Endogenous Fertility in a Stochastic Endogenous Growth Model with Human Capital

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*I am indebted to Alfonso Novales who provided me with some routines for training purposes. The simulations undertaken here are done with MAPLE V. Any comment is very welcome! Correspondence: University of Rostock, Faculty of Economics and Social Sciences, Ulmenstr. 69, D-18057 Rostock, Germany, Phone: ++49-(0)381-4984468, Fax: ++49-(0)381- 4984467, e-mail: andreas.schaefer@wiwi.uni-rostock.de.

Abstract

We simulate a stochastic model with endogenous growth and endogenous fertility of the Lucas-Uzawa-Type. During childhood agents are equipped with education (human capital) and enter the labor force when adult. During adulthood they receive utility from their offspring and consumption. Regarding the children a quality-quantity trade off is generated by changing opportunity costs of child rearing. Unsecurity arises in that model, because of an identical and independent normal distributed shock, which governs the ability to use the human capital in production, hence income. The strength of the correlation between the ability of the parents and their children governs the intergenerational persistence and the oscillation around the deterministic steady state. Because of endogenous labor supply multiple equilibria are generated. It is shown that only one of them is sustainable.
JEL: D31,J1, I2, O0

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1 Introduction

The fertility decline observed in all industrialized countries, since the industrial revolution leading to a fertility level below the replacement level even in countries with traditional rather high fertility rates like Spain and Italy (now belonging with a fertility rate around 1.3 to the group of lowest-low fertility countries; Kohler et.al. (2002)) caused increasing attention for researchers and policy advisers. Obviously, a fertility level much below the replacement level accompanied by an increasing life expectation leads to tensions in the social security systems during the demographic transition and to a not sustainable steady state after fertility has stabilized.

Economic arguments supporting this phenomenon can be found in the increase of wages, especially that of women due to higher education (Galor/Weil (1996) and Greenwood/Seshadri (2002)). In the first macroeconomic models based on Becker (1960) fertility decisions are rather consumption oriented and the investments into children by education are ignored (Barro/Becker (1988,1989), Becker/Murphy/Tamura (1990), Barro/Sala-i-Martin (1995)). As soon as education is considered, the well known interaction between quality and quantity of the offspring comes into play (Becker/Lewis (1973)). Galor/Weil (2000) propose a uniform growth setting, in which the economy escapes from a malthusian trap. Contrary to our framework, the quality-quantity trade off is governed by the technological progress. Opportunity costs do not play any role. The interaction between quality and quantitiy becomes important when families differ in abilities and income. Low income families prefer high fertility and invest less in education per child and the opposite is true for high income families leading to an intergenerational persistence and redistributional pressure because lower income percentiles are growing with a higher fertility rate (Kremer/Chen (2000), Schäfer (2002) and (2003)). If this is true, it is reasonable to examine how fertility and educational decisions are affected by heterogeneities. We build up a frame work in which innate ability is inherited by the parents through an AR(1)-process. The ability governs the productivity in the education sector and in production.

We use an Uzawa-Lucas framework (Uzawa (1965) and Lucas(1988)) to generate an endogenous growth process with transitional dynamics. From Ladron-de-Guevara et.al. (1997), (1999) and Ortiega (2000) it is well known that this framework can generate multiple equilibria if labor supply is endogenous and human capital does not enter the utility function. Contrary to them, we show that due to the non-stationary population only one steady state is optimal. In addition our framework is stochastic. To solve the stochastic non-linear system of difference equations we apply a the method of eigenvalue-eigenvector decomposition based on Sims (2000) and Novales et. al (1999).

2 The Model

We consider an economy which is populated by continuum of two-period overlapping generations. Each individuum lives two periods, childhood and adulthood and derives utility out of consumption c and the number of children n. During childhood $(t-1)$ he or she is equipped with education e and inherited physical capital k. When adult (t) he/she takes all the economic relevant decisions concerning savings, and the amount of time spend for child rearing zn, production u and education $1-u-zn$. The time budget is normalized to one and z represents the time necessary to raise one child.
Individuums are heterogenous in their abilities to accumulate human capital/educate their children and to produce the output. When born, each person inherits the abilities of his/her parents θ_{t-1} which is influenced by a normal and independent distributed shock ϵ with mean zero and standard deviation of σ_ϵ^2. Hence the ability shock follows an AR(1)-process

$$\ln\theta_t = \rho\ln\theta_{t-1} + \epsilon_t. \tag{1}$$

Each generation is altruistic and derives utility out of the discounted period utility of future generations. Consequently the maximization problem of each dynastic head can be described as follows

$$\max_{\{c_t;n_t;u_t;k_{t+1};e_t\}_{t=0}^{\infty}} \quad E_0\sum_{t=0}^{\infty}\beta^t(\ln c_t + bn_t^{\mu}) \tag{2}$$

subject to:

$$n_tk_{t+1} = \theta_{t-1}k_t^{\alpha}(u_te_{t-1})^{1-\alpha} + k_t + c_t \tag{3}$$
$$n_te_t = (1+\theta_tA(1-u_t-zn_t))e_{t-1} \tag{4}$$
$$\ln\theta_t = \rho\ln\theta_{t-1} + \epsilon_t \tag{5}$$
$$\epsilon \sim N(0,\sigma_\epsilon^2) \tag{6}$$

