The correlation dimension of returns with stochastic volatility

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Abstract

One of the well-known stylized facts of financial returns data is that their correlation integrals can exhibit scaling similar to that of deterministic time series. An early report of this fact dates back to Scheinkman and LeBaron (1989), who found scaling (up to a certain length scale) in correlation integrals of returns of the CRSP index.

Historically, the application of correlation integrals to financial data was motivated by the desire to detect the presence of determinism, and hence short-term predictability, in financial data. The correlation integral was expected to show scaling only for deterministic time series, no scaling for IID noise, and approximate scaling for deterministic time series with small noise. Later results indicated that approximate scaling of correlation integrals can be reproduced by stochastic models with conditional heteroskedasticity, even in the absence of a deterministic component. It is therefore not surprising that correlation integrals as means of detecting predictability in financial data were largely abandoned since.

However, the consequence is that some important issues are left unanswered, such as: by which mechanisms can volatility clustering lead to scaling of correlation integrals? In case it does, since randomness is usually thought of as being incompatible with exact scaling, can the the scaling be exact or is it at best approximate? What interpretation does an estimated correlation allow in this context?

In this paper, it is shown that the answers to these questions depend on the time interval between consecutive observations. Clearly, if scaling is to be found, it can occur only at short time scales. Returns over long horizons become independent, so that scaling is no longer present and correlation integrals behave as those of IID observations.

For short horizon log-returns of stochastic volatility models, it is shown analytically that low-dimensional scaling of correlation integrals indeed occurs, under the condition that the marginal distribution of the time dependent volatility is not bounded away from zero. In that case the correlation integrals of short horizon returns behave identical to those of deterministic time series. This implies that the usual model for correlation integrals of deterministic time series apply in this case, and estimation methods from deterministic dynamical systems theory can be used to estimate the scaling exponent (correlation dimension). The scaling exponent is directly related to the behavior of the marginal volatility distribution for small volatilities.

Moreover, in the more general case where the marginal volatility distribution is bounded away from zero, it is shown that approximate scaling occurs in the short horizon limit. In that case, correlation integrals of short horizon returns are found to behave identically to those of deterministic processes with observational noise. The scaling exponent in the general case can be estimated using the gaussian kernel algorithm which was developed in dynamical systems theory to estimate correlation dimensions of deterministic time series with observational noise. The latter estimation method is illustrated with several empirical applications. Although the model is derived only for the limit where the time intervals between consecutive returns tend to zero, the model turns out to describe correlation integrals of financial data well, already for returns on time scales as long as one trading day.