Status and Risk–Taking in a Stochastic Growth Model

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Abstract

This paper is concerned with the effects of status preferences on individual risk–taking in the context of a stochastic growth model with externalities in human capital accumulation. We postulate a tradeoff between status concerns and risk, and between status and the attitude towards risk. The tradeoff is of ambiguous sign, which primarily depends on whether or not the risk-averse agent saves out of precautionary motives. Preferences for social status are able to correct allocative distortions due to the externalities in human capital accumulation, such that the inefficiently low expected growth rate of the decentralized economy is driven towards its Pareto-efficient value. Status preferences affect the effective intertemporal elasticity of substitution. For this reason the expected-utility model is contrasted with a non-expected utility context, which disentangles the effects from risk aversion and intertemporal substitution.

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1 Introduction

It is quite an old idea in economics that agents not only care for the consequences of actions in absolute terms, when evaluating their individual market success, but also draw comparisons with respect to the relative performance of others or the average outcomes in the economy.[1]

Duesenberry (1949) was the first to indicate the importance of relative income for the accumulation process. Supporting empirical evidence for this hypothesis was provided by Easterlin (1974). But only throughout the last decade his thoughts have gained new attraction in macroeconomic theory. An increasing number of contributions combines the *quest for status* or motives for *keeping up with the Joneses* — both reflecting the individual desire to outperform others — with issues of consumption, saving, and growth (cf. Cole *et al.*, 1992, 1998; Konrad, 1992; Zou, 1995; Fershtman *et al.*, 1996; Carroll *et al.*, 1997, 2000; Corneo and Jeanne, 1997, 1998, 2001; Rauscher, 1997; Futagami and Shibata, 1998; Fisher and Hof, 2000). In the majority of contributions either the average consumption level or average wealth in the society serves as the relevant frame of reference. In models with wealth effects, the agents have an additional incentive to accumulate. Wealth holding itself offers a reward in terms of utility, which causes the agents to postpone consume and save more.

Individual decisions on consumption and saving affect the frame of reference within which others evaluate their own actions. This in fact means that the inclusion of such reference variables in individual decision–making generates an externality, which usually implies the inefficiency of the resulting allocation.

Yet, interpretations as to which factors are at work differ. Rauscher (1997), Corneo and Jeanne (1997) as well as Fisher and Hof (2000) agree on the result that direct preferences for relative wealth — or relative consumption respectively — affect the effective intertemporal elasticity of substitution. Contrary to this,

[1]The idea that status concerns help in the explanation of certain aspects of human behavior has increasingly gained attention in the economics discipline throughout the last years. Among others, especially Robert Frank (1985*a*; 1985*b*; 1989; 1997) has contributed to the revival of this argument. But moreover, there seems to be an increasing tendency in many fields of economic research to acknowledge the importance of interpersonal comparisons. For instance, relative success (fitness) also is the major principle and driving force of strategy selection in evolutionary game theory.

Futagami and Shibata (1998) ascribe their results to changes in the effective intertemporal discount rate.

Matters become even complicated if we switch to a stochastic framework. There the results are often explained with a change in the effective degree of risk aversion (cf. Konrad and Lommerud, 1993). Galí (1994), for instance, develops a static CAPM model of conspicuous consumption, where agents care about their relative living standard. He examines the effects of consumption externalities on asset prices and portfolio choice and finds that agents increase the equilibrium share of the risky asset, if the aggregate reference level generates a positive externality on marginal utility. Additionally he shows in a dynamic model that the equilibrium asset prices in an economy with and without consumption externalities only coincide for a properly adjusted coefficient of risk aversion.

Bakshi and Chen (1996) also explore the effects of preferences for wealth status on equilibrium asset prices and portfolio allocation. Like Gong and Zou (2002), who additionally discuss fiscal policy implications, they find that households are more conservative in risk taking, and that consumption is postponed in favor of savings, thereby extending the results from the deterministic to the stochastic context. Yet, their argument follows a different line, by claiming that status concerns alter the effective degree of risk aversion. Their results on asset prices, portfolio choice and intertemporal allocation emerge especially due to the fact that they allow the degree of risk aversion referring to consumption risk to be different from the one of wealth risk.[2]

Hence, on the one hand, some of the results are not robust to preference specification (see Fisher and Hof, 2000), while, on the other hand, especially the comparison between deterministic and stochastic models suffers from one major drawback of expected utility theory. In standard stochastic growth models, the utility over different states of nature is weighted with the associated subjective probabilities by the identical multiplicative manner by which utility is weighted at different instants of time with the discount factor. Therefore it does not distinguish between intertemporal substitution and risk aversion.

