

# Occupational Choice under Risk in an Overlapping Generations Economy with Monopolistic Competition

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## **Abstract**

This paper analyzes a neoclassical overlapping generations model. We discuss a two-sector economy with intermediate and final goods in the spirit of Romer (1990). Agents engage in one of two alternative occupations: either self-employment in the intermediate goods sector, which is characterized by monopolistic competition, or employment as an ordinary worker in this sector. Entrepreneurial profits are stochastic. The occupational choice under risk endogenizes the number of firms and products in the intermediate goods industry. We find that expected profits of monopolists do not vanish in equilibrium. The heterogeneity of incomes also implies heterogeneity in the amounts saved.

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# 1 Introduction

This paper analyzes occupational choice under risk in the context of an overlapping generations neoclassical growth model. We discuss a two-sector economy with intermediate and final goods in the spirit of Romer (1990), but unlike Chou and Shy (1991) there is no endogenous growth. The final good is produced in a perfectly competitive market by employing capital and differentiated intermediate goods. Labor is the single input to production of the intermediate good. The intermediate goods sector is characterized by monopolistic competition. Profits in this sector are stochastic due to a technology shock, which can be interpreted as a measure for entrepreneurial ability. The risk-averse agents live for two periods. When young, they choose between two alternative occupations. They can either be an employee of the monopolistic firm and receive a riskless wage income. Or they set up a monopoly in the intermediate goods sector, thereby receiving a risky profit income. The equilibrium population share of entrepreneurs simultaneously determines the number of intermediate goods used in the production of the final good. Since expected utility from the respective occupation decides upon being a worker or an entrepreneur, expected profits of monopolists do not vanish in equilibrium and exceed riskless wage incomes. When old, agents retire and consume their savings. The heterogeneity of incomes also implies heterogeneity in the amounts saved, such that the mean entrepreneur owns a larger share of the aggregate capital stock than the representative worker.

## 2 The Model

*Households* We consider a discrete time overlapping generations model. The identical households live for two periods. We normalize the population size of each cohort to unity. There is no population growth. Each member of the young generation is endowed with one unit of labor, which she supplies inelastically. At the beginning of their life, citizens choose between two alternative types of occupation. They can decide either to set up a firm and become a monopolistic entrepreneur in the intermediate goods industry, or they become employed in this sector.  $\lambda$  denotes the population share of entrepreneurs. The corresponding population share of workers is given by  $1 - \lambda$ . While employment is paid the

riskless wage income  $w$ , self-employment yields risky profits  $\pi_j$  per monopoly  $j$ . The risk stems from an idiosyncratic technology shock. By the time the households choose between the occupations, they do not know the realization of the shock. By the time they compose their intertemporal consumption profile, the income realization is known and the agents act under perfect foresight. We assume the costs of switching between occupations to be prohibitively high, such that the employment decision once made is irreversible. All individuals retire after the first period. When old, savings and interest payments are used to finance retirement consumption. There are no bequests.

The individuals spend their income on a single final good, which can be consumed or invested respectively. Lifetime utility of a member of a cohort  $i$  is additively-separable and given by

$$U(c_{i,t}, c_{i,t+1}) = \frac{1}{1-\rho} \left[ c_{i,t}^{1-\rho} + \beta c_{i,t+1}^{1-\rho} \right]. \quad (1)$$

The current period utility functions are characterized by constant relative risk aversion, measured by the parameter  $\rho$ . For simplicity, the attitude towards risk is assumed to be identical for all agents, although Kihlstrom and Laffont (1979), Kanbur (1981), and Cramer *et al.* (2002) stress, that the entrepreneurial occupation is more likely to be chosen by agents who are less risk averse.<sup>1</sup> The agents discount future consumption. The discount factor  $0 < \beta < 1$  is related to the intertemporal rate of time preference  $\delta$  via  $\beta = 1/(1 + \delta)$ .

