Demographic and Economic Uncertainties in a large scale computable OLG model.

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Abstract

This paper consider aggregate risks in a large-scale overlapping-generations equilibrium model that is calibrated to France demographic and economic properties. Two distinct sources of uncertainty are assumed : fertility and technological shocks. The paper considers two different issues. The first part of the paper is devoted to a standard RBC analysis. The main finding is that when both sources of shocks are considered together the correlation between hours worked and productivity of hours worked of the model reproduces the one of the French data. In the second part of the paper we compare the effects of adopting different rules of adjustment to insure budget equilibrium of the Pay-as-you-system. This analysis first shows that defined-contribution rules induces more volatility of input but defined-benefits rules imply greater volatility of consumption relative to output. It also indicates that consumption risk-sharing among cohorts vary both with respect to the origins of the surrounding uncertainties and to the the chosen adjustment rule.

Keywords : Stochastic births, Overlapping generations, Business cycles, Social security, CEGM.

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1 Introduction

In most of European continental countries the unfunded public pension system for retirement constitutes the largest part in the government budget.

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Due to this importance and to the major pressures that ageing will impose on it, analysis of social security is now becoming a major concern in economic research. Abstracting from its existence in calibrated overlapping-generation (OLG) models may then lead to some misleading appreciations.
Most recent studies of the future of national pension schemes rely on mechanical projections of demographics, on the one hand, and of the macroeconomic environment on the other hand. Insofar as the present study aims at analysing the interactions between demographic changes and economic variables, it seems appropriate to rely on Modigliani's life-cycle hypothesis of saving and to use the general-equilibrium, OLG framework, as proposed by Samuelson [1958] and amended by Diamond [1965] in a growing economy with production, capital accumulation and government debt. The same classical model is used by Feldstein [1974] to analyse the effect of an unfunded pay-as-you-go (PAYG) system on capital accumulation in a deterministic context. These, by now familiar, theoretical and tractable models have inspired applied developments that have been used increasingly in recent years to study the prospects of national pension schemes in large scale OLG models. However, many such studies, starting with the pioneering work of Auerbach and Kotlikoff [1987] (AK) on the US economy, use a deterministic, multi-cohorts OLG model to analyse more accurately the effects of PAYG on macroeconomic variables as well as accommodation of a foresighted demographic baby boom-baby bust shock.
Subsequent works modify the AK-model by adding others features in order to make it more realistic. More recent research on social security issues now incorporate in this setup various sources of uncertainty, heterogeneity as well as market imperfections in computable OLG models[1].

Recently some authors have developed tractable stochastic OLG models to analyse, in environments with aggregate technological as well as birth rates shocks, the effect of social security rules on risk-sharing between generations (Bohn [1999] or Diamond [1997]), on equity premium (Abel 1999b) or on stock prices (Abel 1999a). This paper considers this issue of macroeconomic risks-sharing between generations within a fully specified stochastic, general equilibrium model populated at each period by 105 cohorts. This large-scale overlapping generation model, *à la* Auerbach and Kotlikoff [1987], is calibrated to reproduce French Economy main characteristics. Two distinct sources of macroeconomic uncertainty are considered in the model : produc-

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[1] For presentations of various issues of this kind and of large scale OLG methods one can see the survey of DeNardi et al. [2001], for formal presentations of various models of this kind one can read the survey of Imrohoroglu et al. [1998].

tivity shocks which affect both the wage rate and the real rate of return on capital and birth rate shocks that affect the relative size of cohorts trough time. No heterogeneity within each cohort is assumed.
Before thinking about risk sharing the paper deals more specifically with standard RBC analysis on replicating and explaining the actual fluctuations of French economy. Our mains original findings are :

1. Fluctuations in Solow residuals can account for more than 90 % of the variance of output whereas births rates shocks only explain the smaller part.
2. Most of the fluctuations in hours worked can be explained by changes on the population structure resulting from birth rates shocks.
3. When both sources of uncertainty are considered together the correlation between hours worked and productivity of hours worked of the model reproduces successfully the actual correlation of the French economy.

The third point can be understand as follow : birth rates shocks implies complex waving dynamic on population structure and then on potential active population. This leads to a strong negative correlation between hours worked and productivity of hours worked because as output does not vary very much labor is much more volatile after demographic changes. As a result because Solow residual shock implies a positive strong correlation between hours and their productivity and because of point 2. the resulting correlation with both shocks is rather small like in the data.

With a PAYG social security system the replacement rate for retirement benefit and the contribution rate are linked each other through the time-to-time balanced budget of the social security and depends of the ratio of active population to retirees. When a demographic shock occurs, some adjustments in replacement and/or contribution rates must then be achieved to restore equilibrium : it is nothing all but a question about burden-sharing (redistribution) between actives and retirees. Before such a shock appears, things are somewhat different, the question is what rules of the pension scheme implies the better risk-sharing between generations? In a similar perspective productivity risks may also be allocated inequitably and inefficiently between cohorts : think about shock on interest rates, because young, mature and retirees have not the same assets accumulation profile there are not affected in the same way. A last example when there is an age-specific productivity of labor profile as in our model such that wage income differs with age for a given unit of time worked an aggregate shock on labor productivity would have different impact on cohorts wage income. In all these instances the nature of the social security rules will have to modify risk sharing (calculus of pension, indexation rule, share of fully funded system, adjustment of social security taxes ...). More globally a connected issue is how changes in public pension system rules affect the properties of the business cycles in the country? Rios-Rull [1996] adopts the financial structure of complete market in such a way that the different generations can collectively pool the cohort specific-risks. Social security in this context is useless regarding to aggregate risks. In this paper we do not have complete financial markets, then risk-sharing is not perfect and social security may have a role to pool consumption risk among cohorts. Rios-Rull [1994] shows that the assumed structure of financial markets is inessential to describe quantitatively the aggregate business cycle when the surrounding uncertainty arise from Solow residuals. As a matter of fact we do not really deals with the virtue of Social Security to ameliorate risk sharing in the absence of complete market. We do not deals either with full elimination of business cycles as in Storesletten et al. [2001] and other welfare aspects associated to imperfect risk-sharing among cohorts. Rather we will just use our model as an illustrative tools to draw some positive conclusions about how different rules of the PAYG system alter the business cycles characteristics and redistribute consumption risks among cohorts.