This aspect provides the starting point of our analysis. Disentangling the effects stemming from intertemporal substitution and risk aversion by assuming

[2]Only in this case status preferences induce a shift in portfolio choice and affect the equilibrium asset prices and the risk premium.

non–expected utility preferences in the spirit of Epstein and Zin (1989, 1991) and Obstfeld (1994*a*,*b*), allows for a more thorough understanding of the forces that affect the intertemporal savings decision and growth.

We are especially interested in the consequences of status preferences for intertemporal risk taking. Here, two counter–acting forces are at work. On the one hand, postponing consumption increases wealth, thereby directly increasing utility. On the other hand the consumer has to decide as to whether or not to expose additional resources to future capital risk by an increase in accumulation. This is less likely, the larger the intertemporal substitution effect. Consequently, the focus of our analysis lies on the interaction between status preferences, risk, the attitude towards risk, and intertemporal substitution in the determination of the growth rate of the economy, and the possibly existing tradeoff relationships between these factors.

The analysis is embedded in a stochastic endogenous growth model of the Romer (1986)–type with externalities in human capital accumulation, which is extended with preferences for relative wealth. Corneo and Jeanne (1997) have demonstrated for the deterministic version of this model that the externality generated by status preferences may compensate the allocative distortion caused by the knowledge spillovers.

We compare the equilibrium values of the macroeconomic variables of the expected utility setting with those derived under the assumption of non–expected utility and show that the latter framework helps to state the tradeoff relationships between the model primitives in a more straightforward way. We find that status preferences and risk may work as substitutes or complements, the results mainly depending on whether or not the risk–averse agent has a motive for precautionary savings.

The paper is organized as follows: In the following section 2, we develop the model and derive the equilibrium conditions. Section 3 presents the corresponding results for the non–expected utility framework, while section 4 is devoted to the comparative dynamic analysis. We examine the impact of a change in risk, the degree of risk aversion as well as in the individual status valuation on the equilibrium economic relationships and on lifetime utility. The implications for an optimal degree of status valuation are discussed. Section 5 concludes.

2 Case 1: Expected Utility

The model The economy is populated by a continuum $[0,1]$ of identical agents who produce a homogeneous good according to the stochastic Cobb–Douglas technology

$$dy(t) = Ak(t)^{\alpha}(K(t)L(t))^{1-\alpha}(dt + dz(t)), \quad \alpha \in (0,1) \tag{1}$$

$k(t)$ denotes the privately owned capital stock. A is a constant productivity parameter. Labor input $L(t)$ is supplied inelastically and normalized to unity. The instantaneous output $dy(t)$ is subject to an aggregate multiplicative productivity shock. $dz(t)$ is a serially uncorrelated increment to a standard Wiener process with zero mean and variance $\sigma^2 dt$. The aggregate capital stock $K(t)$ in individual production denotes a human capital externality in the spirit of Romer (1986), where firms neglect their own contribution to the economywide stock of technical knowledge. For this reason, the production function of a typical producer displays constant returns to scale with respect to capital and labor, while aggregate production is characterized by increasing returns to scale. Physical capital is the single asset of the economy.

The infinitely–lived identical households are characterized by a time–separable utility function in consumption $c(t)$ and status $S(t)$. The intertemporal optimization problem of a typical agent is given by

$$\max_{c} \quad V(0) = \mathrm{E}_0 \int_0^{\infty} U\left[c(t), S(t)\right] e^{-\beta t}\, dt \tag{2a}$$

$$\text{s. t.} \quad dk(t) = dy(t) - c(t)\, dt, \qquad k(0) > 0,\ z(0) = 0. \tag{2b}$$

E_0 denotes the mathematical expectation conditional on time 0 information and $\beta > 0$ is the rate of time preference. The individuals have preferences for social status, which is measured by relative wealth, $S = s(k/K)$. Consumption is supposed to be instantaneously deterministic. The current period utility function $U[c, s(k/K)]$ displays the following properties:

$$U_c > 0, U_{cc} < 0, \quad U_k > 0, U_{kk} < 0, \quad U_K < 0, U_{KK} > 0$$

which means, that instantaneous felicity is increasing and concave in individual consumption and capital, whereas disutility is derived from an increase in the aggregate reference level K. We limit our analysis to the case of risk–averse agents, who are characterized by constant relative risk aversion measured by the parameter ρ, and assume the functional form

$$U[c(t),S(t)] = \frac{1}{1-\rho}\left[c(t)\left(\frac{k(t)}{K(t)}\right)^{\delta}\right]^{1-\rho}. \tag{3}$$

with $\rho, \delta > 0,\ \rho \neq 1,\ \delta(1-\rho) < 1, \rho > \delta/(1+\delta)$ and for logarithmic preferences $U[c(t),S(t)] = \ln c(t) + \delta[\ln k(t) - \ln K(t)]$. Contrary to Bakshi and Chen (1996) and Gong and Zou (2002), the agents of our model are equally risk averse towards consumption and wealth risk.