Forming the Lagrangian and eliminating the Lagrange multiplier leads to the following optimality conditions

$$b\mu = n_t^{1-\mu}\frac{1}{c_t}(zw_te_{t-1} + p_te_t + k_{t+1}) \tag{7}$$

$$w_t = p_t\theta_tA \tag{8}$$

$$c_t^{(-1)}n_t = \beta E_t\{c_{t+1}^{(-1)}(1 + r_{t+1})\} \tag{9}$$

$$\begin{aligned}\frac{n_t^{\mu-1}}{p_tzw_te_{t-1} + p_te_t + k_{t+1}}p_tn_t &= \beta E_t\left\{\frac{n_{t+1}^{\mu-1}}{p_{t+1}zw_{t+1}e_t + p_{t+1}e_{t+1} + k_{t+2}}\right. \\ &\quad p_{t+1}(1 + \theta_{t+1}A(1 - zn_{t+1}))\}\end{aligned} \tag{10}$$

$$E_t\{1 + r_{t+1}\} = E_t\left\{\frac{w_{t+1}}{w_t}\frac{\theta_t}{\theta_{t+1}}(1 + \theta_{t+1}A(1 - zn_{t+1}))\right\} \tag{11}$$

Equations (7) and (8) are the common static optimality conditions, euqalizing the marginal utilities and wages. Equations (9) and (10) are the Euler equations or optimality conditions for investments in physical and human capital. Consequently they can be transformed to an arbitrage condition, equalizing the expected revenue out of physical capital to human capital.

Defining each expectation as W_t^1 and W_t^2 such that

$$W_t^1 \equiv E_t\left\{\left(\frac{c_{t+1}}{e_t}\right)^{(-1)}\left(\frac{e_t}{e_{t-1}}\right)^{(-1)}(1 + r_{t+1})\right\} \tag{12}$$

$$W_t^2 \equiv E_t\left\{\left(\frac{c_{t+1}}{e_t}\right)^{(-1)}\left(\frac{c_t}{e_t}\right)^{(-1)}\left(\frac{e_t}{e_{t-1}}\right)^{(-1)}p_{t+1}(1 + \theta_{t+1}A(1 - zn_{t+1}))\right\} \tag{13}$$

leads to the associated expectation errors

$$W_{t-1}^1 = \left(\frac{c_t}{e_{t-1}}\right)^{(-1)}\left(\frac{e_{t-1}}{e_{t-2}}\right)^{(-1)}(1 + r_t) - \eta_t^1 \tag{14}$$

$$W_{t-1}^2 = \left(\frac{c_t}{e_{t-1}}\right)^{(-1)}\left(\frac{c_{t-1}}{e_{t-1}}\right)^{(-1)}\left(\frac{e_{t-1}}{e_{t-2}}\right)^{(-1)}p_t(1 + \theta_tA(1 - zn_t)) - \eta_t^2 \tag{15}$$

Definition: *A steady state is characterized by an equal and constant growth of the endogenous variables k, c, e, y. The steady state growth rate v^* equals the accumulation rate of human capital $(1 + A(1 - u^* - zn^*))/n^*$. Reallocations across sectors are closed. Hence, u^*, w^*, p^* and n^* are constant $\forall\, t$.*

In the long-run k, c, e are growing at a constant rate v^*. In order to obtain stationary solutions it is reasonable to form relations. We introduce the following relations

$$\chi_t \equiv \frac{k_t}{e_{t-1}}, \tag{16}$$

$$\varphi_t \equiv \frac{c_t}{e_{t-1}}, \tag{17}$$

$$\text{with} \qquad \left(\frac{\chi_{t+1}}{\chi_t}\right)^* = \left(\frac{\varphi_{t+1}}{\varphi_t}\right)^* = 1 \qquad \forall \quad t.$$

Consequently, for any steady state the following conditions are to hold

$$A(1 - zn^*) = aB{\chi^*}^{a-1}{u^*}^{1-a}, \tag{18}$$

$$v^* = \left(\frac{e_t}{e_{t-1}}\right)^* = \left(\frac{c_{t+1}}{c_t}\right)^* = (1 + A(1 - u^* - zn^*)) = \beta(1 + A(1 - zn^*)), \tag{19}$$

$$p^* = \frac{w^*}{A}, \tag{20}$$

$$\varphi^* = B{\chi^*}^{a}{u^*}^{1-a} + \chi^*(1 - n^* v^*), \tag{21}$$

$$\psi^* \equiv -\mu b + {n^*}^{1-\mu}\left[\frac{zw^*}{\varphi^*} + \frac{p^* v^*}{\varphi^*} + \frac{\chi^* v^*}{\varphi^*}\right] = 0. \tag{22}$$

Eliminating φ^*, χ^*, u^* in (22) leads to a function $\psi^* = \psi(n^*)$. Finding the zeros of ψ^* and substituting for n^*, in φ^*, χ^*, u^* yields the steady state solutions.
As it becomes apparent from Fig. 1 the economy exhibits three steady states.

