t. \left| \begin{array}{c} a' = (1+r)\, a + y_i(\rho) - c \\ a' \geqslant 0 \\ \Psi' = \phi(\Psi,\rho) \end{array} \right. \end{aligned}$$

where $V(a,i,\rho,\Psi)$, $y^i(\rho)$ respectively denote the expected intertemporal utility of an agent in state (a,i,ρ,Ψ) and income in employment status i, which depends on the current level of ρ. The voting rule $\nu(\Psi)$ simply reflects the fact that, given Ψ, the median voter chooses a replacement rate $\rho = \nu(\Psi)$. That is,

$$\nu(\Psi) = \arg\max_{\rho} \{V(a_{med}(\Psi), e, \rho, \Psi)\}$$

where $a_{med}(\Psi)$ represents the function associating to each distribution the asset level of the median agent.

As we shall rely on numerical computations, this theoretical formulation is unfortunately of little practical interest. Indeed, the distribution Ψ belongs to a set of infinite dimension. It is therefore impossible to characterize precisely the functions $\nu(\Psi)$ and $\phi(\Psi, \rho)$.

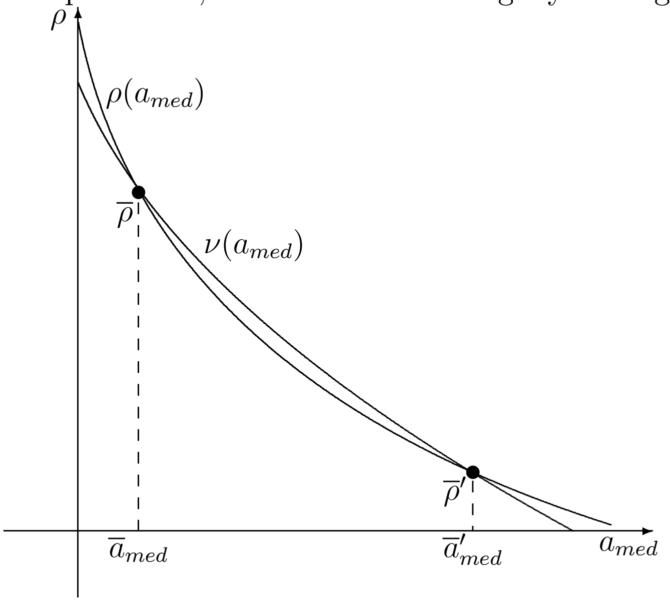
2.2 Handling the dimensional problem

To overcome this difficulty, it is worth noticing that the only information agents need to extract from the distribution Ψ is the state of the median voter. We here make the assumption that unemployed systematically choose higher replacement rates. We *ex post* check that this assumption is justified. Therefore, the median voter is necessarily employed. Following the spirit of Krusell and Smith (1998), we assume that a limited set of statistics of the distribution is enough to predict the law of motion of the state of the median voter. The first statistics we can think of is of course the state of the median voter itself. That is, we assume that tomorrow's median voter can be predicted by today's.

This does not mean that the very same agent is the median voter during two consecutive periods. To understand this, let us admit for the moment that preferences over benefits are monotonously decreasing in the stock of assets. Consequently, the median agent is an employed such that 50% of the total population is employed and richer than her. Pin down a level of asset corresponding to today's median agent. If the agent is still employed tomorrow, she will be above the median agent. Indeed, among all employed above her today, a fraction will be unemployed tomorrow. Therefore, if today, there is exactly 50% of the population which is employed and richer than her, this proportion will be reduced to $50.(1 - \pi_{11})\,\%$ tomorrow. If the agent looses her job, she will not be the median agent either. However, the agent who will be tomorrow the median voter is just below the current median voter in terms of asset holding. Therefore, we have reasons to think that tomorrow's median voter can be statisfactorily guessed out of today's. This means that the state space is reduced to the vector (a, i, ρ, a_{med}) where all components are scalars.

2.3 A characterization of the equilibrium

So far, we have considered dynamic equilibria which were not necessarily stationary. We could for instance have had in mind voting cycles. It turns out that only stationary equilibria have been found numerically. To illustrate such an equilibrium, consider the following stylized figure.