The objective of a typical agent is to select the rate of consumption in order to maximize the expected value of lifetime utility, according to the program as described by (2a) and (2b). In order to solve the optimization problem, we set up the stochastic Hamiltonian[3]

$$H\left(c,k,\lambda,\frac{\partial\lambda}{\partial k}\right) = U(c,S)\,e^{-\beta t} + \lambda\left[Ak^{\alpha}(KL)^{1-\alpha} - c\right] + \frac{1}{2}\frac{\partial\lambda}{\partial k}\sigma_k^2, \tag{4}$$

where $\sigma_k^2 = \mathrm{E}(dk)^2/dt$ denotes the variance of the individual capital stock and the costate variable λ has the standard economic interpretation as the present–value shadow price of wealth.

The associated first–order conditions are:

$$\frac{\partial H}{\partial c} = e^{-\beta t}U_c - \lambda = 0 \tag{5a}$$

$$d\lambda = -\left[e^{-\beta t}U_k + \lambda\alpha Ak^{\alpha-1}(KL)^{1-\alpha} + \frac{1}{2}\frac{\partial\lambda}{\partial k}\frac{\partial\sigma_k^2}{\partial k}\right]dt + \frac{\partial\lambda}{\partial k}\sigma_k\,dz. \tag{5b}$$

together with the transversality condition for a feasible intertemporal program

$$\lim_{t\to\infty}\mathrm{E}_t[H(t)] = 0\,. \tag{5c}$$

Condition (5a) asserts that in an optimal intertemporal program the marginal utility of consumption equals the marginal utility of wealth, which is represented by λ. Condition (5b) describes the stochastic evolution of the costate variable λ over time.

[3] In what follows, we drop the (t) part of the variables for notational convenience.

Macroeconomic equilibrium The conditions from individual optimization can now be employed to determine the market equilibrium. Aggregate and average values coincide given the assumptions of the mass of individuals normalized to unity and all households being identical. Consequently, in equilibrium $k = K$. This result also implies that the status quest subsequently turns out to be a rat race for the single household. With identical agents, no one succeeds in his pursuit of yielding a wealth level above average.

The market clearing condition is

$$dK = [AK - C]\,dt + AK\,dz\,. \tag{6}$$

The solution conjecture usually applied for isoelastic preferences is that the equilibrium value of the propensity to consume out of capital $\mu = C/K$ is constant over time, which means that consumption and capital grow at a common stochastic rate. Differentiating (5a) with respect to time, taking regard of the solution conjecture, the equilibrium condition $k = K$, and market clearing (6), then equating the time derivative of (5a) to (5b) and taking expectations eventually leads to the following equilibrium relationships for the expected propensity to consume out of capital and the expected growth rate of the economy $\psi = \mathrm{E}(dK)/(K\,dt)$

$$\mu = \frac{A\,(\rho-\alpha)+\beta+\rho A^2\sigma^2\left(\alpha-\frac{1}{2}(\rho+1)\right)}{\rho+\delta}, \tag{7}$$

$$\psi = \frac{A\,(\alpha+\delta)-\beta+\rho A^2\sigma^2\left(\frac{1}{2}(\rho+1)-\alpha\right)}{\rho+\delta}. \tag{8}$$

Both economic variables are the sum of two components: the first represents the corresponding equilibrium values of the deterministic model, while the latter, which includes the variance of the technology shock, reflects the agent's response to risk. In order to preserve feasible solutions to the model, we impose an upper bound on the equilibrium risk premium and require the certainty equivalent to capital return $r_s = \alpha A\,(1-\rho A\sigma^2)$ to be positive.[4]

The risk–averse agent has a motive for precautionary savings in the definition of Leland (1968) and Sandmo (1970), if the expected growth rate of the compet-

[4]For details regarding the certainty equivalent to capital return and the size of the risk premium, see Clemens (1999, 2002). A situation of $r_s < 0$ describes a dynamically inefficient allocation, where the competitively chosen growth rate exceeds the Pareto–efficient one; see the statements on optimality below.

itive economy exceeds its deterministic counterpart, that is, if $\frac{1}{2}(\rho+1) > \alpha$. Empirical evidence suggests this to be the relevant case; see Hubbard *et al.* (1994).