Let  $y_{i,t}(k)$  denote the period  $t$  income of a member of generation  $i$  and an occupation generating an income of type  $k = w, \pi_j$ . Then, the intertemporal budget constraint can be written as follows

$$c_{i,t} = y_{i,t}(k) - s_{i,t}, \quad (2a)$$

$$c_{i,t+1} = s_{i,t}(1 + r_{t+1}). \quad (2b)$$

$r_{t+1}$  is the interest rate paid on saving held from period  $t$  to period  $t + 1$ . Define with  $R_{t+1} \equiv 1 + r_{t+1}$  the return factor on saving. Because we assumed the income realizations to be known at the time of intertemporal choice, optimization is performed under certainty and yields the familiar Euler condition

$$U'(c_{i,t}) = \beta R_{t+1} U'(c_{i,t+1}). \quad (3)$$

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<sup>1</sup>Incorporating heterogeneity with respect to the degree of risk aversion is a worthwhile extension of the model, but beyond the scope of this paper.

Given the functional form of utility, and substituting  $c_{i,t} = y(k)_{i,t} - s_{i,t}$ , and  $c_{i,t+1} = R_{t+1} s_{i,t}$  implies the following savings function

$$s_{i,t} = \frac{y(k)_{i,t}}{1 + \beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho}}, \quad (4)$$

and optimal consumption  $c_{i,t}, c_{i,t+1}$

$$c_{i,t} = \frac{\beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho}}{1 + \beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho}} y(k)_{i,t}, \quad c_{i,t+1} = \frac{R_{t+1}}{1 + \beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho}} y(k)_{i,t} \quad (5)$$

Incorporating these relationships into (1) yields the following expression for maximized lifetime utility of a household of generation  $i$  and profession with income of type  $k = w, \pi_j$

$$U(c_{i,t}, c_{i,t+1}) = \frac{\beta R_{t+1}^{1-\rho}}{1-\rho} \left[ 1 + \beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho} \right]^\rho y(k)_{i,t}^{1-\rho}. \quad (6)$$

Occupational choice is related to the labor market equilibrium and will be discussed below.

*Final Goods Sector* The representative firm of the final goods sector produces a homogeneous good  $Q_t$  using capital  $K_t$  and varieties of a differentiated intermediate good  $\{x_{j,t}\}_{j=0}^\lambda$  as inputs. Production in this sector takes place under perfect competition and the price of  $Q_t$  is normalized to unity. We assume a production function of a generalized CES-form; see Spence (1976), Dixit and Stiglitz (1977) and Ethier (1982):

$$Q_t = K_t^{1-\alpha} \int_0^\lambda x_{j,t}^\alpha dj, \quad (7)$$

where  $0 < \alpha < 1$ . The production function displays positive but diminishing marginal productivity for each input  $K$  and  $x_j$ , and constant returns to scale in all inputs together. The capital stock depreciates completely in each period. Additive-separability of (7) in intermediate goods ensures that the marginal product of input  $j$  is independent of the quantity employed of  $j'$ . The intermediate goods are close but not perfect substitutes in production, with the elasticity of substitution between goods  $j$  and  $j'$  given by  $\varepsilon = 1/(1 - \alpha)$ .

The time  $t$  profit of the representative firm in the final goods sector is

$$\Pi_t = Q_t - r_t K_t - \int_0^\lambda p_{j,t} x_{j,t} dj, \quad (8)$$

where  $p_j$  denotes the price of intermediate good  $x_j$ . Optimization yields the standard conditions from marginal productivity theory

$$\frac{\partial Q_t}{\partial K_t} = r_t \quad \implies \quad r_t = (1 - \alpha) \frac{Q_t}{K_t}, \quad (9)$$

$$\frac{\partial Q_t}{\partial x_{j,t}} = p_{j,t} \quad \implies \quad x_{j,t} = K_t \left( \frac{\alpha}{p_{j,t}} \right)^{1/(1-\alpha)} \quad (10)$$