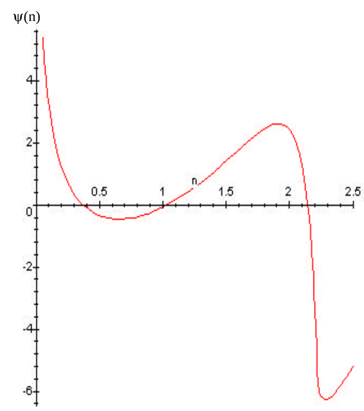

Figure 1:

Steady State I

$$\begin{aligned}
\varphi^* &= 0.027471 \\
\chi^* &= 0.019230 \\
u^* &= 0.119926 \\
n^* &= 0.372407 \\
1 - u^* - zn^* &= .712490 \\
v^* &= 5.555036 \\
W^{1^*} &= 14.735119 \\
W^{2^*} &= 0.0810472
\end{aligned}$$

Steady State II

$$\begin{aligned}
\varphi^* &= 0.035344 \\
\chi^* &= 0.037356 \\
u^* &= 0.096105 \\
n^* &= 1.034117 \\
1 - u^* - zn^* &= 0.438542 \\
v^* &= 1.603118 \\
W^{1^*} &= 31.802596 \\
W^{2^*} &= 0.350399
\end{aligned}$$

Steady State III

$$\begin{aligned} u^* &= 0.034519 \\ n^* &= 2.145513 \\ v^* &< 1 \end{aligned}$$

As expected the three steady states are characterized by an inverse relationship between fertility and education per child and hence growth per capita v^*.
Furthermore, steady state III is a corner solution in the sense that time is only allocated to production and child rearing. In such an economy the accumulation rate of human capital is zero. Given that $n^* > 0$, the initial stock of human capital per capita e_0 is diluted over time and $v^* < 1$. As a consequence, such economies will converge to zero output and steady state III is not maximizing the objective function.
Similar, steady state I is not sustainable in a demographic sense, despite that time is allocated to the three sectors. The long-run fertility falls much below the replacement level. Hence, population is converging to zero and output is converging to zero. Consequently, steady state I cannot maximize the utility of any dynasty, neither.
Therefore, the only feasible steady state, satisfying the first order conditions is steady state II.

3 The Solution Method

The solution method applied here is based on Sims (1998) and Novales et al. (1999). The general problem of solving stochastic linear rational expectation models can be described as follows

$$\Gamma_o y_t = \Gamma_1 y_{t-1} + C + \Psi z_t + \Pi \eta_t, \tag{23}$$

where C is a vector of constants, y is a vector of endogenous variables, z_t is a vector of exogenous variables, for example an exogenous stochastic shock, and η_t represents a vector of rational expectation errors, with $E[\eta] = 0$.
Obviously, the problem in the present context is, (contrary, to the general problem (23)) highly non-linear. Despite that fact the first step of the solution adds a stability condition to the system (23), in order to avoid that unstable paths violate the transversality conditions. This goal is achieved by linearizing the non-linear system around its steady state. After adding the stability condition(s) to the nonlinear system, the solutions of

the endogenous variables are obtained out of the original non-linear system. The gain of this method is that a high degree of non-linearity of the system is maintained and the only source of some numerical errors is introduced by stability conditions. Such errors, due to approximations are absorbed by the expectations errors. As the underlying system is not linearized and the endogenous variables are computed for each period the introduced error is relatively small, because approximation errors do not cumulate over time. Clearly, accuracy of the solutions requires more computational effort. The latter is an important issue, if the errors produced by less demanding methods do cumulate and translate into other endogenous variables. As a consequence, an economy undergoing an instantaneous shock would not converge to the same initial steady state and deviate from rationality.