In European continental countries the connection between an individual's social security mandatory contributions and his subsequent retirement benefit is rather strong with respects for instance to Anglo-saxon countries. Thus the resulting distorsion in individual's labor supply decision over the life-cycle[2] is less accurate than in other countries. But on the opposite this will tend to reinforce the inequity of the system between large and small cohorts.

The plan of the paper is as follows. Section 2 presents the model. Section 3 discusses a number of aspects of the calibration process, performs some briefs steady state analysis and gives the deterministic transition path associated to a Solow residual shock and then to a birth rate shock. Section 4 examines macroeconomic cyclical properties of the model associated to demographic and productivity shocks. Section 5 compares cyclical properties of a variety of public pension system rules. They differ in what instrument is adjusted in order to time-to-time balance social security budget after productivity and/or birth rates shocks.

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[2] Feldstein [1996] also points out that distorsions on labor supply may be increased if the economy with an unfunded social security department will imply a large under-accumulation of capital in long run (with respects to a some kind of *golden rule*).

2 The Model

The model consists of overlapping generations of one-sex individuals with finite lifetimes and an infinitely lived government. Here it is reduced to a social security department, thus we abstract from government purchases as well as other taxes. We also assume a closed economy-framework. The period is set to be one year.

This model is an alternative version of the framework developed by Rios-Rull [1996] which is itself a stochastic variant of AK's model. Contrary to Rios-Rull we take into account social security department, fertility shocks and the costs of child rearing. An another extension is that we have a more realistic demographic structure with 85 adult cohorts instead of 55 as in Rios-Rull [1996] and AK. Individuals in a cohort are assumed to be identical and we abstract from idiosynchratic shocks. While we assume that there is perfect macro-economic risks-sharing within cohort we assume that there is imperfect risk-sharing across cohorts. Contrary to two previously quoted models the financial market is incomplete. First, there is no perfect annuities market to cover the risk of early death, rather we assume that there are accidental bequests that are redistributed back to the agents in a lump-sum fashion. It is know since Hubbard and Judd [1987] that in such context social security may present potential benefit in substituting for private annuity markets. Our second assumption is that trading of asset is sequential and the stock of capital is the only durable asset that can be traded between existing cohort excluding no born yet agents (there is no time-0 trading in Arrow Debreu dated securities).

We assume an exogenous growth rate γ of labor-augmenting technical progress and we abstract from exogenous trend of population growth in the long run. Rather the main concern of the paper is stochastic deviations around these trend. In other words we will examine shocks to fertility as well as shocks to output in order to analyse the cyclical behaviour of the model around a steady-state characterized by a stationary population.

2.1 Demographics

The economy is populated by overlapping generations of one-sex agent who may no live longer than 105 years. The number of people of age a at time t is denoted by $L_a(t)$ (for any variable, a subscript a denotes age and an argument t in parentheses denotes calendar time). At date t the number of births is denoted by $L_0(t)$ while total population is $L(t) = \sum_{a=0}^{105} L_a(t)$.

2.1.1 Mortality

People can die before 105 year ; let s_a the conditional probability of surviving between age a and age $a+1$. The number of age a people at time t evolves according to the standard law of motion :

$$L_a(t) = s_{a-1} \cdot L_{a-1}(t-1) \qquad a = 1, \ldots, 105 \tag{1}$$

Let $\lambda_{a-1} = \prod_{i=0}^{a-1} s_i$ the unconditional probability of being alive at age a, then we also have : $L_a(t) = \lambda_{a-1} L_0(t-a)$. This paper is focused on changes in fertility with time so, for a sake of simplicity, we assume that survival rates remain constant. The changes in cohort sizes $l_a(t) = L_a(t)/L(t)$ then only reflect time variation in population birth rates.

2.1.2 Fertility Process

Like in Rios-Rull [2001] we follow Lee [1974]'s procedure to estimate a process for fertility on the basis of a standard law of motion for births. With deterministic population, the number of births is equal to $L_0(t) = \sum_{a=15}^{50} f_a L_a(t)$, where f_a are the average age-specific fertility rates. We rely on standard assumption that women fertility occurs only between 15 and 50 years old. However here the birth rate is stochastic :

$$l_0(t) = \sum_{a=15}^{50} f_a l_a(t) + \Gamma^f(t) \tag{2}$$

where $\Gamma^f(t)$ is an error term that follows some ARMA process (estimated as an AR(2) below). To examine cyclical properties of the model around a steady-state, we have to specify a population structure that evolves with time around a stable population (as a matter of fact because we abstract from deterministic trend in population growth it is a stationary population). To achieve this stationarity fertility and survival rates have to satisfy the *Lotka condition*. Here this condition is satisfied by normalizing the components of matrix representing the law of motion of the deterministic population (fertility and survival rates) by the biggest eigenvalue of this matrix

