Complex Dynamics in a Simple Model of Economic Specialization*

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1 Introduction

As is well known, one of the main insights of Adam Smith's *Wealth of Nations* was that the most important engine of economic growth is the division of labor. In particular, Smith argued that the division of labor is limited by the extent of the market. Young (1928) made a significant advance by reformulating this statement.

Young claimed that the division of labor is actually limited by the division of labor. He pointed out that, by specializing, an individual agent increases the supply of a certain commodity, but at the same time s/he increases the demand for other commodities. Specialization allows an agent to obtain a surplus of a specific good above her/his consumption: hence s/he can trade the surplus for other specialized agents' surpluses. In other words, the specialization of some agents generates an increase in the extent of the

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market for other agents and can therefore cause their specialization, which in turn may stimulate the specialization of others, etc. Hence, following Young, the dynamics of specialization keeps the economic system in a state of constant change, as the productive activity is continuously reorganized. This mechanism generates economic growth which can therefore be defined as a disequilibrium process.

Starting from the insights of Smith and Young, in this paper we suggest to represent the process of specialization and growth by a network in which nodes are agents and links are potential trade relationships. The network may *evolve* as the state of the network's nodes changes, and new trade relationships are created. That is, agents who are initially nonspecialized may change their state by specializing in the production of one particular good, and establish trading relationships with other agents.

We study the process of economic growth as the diffusion of specialization within an economy, represented by a simple network structure. We consider the Youngian insight that the division of labor is limited by the division of labor, and show that the process may indeed involve a complex network dynamics.

We consider three main variables: the dimension of the neighborhood of each agent, a threshold value for the extent of the market necessary to specialize, and a threshold value for the extent of the market necessary to remain specialized, the latter being related to competition from other specialized agents which may cause de-specialization.

We show that: the diffusion of specialization and economic growth generally increase with the an increase of the dimension of the neighborhood, with a reduction of the extent of the market necessary for specialization and with a reduction of the extent of the market necessary to remain specialized, or with a reduction of the intensity of competition. However, the same parameters affect the *qualitative* features of the dynamics: the network configuration may not settle to a steady state and remain in a condition of constant change.

We also discuss some aspects related to the organization of the economic activity, that is the localization of specialized and nonspecialized agents in the initial conditions of the dynamical system and the activation rules, which represent the sequence in which agents in the model make decisions.

The paper is organized as follows: Section 2 describes the historical background for the model proposed here; Section 3 introduces the model; Section 4 reports the results of the simulations; Section 5 provides a discussion of the results; Section 6 contains some concluding remarks.

2 Historical Background

The theory of growth and division of labor takes a center stage in economics with the publication of Adam Smith's *Wealth of Nations* in 1776.[1] In Smith's view the principal factor affecting growth of per capita income is labor productivity, which increases with the division of labor.

For Smith there exists an important relation between the structure of social and economic interaction, the extent of the market, and the viability of the process of growth. According to him, the important precondition for the development of the division of labor in a society is "a certain propensity in human nature: ... the propensity to truck, barter and exchange one thing for another" (WN I.i.1).

This allows individuals to specialize and benefit from trading their surplus products, that is the production in excess of their own consumption. Such surpluses originate from the increased productivity due to the specialization. In Smith's opinion, individuals choose to specialize, given a structure of social interactions, on the basis of a process of learning. When an individual learns that he enjoys higher consumption levels by exchanging a part of his production, i.e. he has the "certainty of being able to exchange all that surplus part of the produce of his own labour, ..., for such parts of the produce of other men's labour as he may have occasion for", he is encouraged "to apply himself to a particular occupation" (WN I.ii.3).

In particular, economic growth may be spurred even by the creation of a network of similar individuals, that is individuals not endowed with innate or acquired talents, specialized skills, etc. Smith in fact maintains that individuals are similar at birth. The differences "which [appear] to distinguish men of different professions, when grown up to maturity, is not upon many occasions so much the cause, as the effect of the division of labour" (WN I.ii.4). Once connected, individuals can sort themselves out in different occupations, and increase their aggregate production and consumption. The consequential step in Smith's analysis consists in the generalization of the relevant aspect of this reasoning: namely, that the extent of the market limits the division of labor.

Smith's argument is the following: as noted, an agent has the incentive to specialize if he possesses "power of exchanging" the surplus, i.e. if sufficient demand exists, allowing the agent to trade part of his surplus product in exchange of other goods. In particular, as will be emphasized by Young, coordinated specialization of different productive units generates contemporaneous increases in the supply of commodities, since specialization increases

[1] This section draws on Lavezzi (2003).

productivity, and in the demand for other commodities, since specialization implies demanding the goods whose production is given up. The increased supply may provide the necessary means to support the exchange. From this discussion the social dimension of the division of labor and economic progress emerges clearly, as they involve important organizational aspects of economic activities, like the degree of coordination and communication among agents.

It can be argued that Smith anticipated the modern theory of endogenous growth. However, important developments on the *nature* of this process were made by Allyn Young (1928). As noted, Young emphasized some aspects of Smith's argument and specifically reformulated Smith's theory in these terms: the division of labor is limited by the division of labor. This amounts to recognizing that the extent of the market is at least partially endogenous, and that therefore an increase in the extent of the market is not only to be understood as removal to barriers to free trade, construction of roads, railways, etc.

The important implication is that:

> "the counter forces which are continually defeating the forces which make for economic equilibrium are more pervasive and more deeply rooted in the constitution of the modern economic system than we commonly realise. Not only new or adventitious elements, coming in from the outside, but elements which are *permanent characteristics of the ways in which goods are produced* make continuously for change. Every important advance in the organisation of production ... alters the conditions of industrial activity and initiates responses elsewhere in the industrial structure which in turn have a further unsettling effect. Thus change becomes progressive and propagates itself in a cumulative way." (Young (1928), p. 531. Italics added).

Hence for Young, not only economic growth is endogenous, but the endogenous forces generate disequilibrium in the sense that, in the growth process, the structure of the economy and the technological opportunities cannot a priori be considered fixed.[2] Hence, for Young an understanding of the growth process requires to see the economy in its "togetherness"(Young (1999a), p. 45), in particular as a large interactive system made of specialized, interdependent productive units. Young in fact discussed also the process of industrial differentiation which characterizes economic growth. In

[2]This point relates to the strong criticism made by Young to the equilibrium approach advocated by Marshall and the Marshallians.

the economic network, changes originating at a local level, may initiate "responses elsewhere", that is the economy may be characterized by productive and technological feedbacks which propagate across the nodes of the network. Economic growth is associated to an evolution of this system and proceeds as a disequilibrium, or complex in modern terms, process.

3 Economic Specialization in a Network

To take into account Smith and Young's insights, we consider a simplified economy in which agents are located on a one-dimensional circular lattice. We assume that there are two goods, denoted as good 1 and good 2. Agents's utility is a function of consumption of both goods. Agents can either produce two goods in a nonspecialized fashion, or specialize in the production of one good. If the agent is nonspecialized, then s/he may produce small quantities of both goods, if specialized s/he can produce a large quantity of a single good.