3 Case 2: Non–Expected Utility

In what follows, we disentangle the effects stemming from intertemporal substitution and risk aversion by assuming non–expected utility preferences as developed by Epstein and Zin (1989), implemented in models of stochastic growth for instance by Obstfeld (1994*b*), Smith (1996) and Clemens and Soretz (1999). We assume iso–elastic recursive preferences of the form

$$G[(1-\rho)V(t)] = \frac{1-\rho}{1-1/\varepsilon}\, c(t)^{1-1/\varepsilon}\, h + e^{-\beta h}\, G[(1-\rho)\,\mathrm{E}_t\, V(t+h)]\,. \qquad (9)$$

The parameter ρ still denotes the coefficient of relative risk aversion, while $\varepsilon > 0$ represents the intertemporal elasticity of substitution. Without loss of generality we do not discuss the limiting cases where $\rho = 1$ and / or $\varepsilon = 1$. The function $G(\centerdot)$ is given by

$$G(\centerdot) = \frac{1-\rho}{1-1/\varepsilon}\,[(1-\rho)V(t)]^{\frac{1-1/\varepsilon}{1-\rho}}\,. \qquad (10)$$

We will now skip the solution procedure (see Appendix) and proceed directly to the equilibrium relationships of the macroeconomic variables. The expected growth rate of the economy and the equilibrium propensity to consume out of capital can be derived as follows:

$$\tilde{\mu} = \frac{\varepsilon}{1+\varepsilon\delta}\left[\frac{A(1-\varepsilon\alpha)}{\varepsilon} + \beta + \rho A^2\sigma^2\left(\alpha - \frac{1}{2}\frac{\varepsilon+1}{\varepsilon}\right)\right], \qquad (11)$$

$$\tilde{\psi} = \frac{\varepsilon}{1+\varepsilon\delta}\left[A\,(\alpha+\delta) - \beta + \rho A^2\sigma^2\left(\frac{1}{2}\frac{\varepsilon+1}{\varepsilon} - \alpha\right)\right]. \qquad (12)$$

4 Comparative Dynamics

Risk aversion versus intertemporal substitution If we now compare the equilibrium values of the macroeconomic relationships of the expected utility model (7) and (8) with those of the non–expected utility approach (11) and (12), we see our presumptions on the deviating interpretations of the consequences of status preferences confirmed.

A look at the expected growth rate (8) makes obvious that the elasticity of marginal utility — represented by the coefficient ρ in the expected utility setting — plays a dual role. First, its reciprocal measures the households willingness to substitute current against future consumption. On the one hand, this induces a size effect on the expected growth rate (8), which is altered by status preferences. The coefficient now amounts to $(\rho+\delta)^{-1}$, and very clearly shows the origin of the argument, that status preferences change the *effective* intertemporal elasticity of substitution (*e*IES). The lower this expression, the less the agents are willing to deviate from a uniform consumption pattern over time

On the other hand, the size of ρ decides on the degree of intertemporal risk taking in a stochastic environment, where preferences are characterized by positive third derivatives; see Leland (1968) and Sandmo (1970). Changes in risk give rise to intertemporal income and substitution effects. If current and future consumption are normal goods, the income effect is positive, thus increasing the demand for both and eventually promoting growth. The intertemporal substitution effect is negative, because a rise in savings exposes additional resources to capital risk, thereby increasing the volatility of future consumption flows. In a model of pure capital risk, the question of dominance of either of the effects depends on ρ to exceed or fall below unity.[5] In our model, the intertemporal income effect dominates in case of $\frac{1}{2}(\rho+1)>\alpha$, which reflects a motive for precautionary savings. The status parameter δ does not reappear here, since we assumed the household to be equally risk averse towards consumption and wealth risk.

Second, if we focus on the entire risk component of the expected growth rate, $\rho A^2\sigma^2(\centerdot)$, we observe that the degree of risk aversion additionally has a effect on the intensity of precautionary saving. The higher the degree of risk aversion, the stronger is the household's overall response to risk.