The $\rho(a_{med})$ curve depicts the relation between the level of asset of the median agent and the level of the replacement rate in an economy without votes and where a unique replacement rate is held indefinitely constant. As is common in such a setup, the aggregate wealth and the wealth of the median agent are both increasing in the income risk. The lower the replacement rate, the higher a_{med}.

The $\nu(a_{med})$ represents the voting rule. It is decreasing in the wealth of the median voter. Indeed, as agents become richer, they can better insure themselves against future employment shocks. Since the voting rule is unambiguously decreasing in the asset holdings, the median voter can be easily identified as an employed with 50% of the population richer than her.

A stationary equilibrium must necessarily belong to both curves. That it should belong to $\nu(a_{med})$ is obvious. As for $\rho(a_{med})$, note that once a stationary equilibrium is reached, agents expect the replacement rate $\overline{\rho}$ to be held constant in the future. Therefore, their saving behavior is identical to that of agents in an economy without vote with a replacement rate worth $\overline{\rho}$.

The curves may intersect many times. Numerically, we have looked for a single intersection, namely, that corresponding to the lowest level for a_{med}

and consequently, the highest level for ρ. This equilibrium is found to be stable. For various initial conditions close to this stationary equilibirum, the economy converges toward it. The relative position of the two curves enables us to understand why this is the case.

Consider first an economy where the median agent would be somewhat richer than $\overline{a}_{med}$, and where a vote would occur at date $t = 0$. Since the ν curve is above the ρ one, this means that the replacement rate ρ_0 will be set at a higher level than that which would guarantee the constancy of a_{med}. In other words, agents are better insured than if ρ_0 were equal to $\rho(a_{med})$. They will save relatively less, which means that the median agent will start to dissave. When a new vote shall occur in the future, the outcome shall be again higher, and shall converge toward the intersection of the two curves. A similar reasoning would apply for the symmetric case where the median agent would lie initially below $\overline{a}_{med}$.

If we now consider the possibility of multiple intersections it must be that the second one yields the opposed relative position of the two curves. The equilibrium would be unstable. For any initial a_{med} below $\overline{a}'_{med}$, the economy would converge toward the first intersection. If $a_{med} > \overline{a}'_{med}$, the economy would diverge from $\overline{a}'_{med}$. It would converge to a third equilibrium, characterized by a lower replacement rate, possibly equal to zero. This could be explained as follows. In an economy where agents have high asset holdings, the median agent does not fear much being unemployed in the near future. She may therefore wish to suppress unemployment benefits. In turn, the absence of unemployment benefits makes the income risk quite high, which induces agents to keep high asset holdings. This self-enforcing mechanism suggests that such an equilibrium could be stable. Finally, this argument shows that the equilibrium which we have found may not be unique.

Note also that the regression implemented on the law of motion of a_{med} is only locally valid. The true law of motion $\phi(\Psi)$ is of course globally true, but the approximated law of motion $a'_{med}(a_{med})$ is a projection of this law. There is no reason for this projection to be globally a good approximation.

3 Computation

3.1 Discretization method

The state space consists of the Cartesian product $[a_{\min}; a_{\max}] \times \left[a_{med}^{\min}; a_{med}^{\max}\right] \times [\rho_{\min}; \rho_{\max}] * \{e, u\}$. Each of the first three components is discretized into respectively N^a, $N^{a_{med}}$ and N^b uniformly distributed points. $a_{\min}$ is set to 0. In practive, we use $N^a \simeq 400$, $N^{a_{med}} \simeq 20$ and $N^b \simeq 30$. The minimum level acceptable for $a_{\max}$ depends on the lower bound on ρ, $\rho_{\min}$. We also set $a_{med}^{\min} = 0$, and $a_{med}^{\max} = \frac{a_{\max}}{2}$. We need to have an idea of the equilibrium level of benefits to determine the bounds $\rho_{\min}$ and $\rho_{\max}$. $\rho_{\max}$ must be high enough for the highest level of benefits chosen to be interior. In practice, this level is that of an employed agent with no wealth : $\nu(0)$. It can be quite high, which means that the length of the interval $[\rho_{\min}; \rho_{\max}]$ can reach $15 - 20$ points.