4 The Stability Conditions

The economy is described by the following system derived out of the first order conditions:

$$\psi_t = -\psi_{t+1}(1+\theta_t A(1-u_t-zn_t)) + \theta_{t-1}B\chi_t^{\alpha}u_t^{1-\alpha} + \chi_t, \tag{24}$$

$$b\mu = n_t^{1-\mu}\frac{1}{c_t}(zw_t e_{t-1} + p_t e_t + k_{t+1}) \tag{25}$$

$$\varphi_t^{-1} = \frac{\beta}{n_t}W_t^1 \tag{26}$$

$$w_t = \frac{\beta}{n_t}W_t^2 \tag{27}$$

$$W_{t-1}^1 = \left(\frac{c_t}{e_{t-1}}\right)^{(-1)}\left(\frac{e_{t-1}}{e_{t-2}}\right)^{(-1)}(1+r_t) - \eta_t^1 \tag{28}$$

$$W_{t-1}^2 = \left(\frac{c_t}{e_{t-1}}\right)^{(-1)}\left(\frac{c_{t-1}}{e_{t-1}}\right)^{(-1)}\left(\frac{e_{t-1}}{e_{t-2}}\right)^{(-1)} p_t(1+\theta_t A(1-zn_t)) - \eta_t^2 \tag{29}$$

$$\ln\theta_t = \rho\ln\theta_{t-1} + \epsilon_t. \tag{30}$$

Hence, each equation can be expressed as a function f, such that

$$f(\varphi_t, \chi_{t+1}, u_t, n_t, W_t^1, W_t^2, \eta_t^1, \eta_t^2, \epsilon_t, \ln(\theta_t)) = 0. \tag{31}$$

Defining the vectors y_t and y_{t-1} which contain the deviations from the steady state, the vector of expectations errors η_t, and the (1×1) $z-$ vector, yields

$$y_t = (\varphi_t - \varphi^*, \chi_{t+1} - \chi^*, u_t - u^*, n_t - n^*, W_t^1 - W^{1^*}, W_t^2 - W^{2^*}, \ln(\theta_t))', \quad (32)$$

$$\eta_t = (\eta_t^1, \eta_t^2)' \quad (33)$$

$$z_t = (\epsilon_t)'. \quad (34)$$

Developing a first-order approximation around the steady state leads to:

$$\left.\frac{\partial f}{\partial y_t}\right|_* y_t + \left.\frac{\partial f}{\partial y_{t-1}}\right|_* y_{t-1} + \left.\frac{\partial f}{\partial \eta_t}\right|_* \eta_t. + \left.\frac{\partial f}{\partial \epsilon_t}\right|_* \epsilon_t = 0, \quad (35)$$

$$\text{with} \qquad \eta^{1^*} = \eta^{2^*} = \epsilon^* = C = 0 \quad (36)$$

the linearized problem can be written as

$$\Gamma_o y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t. \quad (37)$$

Given that Γ_0 is not singular and hence invertible multiplying the system from the left side with Γ_0^{-1} and defining

$$\begin{aligned} \tilde{\Gamma_1} &\equiv \Gamma_0^{-1}\Gamma_1, \\ \tilde{\Psi} &\equiv \Gamma_0^{-1}\Psi, \\ \tilde{\Pi} &\equiv \Gamma_0^{-1}\Pi, \end{aligned}$$

leads to

$$y_t = \tilde{\Gamma_1} y_{t-1} + \tilde{\Psi} z_t + \tilde{\Pi}\eta_t. \quad (38)$$

By Jordan decomposition of $\tilde{\Gamma_1}$ we receive $\tilde{\Gamma_1} = P\Lambda P^{-1}$ and through multiplying by P^{-1} and defining $\varpi_t = P^{-1} y_t$

$$\varpi_t = \Lambda w_{t-1} + P^{-1}(\tilde{\Psi} z_t + \tilde{\Pi}\eta_t). \quad (39)$$

The stability conditions are obtained by imposing orthogonality between each eigenvector associated with an unstable eigenvalue and the vector of variables in it. A general restriction can be approximated by an upper bound ϕ on the growth rate of a linear combination νy_t of the variables of the model

$$\lim_{s\to\infty} E_t[\nu y_t \phi^{-s}] = 0 \tag{40}$$

$$(\nu P) \lim_{s\to\infty} E_t[\varpi_{t+s}\phi^{-s}] = 0 \tag{41}$$

$$(\nu P) \lim_{s\to\infty} E_t[\Lambda^s \varpi_t + P^{-1}(\tilde{\Psi} z_t + \tilde{\Pi}\eta_t)\phi^{-s}] = 0 \tag{42}$$

$$(\nu P)\left\{ \lim_{s\to\infty} E_t[\Lambda^s \varpi_t \phi^{-s}] + \underbrace{\lim_{s\to\infty} E_t[P^{-1}(\tilde{\Psi} z_t + \tilde{\Pi}\eta_t)\phi^{-s}]}_{0} \right\} = 0 \tag{43}$$

$$(\nu P) \lim_{s\to\infty} E_t[\Lambda^s \varpi_t \phi^{-s}] = 0. \tag{44}$$

From the last equation it becomes apparent that each of the $\varpi_{j,t}$ elements of ϖ_t corresponding to an unstable eigenvalue, such that $|\lambda_{jj}| > \phi$ and $\nu P \neq 0$ must be equal to zero for all t

$$\varpi_{jt} = P^{j\bullet} y_t = 0 \qquad \forall \quad t. \tag{45}$$

The usual imposed upper bound is that the product out of shadow prices and state variables cannot grow faster than β^{-1}, so that both shadow prices and state variables cannot grow faster than $\beta^{-\frac{1}{2}}$.