2.2 The household sector

Individuals are assumed to become adults when they turn a_0. During any period, the household sector is then made of $105 - a_0$ overlapping cohorts of "adults", of age between a_0 and 105, and a_0 cohorts of "young". Adults may no stay in the labor force after a legal maximal mandatory retirement age $\bar{r}_a$ but in order to reproduce a realistic scheme of retirement decisions we

also suppose that they can partly be retired from a minimal retirement age of $\underline{r}_a$. Economic decisions are on consumption, leisure and saving, there are made under a rational expectation hypothesis at the beginning of the adult life. Between 15 and 50 yrs. adults are supposed to give birth to children, according to the previously defined fertility calendar. Children are dependent until they turn age a_0. Before a_0 they consume with a cost per child that is supposed to be proportional to the parents consumption. For simplicity we also assume that people under age of a_0 do not work.
Each new working generation can be represented by the behaviour of a representative household. Here agents are endowed with one unit of time per period that can be enjoyed as leisure $(1 - h_a)$ or can be supplied as worked hours $h_a \in [0, 1[$. The intertemporal preferences of a new entrant on working-life are given by the following life-time utility function over uncertain streams of consumption and leisure demands for its expected life[3] :

$$U(t) = E(t) \sum_{a=a_0}^{105} \rho^{a-a_0} \frac{\lambda_a}{\lambda_{a_0}} \frac{\eta}{\eta - 1} \left(C_a(t + a - a_0)^{\xi} \cdot (1 - h_a(t + a - a_0))^{1-\xi} \right)^{\frac{\eta-1}{\eta}} \tag{3}$$

where E(t) is the mathematical expectations operator conditional on age a_0 and time t information , ρ is the psychological discount factor[4], C_a is consumption at the age a and η is the intertemporal substitution rate (or the inverse of coefficient of relative risk aversion). There is no bequest motive but due to life uncertainty there are unintended bequests $B_a(t)$ that are taken as lump-sum by individuals[5]. At any given period, the budget constraint facing an age-a representative individual is (with additional constraints $S_{a_0-1} = 0$ and $S_{105} \geq 0$) for a = a_0, . . . , 105 :

$$\tau_a(t)C_a(t) + A_a(t) = Y_a(t) + (1 + r(t))A_{a-1}(t - 1) + B_a(t) \tag{4}$$

$$Y_a(t) = \begin{cases} W(t)(1 - \theta(t))h_a(t)\epsilon_a & \text{for} \quad a < \underline{r}^a \\ W(t)(1 - \theta(t))h_a(t)\epsilon_a + P_a(t)(1 - h_a(t)) & \text{for} \quad \underline{r}^a \leq a < \bar{r}^a \\ P_a(t) & \text{for} \quad a \geq \bar{r}^a \end{cases}$$

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[3] This class of utility function has been extensively used in similar context by Rios-Rull [1996] and [2001], DeNardi et al. [2001] or Imrohoroglu et al. [1998] because economy may be easily rewritten as stationary.

[4] Notice that the effective discount rate is equal to $\lambda_a \rho^{a-a_0}$, meaning that agents only care of their future as long as they stay alive. In other words the expectation takes into account that the agent can die before 105 yrs. old

[5] The lack of explicit bequest motive as well as preferences over children consumption may appear as a restriction but as a matter of fact empirical studies like Altonji et al. [1996] find very imperfect intergenerational links and risk sharing within families. Other assumptions will be examined in the last part of the paper.

where A_a denotes the stock of assets held by the individual at the end of age a and time t, $(1+r) \cdot A_a(-1)$ is financial income (real return on assets holdings times wealth), τ_a is the age-specific equivalence scale that takes into account the direct and indirect private costs of child-rearing, and Y_a is the non-assets net disposal income. For full-time active years ($a \in [a_0, \underline{r}^a[$) it is simply equal to the net labor income after social security taxes (at rate θ), where W is the real wage rate per efficient unit of labour at time t. When agent is partly retired ($a \in [\underline{r}^a, \bar{r}^a[$) he also receives a pension benefit P_a for the unworked hours. And when he is full-time retired ($a \in [\bar{r}^a, a_T)$) he only receives the pension benefit . In this paper, pension benefit is assumed to be age dependant first in order to take into account the indexing pension rule and second to specify some kind of specific rule for pension before $\bar{r}^a$ the maximum mandatory retirement age (see after the description of the public retirement system).

In order to calculate the relative cost of child-rearing τ_a for each cohort we use the age distribution of children for each parent (from their past fertility behaviour) and weighted it by the age-^{c} equivalence scale of children β^c, which will be assumed to be constant :

$$\tau_a(t) = 1 + \sum_{c=\max(0,a-50)}^{\min(a_0-1,a-15)} \beta^c \cdot l_a^c(t) \qquad a = a_0, \ldots, 50 + a_0 - 1 \tag{5}$$

where the average number $l_a^c(t)$ of children of age c raised by cohort of age a can be recover from past fertility evolutions and the early deaths :

$$l_a^c(t) = \frac{\lambda_{c-1} \cdot \lambda_{a_0}}{\lambda_{a-1}} \cdot f_{a-c} \cdot \frac{l_0(t-c)}{l_0(t-c) - \epsilon^f(t-c)} \tag{6}$$

The last term indicates that we assume that unexpected births are allocated between parents from 15-50 according to the same distribution that age specific fertility. For simplicity, the children depending of parents younger that a_0 years old are assumed to be “allocated” between the adults that have same age children (allocation with age-specific weights)[6]. Notice that for simplicity the model abstracts from time allocated to child-rearing.