These effects are more carefully separated in the non–expected utility setting. There, all matters concerning the timing of consumption are captured by the intertemporal elasticity of substitution. Again, we have to distinguish between the standard IES, denoted by the parameter ε, and the effective IES, the latter modified by status preferences and measured by $\varepsilon/(1+\varepsilon\delta)$. Besides this level effect on expected growth (12), the intertemporal elasticity of substitution determines

[5] The *AK*–type model of endogenous growth is a member of this class.

whether or not the consumer saves out of precautionary motives. This kind of self–insurance on capital markets can be observed for $\frac{1}{2}(\varepsilon+1) > \varepsilon\alpha$.

The coefficient of risk aversion ρ is left with its magnifying role on the risk component of the expected growth rate $\rho A^2\sigma^2(\centerdot)$. Independent of how the household responds to aggregate risk — be it with an increase or a decrease in accumulation — the degree of risk aversion amplifies this response.

Status preferences and expected growth We now turn to the question of how precisely status preferences influence growth and risk taking. From (8) and (12), we see that the status parameter δ affects the expected growth rate of the economy twofold. On the one hand, we have the modifying effect on the IES, already mentioned above. If we look at the two expressions derived for the expected utility and the non–expected utility framework, $(\rho+\delta)^{-1}$ and $\varepsilon/(1+\varepsilon\delta)$, we find that in both models an increase in the status parameter δ leads to a lower value of the effective intertemporal elasticity of substitution, thereby reducing expected growth.

On the other hand, we have a positive impact of status preferences on the expected growth rate. An increase in accumulation yields a greater relative wealth in the future, which is rewarded with a higher social status. In addition to the market return αA, capital holdings also receive a marginal payoff in terms of status, such that net returns in total amount to $A(\alpha+\delta)$. This incentive to postpone consumption and save more is preserved, despite the fact that in equilibrium no one succeeds in his desire to outperform the others.

The overall effect of a change in the status parameter on the expected growth rate and the propensity to consume is unambiguous in the two settings:

Proposition 1 (Expected growth and status concerns) *A rise in the marginal valuation for social status promotes growth. In both settings, the expected growth rate converges asymptotically to a constant, which is equal to the productivity parameter A and independent of the underlying risk*

$$\frac{\partial\psi}{\partial\delta} = \frac{\mu}{eIES} > 0, \qquad \textit{and} \qquad \frac{\partial\tilde{\psi}}{\partial\delta} = \frac{\tilde{\mu}}{eIES} > 0, \tag{13}$$

$$\lim_{\delta\to\infty}\psi = \lim_{\delta\to\infty}\tilde{\psi} = A, \qquad \textit{and} \qquad \lim_{\delta\to\infty}\mu = \lim_{\delta\to\infty}\tilde{\mu} = 0\,. \tag{14}$$

Proof of (14)*: By application of L'Hôpital's rule.* □

Despite the diminishing effect on the *e*IES, the reward of an increase in savings in terms of status utility always predominates in the determination of the expected growth rate.

Figure 1 illustrates this augmenting effect of status preferences on expected growth for precautionary savings, although the results of Proposition 1 are invariant with respect to this specific case. The solid black line depicts the expected growth rate of the stochastic economy with status concerns, while the dotted line shows expected growth of a status–neutral society. The figure also compares the stochastic economy with the non–stochastic one, the latter represented by the grey lines. Due to the effect of precautionary savings, expected growth of the stochastic economy exceeds deterministic growth for all values of the status parameter, although, asymptotically, both growth rates converge towards the identical constant value A.

Tradeoff relationships In what follows, we are interested in the interaction of the model parameters in the determination of the expected growth rate. Totally differentiating (8) and (12) with respect to the intertemporal elasticity of substitution ε and/or the degree of risk aversion ρ, the marginal valuation of status δ, and risk, measured by the variance of the technology shock σ^2, leads to the following tradeoff relationships:

Expected utility $(d\psi = 0)$

$$\frac{d\rho}{d\delta} = \frac{\mu}{\psi - A^2\sigma^2\left(\rho - \alpha + \frac{1}{2}\right)} \tag{15}$$

$$\frac{d\sigma^2}{d\delta} = -\frac{\mu}{\rho A^2\left(\frac{1}{2}(\rho+1) - \alpha\right)} \tag{16}$$