The value functions and the policy rules are computed by applying the Euler condition, which writes :

$$u'(c_t^i) = \beta\,(1 + r) \left(\sum_j \pi_{ij} u'(c_{t+1}^j) \right)$$

The choice of a' is not restricted to belong to the grid. By interpolating the value function, we determine the level of savings a' such that the Euler equation holds. Interpolation is also required because there is no reason for the law of motion $a'_{med}(a_{med})$ to take values on the grid for a_{med}. In other words, we restrict the calculations over the grid for the current level of a_{med}, but we use the law of motion $a'_{med}(a_{med})$ which we interpolate to get the contribution of tomorrow's value for a_{med}. The voting rule $\nu(a_{med})$ is not restricted to be on the grid for ρ either, therefore we also interpolate the utility. Again, the calculations are performed on the grid for the current levels of benefits ρ, but the interpolation is required since a vote can occur tomorrow, which affects the conditional expectation. In the end, we use a tri-linear interpolation along the three dimensions of the state space.

The algorithm rests on finding a fixed point $(a'_{med}(a_{med}), \nu(a_{med}))$. We assume that the evolution of the median voter's asset can be described by the following linear relation:

$$a'_{med} = \alpha_0 + \alpha_a . a_{med} + \alpha_\rho . \rho$$

The voting rule $\nu(a_{med})$ is numerically computed over the grid for a_{med}. To compute the voting rule, we simply need to maximize the agent's utility, at any given level of asset a, with respect to ρ :

$$\nu(a) = \arg\max_\rho \left\{ V\,(a, a, \rho) \right\}$$

In this writing, we impose $a_{med} = a$, because we only need to know the choices of the median voter. Numerically, we dispose of the value functions over the grid for ρ. We use a Lagrange interpolation to find the level of ρ which precisely maximizes the agent's utility. We then use a third-degree polynomial fit for the curve $\nu(a_{med})$ to ensure smoothness throughout the iterations. Without such a fit, a very small perturbation in the smoothness of the curve at the first iteration, due to numerical approximations, would most probably grow bigger and prevent from convergence.

We proceed in two steps : for a given law of motion $a'_{med}(a_{med})$, we find the voting rule $\nu(a_{med})$. This voting rule is both an input into the agent's program through her expectation of future vote outcomes and an output revealing the choices of the agent. Therefore, we have to find a voting rule $\nu(a_{med})$ such that when agents expect $\nu(a_{med})$, they choose $\nu(a_{med})$. Convergence of the voting rule is time-consuming, since the fixed point consists of a whole curve, and not a single parameter. Besides, we have not formally proved the uniqueness of the fixed-point $\nu(a_{med})$. However, after some $10-15$ iterations, the *ex ante* and *ex post* curves can hardly be distinguished.

3.2 Determining the law of motion of the economy

As mentioned above, the law of motion for the state of the economy consists of the 3 coefficients α_0, α_a and α_ρ. To obtain values for these 3 coefficients, we need to describe the actual evolution of the economy. The problem here is that there is a great variety of possible transitions. If a stationary equilibrium exists, the economy will, in some cases at least, converge toward it. Since the projection may be only locally valid, the idea is to simulate the path of the economy around the stationary equilibrium. We therefore need to guess where this equilibrium is. Besides, once the equilibrium is known, we dispose of the equilibrium distribution of assets, but there are still a great number of distributions close to this one. The idea is to simulate the path of the economy around the equilibrium. The initial conditions correspond to the distribution of assets which would prevail if a particular level of benefits had been indefinitely chosen in the past. By choosing levels of benefits near that corresponding to the stationary equilibrium, the initial distributions will be close to the stationary one, at least in the sense that the transition toward the stationary equilibrium will be completed in a short length of time.

Since votes occur at random, paths of the economy are not deterministic. This is why, for a given initial distribution, we simulate several transitions, and we repeat this for different initial conditions. During the simulations, the actual state of the median voter is determined each period. Votes occur at random, and whenever they do, the outcome is obtained from the state of the current median voter a_{med} and the voting rule $\nu(a_{med})$.