Hence, agents can be in one of three possible states: *state 0*: the agent is nonspecialized and produces both goods; *state 1*: the agent is specialized in the production of good 1; *state 2*: the agent is specialized in the production of good 2.

Every agent i interacts with a certain number of other agents in a neighborhood denoted as N_i. The dimension of the neighborhood is given by D, and includes i.[3] An agent decides to become or remain specialized if there is sufficient demand in her/his neighborhood. However, to capture Young's insight, we posit that demand for a certain good comes mostly from specialized producers of the other good, as specialized production generates a large surplus available for trade. Nonspecialized agents can exert only a low demand for both goods, as their production of both goods is low.

In addition, we assume the existence of counter-forces to the process leading to specialization. Namely, agents de-specialize if there is insufficient demand in their neighborhood, in particular if there are "too many" agents of the same type in their neighborhood. We broadly define this effect as the result of competition, which causes the "elimination" of some specialized agents, who then become nonspecialized.[4]

In every period t one agent is "active" and has the possibility to make

[3]This means that the radius of the neighborhood is $(D-1)/2$: every agent is connected to $(D-1)/2$ agents on the right and $(D-1)/2$ agents on the left.

[4]There is a similarity with the death of agents for overpopulation in Conway's classic game of life. For simplicity we assume that agents specialized in the production of one good cannot specialize in one period in the production of the other good.

a transition across states. The transition rules which govern the change of state are of the following type:

$x_i(t+1) = F\{x_i(t), D(N_i(t))\}$,

where $x_i(t)$ is the state of agent i in period t, $F\{.\}$ is the transition function and $D(N_i(t))$ is a vector indicating the demand for both goods expressed by agents in the neighborhood of i in period t. Thus, this is a "totalistic" rule (see, e.g. Wolfram (2002)[p. 40]): the agent considers an aggregate indicator of the neighborhood, and is not interested in the exact location of the agents in N_i.

$D(N_i)$ depends on a set of parameters: $d_{10} > 0$ and $d_{20} > 0$, representing the demand of good 1 and 2 coming from 0-agents; $d_{21} > 0$ and $d_{12} > 0$ respectively referring to the demand of good 2 from 1-agents and for good 1 from 2-agents; finally, $d_{11} < 0$ and $d_{22} < 0$, representing demand for good 1 from 1-agents and for good 2 from 2-agents. The latter parameters take on negative values as specialized agents evaluate the presence of agents of the same type in their neighborhood as negative potential demand.[5]

We define two threshold values for demand related to the decision to specialize and to despecialize. To capture the role of the extent of the market on the decision to specialize, we introduce $t_{01} > 0$ and $t_{02} > 0$ which, respectively, denote the threshold levels of demand for good 1 and good 2 which may induce a 0-agent to specialize. The decision to despecialize instead requires a demand above $t_{11} > 0$ and $t_{22} > 0$, which respectively represent the threshold levels of demand for good 1 or good 2 in the neighborhood of a 1-agent or a 2-agent, that causes the decision to remain specialized. In other words, if the demand for good 1 (good 2) is below t_{11} (t_{22}), then a 1-agent (2-agent) becomes a 0-agent. Even if both thresholds are referred to production decisions on the part of the agents, we keep them distinct as we want to include in the decision to remain specialized the effect of the competition of other agents, which we represent as negative demand. [6]

Therefore, each agent evaluates her/his neighborhood in terms of the production and consumption possibilities, represented by the vector of potential demand for both goods, and chooses a state.[7]

[5]A fully specified model would feature the consideration on the part of the active specialized agent of the perspective losses of trade possibilities due to the proximity to other agents of the same type. Here this aspect is left on the background, and it is synthesized as negative demand.

[6]For a given number of identical agents in N_i, a low value of t_{11} and t_{22} implies that the active agent can remain specialized more easily, and therefore we consider low values of t_{11} and t_{22} as "weak competition".

[7]In the simulations in the next section, we assume that if the demand for both goods is above the threshold, than the active 0-agent chooses to specialize in the production of the good with the higher demand.

4 Simulations

With the simple rules defined in the previous section, it is possible to simulate the dynamics of this economy. Our aim is to study the characteristics of the growth process, namely whether the system is led toward complete or incomplete specialization, and whether growth occurs with or without fluctuations. As it will be clear, the conditions for economic growth are to be found in the interaction among the critical parameters: the dimension of the neighborhood D, the threshold levels for specialization t_{01} and t_{02}, and the threshold levels for despecialization t_{11} and t_{22}. The same parameters determine the emergence of fluctuations. We will also see that initial conditions, that is the initial localization of specialized and nonspecialized agents, and the activation rules may play a role.

We discuss two cases in which the activation of the agents is asynchronous, that is one agent is activated in each period t. This aims to capture the Youngian insight on the possible presence of “waves” of specialization which arise in some part of the network and then propagate in the economy. Moreover, as Axtell (2001), p. 241, points out, in real socio-economic systems agents' activations are not likely to be regulated by the same internal clock. Therefore, the use of asynchronous activation also appears to be a way to add realism to the model, with respect in particular to a standard cellular automata (CA), in which all cells are activated contemporaneously.

We analyze the cases of deterministic and random activation. In both cases the initial values of the parameters generating the entries of vector $D(N_i)$ are: $d_{10} = 1$; $d_{20} = 1$; $d_{21} = 3$; $d_{11} = -2$; $d_{12} = 3$; $d_{22} = -2$, while the initial values of the thresholds are: $t_{01} = 5$; $t_{02} = 5$; $t_{11} = 4$; $t_{22} = 4$.

Following our introductory discussion, we have assumed that $d_{12} > d_{10}$ and that $d_{21} > d_{20}$, that is specialized agents generate a higher demand than nonspecialized agents. Notice that d_{11} and d_{22} are negative: a 1-agent attaches a negative value to the presence of other 1-agents in N_i, which certainly do not demand good 1 and represent competitors, and the same holds true in the case of 2-agents.[8]

In the simulations we consider two types of initial conditions. In one case all agents are randomly assigned a state. We can think of a situation in which, in Smithian terms, agents are potentially specialized in the production of one good or not specialized. When activated, they start a learning process

[8] We also assume that 0-agents evaluate the presence of 1-agents (2-agents) negatively for what concerns the potential demand for good 1 (good 2). In particular, they attach a weight (0.5 for now) to $d11$ ($d22$), representing the expectation that, if they specialize in good 1 (good 2), they find a 1-agent (2-agent) in their neighborhood in the following period.

based on the evaluation of the real possibilities given by the structure of their neighborhood, and then make a decision. Alternatively, we consider the case in which there is an initial small cluster of specialized agents, possibly one, and analyze a typical diffusion process. Again, the initial condition may be considered an instance in which one or a few contiguous agents evaluate the possibility to specialize. We will see that the initial conditions, that is the initial localization of specialized agents may be important.