As in Yaari [1965] we assume that, though individuals are uncertain about the length of their life, the population is large enough to ensure aggregate certainty over the population of each cohort. Contrary to Yaari [1965] there

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[6] Being more precise will need to conserve the distribution of child with respects to their grand-parents and will complicated in an useless way the number of *state variables.*

is no insurance companies or perfect and fair annuities market such that mortality risks are pooling within the same cohorts to cover the eventuality of early death[7]. We do not retain this assumption because it seem rather unrealistic for France first with regards to the actual volume of such contracts as documented in Gaudemet [2001], a weakness consecutive to the “imperfect nature” of such contracts due mainly to “self-selection problems” (Mahieu and Sédillot 2000). We then have to precise how is distributed the assets of dying people. Imrohoroglu [1998] choose that accidental bequests are taxed at 100 % and lump-sum rebated among all the survivors by government. For simplicity we also retain this assumption for the baseline model.
As discussed in Bohn [1999] the assumptions on unintended bequest, their distribution, and perfect annuities markets are a much more important element of the model when one considers random life survival rates which is not the case here. Even if the effects are small we have to precise that the distribution of accidental bequests will nevertheless matter, for instance if part of bequest are distributed to younger active cohorts they appear to be more sensitive to capital return risk than they would be in their absence.

An agent’s earning ability is assumed to be an exogenous function of its age. These skill differences by age are captured by the efficiency parameter ϵ_a which changes with age in a hump-shape way to reflect the evolution of human capital. For simplicity, we assume that this age-efficiency profile is time-invariant. With this specification shocks that will influence wage rate will then have different impacts on effective disposal income for different age. Here children matter for the analysis first because they provide notice about the size of the future labour force (survival table being fixed) and second because they affect the net resources needs available to their parents. In the following we assume that age-distribution of relative costs of child-rearing are constant but as long as cohorts size will change with fertility shocks the total costs of child-rearing $\tau_a(t)$ will vary with time. So for a given period part of this extra cost (i.e. changes in child costs relative to new birth) is unexpected and plays like unexpected changes in a ”consumption tax”.

2.3 Production side

Aggregate output, $Y(t)$, is produced according to a Cobb-Douglas, constant-return-to scale, technology (7) that combines aggregate capital stock installed at the beginning of time t ($K(t-1)$) and aggregate labor input ($N(t)$).

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[7] The Yaari’s assumption is retained in most of EGCM analysis for France (Chauveau and Loufir [1997] or Docquier et al. [2002]) as well as for other countries (Rios-Rull [1996] for US, Broer and Westerhout [1997] for Dutch, ...

$$Y(t) = e^{\Gamma(t)} K(t-1)^{\alpha} (N(t)(1+\gamma)^{t})^{1-\alpha} \tag{7}$$

where γ is the constant and exogenous rate of growth of labour productivity, $\alpha \in (0,1)$ is capital's share of output, and $\Gamma(t)$ is a multiplicative shock to technology that is observed at the beginning of the period. It consists of a simple persistent component, following standard RBC literature, that evolves according to an AR(1) process :

$$\Gamma(t) = \rho^{\Gamma} \Gamma(t-1) + \epsilon^{\Gamma}(t) \tag{8}$$

The random variable ϵ^{Γ} is assumed to be normally distributed with mean zero and standard deviation σ^{Γ}. Output can be used either for current consumption or for increasing next period capital stock. Aggregate capital is assumed to depreciate at the constant rate δ.

2.4 The public sector

The public sector is reduced to a social security department; it is an unfunded pay-as-you-go (PAYG) public pension scheme. The department collects payroll taxes on all labor incomes and pays pension benefits to retired households. We assume that retirement benefits may be divided in two parts : a lump-sump fashion benefit and an earnings-dependant pension there is an imperfect linkage between an individual's social security contribution and the present value of retirement benefits that may have distorsives effect on labour supply decision as in Auerbach and Kotlikoff [1987]. As a matter of fact in France retirement benefit is nearly contributive for most of people who have worked a full-time career. But the system is more complex part of this contributive pension is defined-benefit (the base pension) and part is defined-contribution (the complementary pension). Because of the existence of some bounding cells for contributive pension, of minimum income and disability pension for old people who are not eligible for full pension and also for child care benefits we can consider that the representative agent of a given old-age cohort has a two component old-age pension.

2.4.1 Brief description of the French Pension System

The model aims at reproduce the main French characteristics so it is important to describe as much as we can how the French pension system operates. Let briefly recall major institutional facts about it (see Blanchet and Legros [2002] for more details) : (i) its almost exclusive reliance on PAYG financing ; (ii) it is a very complex and un-unified system (according to origins of wage earners, and average level of income) ; (iii) its large generosity both in terms of replacement rates and low mandatory age of legal retirement. Two figures given in the official report Charpin [1999] help to explain this last point ; first the net replacement rate of the pension benefit on the first year of retirement is on average equal to 80 % of the last year of activity net-of-taxes wage rate (for a full-time career), and, second, the actual medium age of retirement is 59 yrs. old. This induce that public pensions actually accounts for 12.1 % of GDP in 1998 and it is expected to attains approximatively 16.5 % in 2040 with the maintains of actual rules and reforms (Charpin 1999).