With all these simulated time series, we determine the coefficients of the equation describing the law of motion $a'_{med}(a_{med})$ by ordinary least squares. With this revised law of motion, we go back to the step computing the voting rule, and iterate until two consecutive sets of coefficients for the law of motion are close enough.

The core of the program, computing the value functions and the simulations of the economy, is written in C++ langage. Whenever possible, tabulations are computed, in order to gain time. Recall indeed that the state space consists of roughly 240000 points. The regressions and the polynomial fit are computed on Matlab.

4 Calibration

There are only 5 parameters to determine: π_{11}, π_{22}, λ, μ and r. With an unemployment rate set around 13.5%, and an unemployment duration of 4 quarters, we get $\pi_{22} = 0.75$ and $\pi_{11} = 0.96$. The relative risk aversion $\lambda = 2.0$, which is common in the heterogeneous-agents literature. As for μ, it is rather difficult to quantify the frequency at which unemployment benefits are revised. Since they are associated with a radical change in politics, one could assume that their periodicity is that of nation-wide votes, which occur in France every 5 years. Therefore, the benchmark value for μ is $\frac{1}{20} = 0.05$. Finally, $r = 1.3\%$.

5 Simulations

5.1 The impact of expectations on the equilibrium

What makes recursive politico-equilibrium differ from ealier version of politico-equilibria is the nature of expectations (see Krusell, Quadrini and Rios-Rull, 1997). Here, agents are fully rational, in that they correctly predict the evolution of the state of the economy, and therefore can correctly predict the potential outcomes of future votes. This is true at any time, whether a vote is currently going on or not. Besides, when votes occur, agents choose their prefered level of unemployment benefits by taking into account the effect of such a choice on the evolution of the economy.

To assess in what way these expectations affect the equilibrium, we compare two economies. In the first one, agents have rational expectations, as described in this paper. In the second, agents systematically expect a constant replacement rate as the outcome of future votes. With such parameterized expectations, the law of motion of the median voter's level of asset a_{med} provides no information, since agents expect a unique replacement rate to be chosen in the future, regardless of a_{med}. The expected level of the replacement rate is set at the equilibrium value for the recursive equilibrium.

The figure below plots the voting rules for these two economies.

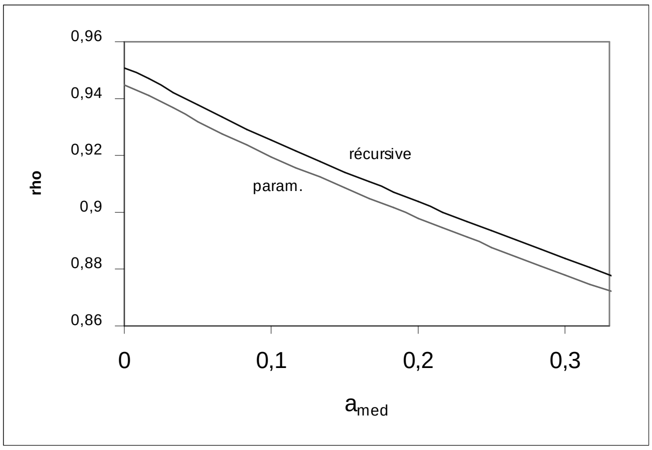


Figure 2 : Voting rules for the recursive equilibrium and the parameterized expectations economies

The voting curve in the parameterized expectation case always lies below that of the recursive equilibrium. To understand the implications of this finding, consider the first economy at the equilibrium. There, the replacement rate and the assets of the median voter are tied by the relation $\rho = \nu(a_{med})$. Holding the distribution of agents unchanged, what would happen if agents expected a constant outcome of future votes? Nothing, as long as no vote occurs. Indeed, agents expect the replacement rate to be held indefinitely constant, and their saving behavior is such that their distribution -and, consequently, a_{med}- remains the same. However, as soon as a vote takes place, agents will choose a lower replacement rate. They will therefore start to save, and the median voter's assets will rise. Therefore, what was a stationary recursive equilibrium is not stable in the parameterized expectation economy.