4.1 Asynchronous activation: deterministic, Mobile Automata

We consider the framework described above in the context of a Mobile Automata (MA) (see, e.g. Wolfram (2002), pp. 71-77). A MA is a particular CA, in which only one cell is active in every period. A MA needs a rule which describes the movement of the active cell across the lattice. As noted, a MA is perhaps a better framework to introduce *feedback effects*. A MA allows for the presence of "waves" of specialization, that originate in some part of the economy and then possibly propagate.

In a "round" of the program every agent is activated exactly once, and every round lasts a number of periods given by the number of agents, denoted by N. In the figures that follow we represent the network as a row of agents, being understood that the last agent on the right is connected to the first on the left. The active agent at $t = 1$ is the first on the left (agent 1), then the active cell moves to the right (the active agent at $t = 2$ is agent 2, etc.) The color white indicates 0-agents, while gray and black respectively indicate 1-agents and 2-agents. The figures should be read from the top to the bottom to see the evolution over time of the configuration of the network, where a configuration is the network structure in terms of the type of agents. We report the configurations corresponding to successive rounds, that is after allowing each agent to make a transition.[9] Also, we compute the aggregate per capita output at the end of every round, and plot its dynamics over time.[10]

[9] For example, if there are 200 agents, we represent the configurations at periods 1, 201, 401, etc.

[10] We compute output in the following way: the production of good 1 and 2 by an unspecialized agent counts 1 (hence their total production counts 2), while production by specialized agents counts 3. Therefore, the range for per capita output is $[2, 3]$

Figure 1: MA, N=200, t=30000, D=5, $t_{01} = t_{02} = 5$; $t_{11} = t_{22} = 4$, i.c.: IC1

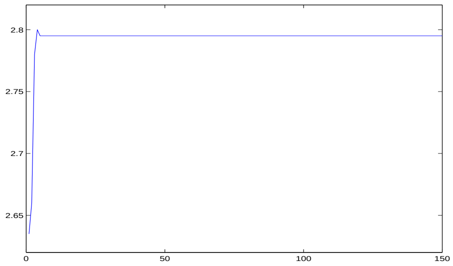

Figure 2: Time series of per capita output

Figures 1 and 2 represent an example in which the system converges to a steady state, and the steady-state output stabilizes at 2.79.[11] For comparison with subsequent cases, we define the randomly generated initial conditions in Figure 3 as $IC1$.[12]

In Figures 3 and 4, we increase the dimension of the neighborhood to $D = 7$. For given threshold values, this corresponds to an increase of the potential market for each agent, which has the possibility to interact with a larger number of agents.

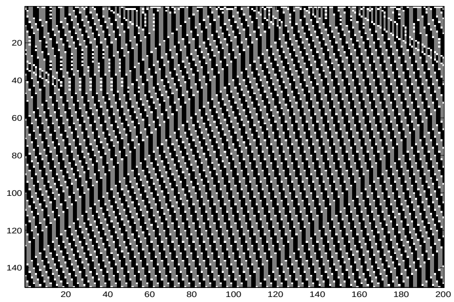

Figure 3: MA, N=200, t=30000, D=7, $t_{01} = t_{02} = 5$; $t_{11} = t_{22} = 4$, i.c.: IC1

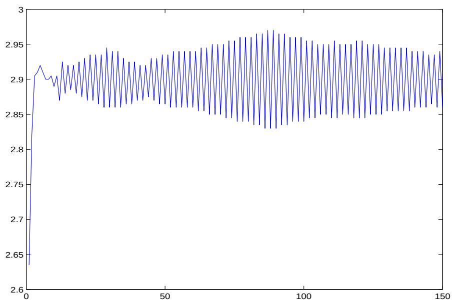

Figure 4: Time series of per capita output

We can observe that the dynamics changes radically: the system does not stabilize, and fluctuations appears. The instability of the system is reflected in the continuous changes which take place in the configurations of the network, and in the fluctuations in the simulated time series of aggregate output. The aggregate output increases, as the theory suggests, and now

[11]With $D = 3$, the system reaches a steady state with much lower specialization and output stabilizes at 2.1.

[12]These and the following figures are representative examples chosen from numerous simulations.

has an average of 2.89. The appearance of fluctuations may be explained by the increase in the probability of transition which accompanies the increase of the neighborhood in relation to the thresholds, as any individual agent is exposed to more variability, being affected by a higher number of agents and to their possible transitions.

In the last examples the threshold levels t_{01} and t_{02} are quite low if compared to the dimension of the neighborhood ($D = 7$). Increasing these values to $t_{01} = 7$ and $t_{02} = 7$ stabilizes the system, which now converges toward a steady state with incomplete specialization, and eliminates fluctuations. Figures 5 and 6 report the results starting from the same initial conditions of Figure 3.

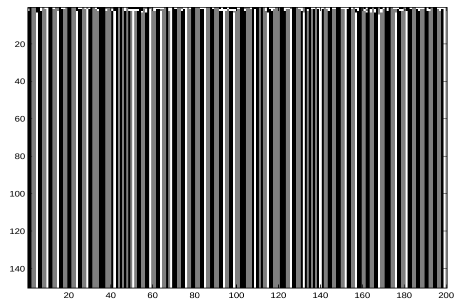

Figure 5: MA, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$; $t_{11} = t_{22} = 4$, i.c.: IC1

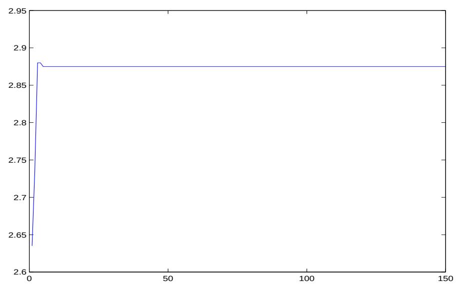

Figure 6: Time series of per-capita output

Now output stabilizes at 2.87, slightly below the average output in Figure 3. This is due to the higher level of demand necessary to induce specialization. The reason for the elimination of fluctuations is the following: when the threshold is low with respect to D, as noted, agents are exposed to the variability of more agents. This variability is high as the threshold for specialization is low. For a given D the increase in t_{01} and t_{02} reduces the variability of the system.

Also, the increase in t_{01} and t_{02} reduces aggregate output.[13] Intuitively, an increase in t_{01} and t_{02} should always reduce the diffusion of specialization, for a given value of D. However, this happens in general but, from various simulations, it appears that there can be exceptions. In fact, with a low threshold for specialization, more agents specialize but this may cause more "competition", which causes the despecialization of others. The final effect is likely to be nonlinear, and will in general depend on the balance between the two forces and the initial conditions.[14]

[13] If we set $t_{01} = t_{02} = 9$, output stabilizes at 2.67

[14] We will see in the next section a case in which an increase in t_{01} and t_{02} actually

In general, a higher instability appears to be positively associated to the "easiness" of changes of state. In fact, if we maintain the threshold level for specialization at $t_{01} = t_{02} = 7$, and increase the level of the demand necessary to keep a specialized agent in the same state, for instance by putting $t_{11} = t_{22} = 7$, then we obtain the dynamics in Figures 7 and 8 where instability reappears. A higher threshold for despecialization (t_{11} or t_{22}) means that it is more difficult for a specialized agent to "survive" (and then it is easier to change state), as s/he needs a "high" demand, and therefore it proxies a case of stronger competition. In fact, if an agent has many identical agents in her/his neighborhood, this reduces the demand s/he may have from the neighborhood. With a low threshold, the agent may remain specialized with low demand, hence with many competitors.