Our model deals with cyclical behaviour around a long run stable path characterized by a stationary population, so we implicitly assumed that the demographic transition resulting from the actual ageing consecutive to the increase in life expectancy and to the 2^{nd} WW baby-boom has been achieved. So to be consistent with this framework we assume that the gradual reforms of the pension system put in place in the early 1990's have also attained maturity. We also assume that all workers are liable to the two-pillar scheme for the wage earners of the private sector[8].

2.4.2 The general basic scheme

The *general basic scheme* (CNAVTS) operate according to a defined-benefit rule : wage-earners contribute the fraction of their gross wage below the *social security contribution ceiling* and receive when they retire a pension benefit proportional both to the number of year they have contributed to the scheme and to a *reference wage*. As a matter of fact since 1993 only the “best” 25 years are retained in the calculus of this reference wage. The pension has a maximum of 50 % of the *reference wage*. Moreover, under the 1993's reform both worker's past contributions and pension benefit are indexed on prices instead of average wages. In practice, people may receive this pension once they attained the age of 60 but the full pension benefits necessitate a contribution record of at least 40 years. Because we assume that people enter in the labor market at the age of 20 and there is no unemployment everybody fulfills the latest condition in our model. This very close to 60.5, the observed average age of new beneficiaries of *basic scheme* in 2000 (COR 2002). More-

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[8] Actually these worker account for almost 70 % of the labor force, public sector employees account for 20 % and self-employed workers for 10 % (Blanchet and Legros 2002). As a matter of fact, civil servant pension system is more generous than private sector employees one is. But the system of self-employed workers is less generous, so roughly the differences are offsetting each others. The main drawback of this assumption is that no change is actually planned for civil servants replacement rates but it is now a major part of the political debate about pensions since 2003.

over because of the economic growth and human capital increasing profile the "best" 25 years used to compute the *reference wage* are simply given in our model by the last twenty-five years before eligible retirement age. At last because in France the *social security contribution ceiling* is very close to the average wage of the economy (5 % higher in 1998) and because in our model wages between 35 and 60 yrs. old are closed or over the average wage over the whole active population, we will take the average wage for the calculus of the basis of pension. All these elements give to us the following equations for the *general basic pension* P^b:

$$P_a^b(t) = \begin{cases} 0 & \text{for} \quad a < 60 \\ \pi \cdot \frac{1}{25} \sum_{i=1}^{26} \frac{\widetilde{W}(t-i)}{(1+\phi\gamma)^i} & \text{for} \quad a = 60 \\ \frac{P_{a-1}^b(t-1)}{(1+\Phi\gamma)} & \text{for} \quad a \geq 60 \end{cases} \tag{9}$$

where $\widetilde{W}(t) = \frac{W(t)N(t)}{\sum_{a=a_0}^{\bar{r}^a} L_a(t)}$ is the weighted average gross-of-contribution-wage income and Φ is the coefficient of indexation of benefit ($\Phi = 1$ indexation on price as in actual system since 1987). Notice that in accordance to the French *general basic scheme* people can not gain more than a fraction π of their *reference wage* even if they work more than 40 years.

2.4.3 The complementary scheme

The French *complementary schemes* (ARRCO plus AGIRC for executives) for private sector wage earners are purely contributive and organized as systems of notional accounts[9] : complementary pension benefits (P^c) are computed by multiplying the number of "*earning point*" (EP) that workers have accumulated during their whole career with the *actual value of the point* (VP). Because complementary benefit may be received once an individual fulfills the conditions for full *basic pension* we also retain the age of 60 for the minimal age to be eligible The point-basis is the gross wage income, contrary to the basic pension, individuals who work after 60 years still accumulate points, until they attain the maximum mandatory age of retirement and conserve the same number of points :

$$P_a^c(t) \;=\; \begin{cases} 0 & \text{for} \quad a < 0 \\ VP(t) \cdot EP_a(t) & \text{for} \quad 60 \leq a < \underline{r}^a \\ \frac{P_{a-1}^c(t-1)}{(1+\phi\gamma)} & \text{for} \quad a \geq \underline{r}^a \end{cases}$$

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[9] For each unit of work an active agent accumulates the right to an annuity called "point".

$$\text{where} \qquad EP_a(t) = \sum_{i=20}^{a-1} \frac{W(t-a+i)h_i(t-a+i)\epsilon_i}{\lambda(t-a+i)(1+\phi\gamma)^{t-a+i}} \tag{10}$$