Condition (10) represents the demand function, which the producer of the intermediate good  $x_j$  faces. It is isoelastic, with the direct price elasticity of demand given by

$$\eta_{x_j, p_j} = \frac{\partial x_j}{\partial p_j} \times \frac{p_j}{x_j} = -\frac{1}{1-\alpha} = -\varepsilon.$$

*Intermediate Goods Sector* The intermediate goods sector is populated by a large number  $\lambda$  of small firms, each producing a single variety  $j$  of a differentiated good. The producers engage in monopolistic Bertrand competition. Labor  $L_t$  is the single input of production. We assume that all the monopolists of the intermediate sector produce according to the identical constant returns to scale technology of the form

$$x_{j,t} = \theta_{j,t} L_{j,t}. \quad (11)$$

Firms differ only with respect to the realization of the idiosyncratic (firm specific) productivity shock  $\theta_j$  with density  $\theta_j \in \Theta \subset \mathbb{R}^{++} : f(\theta)$ , which is assumed to be non-diversifiable, uncorrelated across firms and lognormally distributed, with mean  $E[\ln \theta] = \bar{\theta}$  and variance  $\text{Var}[\ln \theta] = \sigma^2$ . Similar to Kanbur (1979), we posit that the entrepreneurs hire labor after the draw of nature has occurred. Recall that earlier we assumed the costs of changing occupations to be prohibitively high, such that agents are prevented from switching between groups in case of unfavorable realizations of the shock.

Given (10) and (11), the time  $t$  profit of a typical producer in this sector then reads as

$$\pi_{j,t} = K_t \left( \frac{\alpha}{p_{j,t}} \right)^{1/(1-\alpha)} \left[ p_{j,t} - \frac{w_t}{\theta_{j,t}} \right]. \quad (12)$$

The firm problem essentially is a static one. Under perfect competition on the labor market, the producer treats the wage rate  $w_t$  as exogenously given. Price setting behavior implies the following solution for the monopoly price

$$p_{j,t} = \frac{w_t}{\alpha \theta_{j,t}}. \quad (13)$$

The profit maximizing price of a typical entrepreneur in the intermediate goods market is the markup  $1/\alpha > 1$ ,  $\forall \alpha \in (0, 1)$  over the marginal costs of production.

### 3 Market Equilibrium

*Market for intermediate good of type  $j$*  The demand for intermediate good  $j$  is given by equation (10) and can be rewritten as follows  $p_{j,t} = \alpha(K_t/x_{j,t})^{1-\alpha}$ . Equating this expression with condition (13) yields the market clearing amount of good  $j$

$$x_{j,t} = \left( \frac{\alpha^2 \theta_{j,t}}{w_t} \right)^{\frac{1}{1-\alpha}} K_t. \quad (14)$$

By substitution into (11), we derive the labor demand of entrepreneur  $j$  as follows

$$L_{j,t}^D = \left( \frac{\alpha^2 \theta_{j,t}^\alpha}{w_t} \right)^{\frac{1}{1-\alpha}} K_t. \quad (15)$$

*Labor Market* The labor market is characterized by perfect competition. The equilibrium wage rate can then be derived by equating the aggregate labor supply with expected labor demand. The aggregate labor supply equals the population share of workers,  $L_t^S = 1 - \lambda_t$ , due to the normalization of population size. If we take account of the i. i. d. property of the firm-specific technology shock and the characteristics of the underlying distribution, the expected aggregate labor demand is given by

$$\begin{aligned} L_t^D &= K_t \left( \frac{\alpha^2}{w_t} \right)^{\frac{1}{1-\alpha}} \int_0^{\lambda_t} \int_{\theta \in \Theta} \theta^{\frac{\alpha}{1-\alpha}} f(\theta) d\theta dj \\ &= \lambda_t K_t \left( \frac{\alpha^2}{w_t} \right)^{\frac{1}{1-\alpha}} \exp \left[ \frac{\alpha}{1-\alpha} \left( \bar{\theta} + \frac{1}{2} \frac{\alpha \sigma^2}{1-\alpha} \right) \right] \end{aligned} \quad (16)$$