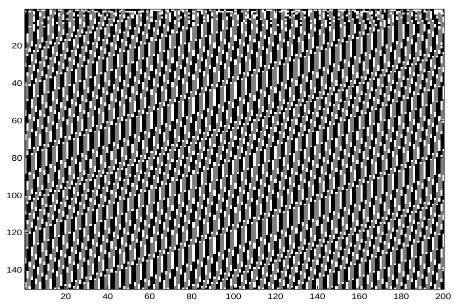

Figure 7: MA, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$; $t_{11} = t_{22} = 7$, i.c.: IC1

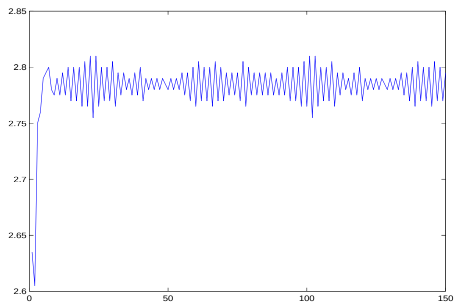

Figure 8: Time series of per-capita output

Notice that per capita output stabilizes at a lower level than in Figures 6, and is equal to 2.78. In this case, *ceteris paribus*, an increase in the intensity of competition reduces aggregate output as we are focusing here on the "destructive" role of competition, and abstract from its positive role for instance in stimulating productivity, innovativeness, etc.

For a comparison with Figures 3 and 4, in which instability was already present, we can consider the case of an identical value for the thresholds for specialization, $t_{01} = t_{02} = 5$, and a higher value for the threshold for despecialization, $t_{11} = t_{22} = 7$. Figures 9 and 10 show that instability increases, as the landscape of the configurations appears more complex, and average output decreases, as the average now is 2.85 compared to 2.89 in Figure 4.

increases the degree of specialization and aggregate output. This is related to the initial conditions and to the pattern which may be established in the initial periods. If this pattern features high specialization, the fact that 0-agents find more difficult to specialize may keep the system stable and highly productive.

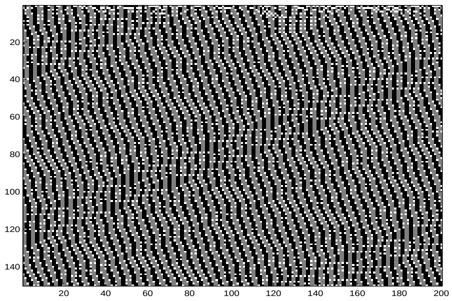

Figure 9: MA, N=200, t=30000, D=7, $t_{01} = t_{02} = 5$; $t_{11} = t_{22} = 7$, i.c.: IC1

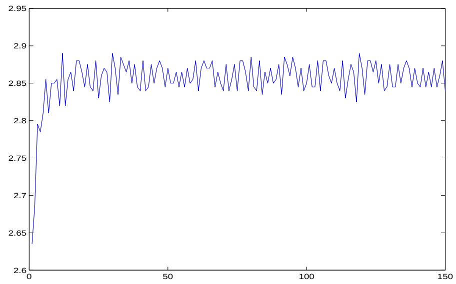

Figure 10: Time series of per-capita output

4.2 Diffusion of specialization with Deterministic Activation

To study the process of economic growth, we also consider the case in which there exists an initial small cluster of specialized agents. In this case we are interested in the conditions which favor the *diffusion* of specialization in the economy and hence economic growth. As a first case we use the parameters of Figures 1 and 2, that is $t_{01} = t_{02} = 5$ and $t_{01} = t_{02} = 4$, when only one specialized 1-agent is present in the initial period. When the dimension of the neighborhood is very small, $D = 3$, no diffusion takes place, and the system converges to a steady state with no specialized agents. The aggregate output is 2, the minimum (we omit the figures).[15]

If we increase the dimension of the neighborhood to $D = 5$, with only one 1-agent in the initial period, specialization diffuses and the system reaches a steady state with incomplete specialization (output stabilizes at 2.67). See Figures 11 and 12. Notice that growth appears suddenly, but this is due to the representation of the dynamics and to the activation rule.[16]

[15]The same result obtains with two contiguous 1-agents. When the initial cluster features one 2-agent between two 1-agents, then they remain specialized but no diffusion takes place.

[16]Recall that we are representing periods 1, 201, 401, etc., since we want to reproduce only the configurations after each round. With the activation rule of MA, when the active cell reaches the initially specialized agent, coming from the left of the figure, then a regular pattern is established on the right of the agent and follows the movement to the right of the active cell. We will see that there is a difference when we use random activation.

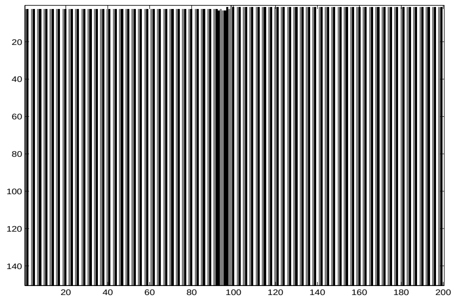

Figure 11: MA, growth, N=200, t=30000, D=5, $t_{01} = t_{02} = 5$; $t_{11} = t_{22} = 4$, i.c.: one 1-agent (99)

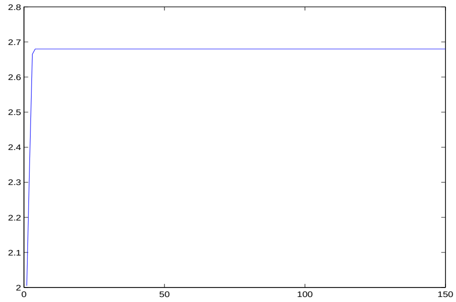

Figure 12: Time series of per-capita output

Hence, *ceteris paribus*, an increase in the potential extent of the market proxied by the dimension of the neighborhood favors the diffusion of specialization and economic growth. Notice that, with these particular initial conditions, the level of steady-state output is lower than in Figure 2, obtained with the same parameters.[17]

Now we keep the threshold level for specialization low ($t_{01} = t_{02} = 5$) and increase the dimension of the neighborhood to $D = 7$. With randomly generated initial conditions this change produced important modifications in the dynamics. In this case, if we start from only one 1-agent, the system does not converge to a steady state, but reaches quasi-full specialization and displays very little instability, and output has an average of 2.98.[18] In the last two examples, as in the previous section, we can see that *ceteris paribus*, an increase in D causes an increase in aggregate output.