5 Simulating the Model

This section serves to explore the dynamic behavior of the economy under a given parameter set

$\alpha = 0.5\ A = 1.5;\ B = 1;\ \mu = 0.2;\ b = 77.5;\ z = 0.45.\rho = 0.95; \sigma_\epsilon = 0.005$

We will restrict our simulation to Steady State I and II. Steady State III is characterized by a steady state growth rate smaller than one and is highly unstable. Staying in this steady state is not optimal and leaving this steady state would either lead to a convergence to Steady State II or permanent divergence from all steady states. Obviously, a social planer would prefer the former option. As mentioned above, Steady State I is not sustainable in the demographic sense and would violate the transversality conditions, but contrary to Steady State III it will be characterized by the same stability propertiers as steady state I.

5.1 Steady State I

Applying the solution method described above, we receive the following matrices of eigenvalues Λ and and left eigenvectors P^{-1}

$$P^{I^{-1}} = \tag{46}$$

$$\begin{vmatrix} 1 & 14.3732 & -4.106 & 4.1871 & -0.0119 & -7.4701 & -0.1432 \\ 0 & 26.0842 & -0.2169 & -0.0976 & 0.0114 & -0.7027 & 0.3883 \\ 0 & -50.1807 - 39.0195i & 10.6908 + 11.4196i & 4.8109 + 5.1388i & -0.4755 - 0.0082i & 18.7063 - 70.8734i & 2.4835 - 1.1388i \\ 0 & -50.1807 + 39.0195i & 10.6908 - 11.4196i & 4.8109 - 5.1388i & -0.4755 + 0.0082i & 18.7063 + 70.8734i & 2.4835 + 1.1388i \\ 0 & -13.6545 & 3.9023 & -4.4235 & 0.0132 & 7.1207 & 0.1369 \\ 0 & 76.3919 & -21.6905 & -8.9556 & -0.0621 & -37.6713 & -0.6939 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4.7816 \end{vmatrix}$$

$$\Lambda^I = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.7259 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.2687 + 0.2393i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2687 - 0.2393i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.95 \end{vmatrix}. \tag{47}$$

Obviously, the system is generating real and complex eigenvalues. Whereas the complex eigenvalues are within the unit circle instability arises by one real eigenvalue. Hence, one stability condition is given by the second row of $P^{I^{-1}}$

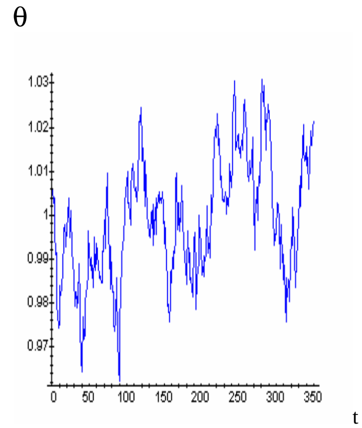

Figure 2: *Ability shock θ*

$$\varpi_{2,t} = P^{2\bullet} y_t = 0 \quad \forall \qquad t \tag{48}$$

$$26.0898(\chi_{t+1} - \chi^*) - 0.2169(u_t - u^*) - 0.0976(n_t - n^*) \\ +0.0114(W_t^1 - W^{1^*}) - 0.7027(W_t^2 - W^{2^*}) + 0.3883 \ln \theta_t = 0. \tag{49}$$

A second stability condition needed for the solution is generated out of zero eigenvalue, which mean that this condition is redundant and not adding any further restriction

$$\varpi_{6,t} = P^{6\bullet} y_t = 0 \quad \forall \qquad t \tag{50}$$

$$76.3919(\chi_{t+1} - \chi^*) - 21.6905(u_t - u^*) - 8.9556(n_t - n^*) \\ -0.0621(W_t^1 - W^{1^*}) - 37.6713(W_t^2 - W^{2^*}) - 0.6939 \ln \theta_t = 0. \tag{51}$$

In Figure 2 the sample realization of θ is shown, which is assumed to be the same for all the simulations undertaken, here. Obviously, all the endogenous variables are oscillating around their steady state values. As expected, current innate ability and wage

rate are positively correlated and both are negatively correlated to fertility. Increasing opportunity costs lead to a lower fertility. The time fraction u allocated to production **and** the time fraction allocated to education $(1 - u - zn)$ are also positively correlated to the ability shock (see Figures 7 and 8). Obviously, the considered stochastic dynamics of the model generates a quality-quantity trade-off, in the sense that high abilities are generating a high wage rate, high opportunity costs of child bearing and hence a low fertility traded against higher education per child. The latter increases in conjunction with the persistence of the inherited ability the opportunity costs of child rearing for the next generations even more. Leading to an intergenerational persistence in those variables (n, w, e) over some periods, until the ability shock may reverse the dynamics.
The expectations errors η^1 and η^2 are oscillating around the zero line and rational expectations are full filled (see Figures 10 and 11), such that $E[\eta 1] = E[\eta 2] = 0$.