with $\lambda(t)$ the "purchasing-price" of points which is different from the nominal Value of Points VP(t). According to ARRCO's projection for 2000-2040 done for the Charpin [1999] report this purchasing price is assumed to be proportional to the average wage in the future : $\lambda(t) = 1.25 \cdot \widetilde{W}(t)$[10].
Notice that according to the 1996's reform when an individual will retire each point it has purchased in the past will be valued from its date of purchase according to prices evolution and not on average wage evolution ($\phi = 1$). As a matter of fact indexation rules in complementary schemes fluctuates a lot, in 2001 the "purchasing-price" of point is now also indexed on prices, thus the wage income as well as the purchasing-price of points are again both indexed in the same way. Here for symmetry, we assume in the baseline case the same indexing rule for complementary pension benefit once maximum mandatory age is attained, than for basic pension benefit (as it was assumed in Charpin [1999]'s official projections). In the baseline case we will assume that VP remain constant.
When indexing rules are similar as in our baseline case the distinction between of between complementary and basic pension may appear to be quite useless. But we can see that a decrease in "VP" affects instantaneously both the current pension benefit at 60 yrs. and the past pension benefits whereas an increase in λ only affect future pension. In the basic scheme a decrease in π would reduce pension benefit at 60 yrs as well as future benefits but not the past one. So the effects of these two reforms are quite different. Moreover when the main adjustment variable to balance accounts of the *complementary schemes* is the current value of points as it has been the case in the 1990's, instead of the contribution rate, this scheme appear to be closed to be a "defined- contribution" scheme. In the theoretical literature such a system is shown to be more efficient in deterministic setting because it reduces the distorsions on intertemporal allocation of labor supply.

2.4.4 The minimal retirement (non contributive) scheme

At last, we assume that there exist a non contributive and independent of age pension benefit P^n proportional (with coefficient Ψ) to current average net-of-tax wage income. People can receive this kind of benefit for their fraction of time spent in non-working activities since the time they have reached the minimum legal age of retirement :

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[10] The coefficient 1.25 is called "*taux d'appel*". It traduces that since 1996 a contribution of 1 gives a corresponding "right to benefit" of only 1/1.25.

$$P_a^n(t) = (1-\theta(t))\widetilde{W}(t) \cdot \begin{cases} \Psi^{\underline{r}^a} & \text{for} \quad \underline{r}^a \leq a < 60 \\ \Psi^{60} & \text{for} \quad a \geq 60 \end{cases} \tag{11}$$

This kind of pension benefit is included in our model for two distinct purposes. First, it helps to reproduce intra-cohorts redistributive instruments and, second, it allows income granted to worker between the minimum age of retirement and the age of 60 years like : the various credits given by the basic and complementary schemes for non-contributory periods (child rearing plus time bonus for children, military service, early retirement and unemployment) ; there also exist non contributory financial benefits minimum retirement income and invalidity benefit[11] schemes (RMI, ASV, ...) as well as pre-retirement schemes.

Because the origins of the two kind of non-contributory benefits are different for people with age before and after 60 years old, the ratio of replacement of pension to average-net-of-contribution wage are assumed to be different before and after 60 : $\Psi^{\underline{r}^a} \neq \Psi^{60}$.

2.4.5 Social security budget constraints

The total pension benefit granted to an age-a individual is then given by the sum of the three kind of pension : $P_a(t) = P^b(t) + P_a^c(t) + P^n(t)$.

We assume as in French system first that the budget of the distinct schemes are time-to-time balanced, and second that the minimal and basic schemes are integrated, so let θ^b and θ^c be the specific contribution rates to basic and complementary scheme ($\theta \equiv \theta^b + \theta^c$), respectively, we have :

$$\theta^b wN = \sum_{a=\underline{r}_a}^{a_T} (1-h_a)(P_a^b + P_a^n)L_a \tag{12}$$

$$\theta^c wN = \sum_{a=60}^{a_T} (1-h_a)P_a^c L_a \tag{13}$$

In the baseline case, we will assume that the contribution tax rates to both retirement scheme[12] are adjusted at each time to insure a time-to-time balanced-budget rule (12)-(13) given the ratios of replacement, the value of the point and the indexation rules.

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[11] In practice invalidity pension benefit and preretirement are granted for people younger than 60 and are financed respectively by Health system, and Unemployment system but as it mainly concern older worker there are economically nothing than early retirement.

[12] Recall than in practice the complementary schemes adjust very frequently the value of the point instead of the contribution rate.

2.5 Equilibrium

We assume that productive capital is the only asset in this economy where its return is contingent upon to the realization of aggregate shocks. There is neither risk-less bonds and portfolio allocation problem as in Abel [1999b] nor a full set of *Arrow Debreu* securities as in Rios-Rull [1996].

Definition : Given the initial stock of capital $K(0)$, the initial distribution $\{A_a(0)\}_{a=a_0,..,104}$ of asset holdings, the initial structure of the population $\{P_a(0)\}_{a=a_0,..,105}$, the initial adult descendance $\{P_a(0)^c\}_{a=a_0,..,105;c=0,..,a_0}$, the technical progress $\{\Gamma_t\}_{t\geq 1}$ and a given social security policy $\{\theta^c, \theta^b, \pi, \Phi, VP, \Psi^{\underline{r}^a}, \Psi^{60}\}_{t\geq 0}$ that satisfy (12)-(13), a competitive equilibrium with an unfunded PAYG system for retirement is a set of sequences for factor prices $(\{W(t); r_t\}_{t\geq 1}$, of pension benefits $(\{P_a^c(t), P_a^b(t), P_a^n(t)\}_{t\geq 1; a\geq \bar{r}^a}$ and an allocation of quantities $(\{A_a(t), C_a(t), h_a(t)\}_{t\geq 1, a=20,..,105}$; $\{K(t), B(t), N(t)\}_{t\geq 1}$ such that :