If we start from an initial cluster of two 1-agents, we obtain Figures 13 and 14.

[17] Basically the same results obtain with two contiguous 1-agents.

[18] A similar result obtains when there is an initial cluster of one 1-agent and one 2-agent. We omit the figures.

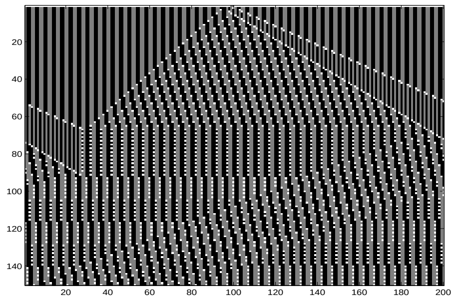

Figure 13: MA, growth, N=200, t=30000, D=7, $t_{01} = t_{02} = 5$; $t_{11} = t_{22} = 4$, i.c.: two 1-agents (99,100)

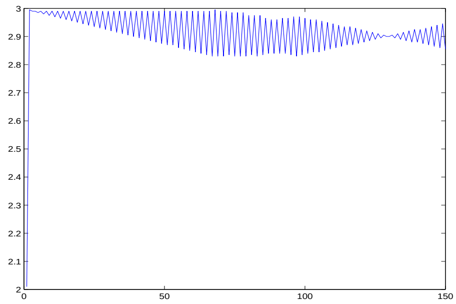

Figure 14: Time series of per-capita output

With these parameters, we observe that the system does not settle at a steady state. In particular, in initial periods there is a regularity emerging in areas of the network around the center. However, a "wave" spreads from the location of the initial cluster and breaks down the regular pattern, generating persistent instability. The average output is 2.91, hence very close to the maximum. Clearly, we see the importance of initial conditions, as the small number of agents in the central position in conjunction with certain values of the parameters, brings the system toward quasi-maximum output although with fluctuations. The addition of only one 1-agent causes a remarkable change in the dynamics although the appearance of instability is not very robust to a change of the localization of the initially specialized 1-agents.[19] The reason is that, with these parameters, the "admissible" regular grid of specialized agents features an alternation of two 1-agents and two 2-agents. However, it may happen that, depending on the location of the initial cluster in relation to the first agent activated, a series of three agents of the same type appears. This causes the despecialization of some agent which in turn generate feedbacks to other agents.

This wide instability disappears if we increase the threshold for specialization which, as noted in the previous section, reduces the probabilities of transition of 0-agents. With $t_{01} = t_{02} = 7$ and $t_{12} = t_{21} = 4$ the system converges toward a stationary state with with (almost) full specialization (output converges to 2.99). See Figures 15 and 16.

[19] In fact, with the parameters of Figures 13 and 14 it is possible to obtain an almost regular pattern with output stabilizing at 2.98 (we omit the figures), when the initial cluster is located at positions 98 and 99, instead that at 99 and 100.

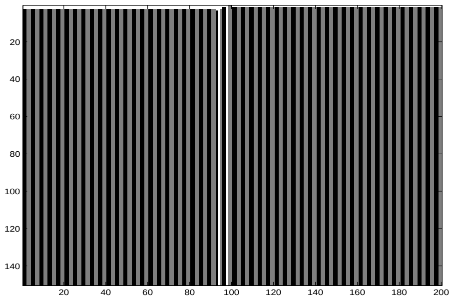

Figure 15: MA, growth, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$; $t_{11} = t_{22} = 4$, i.c.: two 1-agents (99,100)

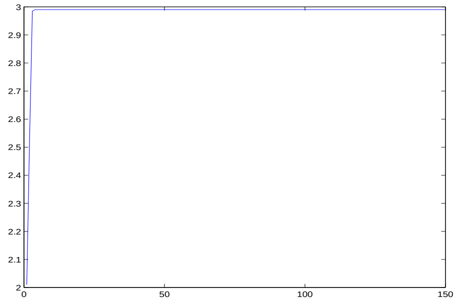

Figure 16: Time series of per-capita output

Also in this case an increase in the thresholds $t_{01} = t_{02}$ stabilizes the system, but does not reduce the final output, as it may have been predictable. In this case the combination of the parameters and the localization of the initial agents is such that an "ordered" structure emerges, in which almost only specialized agents exist and alternate in the network. However, if we further increase the threshold for specialization and set $t_{01} = t_{02} = 9$, then the threshold is so high to check the diffusion process (figures omitted). Therefore, the effect of $t_{01} = t_{02}$ on the level of output and its fluctuations may depend on initial conditions. For given parameters, as noted, the effect of the increase in t_{01} and t_{02} is likely to be nonlinear.

With $t_{01} = t_{02} = t_{11} = t_{22} = 7$, the system converges to a steady state with much lower specialization (output converges to 2.5) (we omit the figures). This is a different result with respect to the previous section when these parameters produces a high instability (although in both cases an increase of t_{11} and t_{22} reduces output). However, also in this case this depends on initial conditions. When we have two initial clusters of two 2-agents, then these parameters are associated to high instability (the average output is 2.74). See Figures 17 and 18.

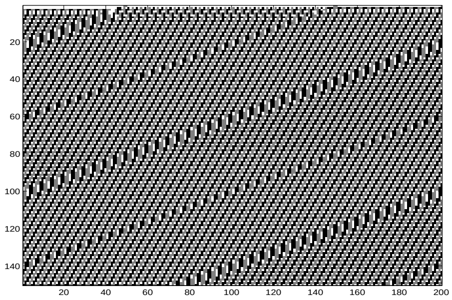

Figure 17: MA, growth, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$; $t_{11} = t_{22} = 4$, i.c.: four 1-agents (49,50,149,150)

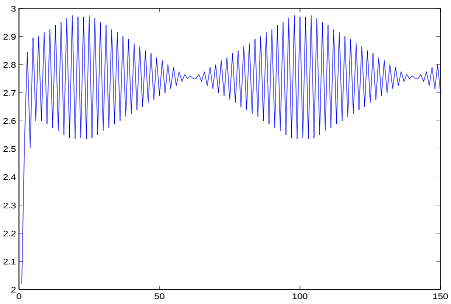

Figure 18: Time series of per-capita output

Finally, if we return to a lower dimension of the neighborhood, $D = 5$, and keep the same parameters and initial conditions of Figures 15 and 16, where the threshold for specialization is higher than in Figures 13 and 14, the diffusion process does not take place, as only few agents in the neighborhood of the initially specialized agents specialize, and output stabilizes at 2.01. In addition, with these parameters, when there is only one 1-agent in the initial conditions, no other agent specializes (we omit these figures).[20]

4.3 Asynchronous activation: random activation

To check the sensitivity of the results in the previous section to the activation rule, here we maintain the hypothesis that in every period only one agent is active, but we introduce random activation.