5.2 Steady State II

Similar to the above described scenario we reach to the following matrices and stability conditions

$$P^{II^{-1}} = \tag{52}$$

$$\begin{vmatrix} 1 & -0.2467 & 0.1652 & -0.4274 & 0.0014 & 0.0729 & 0.01 \\ 0 & 41.6338 & -1.7360 & -0.7812 & 0.065 & -0.2332 & 0.8016 \\ 0 & 40.1605 + 3.4177i & 13.2252 - 10.1408i & -5.9513 - 4.5634i & +0.5227 + 0.0073i & -4.3006 + 32.4106i & -3.1577 + 0.2908i \\ 0 & 40.1605 - 3.4177i & -13.2252 + 10.1408i & -5.9513 + 4.5634i & +0.5227 - 0.0073i & -4.3006 - 32.4106i & -3.1577 - 0.2908i \\ 0 & -1.6740 & 1.1549 & -4.6850 & 0.0031 & 0.5804 & 0.0780 \\ 0 & 37.1256 & -23.6064 & -15.6800 & -0.0166 & -7.8085 & -1.1139 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6.0110 \end{vmatrix}$$

$$\Lambda^{II} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.5522 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.7749 + 0.5632i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -07749 - 0.5632i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.95 \end{vmatrix} \tag{53}$$

$$\varpi_{2,t} = P^{2\bullet} y_t = 0 \quad \forall \qquad t \tag{54}$$

$$\begin{aligned} &41.6338(\chi_{t+1} - \chi^*) - 1.7360(u_t - u^*) - 0.7812(n_t - n^*) \\ &+0.0650(W_t^1 - W^{1^*}) - 0.2332(W_t^2 - W^{2^*}) + 0.8016 \ln \theta_t = 0. \end{aligned} \tag{55}$$

$$\varpi_{6,t} = P^{6\bullet} y_t = 0 \quad \forall \qquad t \tag{56}$$

$$\begin{aligned} &37.1256(\chi_{t+1} - \chi^*) - 23.6064(u_t - u^*) - 15.6800(n_t - n^*) \\ &-0.0166(W_t^1 - W^{1^*}) - 7.8085(W_t^2 - W^{2^*}) - 1.1139 \ln \theta_t = 0. \end{aligned} \tag{57}$$

As can be verified easily from the Figures 2, 12 and 13 fertility, ability and wages are again negatively correlated. But and especially therefore, the question arises what makes this steady state different from the other ones ?

The key to the answer seems to lie in the way how time is allocated to the three activities child rearing, education and production. From considering Figures 16 and 17 it becomes apparent that contrary to Steady State I, u and $(1 - u - zn)$ are not negatively correlated. High innate abilities lead to a high labor supply, but individuums increase only $(1-u-zn)$, moderatly. Consequently opportunity costs are not increased so dramatically as in Scenario I. This allows the economy to sustain a long-run fertility above the replacement level accompanied by a moderate growth rate. The ability effect is internalized into the decision to educate children, that allows for fertility above the replacement level. The opposite was true in the former scenario, so that opportunity costs for child rearing increased even more in periods of high ability shocks.

6 Concluding Remarks

In a dynastic endogenous growth setting with endogenous fertility we generated multiple equilibria. Contrary to the existing literature with endogenous labor supply we have shown that only one steady state is optimal. Zero output in education leads to a negative growth rate, because the existing stock of human capital has to be distributed over a growing population. A high growth equilibrium with a fertility rate below the replacement level leads to zero population. Consequently, the only remaining outcome is a steady state with moderate investments in education per child and a fertility above the replacement level.

Responsible for the described outcome in the later scenarios was the way in which time was allocated to child rearing, education, and production. High ability shocks lead in both scenarios to a positive impact on labor supply. The fast growth scenario instead was characterized by an increase in the time allocated to production and to education, whereas the sustainable growth scenario is characterized by a only moderate increase in time allocated to education. This is exactly the underlying key mechanism which weakens the quality-quantity trade off and allows for a fertility above the replacement level and moderate growth rates per capita, in the long-run.