Equating this expression with  $1 - \lambda_t$  and integrating, allows us to solve for the market clearing wage rate  $w_t$

$$w_t = \alpha^2 K_t^{1-\alpha} \left( \frac{\lambda_t}{1-\lambda_t} \right)^{1-\alpha} \exp \left[ \alpha \bar{\theta} + \frac{1}{2} \frac{\alpha^2 \sigma^2}{1-\alpha} \right]. \quad (17)$$

The equilibrium wage rate still is a function of the population shares of workers and entrepreneurs. It is increasing with a rise in  $\lambda$ . Since we are dealing with a general equilibrium model, an increase in the number of monopolistic firms on the intermediate goods market is accompanied by a decrease in aggregate labor supply, which drives the market clearing wage rate upwards.

Given the equilibrium wage rate, it is now possible to determine the profit of monopolist  $j$  in the intermediate goods market. Substituting (17) and (13) into (12) yields

$$\pi_{j,t} = \theta_{j,t}^{\frac{\alpha}{1-\alpha}} \alpha (1-\alpha) K_t^{1-\alpha} \left( \frac{1-\lambda_t}{\lambda_t} \right)^\alpha \exp \left[ -\frac{\alpha}{1-\alpha} \left( \alpha \bar{\theta} + \frac{1}{2} \frac{\alpha^2 \sigma^2}{1-\alpha} \right) \right]. \quad (18)$$

The profit income of a typical producer  $j$  in the intermediate goods industry also depends on the yet undetermined equilibrium distribution of agents over occupations. Moreover, the idiosyncratic realization of the technology shock is another important determinant of entrepreneurial income.

*Equilibrium occupational choice* An equilibrium distribution of households over the two types of occupation is characterized by a situation, where the pivotal agent's utility gain from switching between occupations is zero, or, in short, if expected lifetime utility from being an entrepreneur equals lifetime utility of a worker.

Since the equilibrium wage rate is safe, lifetime utility  $U_W(c_{i,t}, c_{i,t+1})$  of a worker of generation  $i$ , can simply be derived by substituting (17) into (6)<sup>2</sup>

$$U_W(c_{i,t}, c_{i,t+1}) = A \left( \alpha^2 K_t^{1-\alpha} \left( \frac{\lambda_t}{1-\lambda_t} \right)^{1-\alpha} \exp \left[ \alpha \left( \bar{\theta} + \frac{1}{2} \frac{\alpha \sigma^2}{1-\alpha} \right) \right] \right)^{1-\rho} \quad (19)$$

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<sup>2</sup>For notational simplicity, we define  $A \equiv \frac{\beta R_{t+1}^{1-\rho}}{1-\rho} \left[ 1 + \beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho} \right]^\rho$ , such that lifetime utility is given by  $U(c_{i,t}, c_{i,t+1}) = A y(k)_{i,t}^{1-\rho}$ .

The entrepreneurs of the intermediate goods industry receive risky profits, due to the technology shock. Expected lifetime utility  $E[U_M(c_{i,t}, c_{i,t+1})]$  of being a monopolist of cohort  $i$  in this sector can be determined as follows