In every round, given by a number of periods equal to N, every agent is activated once, but the order of activation is random and changes in every round. Basically, the program generates a random permutation of the indices of all agent. Then, the activation starts from the agent whose index appears first in this sequence, then moves to the second element, etc. As before, the round ends when the Nth agent has been activated. Clearly, by introducing randomness and we are not in the realm of standard CA, which are completely deterministic.

If we use the same parameters and the same initial conditions $IC1$ of Figures 1 and 2, then the system still reaches a steady state (we omit the

[20]The latter case is an example in which, for a given D, t_{01}, and t_{02}, the extent of the market is low because the number of specialized agents in the neighborhood is low. Adding one 1-agent causes the specialization of some other agent in the neighborhood, even if it is not sufficient to activate a diffusion process involving the whole economy.

figures). The average aggregate output from many simulations is 2.81, a value very similar to the stable value of 2.79 reached with MA.

When we increased D with MA, we generated instability in the system and an increase in (average) aggregate output. Here we obtain a similar result. Figures 19 and 20 contain an example of the dynamics with the same parameters and the same initial conditions of Figures 3 and 4. RA stands for random activation.

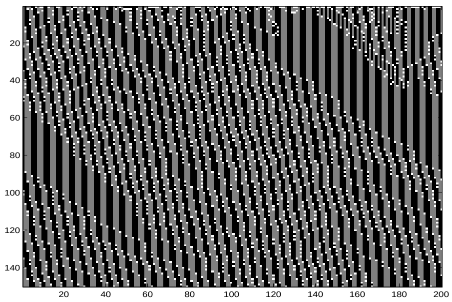

Figure 19: RA, N=200, t=30000, D=7, $t_{01} = t_{02} = 5$; $t_{11} = t_{22} = 4$, i.c.: IC1

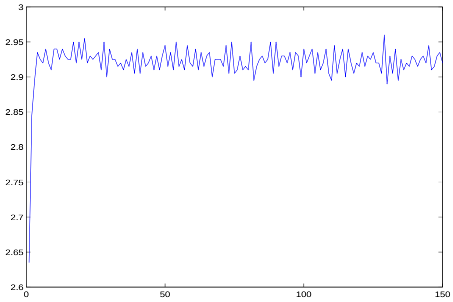

Figure 20: Time series of per-capita output

We can observe that the system displays instability without the somewhat regular patterns found in Figure 3. The time series is more irregular than in Figure 4, although in both cases output stabilizes around 2.9 (other simulations confirm this result).

This allows for the following consideration: (i) the parameters of the model are able to determine the average of the aggregate output; (ii) they are also responsible for the presence of fluctuations: in particular D in relation to the thresholds; (iii) different types of fluctuations originate from different activation rules. In the case of MA, the fluctuations are more regular than in the case of random activation.

Putting $t_{01} = t_{02} = 7$ eliminates the fluctuations and stabilizes the system at a lower per capita output as in the case of MA (from many simulations in which output stabilizes at some level, the modal value of this stable level is 2.875). See Figures 21 and 22.

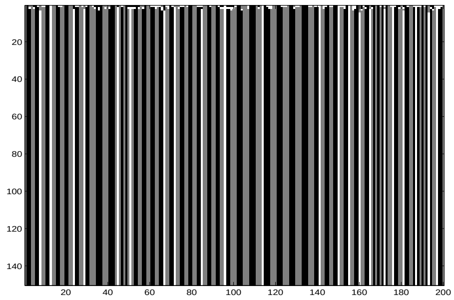

Figure 21: RA, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$; $t_{11} = t_{22} = 4$, i.c.: IC1

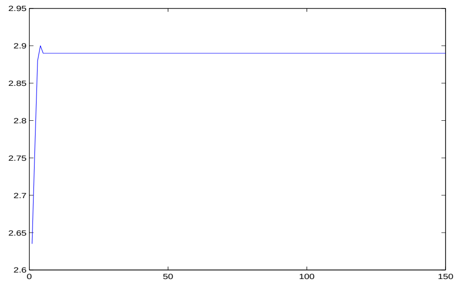

Figure 22: Time series of per-capita output

By putting $t_{01} = t_{02} = t_{11} = t_{22} = 7$ we re-obtain fluctuations (more irregular), and output stabilization at an even lower level (from many simulations, the average output is about 2.79, similar to the output with MA, even if the standard deviations of the time series differ). See Figures 23 and 24.

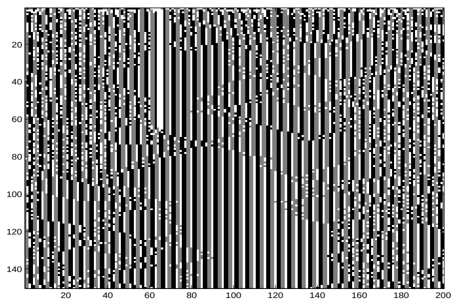

Figure 23: RA, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$, $t_{11} = t_{22} = 7$, i.c.: IC1

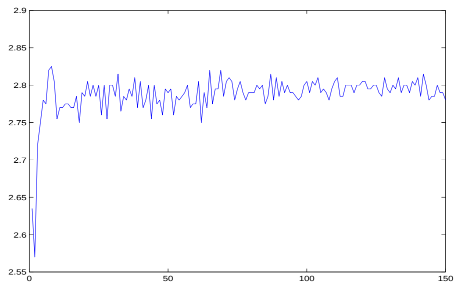

Figure 24: Time series of per-capita output

If we maintain the parameter values of Figures 23 and 24 and try different randomly generated initial conditions, we generally obtain a stationary time series of per capita output with mean equal to 2.78/2.79. However it seems that the standard deviation of the series is more volatile, with a range from 0.020 to 0.029. In general, the variation in the time series depends on the initial conditions and, for given initial conditions, on the randomness of the activations across simulations.

The same parameters may in fact be associated with a stable configuration. Figures 25 and 26 present the case of initial conditions indicated as $IC2$.

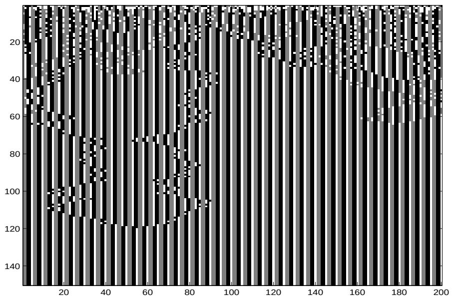

Figure 25: RA, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$, $t_{11} = t_{22} = 7$, i.c.: IC2

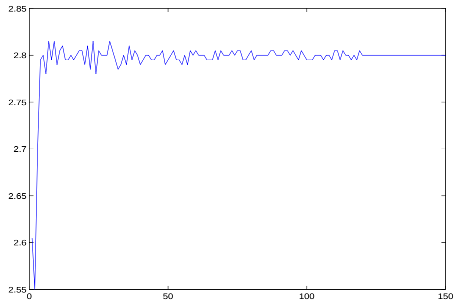

Figure 26: Time series of per-capita output

From other simulations with the same initial conditions $IC2$, it is possible to obtain fluctuations caused by the random activation. However, it is also possible to re-obtain a stable configurations with a different dynamics from the one in Figures 25 and 26. See Figures 27 and 28.