What causes this divergence? Of course, the reason has something to do with agents' expectations. Consider the first economy. As we have reached a stable equilibrium, agents will always choose the same replacement rate whenever votes take place. This means that the median voter has no temptation to deviate toward lower replacement rates. When making her choice, however, the median voter contemplates the effects of such a deviation. She then takes into account the fact that, with a lower replacement rate, agents will save more, a_{med} will rise and the next vote result will be lower. This follows from the law of motion of a_{med}: at the equilibrium, (a_{med}, ρ) is stable in the sense that $a'_{med}(a_{med}, \rho) = a_{med}$. Since the coefficient α_ρ is always found to be negative in the simulations, holding a_{med} constant, a decrease in ρ induces an increase in a'_{med}. Consequently, the median agent, who is employed, does not want to choose a lower replacement rate, because future unemployment benefits would be lower as well.

The situation is quite different in the parameterized expectation economy. Regardless of the law of motion for a_{med}, agents expect a constant future level of benefits. The temptation to deviate and choose a lower replacement rate is then much higher. Indeed, employed find it worth to reduce benefits today, given that benefits will be reset at their initial level in the future. For them, unemployment insurance is not necessary in the short run, but only in the median and long run.

To assess the quantitative importance of this mechanism, we have looked for a stationary equilibrium in the second economy. The equilibrium is reached when the state of the economy remains identical as time passes. By considering the intersection between the previous $\nu(a_{med})$ curve and the unique $\rho(a_{med})$ one, we would get such an equilibrium. However, the $\nu(a_{med})$ curve is constructed conditional on agents' expectations. Therefore, the equilibrium would be such that agents always expect a given outcome of future votes, and are always mistaken. To avoid this undesired feature, we look for a situation where agents' expectations are correct. We then need to look for a lower expected level of benefits, such that when agents expect it, they indeed choose it once the transition is over. Table 1 below presents the two equilibrium levels of benefits for the two economies investigated, for different voting frequencies.

$\frac{1}{\mu}$	14	20	100
Benchmark (recursive)	75.2%	80.6%	94.6%
Parameterized expectations	34%	54%	94.0%

When votes are rare ($\frac{1}{\mu} = 100$), the quantitative difference is rather small. The temptation to deviate is only slightly higher with parameterized expectations. This is due to the fact that, for both economies, the median agent considerably values unemployment insurance, and choose a replacement rate close to one. Another explanation is that, as μ gets smaller, the impact of the expectations of future vote outcomes is less decisive. If the next vote is expected to take place in some $\frac{1}{\mu}$ periods, agents are less concerned with the next vote result because of the discounting factor.

However, for higher voting frequencies, the two equilibria appear very different. Indeed, the temptation to deviate is then higher, because the current choice is to be implemented over a shorter horizon. When $\frac{1}{\mu} = 20$, the equilibrium replacement rate for the recursive case is 80.6%, while it drops down to 54% for the parameterized case.

In a word, the impact of expectations on the equilibrium is highly significant. It was not that obvious at first glance. Indeed, at a stationary equilibrium, the vote outcomes are constant. We could therefore view the second economy as equivalent to the first one, at least locally around this equilibrium. This does not hold, since the voting preferences are obtained by computing the effects of small deviations. Depending on how agents relate this small current deviation with their expectations of future votes, their choices will be quite different.

Now that the importance of how agents anticipate the effects of small deviations is made clear, we can turn to the law of motion of a_{med}:

$$a'_{med} = \alpha_0 + \alpha_a.a_{med} + \alpha_\rho.\rho$$

So far, the intuition has suggested to take both a predetermined state variable into account (a_{med}) and a policy variable (ρ). That a predetermined state variable is required is quite natural. However, it was not clear that the additional policy variable ρ would modify substantially the nature of the equilibrium. The previous reasoning shows that it is the case. Indeed, consider the following law of motion;

$$a'_{med} = \alpha_0 + \alpha_a.a_{med}$$

For $a_{med} = \frac{\alpha_0}{1-\alpha_a}$, $a'_{med} = a_{med}$. In other words, if $a_{med} = \frac{\alpha_0}{1-\alpha_a}$, agents anticipate $a'_{med} = a_{med}$, regardless of their current choice on ρ. They would therefore anticipate a unique future voting outcome $\nu(a_{med})$, no matter what they choose today. When assessing the impact of small deviations, they would anticipate the replacement rate to be reset at $\nu(\frac{\alpha_0}{1-\alpha_a})$ in the future. Should we find one stable stationary equilibrium, it would coincide with that of the second economy. We have seen that this equilibrium may be very different from that of the first economy. In a word, although the law of motion is only a projection of the true law, it contains enough information to let agents take small deviations into account.