$$\begin{aligned}
E[U_M(\cdot, \cdot)] &= A E \left[ \pi_{j,t}^{1-\rho} \right] \\
&= A \left( \alpha(1-\alpha) K_t^{1-\alpha} \left( \frac{1-\lambda_t}{\lambda_t} \right)^\alpha \exp \left[ -\frac{\alpha^2 \bar{\theta}}{1-\alpha} - \frac{1}{2} \frac{\alpha^3 \sigma^2}{(1-\alpha)^2} \right] \right)^{1-\rho} \\
&\quad \times \int_{\theta \in \Theta} \theta^{\frac{\alpha(1-\rho)}{1-\alpha}} f(\theta) d\theta \\
&= A \left( \alpha(1-\alpha) K_t^{1-\alpha} \left( \frac{1-\lambda_t}{\lambda_t} \right)^\alpha \exp \left[ \alpha \bar{\theta} + \frac{\alpha^2 \sigma^2 (1-\rho-\alpha)}{2(1-\alpha)^2} \right] \right)^{1-\rho}
\end{aligned} \tag{20}$$

Equating (19) with (20) finally yields the equilibrium population share of monopolists in the intermediate goods industry

$$\lambda_t^* = \frac{1-\alpha}{1-\alpha + \alpha \exp \left[ \frac{1}{2} \frac{\rho \alpha^2 \sigma^2}{(1-\alpha)^2} \right]}, \tag{21}$$

and  $1 - \lambda_t^*$  residually. The populations shares are constant in equilibrium and depend on the primitives of the model, which are the degree of risk aversion  $\rho$ , the variance of the technology shock  $\sigma^2$  and the elasticity of substitution between two arbitrary intermediate goods  $j$  and  $j'$ , implicitly measured by  $\alpha$ . We find  $0 < \lambda_t^* < 1$ ,  $\forall \alpha, \sigma, \rho$ . Note that  $\lambda_t^*$  is independent of the mean  $\bar{\theta}$  of the productivity shock. This results can be ascribed to the assumption of CRRA preferences, where the degree of risk aversion does not depend on the level of income.

**Proposition 1** *The occupational choice of risk-averse households endogenizes the number of firms in the intermediate goods industry in terms of a population share, thereby simultaneously determining the range of intermediate goods employed in the production of the final good.*

**Proposition 2** *Due to the endogenized firm number, profits in monopolistic competition do not vanish. If agents are risk-averse, the expected profit in the intermediate goods industry exceeds the riskless wage rate. The entrepreneurs demand a positive risk premium for bearing the production risk.*



*Proof:* Expected profits in the intermediate sector are given by

$$\pi_t = \alpha(1-\alpha)K_t^{1-\alpha} \left( \frac{1-\lambda}{\lambda} \right)^\alpha \exp \left[ \alpha\bar{\theta} + \frac{1}{2} \frac{\alpha^2 \sigma^2}{1-\alpha} \right]$$

Subtracting  $w_t$  from  $\pi_t$  yields the following expression for the expected risk premium

$$\pi_t - w_t = \alpha^{1+\alpha}(1-\alpha)^{1-\alpha} K_t \exp \left[ \alpha\bar{\theta} - \frac{1}{2} \frac{\alpha^2 \sigma^2 (1-\rho)}{1-\alpha} \right] \left( \exp \left[ \frac{1}{2} \frac{\rho \alpha^2 \sigma^2}{(1-\alpha)^2} \right] - 1 \right).$$

From this follows immediately that  $\pi_t \stackrel{\geq}{\leq} w_t$ , if

$$\exp \left[ \frac{1}{2} \frac{\rho \alpha^2 \sigma^2}{(1-\alpha)^2} \right] \stackrel{\geq}{\leq} 1 \iff \rho \stackrel{\geq}{\leq} 0 \text{ for } \alpha \in (0,1), \sigma > 0. \quad (22)$$

□

*Final goods sector* The market for intermediate goods is cleared, if aggregate expected demand for goods  $x_{j,t}$  equals expected aggregate supply. By utilizing the demand function (14) for intermediate goods  $j$ , the equilibrium output of the final good can be derived as follows

$$Q_t = \lambda^{*1-\alpha} (1-\lambda^*)^\alpha K_t^{1-\alpha} \exp \left[ \alpha\bar{\theta} + \frac{1}{2} \frac{\alpha^2 \sigma^2}{1-\alpha} \right], \quad (23)$$

with  $\lambda^*$  given by (21). The representative firm of the final goods sector then spends the amount of

$$\alpha Q_t = \left( \frac{\alpha}{w_t} \right)^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} K_t \int_0^\lambda \int_{\theta \in \Theta} \theta^{\frac{\alpha}{1-\alpha}} f(\theta) d\theta dj$$

on the purchase of intermediate goods, and

$$(1-\alpha)Q_t = r_t K_t$$

on physical capital, thereby showing the well-known result of zero profits in perfect competition.