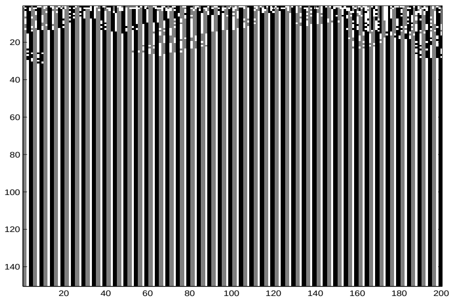

Figure 27: RA, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$, $t_{11} = t_{22} = 7$, i.c.: IC2

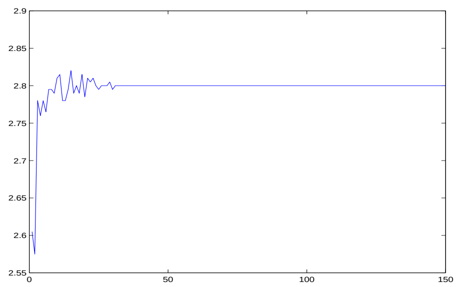

Figure 28: Time series of per-capita output

Hence, also in this case there exists sensitivity from the initial conditions, which is also related to the activation rule. As noted, the parameters of the model are however able to determine some broad characteristic of the dynamics, as the level of diffusion of specialization and of aggregate output, but the emergence of fluctuations may in this case be related to the parameters *and* to the activation rule and initial conditions.[21]

4.3.1 Diffusion of specialization with random activation

In this section we study the diffusion process starting from one or a few specialized agents. We follow the same steps of Section 4.2, and begin with

[21] At this stage we do not try to disentangle the role of initial conditions from the role of the activation rule. This is left for future work.

the usual parameters, $d_{10} = 1; d_{20} = 1; d_{21} = 3; d_{11} = -2; d_{12} = 3; d_{22} = -2; t_{01} = 5; t_{02} = 5; t_{11} = 4; t_{22} = 4$. If there is only one 1-agent in the initial period, then with $D = 3$ the diffusion process does not start, as in the case of MA.[22]

If we increase the dimension of the neighborhood, and set $D = 5$, then the diffusion process takes place, as shown in Figures 29 and 30.

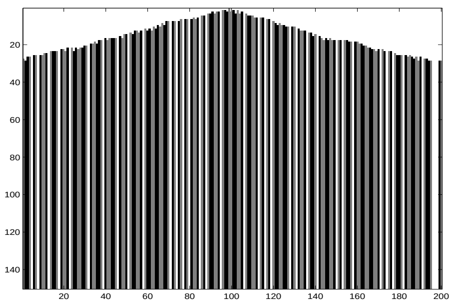

Figure 29: RA, N=200, t=30000, D=5, $t_{01} = t_{02} = 5$, $t_{11} = t_{22} = 4$, i.c.: one 1-agents at (99)

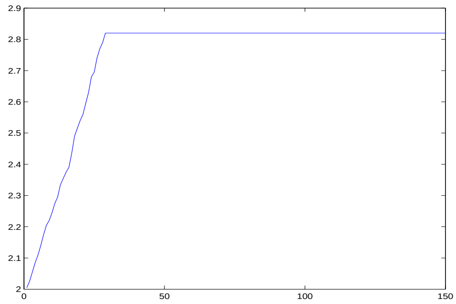

Figure 30: Time series of per-capita output

We may observe that, the diffusion takes place gradually, and that the system stabilizes at 2.76 (this is confirmed from other simulations).[23] The diffusion is gradual because of the activation rule: the active cell moves randomly, and not from the left to the right as with MA. This means that, as before, the first agents who can make a transition are those near the initial specialized agent. However, now it may not be the case that the more regular pattern of MA is established.[24]

If we increase further D to 7, then we obtain the results reported in Figures 31 and 32.

[22]Also, as with MA, the process does not start with two contiguous 1-agents, and with a small cluster given by one 2-agent between two 1-agents. In the latter case they are the only specialized agents in the steady state. We omit the figures.

[23]With two 1-agents the result is similar.

[24]With the active cell moving from left to right, this pattern featured the specialization of some agents on the right of the initial 1-agent, which allowed other agents on their right to specialize and so on. Now it may be the case that, those agents who were activated exactly after the specialization of some agents on their left, are activated when there are no specialized agents in their neighborhood, and therefore they do not make transitions. The agents on their right also do not make transitions, etc.

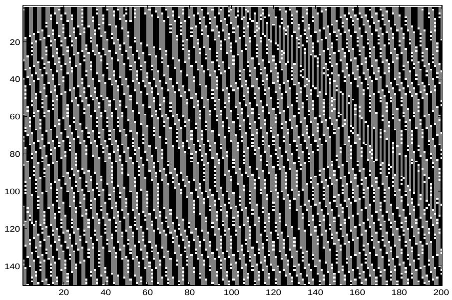

Figure 31: RA, N=200, t=30000, D=7, $t_{01} = t_{02} = 5$, $t_{11} = t_{22} = 4$, i.c.: one 1-agents at (99)

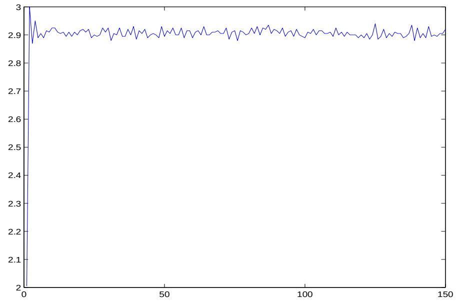

Figure 32: Time series of per-capita output

Output does not stabilize and reaches the average level of 2.80 (hence output increases with an increase in D). In this case, the threshold for specialization is so low compared to D, that it is not necessary to be close to the initial 1-agent to become specialized. Therefore growth appears suddenly as with MA. Notice that here putting $D = 7$ generates instability, while with MA it was necessary to have two 1-agents (although this result was sensitive to their localization). If we put two agents in the initial cluster of specialized agents results do not change.

If we increase the thresholds for specialization to $t_{01} = t_{02} = 7$ and leave $t_{11} = t_{22} = 4$, then we have the results in Figures 33 and 34.