5.2 The impact of savings on the equilibrium

To assess how the possibility to save assets affects the political equilibrium, we can analytically derive the equilibrium replacement rate in an economy similar in every respect, but in which agents cannot store assets and consume their entire income at each period.

Let us first remark that there are only two types of agents: employed and unemployed. Within a category, all agents are alike, and they will therefore choose the same replacement rate whenever a vote is organized. Besides, the state of the economy is not time-dependent. Indeed, the state of the economy consists only in the unemployment rate, which is constant. Regardless of the previous choices over the replacement rate, the economy consists of $1-u$ employed and u unemployed. This trivial remark greatly simplifies the computation of the equilibrium. Indeed, rational agents will expect the future outcome to be independent of their current choices. Let us denote this expectation by $\overline{\rho}$. The value function of an agent then depends on her current employment status and on the current level of benefits, which will apply until the next vote. It writes:

$$\begin{aligned} V_e(\rho) &= u\left(w\left(1-\tau(\rho)\right)\right)+\beta\left[(1-\mu).\left[\pi_{ee}V_e(\rho)+\pi_{eu}V_u(\rho)\right]+\mu.\left[\pi_{ee}V_e(\overline{\rho})+\pi_{eu}V_u(\overline{\rho})\right]\right] \\ V_u(\rho) &= u\left(\rho.w\left(1-\tau(\rho)\right)\right)+\beta\left[(1-\mu).\left[\pi_{ue}V_e(\rho)+\pi_{\text{uu}}V_u(\rho)\right]+\mu.\left[\pi_{ue}V_e(\overline{\rho})+\pi_{\text{uu}}V_u(\overline{\rho})\right]\right] \end{aligned}$$

Given $\overline{\rho}$, the median agent (employed) will choose ρ in order to maximize her intertemporal utility $V_e(\rho)$. After a few calculations (available upon request), we get:

$$\rho=\left(\frac{1-\pi_{\text{uu}}}{\frac{1}{\beta(1-\mu)}-\pi_{\text{uu}}}\right)^{\frac{1}{\lambda}}$$

Table 2 below reproduces the previous computations and that for the no savings economy.

$\frac{1}{\mu}$	14	20	100
Benchmark (savings)	75.2%	80.6%	94.6%
No savings	85.5%	88.6%	95.3%

For very long voting cycles ($\frac{1}{\mu}=100$), the results are hardly different. Consider the benchmark economy with savings. When agents know that their current decision will be implemented over a long horizon, they choose generous benefits. This, in turn, reduces their savings, since the precautionary motive is quite low. Consequently, the median voter (employed) has a very small asset buffer, which means that her behavior will closely match that of an employed in the no savings economy. However, as the voting cycle gets smaller, the gap between the two equilibrium replacement rates increases. The same argument applies. When $\frac{1}{\mu}$ decreases, employed choose lower replacement rates (in both economy). In the benchmark economy with savings, this implies that the median agent will hold more assets. Her temptation to deviate toward lower replacement rates is then much higher than in the economy without savings, since she can afford an unemployment spell in the short run.

In terms of mechanisms at work, the two models are quite different. We have seen that in the benchmark economy, the presence of savings has two major effects. First, as mentioned above, that the median voter holds some assets makes her temptation to choose lower replacement rates somewhat higher than in the no-savings economy. Second, however, the presence of savings creates a dynamic link between today's choices and tomorrow's expected ones. This effect has been discussed in the previous sub-section. It tends to make rather generous levels of benefits sustainable. This mechanism is absent from the no-savings economy. Whatever their current choices, agents know that they will not affect the next vote result. This is straightforward since the dynamic link is operated by the saving rate which is increased whenever the replacement rate is reduced. This affects the distribution of agents. In the no-savings economy, the distribution of agents is exogenous.