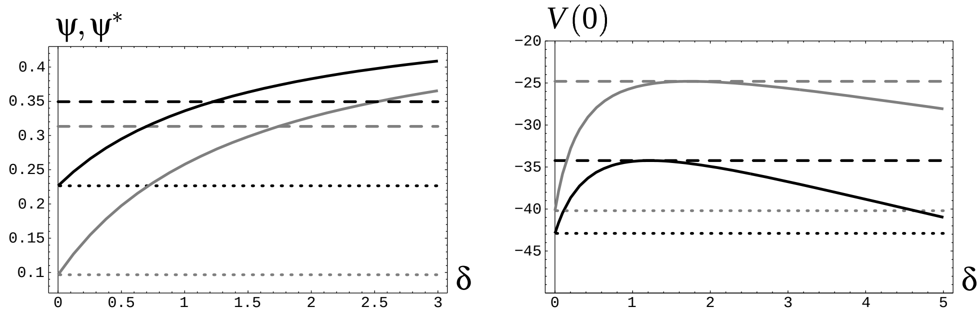
Non–expected utility $(d\tilde{\psi} = 0)$

$$\frac{d\rho}{d\delta} = -\frac{\tilde{\mu}}{A^2\sigma^2\left(\frac{1}{2}\frac{\varepsilon+1}{\varepsilon} - \alpha\right)} \tag{17}$$

$$\frac{d\sigma^2}{d\delta} = -\frac{\tilde{\mu}}{\rho A^2\left(\frac{1}{2}\frac{\varepsilon+1}{\varepsilon} - \alpha\right)} \tag{18}$$

$$\frac{d\varepsilon}{d\delta} = -\frac{\tilde{\mu}\varepsilon^2}{\tilde{\mu} - \frac{1}{2}\rho A^2\sigma^2} \tag{19}$$

All equations are of ambiguous sign. In so far, the desire for social status may serve as a substitute as well as a complement to risk, risk aversion and intertemporal substitution. Nevertheless, we are able to provide an easy understanding of the economic intuition behind the relationships (16) (17), and (18), which all have a negative sign in case of precautionary savings. Here, status preferences



(a) Expected growth rate (b) Expected utility

Figure 1: *Macroeconomic response to changes in the status parameter*

serve as a substitute for risk and the degree of risk aversion (the latter only for the case of non–expected utility).

Precautionary savings provide a self–insurance against stochastic future income flows and are positively correlated with risk. The agent favors a smooth consumption flow over time. A rise in status needs mitigates the growth effects stemming from the precautionary motive. Individual risk–taking increases in terms of the agent demanding less intertemporal insurance and accepting a more volatile consumption flow over time.

The signs of (15) and (19) cannot be assessed in such an intuitive way, because these expressions capture all the previously described partly counter–acting effects from risk aversion and intertemporal substitution. For (15) we can state that a negative tradeoff between ρ and δ also is more likely in case of precautionary savings.

Optimal degree of status desire We now recall the well–known result from the learning–by–doing framework with knowledge spillovers, namely, that the competitively chosen growth rate falls short of the socially optimal one. Given the statements of Proposition 1, it is a natural question to ask, whether there exists an optimal degree of status needs, which implies an efficient allocation and maximizes intertemporal expected utility

$$V(0) = \frac{k(0)^{1-\rho}}{1-\rho} \cdot \frac{\mu^{1-\rho}}{\beta - (1-\rho)\left(\psi - \frac{1}{2}\rho A^2 \sigma^2\right)}. \tag{20}$$

A Pareto–efficient allocation would then be characterized by a situation, where the macroeconomic variables of the competitive economy with status concerns equal their efficient counterparts, which a benevolent planner chooses, who takes account of the two relevant distortions: (a) the externality from technical knowledge, and (b) the result that the status game is a game nobody wins in equilibrium. Under these considerations, efficient expected growth in the two settings is given by

$$\psi^* = \tfrac{1}{\rho}(A-\beta) + \tfrac{1}{2}(\rho-1)A^2\sigma^2, \tag{21}$$

$$\tilde{\psi}^* = \varepsilon(A-\beta) + \tfrac{1}{2}(1-\varepsilon)\rho A^2\sigma^2 \tag{22}$$

while the propensities to consume out of capital can be determined residually with $\mu^* = A - \psi^*$

$$\mu^* = \frac{1}{\rho}\left[\beta + A(\rho-1)\left(1-\tfrac{1}{2}\rho A\sigma^2\right)\right]$$

$$\tilde{\mu}^* = \varepsilon\beta + A(1-\varepsilon)\left(1-\tfrac{1}{2}\rho A\sigma^2\right) .$$