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7 Appendix

7.1 Simulation Results for Scenario I

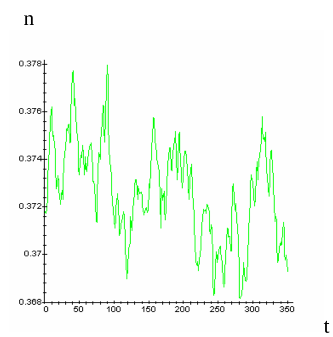

Figure 3: *Behavior of the fertility in scenario I*

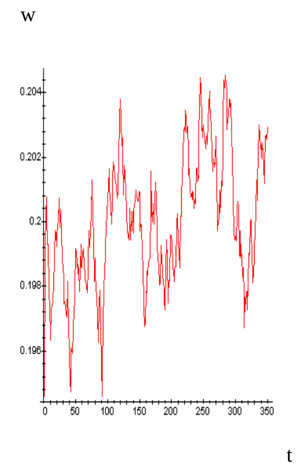

Figure 4: *Behavior of the wage rate in scenario I*

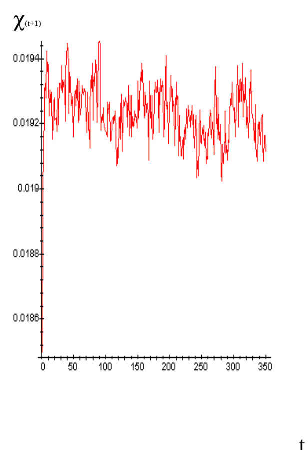

Figure 5: *Behavior of the capital/human capital relation in scenario I*

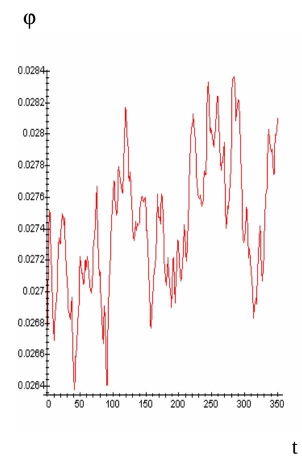

Figure 6: *Behavior of the consum/human capital relation in scenario I*

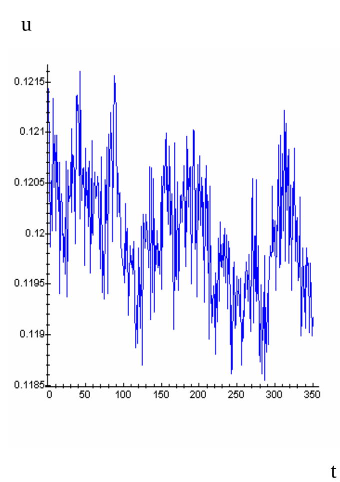

Figure 7: *Behavior of the time fraction u allocated to production in scenario I*

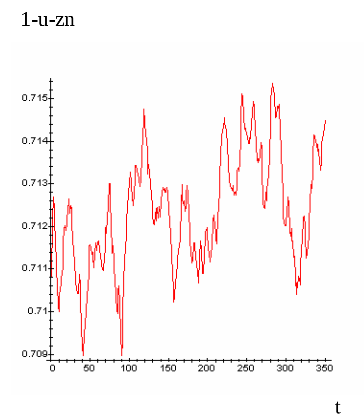

Figure 8: *Behavior of the time fraction* $1 - u - zn$ *allocated to education in scenario I*