*Capital market* The market clearing interest rate on capital is determined by marginal productivity theory and can be derived as

$$r_t = (1-\alpha) K_t^{-\alpha} \lambda^{*1-\alpha} (1-\lambda^*)^\alpha \exp \left[ \alpha\bar{\theta} + \frac{1}{2} \frac{\alpha^2 \sigma^2}{1-\alpha} \right]. \quad (24)$$

The capital stock was assumed to depreciate completely at the end of each period. Intertemporal market clearing requires the capital stock of time  $t + 1$ , which equals investment  $I_t$ , to be equal to aggregate expected savings  $S_t$  of period  $t$ , that is

$$K_{t+1} \equiv I_t = S_t . \quad (25)$$

By taking account of (4), aggregate savings of period  $t$  can be determined as the with the associated population shares weighted average of savings out of the two types of income

$$\begin{aligned} S_t &= \frac{1}{1 + \beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho}} \left[ (1 - \lambda^*) w_t + \int_0^{\lambda^*} \int_{\theta \in \Theta} \pi_{j,t} f(\theta) d\theta dj \right] \\ &= \frac{1}{1 + \beta^{-1/\rho} R_{t+1}^{(\rho-1)/\rho}} \left[ (1 - \lambda^*) w_t + \lambda^* E[\pi_{j,t}] \right] \\ S_t &= s[r(K_{t+1})] Y_t , \end{aligned} \quad (26)$$

where  $s[r(K_{t+1})]$  denotes the propensity to save, which depends on the next period's capital stock and preference parameters, and  $Y_t$  is the aggregate mean income in the intermediate sector.

Equation (25) can then be rewritten to the yield the first order nonlinear difference equation for the evolution of the capital stock over time

$$K_{t+1} = \frac{\alpha \lambda^{*1-\alpha} (1 - \lambda^*)^\alpha \exp \left[ \alpha \bar{\theta} + \frac{1}{2} \frac{\alpha^2 \sigma^2}{(1-\alpha)^2} \right]}{1 + \beta^{-1/\rho} [1 + r(K_{t+1})]^{(\rho-1)/\rho}} K_t^{1-\alpha} . \quad (27)$$

Equation (27) implicitly defines  $K_{t+1}$  as a function of  $K_t$  and reflects the well-known dynamics of the capital stock of the neoclassical growth model. A stationary point or steady state is characterized by  $K_t = K_{t+1}$ . In the special case of logarithmic utility ( $\rho = 1$ ), the propensity to save is independent of the interest rate. For this reason, (27) can be solved explicitly for the steady state value of the capital stock

$$K^* = \left( \alpha \lambda^{*1-\alpha} (1 - \lambda^*)^\alpha \frac{\beta}{1 + \beta} \exp \left[ \alpha \bar{\theta} + \frac{1}{2} \frac{\alpha^2 \sigma^2}{(1-\alpha)^2} \right] \right)^{1/\alpha} . \quad (28)$$

## 4 Comparative Static Results

**Proposition 3** *The equilibrium population share of entrepreneurs decreases with a rise in the elasticity of substitution  $\epsilon$  between two intermediate goods  $j$  and  $j'$ , a rise in the degree of risk aversion, and a rise in risk, the latter measured by the variance of the technology shock*

$$\frac{\partial \lambda^*}{\partial \alpha} < 0, \quad \frac{\partial \lambda^*}{\partial \rho} < 0, \quad \frac{\partial \lambda^*}{\partial \sigma^2} < 0. \quad (29)$$