If we additionally recall the definition of the certainty equivalent to capital return $r_s = \alpha A(1-\rho A\sigma^2)$ from page 6, which we required to be positive for feasible solutions of the model, the following condition on the optimal intensity of status preferences can be derived:

Proposition 2 (Optimal degree of status) *Optimal preferences for social status are characterized by* $\psi = \psi^*$ *or* $\tilde{\psi} = \tilde{\psi}^*$

$$\delta^* = \frac{1-\alpha}{\alpha}\frac{r_s}{\mu^*} \quad \textit{and} \quad \tilde{\delta}^* = \frac{1-\alpha}{\alpha}\frac{r_s}{\tilde{\mu}^*} \tag{23}$$

Proof: Taking the derivative of (20) with respect to δ *yields*

$$\frac{\partial V(0)}{\partial\delta} = \frac{k(0)^{1-\rho}\mu^{1-\rho}}{\left[\beta-(1-\rho)\left(\psi-\tfrac{1}{2}\rho A^2\sigma^2\right)\right]^2} \times \frac{\rho}{\rho+\delta} \times (\psi^*-\psi) .$$

The first two expressions are positive for feasible solutions of the model and given the assumptions stated on the primitives. Welfare is maximized, if the last term on the RHS is equal to zero. The same argument applies for the non–expected utility setting. Lifetime utility is maximized by equating the expected growth rate $\tilde{\psi}$ *with* $\tilde{\psi}^*$*, which can then be solved for the optimal* $\tilde{\delta}^*$*.* □

Status preferences are able to compensate for the inefficiency arising from the technological spillover effects. As long as growth in the competitive economy is suboptimally low, welfare grows with an increase in the status parameter. In this our results for the stochastic economy coincide with the ones of Corneo and Jeanne (1997), but contrary to the deterministic model, we have to impose a non–negativity condition on the certainty equivalent to capital return, or an upper bound on the size of the risk premium respectively.

The dashed lines in Figures 1(a) and 1(b) represent efficient growth and welfare of the stochastic (black) and the deterministic economy (grey). While the growth rate increases with a rise in δ according to Proposition 1, welfare reaches its maximum in δ^*, where $\psi = \psi^*$. Growth is dynamically inefficient for higher values of the status parameter, and welfare in the competitive economy declines.

Moreover, it becomes obvious that preferences for status cannot compensate the utility loss resulting as the consequence of an uncertain environment. Welfare in the deterministic society exceeds welfare of the stochastic one. We can now compare the results referring to the optimal status degree of the stochastic economy with the corresponding result under certainty ($\sigma^2 = 0$) and find

Proposition 3 (Optimal status degree under risk and certainty) *The welfare maximizing degree of status preferences in a stochastic economy* $\delta^*, \tilde{\delta}^*$ *is smaller than the associated value* δ_s^* *of a deterministic environment, if*

$$\begin{aligned} &(i) \qquad \beta + \tfrac{1}{2}A(\rho - 1) > 0 \quad \textit{for} \quad \rho = \tfrac{1}{\varepsilon}\,, \\ &(ii) \qquad \varepsilon\beta + \tfrac{1}{2}A(1-\varepsilon) > 0 \quad \textit{for} \quad \rho \neq \tfrac{1}{\varepsilon}\,. \end{aligned} \tag{24}$$

Condition (i) is satisfied for all $\rho \geqslant 1$, *while condition (ii) is satisfied for all* $\varepsilon \leqslant 1$, *both denoting the empirically more plausible cases.*

Proof: By taking differences $\delta_s^* - \delta^*$ *and* $\delta_s^* - \tilde{\delta}^*$ *respectively.* □

The result of Proposition 3 can be easily explained, if we recall the second component of the expected growth rate, which captures the consumer's response to risk. This expression is always larger in the suboptimal allocation of the market economy, compared to the Pareto–efficient one. The reason for this lies in the fact, that the households of the decentralized economy not only underestimate capital productivity but also the risk associated with capital incomes. A correct perception of the volatility of capital returns would give rise to a stronger intertemporal substitution effect, thereby causing lower expected growth. So the, risk–induced component already takes on a larger value than necessary in case of precautionary savings, thus driving the expected growth rate closer to its efficient value. Consequently, a lower degree of status desire is sufficient to achieve optimal growth.

5 Summary of Results

In this paper we discussed a stochastic endogenous growth model with externalities in human capital accumulation extended by a preference for social status, which is measured by relative wealth. Although, in an macroeconomic equilibrium with identical households, no one succeeds in his desire to outperform others, the agents ignore this aspect of the status game, which introduces a second externality into the model. The prospect of yielding a higher social status in the future by postponing current consumption, provides an incentive for the single household to increase accumulation.