It may seem intuitive that the second effect, which tends to increase the replacement rate, does not dominate the first one. Indeed, it seems natural to think that in the presence of self-insurance, agents will choose lower benefits. At the limit, should agents vote for very generous benefits, they would not be incited to save, and the equilibrium would be the same as that of the no-savings economy.

5.3 Other comments

As could be expected, the replacement rate is a decreasing function of μ. When votes occur frequently (μ high), employed do not wish to pay for a generous insurance system. They indeed know that there is little chance for them to become unemployed before the next vote. Conversely, for a low frequency, employed start to think of unemployment benefits as a true insurance and not simply as a transfer to current unemployed. Quantitatively, it appears that the replacement rate is quite high for plausible voting frequencies ($\frac{1}{\mu} \geqslant 15$).

As compared to Hassler and Mora (1999), this model does not exhibit low equilibrium replacement rates. The presence of the liquidity constraint has therefore a crucial impact on the vote outcome. Recall in addition that Hassler and Mora (1999) work with a CARA utility function. This enables the authors to distinguish only two different votes : that of the employed, and that of the unemployed. All employed indeed choose the same replacement rate regardless of the current level of wealth. More important, this means that the agents are less affected by low levels of consumption. Here, the presence of a constant relative risk aversion makes low levels of consumption particularly costly in utility terms. Combined with the liquidity constraint, this implies that all agents want to protect themselves against a situation where the replacement rate would be very low and where they would have depleted their asset buffer. In the end, the replacement rate is below the observed level in France only for unplausible periodicities. What apperared as a counterfactual property of Hassler and Mora's (1999) model does not carry to an economy with a liquidity constraint and a constant relative risk aversion.

To assess the impact of job turnover on the equilibrium level of benefits, let us compute the model for a higher job turnover. I choose $\pi_{22} = 0.5$ and $\pi_{11} = 0.92$. The unemployment rate is unchanged, while the unemployment duration is half its previous value. Table 3 below reports the results.

$r \backslash \frac{1}{\mu}$	14	20	100
Benchmark (low turnover)	75.2%	80.6%	94.6%
High turnover	/	86.6%	97.2%

Like Hassler and Mora (1999), unemployment benefits are higher in a high turnover economy. This qualitative feature implies that differences in turnover cannot explain the gap between European and U.S. replacement rates. There are of course many dimensions which this highly stylized model does not take into account. A potential mechanism which could solve this apparent paradox would be to endogenize the unemployment duration by letting agents refuse job offers. Agents in the high-duration economy would be more choosy, and they would prefer higher benefits.

6 Concluding remarks

In this paper, we build a recursive politico-economic equilibrium of unemployment insurance. Agents face exogenous idiosyncratic employment shocks, and must decide on the level of the replacement rate. Unlike previous studies, agents can save, but are submitted to a liquidity constraint. The definition of the equilibrium is somewhat complexified, since the whole distribution of agents would theoretically be a state variable. The strategy adopted to overcome this difficulty is inspired from Krusell and Smith (1998): it consists of using a limited set of statistics of the distribution.

We first explore the implications of such an equilibrium by assessing how expectations affect the agents' behavior and the equilibrium level of unemployment benefits. It appears that the nature of expectations may have a strong quantitative impact on the equilibrium, since it is central as to how agents evaluate small deviations. The qualitative and quantitative results tend to show that when agents are not myopic and take into account the effects of small deviations, the stable equilibrium replacement rate is much higher than when agents are myopic. Besides, we compare the equilibrium with that of an economy without savings. We show that the two equilibria may significantly differ for plausible voting frequencies. In the no-savings economy, the temptation to deviate toward lower replacement rates is smaller for employed, since they cannot protect themselves against an unemployment spell. In the benchmark economy, employed holding moderate levels of assets can better endure a future unemployment spell. When voting cycles are short, they value less the public insurance system.

We then assess the impact of various parameters on the equilibrium, and among them, the frequency of the votes. Preliminary results tend to show that (i) the replacement rate is a decreasing function of the voting frequency and (ii) the equilibrium replacement rate can match its observed level in France for plausible voting frequencies. Further work will aim at understanding how low (resp. high) replacement rates can be sustained in high (resp. low) turnover economies.

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