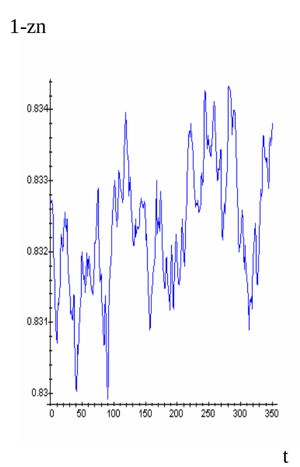

Figure 9: *Behavior of the labor supply in scenario I*

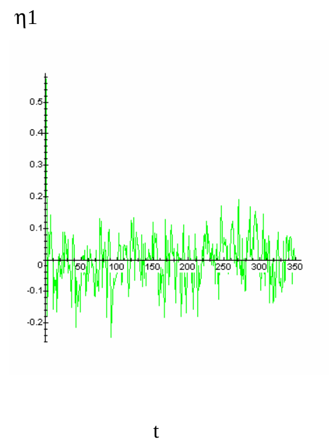

Figure 10: *Behavior of the expectation error η^1 in scenario I*

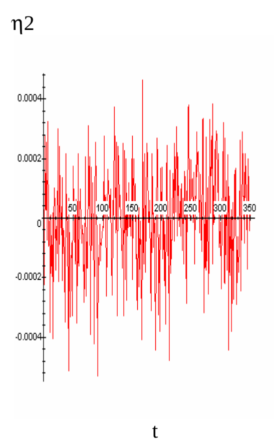

Figure 11: *Behavior of the expectation error η^2 in scenario I*

7.2 Simulation Results for Scenario II

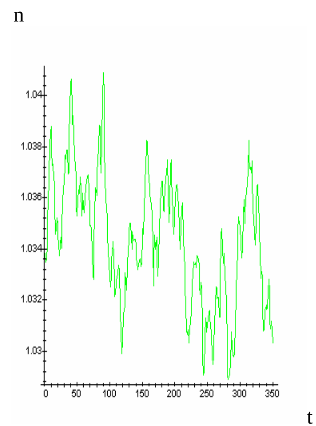

Figure 12: *Behavior of the fertility in scenario II*

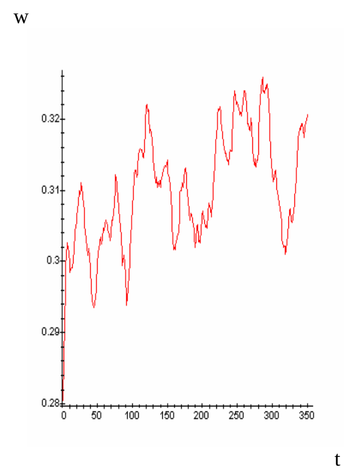

Figure 13: *Behavior of the wage rate in scenario II*

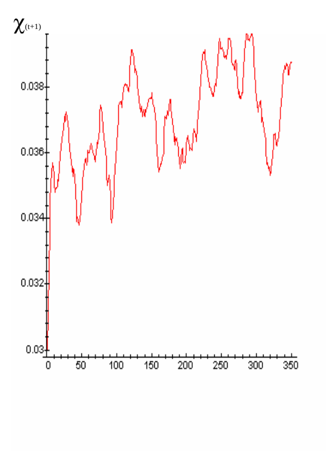

Figure 14: *Behavior of the capital/human capital relation in scenario II*

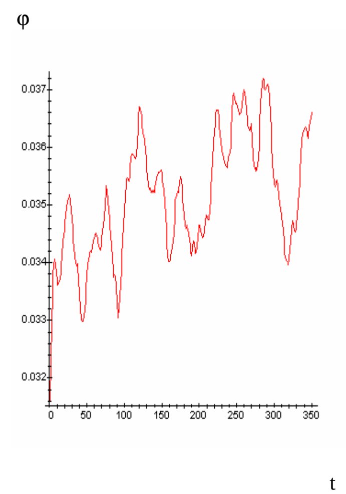

Figure 15: *Behavior of the consum/human capital relation in scenario II*

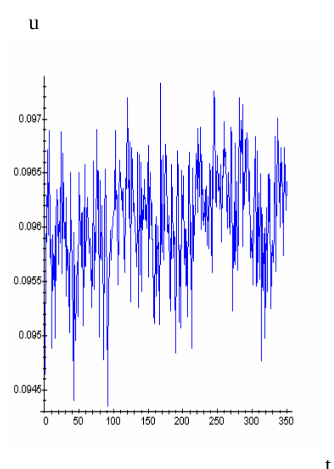

Figure 16: *Behavior of the time fraction u allocated to production in scenario II*

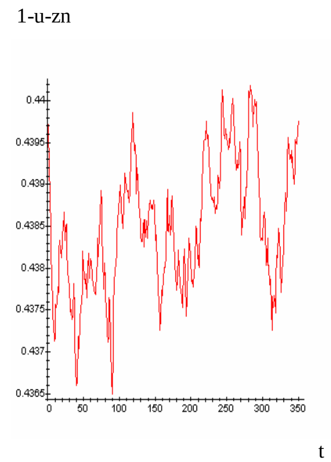

Figure 17: *Behavior of the time fraction* $1 - u - zn$ *allocated to education in scenario II*

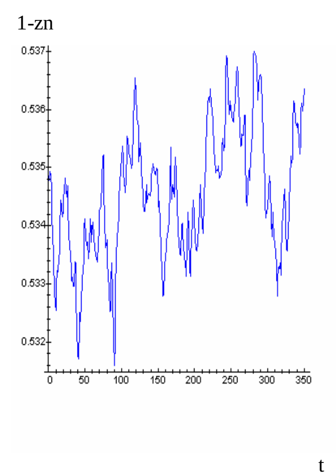

Figure 18: *Behavior of the labor supply in scenario II*

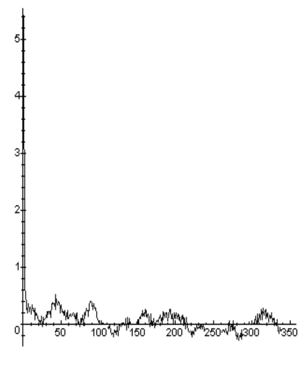

Figure 19: *Behavior of the expectation error* η^1 *in scenario II*

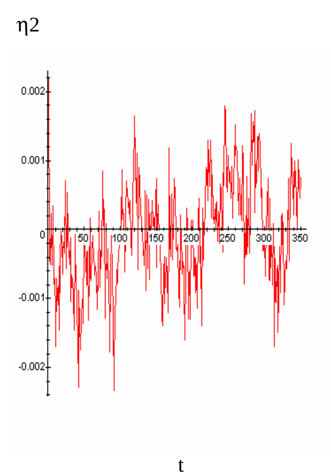

Figure 20: *Behavior of the expectation error* η^2 *in scenario II*