(i) Individuals choose contingency plans for future consumption and leisure that maximize the expected value of life-time utility (3) under their intertemporal budget constraint (4), taking as given factors prices, social security instruments and unintended bequests. First order conditions yield, together with budget constraint, (time arguments are suppressed for ease exposition and primes are used instead to denote the next period's variables):

$$\begin{array}{rcll}
\dfrac{C_a^{1/\sigma}(1-h_{a-1})^{-(1/\sigma+1/\eta)}}{\tau_a} & = & E\left[\dfrac{\rho s_a R' C_{a+1}'^{1/\sigma}(1-h_{a+1}')^{-(1/\sigma+1/\eta)}}{\tau_{a+1}'}\right] & a\in]a_0, 105] \\
(1-\xi)\tau_a C_a & = & \xi(1-h_a)(1-\theta)w\epsilon_a & a\in [a_0, \underline{r}_a[\\
(1-\xi)\tau_a C_a & = & \xi(1-h_a)[(1-\theta)w\epsilon_a - P_a & a\in [\underline{r}_a, \bar{r}_a[\\
1-h_a & = & 0 & a\in [\bar{r}_a, 105]
\end{array}$$

where for simplicity we denote $\sigma = 1/(\xi(1-1/\eta)-1)$ as the intertemporal elasticity of consumption demand in the steady state[13].

(ii) where at the equilibrium the lump-sum accidental bequest received by any adult cohorts at the end of time t is given by :

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[13] As in Auerbach and Kotlikoff [1987] the retirement constraint (last line) implies to define shadow cost of leisure so as to set leisure to unity when retirement age is reached as well as non-negativity conditions on labor supply before maximum mandatory retirement age. For ease exposition we do not present these condition here but as a matter of fact we will always check in computations that such interior solutions hold as it will be in our baseline calibration.

$$B(t) = \frac{\sum_{a=a_0}^{104}(1-s_a)L_a(t)A_a(t)}{\sum_{a=a_0}^{104}(1-s_a)L_a(t)} \tag{14}$$

(iii) The profit-maximizing behaviour of the firm gives rise to first-order conditions which determine the real net-of-depreciation rate of return to capital and the real wage rate, respectively :

$$\begin{aligned} r(t) &= e^{\Gamma(t)}\alpha\left(\frac{k(t-1)}{(1+\gamma)N(t)}\right)^{\alpha-1} - \delta & (15) \\ w(t) &= e^{\Gamma(t)}(1-\alpha)\left(\frac{k(t-1)}{(1+\gamma)N(t)}\right)^{\alpha} & (16) \end{aligned}$$

where for convenience aggregate quantities in minuscule characters are variables adjusted for the exogenous trend of labour productivity.

(iv) The labour market is equilibrated (i.e. the aggregate labor supply is taken to be an efficiency and cohort-size weighted average of labor supply across households):

$$N(t) = \sum_{a=a_0}^{\bar{r}_a} L_a(t)h_a(t)\epsilon_a \tag{17}$$

(iv) as well as the aggregate capital market is :

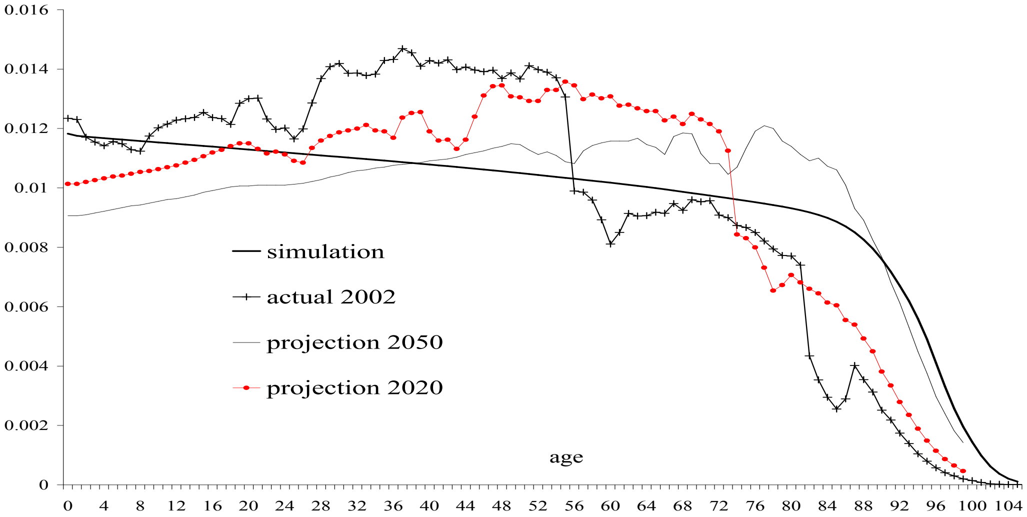
$$K(t) = \sum_{a=a_0}^{104} s_a L_a(t)A_a(t) \tag{18}$$

In this economy *Walras' law* insures that the good market is also equilibrated at the equilibrium : $I + C = Y$, where I is the gross-of-depreciation investment and $C = \sum_{a=20}^{105} \tau_a C_a L_a$ is the aggregate consumption.

3 Parameterization and Deterministic Simulations

Contrarily to Auerbach and Kotlikoff [1987], we do not assume that the economy is *dynamically efficient* in the absence of social security. We calibrate the economy in order to match French data do not care about if the resulting economy with the actual rate of growth of Harrod's technical progress together with the actual size of social security is *dynamically efficient* or not in a deterministic setting (we let this task for a further work).

Figure 1: Population Structure



Sources : Ined for “actual” structure and Insee for “projections”.

3.1 Calibration and simulation

3.1.1 Demographics

With respect to demographic variables we choose $a_0 = 20$, as a matter of fact the average age of entering in labor market is 22 in France (calculated on the basis of 1999’s Census data) but because in our model nobody is assumed to work before being adult it appears to be more convenient to lower this age.