A large part of the paper was devoted to the analysis as to what determinants of long–run expected growth are effected by status concerns, since this still is a point of disagreement in the literature. While deterministic approaches claim, that status preferences alter the effective intertemporal elasticity of substitution, the change is often assigned to the effective degree of risk aversion in a stochastic context.

The disagreements arise from a major characteristic of expected utility theory in time–separable dynamic models, which is, that it does not allow for a separate treatment of the individual attitude towards risk and the willingness to substitute consumption over time. This problem was avoided in our paper by assuming the recursive form of preferences developed by Epstein and Zin (1989), which disentangle the effects from risk aversion and intertemporal substitution. We found that status preferences do not change the individual degree of risk aversion, if the household is uniformly risk averse towards consumption risk and wealth risk. Apart from the net return to physical capital, the desire for social status changes the intertemporal elasticity of substitution.

In this context we examined the tradeoff relationships between the parameters which determine the expected growth rate of the economy. We were especially interested in the question whether the status parameter serves as a complement or a substitute for the degree of risk aversion and risk itself. The latter is true for the case of precautionary saving.

Since our framework exhibits two externalities — the first from the production technology implying suboptimal low growth, the second from the status quest, implying suboptimal high growth — the last part of the paper examined the question, if there is an optimal degree of desire for social reputation. We showed that the presence of status preferences can correct the distortion arising from knowledge spillovers. There exist an interior solution for the marginal valuation of status needs, such that the Pareto–efficient allocation is supported.

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Appendix

Let $J[k(t),t]$ denote the maximum feasible level of lifetime utility for a time t capital stock of $k(t)$. Given (9), by application of Itô's lemma, the objective function of a typical household is

$$\max_{c} \quad \frac{1-\rho}{1-\frac{1}{\varepsilon}}\left[c\left(\frac{k}{K}\right)^{\delta}\right]^{1-\frac{1}{\varepsilon}} - \beta G(.) + G'(.)\left[J'(k)\frac{\mathrm{E}(dk)}{dt} + \frac{1}{2}J''(k)\frac{\mathrm{E}(dk)^2}{dt}\right] \quad \text{(A.1)}$$

Differentiating (A.1) with respect to c and k yields

$$0 = (1-\rho)\, c^{-\frac{1}{\varepsilon}} \left(\frac{k}{K}\right)^{\delta\left(1-\frac{1}{\varepsilon}\right)} - G'(.)\, J'(k)\,, \tag{A.2}$$

$$\begin{aligned} 0 = G'(.)\, J'(k) \Bigg\{ & \frac{\delta(1-\rho)}{k G'(.)\, J'(k)} \left[c \left(\frac{k}{K}\right)^{\delta} \right]^{1-\frac{1}{\varepsilon}} - \beta + \alpha A \left(\frac{k}{K}\right)^{\alpha-1} + \frac{\rho - 1/\varepsilon}{1-\rho} \times \\ & \times \left[\frac{J'(k)}{J(k)} \frac{\mathrm{E}(dk)}{dt} + \frac{\sigma_k^2}{2} \frac{J''(k)}{J(k)} \right] + \frac{J''(k)}{J'(k)} \frac{\mathrm{E}(dk)}{dt} + \frac{1}{2} \frac{J''(k)}{J'(k)} \frac{\partial \sigma_k^2}{\partial k} + \frac{\sigma_k^2}{2} \frac{J'''(k)}{J'(k)} \Bigg\}\,, \end{aligned} \tag{A.3}$$

where the derivative of the function $G(.)$ with respect to $J(k)$ is given by

$$G'(.) = (1-\rho)\, [(1-\rho) J(k)]^{\frac{1-1/\varepsilon}{1-\rho} - 1}$$

We postulate a value function of the form

$$J[k(t), t] = \frac{\mu^{\frac{1-\rho}{1-\varepsilon}}\, k(t)^{1-\rho}}{1-\rho}, \tag{A.4}$$

which takes account of the fact, that in equilibrium $k = K$. Substitution of (A.4) and the associated derivatives $J''(k)$ and $J'''(k)$ into (A.2) results in the familiar relationship

$$c = \mu k \tag{A.5}$$

Substitution of (A.5) and (A.4) into (A.3) leads to the equilibrium value (11) of the propensity to consume, which, employed in the market clearing condition (6), finally implies the expected growth rate of the economy (12).