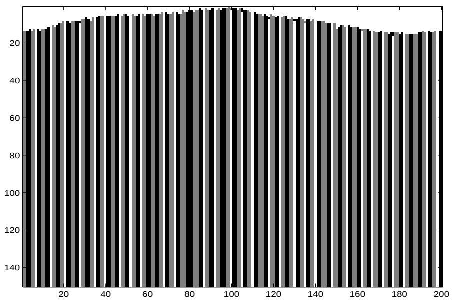

Figure 33: RA, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$, $t_{11} = t_{22} = 4$, i.c.: one 1-agents at (99)

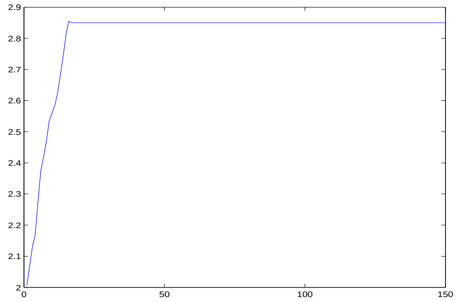

Figure 34: Time series of per-capita output

We can see that the system stabilizes at a lower level of output which, from different simulations, is about 2.81. Then, as with MA, the increase in t_{01} and t_{02} contributes to the stabilization of the system and, again, the increase in the thresholds for specialization does not reduce aggregate output.

If we increase the threshold levels for despecialization and set $t_{11} = t_{22} = 7$, then we obtain the Figures 35 and 36.

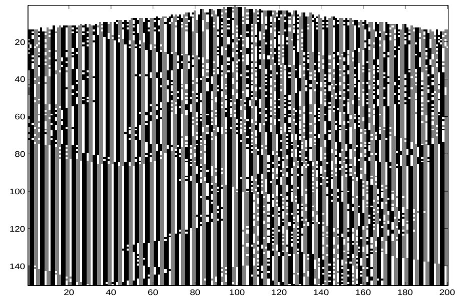

Figure 35: RA, N=200, t=30000, D=7, $t_{01} = t_{02} = 7$, $t_{11} = t_{22} = 4$, i.c.: one 1-agents at (99)

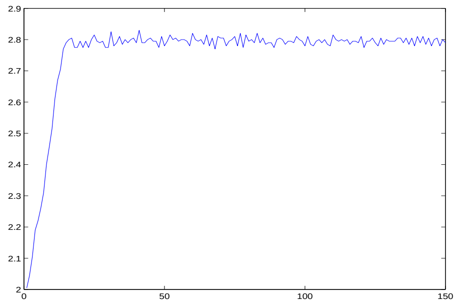

Figure 36: Time series of per-capita output

In the last case, the system generally does not stabilize (with the exception of few simulations), and output reaches an average of 2.74, that is it decreases with an increase of "competition".

However, with $t_{01} = t_{02} = t_{11} = t_{22} = 7$ and $D = 7$, one interesting case may arise with random activation. It may happen that the initial 1-agent is activated when there are only 0-agents in her/his neighborhood. Then, s/he despecialize and no other agent specializes in the economy, for the given thresholds (however, from various simulations this case seems rare). In the latter case, this probability may be further reduced if more than one agent is present in the initial cluster of specialized agents (notice also that the probability for the initial 1-agent to be activated when surrounded by zero agents, increases when D is smaller). This can be considered an example in which the random activation precludes the economy from taking off. More coordination, with agents in the neighborhood of the initial 1-agent specializing before her/him, could profoundly alter the ultimate results and generate economic growth.

5 Discussion

From the simulations in the previous section, we may draw the following conclusions: the diffusion of specialization and the level of per capita output increase in general with: i) an increase in the dimension of the neighborhood; 2) a reduction in the threshold levels for specialization, represented by parameters t_{01} and t_{02}; 3) a reduction in the threshold levels for despecialization, proxing for the strength of competition, that is reduction in t_{11} and t_{22}.

These results are quite predictable, given the assumptions of the model. However, we have found that these parameters influence also the qualitative features of the dynamics, in particular the convergence toward a steady state or the emergence of more complex types of dynamics, in which the system does not settle to a steady configuration but remains in a state of constant change. Also, we have found that the initial conditions and the activation rules may in some cases be relevant.

Let us discuss the issue of complexity. As remarked, the instability or complexity in the network dynamics depends on the easiness of making transitions. In this respect, we have observed that, for example, an increase in the dimension of the neighborhood which, given the assumptions, predictably favors specialization and growth, may also increase the instability of the system. This happens when the dimension of the neighborhood interacts with a relatively low value of the threshold for specialization or with a relatively high level of competition, proxied by a high level of the threshold for despecialization. Hence, a complex dynamics may arise from a particular combination of the parameters of the model.

The model proposed is highly simplified, but we argue that it may nonetheless provide some support to the intuition of Ally Young on the nature of the process of economic growth based on specialization. The simple feedback mechanism in a network proposed in this paper shows that, indeed, the aggregate dynamics may look similar to the one described by Young. At this stage, we suggest that this approach may represent a step toward a theory which is alternative to that advanced in the modern theory of endogenous growth, where growth and specialization are represented as an equilibrium process.[25]

As noted, these results hold in general, but in some cases the initial conditions and the activation rule may have a role although, broadly, the results listed above do not seem to be strongly affected by the activation rule.[26] This is related to the more general issue of the organization of the economic activity. By organization we refer to the issue of the localization of economic agents in the initial period and to the dynamics of agents'activation.

For instance, we have observed that steady states consist in regular patterns in which specialized and nonspecialized agents alternate. That is, given the parameters, one pattern could feature the alternation of couples of agents specialized in different goods (with the possible presence of nonspecialized agents. See for example Figure 15).

Given the local interaction and the feedbacks among agents, it may happen that the regular pattern, possibly corresponding to complete specialization, is never established. However, it may be the case that different initial

[25]See for instance the model of growth and specialization in Romer (1987).

[26]There is also a difference in the type of fluctuation that emerge, with fluctuations in the deterministic case appearing more "regular".

conditions or the order of activation of the agents is such that the regular pattern is established in one case and is not established in another, given the same set of parameters (including the dimension of the network N) (see for instance Figure 13 and the related discussion).

In general, growth in this context appears essentially as a nonergodic process. Given the same set of parameters and different initial conditions, the system may have a similar dynamics from the qualitative point of view, e.g. it may converge to a steady state, but it may differ from the quantitative point of view, i.e. it may converge to different levels of specialization and aggregate output (compare Figures 1 and 2 with Figures 11 and 12). Also, for what concerns the qualitative features of the dynamics, in the case of random activation we have seen that with the same initial conditions and the same set of parameters, the dynamics may be qualitatively different (see Figures 25 and 27).

Summing up, as briefly mentioned in the discussion of Smith, we argue that an understanding of the mechanism of growth based on specialization requires the consideration of important organizational aspects of the economy. As economic growth is often characterized by take-offs, we suggest that successful economies have not been only characterized by favorable conditions in terms of resources and productive factors, but may also have had a favorable organization of the productive activity.

6 Concluding Remarks

In this paper we have explored the possibility to study growth based on division of labor in a simple network structure. We highlighted the factors which can be at the roots of economic growth, as the extent of the market, but also showed that the same parameters affecting the diffusion of specialization and the level of output can be responsible for the emergence of complex dynamics of the network and output fluctuations at the aggregate level.

We showed that, indeed, economic growth is likely to take the shape of a complex process, and be characterized by constant change in the structure of the economy. We take this as a partial confirmation of some of Smith and Young's insights, on the nature of the growth process.

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