A rise in the parameter  $\alpha$  corresponds to an increase in the elasticity of substitution between intermediate goods. This increase in competition is accompanied by a decrease in expected profits and induces agents to switch away from entrepreneurship. We observe an equivalent effect on the population share of entrepreneurs for rises in  $\rho$  or  $\sigma^2$ . In the first case, households either develop a greater disliking for risk in the latter the riskiness of profit incomes increases, both causing agents to switch towards safe wage incomes. Since we deal with a general equilibrium framework, this leads to a fall in the market clearing wage rate and a rise in expected profits, thereby increasing the risk premium of entrepreneurs.

**Proposition 4** *A rise in the population share of entrepreneurs reduces profits for the single firm as well as expected profits in the intermediate goods industry and leads to an increase in the market clearing wage rate.*

$$\frac{\partial \pi}{\partial \lambda} < 0, \quad \frac{\partial \pi_j}{\partial \lambda} < 0, \quad \frac{\partial w}{\partial \lambda} > 0. \quad (30)$$

An increase in the number of firms in the intermediate sector, reduces the market share falling to the single firm and thereby individual profits. If more households choose to be an entrepreneur, this goes along with a decrease in aggregate labor supply and a situation of excess demand on the labor market. A new equilibrium is characterized by a higher market clearing wage rate.

In what follows, it will be convenient to employ the notion of expected income shares in the intermediate goods sector. Let  $\gamma = \pi_t/Y_t$  define the expected income share of profit incomes and  $1 - \gamma = w_t/Y_t$  the income share of workers respectively. These income shares can be derived as

$$\gamma = \frac{1 - \alpha}{\lambda} \quad \text{and} \quad 1 - \gamma = \frac{\alpha}{1 - \lambda}. \quad (31)$$

**Proposition 5** *Mean income in the intermediate goods sector and aggregate savings decreases with a rise in the population share of entrepreneurs*

$$\frac{\partial Y}{\partial \lambda} = -\frac{(1-\gamma)Y}{1-\lambda} < 0 \quad \text{and} \quad \frac{\partial S}{\partial \lambda} = -\frac{(1-\gamma)S}{1-\lambda} < 0. \quad (32)$$

Here we observe the well-known inefficiency property of monopolistic markets, which eventually implies welfare losses. An increase in the number of entrepreneurs aggravates this effect. Aggregate income and consequently savings, as a constant fraction of mean income, are larger under perfect competition.

**Proposition 6** *Entrepreneurs own a larger expected share of the aggregate capital stock than workers.*

By (22), expected profits exceed wage incomes, when households are risk averse. With identical propensities to save, entrepreneurs contribute a larger amount to the aggregate capital stock.

**Proposition 7** *The market clearing interest rate decreases with a rise in the population share of entrepreneurs and increases with a rise in the degree of risk aversion or a rise in risk respectively*

$$\begin{aligned} \frac{\partial r}{\partial \lambda} &= -\frac{(1-\gamma)r}{1-\lambda} < 0, & \frac{\partial r}{\partial \rho} &= \frac{\partial r}{\partial \lambda} \times \frac{\partial \lambda}{\partial \rho} > 0, \\ \frac{\partial r}{\partial \sigma^2} &= r \left[ \frac{\alpha^2}{2(1-\alpha)} - \frac{1-\gamma}{1-\lambda} \times \frac{\partial \lambda}{\partial \sigma^2} \right] > 0. \end{aligned} \quad (33)$$

These results can be explained via the savings channel. As we have already shown above, aggregate savings decline with an increase in the population share of entrepreneurs and consequently with all factors that let  $\lambda$  rise. Lower savings imply a lower future capital stock and increase in the marginal productivity of physical capital.

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