The probabilities of surviving at each age are taken to be those of French female for 1996 from the estimates of Meslé and Vallin [2001]. These data give the mortality table for the cohort born in 1996 (extrapolated longitudinal data). The age-specific fertility rates f_a used to estimate the process for fertility (2) are the average rates over the period 1901-1997 calculated with Daguet [2002]’s data. To estimate the process with the underlying assumption of a stationary population these fertility rates have been corrected by the biggest eigenvalue of the population transition matrix (see above) jointly implied by average fertility rates and 1996 female mortality rates. As a matter of fact, the implicit assumption of a constant and zero rate of population growth assumed in our model in the long run matches the Insee latest official population forecast over the years 2000-2050.
The figure 1 reports the stationary population structure implied by both these adjusted fertility and mortality rates together with a zero growth rate of population. For comparisons, we also report the actual structure of female

population structure in 2002 (INED data) and the corresponding "official" forecasted structures for 2020 and 2050 (Insee). It appears that the model structure obtained on the basis of longitudinal data look like to the projected structure for 2020 (excepted for younger age because official projections assume a lower fertility rate than we have done). So for the remainder of the paper we think as if the initial steady state were the year 2020. In other words variables very depend of the demographic structure like PAYG instruments will be considered as if we were in 2020.

The residual process $\Gamma^f(t)$ for fertility is estimated as a univariate process. With previous adjusted fertility rates and historical data on 1901-2001 (from Insee) we find an AR(2) process (as Lee [1974] found for USA and Rios-Rull [2001] found for spain)[14] : $\Gamma^f = \rho_1^f \Gamma^f(-1) + \rho_2^f \Gamma^f(-2) + \epsilon^f$. The coefficients estimated (std. errors in parenthesis) are ρ_1^f = 1.067048 (0.099512) and ρ_2^f = -0.176581 (0.099747) with a R^2-statistic equals to 0.7845. The implied standard deviation of the innovation to the fertility shock is $\sigma^f = 0.001192$.

The equivalence-scale coefficient β^c that takes into account the direct and indirect costs of child-rearing are taken from estimation done by Hourriez and Olier [1997]. These figures are only given for 5-years-age-groups, we simply assume that these costs are identical across the different years of the age-group. Hourriez and Olier [1997] give the private costs induced by an extra-individual according to its age, so in order to have the good relative costs we have to adjust the cost of an extra children with respect to the cost of an extra adult. This calculus give to us the following constant parameters : $\beta^{c=0,..,4} = 0.27$, $\beta^{c=5,..,9} = 0.25$, $\beta^{c=10,..,14} = 0.41$ and $\beta^{c=15,..,19} = 0.64$[15].

3.1.2 Economic Parameters and steady-state values

The rate of growth of technical progress γ is set to 1.7 % per year as in Charpin [1999]. In order to calibrate the model in a consistent way we have next to match the capital and its accumulation as they are conceived in our one sector-growth model to those measured in the French Annual National Accounts (FANA), we use the new Insee database on ESA-95 basis. For doing this we follow Cooley and Prescott [1995]'s procedure : we determine capital accumulation as the sum of private and public gross investment but

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[14] As a matter of fact, an ARMA(1,1) process also gives good and very similar results but for symmetry with other studies we choose the AR(2) process.

[15] A more rigourous treatment of data would take into account public relative cost of children-rearing like education here this consumption is uniformly distributed over all individuals.

we also add as in the changes in inventories, the consumption of durable goods and the net export. To be consistent with this implicit notion of the stock of capital we have then to impute in output the flow of service from durable goods. As we lack of data concerning the stock of durable and its specific rate of depreciation we simply retain from Cooley and Prescott [1995] calculus on US economy that the rate of depreciation of durable is equal to 0.21 with the assumption that we are initially at the steady state (i.e. St. durables = $(1+\gamma)\times$ Cons. durables $/(\gamma+0.21)$)) give to us that the stock of durables to output ratios is equal to 4.97 % on average over the year 1978-2001. To impute the corresponding flow of service of durable in output we simply assume that in the spirit of our model the net return to investment in durables is the same that to other capital investment (equal to 7.044 % on average over the years 1978-2001). With these elements we obtain on the basis of FANA's data a share of GNP going to investment of 24.7 %, an output share of labor income of 41.3 % and a capital to output ratio of 2.95 (on average over the years 1978-2001). In this study we take $1-\alpha$ = 0.59, notice that this value also correspond to this calculated by Prigent [1999] over 1960-1997, It also roughly corresponds to the value calculated by Cooley and Prescott [1995] for the US economy (0.6).

Here the depreciation rate of the capital δ and the subjective discount factor ρ are set to get values similar to those given by data for the previous capital to output ratio and investment to output ratio. One can also consider to fix the value of the discount factor estimated by Hurd [1989] on US economy where mortality risk is accounted separately is 1.011 as in Rios-Rull [1996] as a matter of fact it is a better way to let this parameter adjust for a French economy because Hurd's estimates do not appear to be adapted for a country with a large public PAYG pension system. As a matter of fact, the resulting calculated parameter shown in Table 1 is also what Hairault et al. [2003] have taken. The resulting depreciation rate and the rate of return of assets holdings in the baseline steady state appear to be only slightly higher than those calculated directly from the data, respectively 6.05 % and 7.04 % (1978-2001 average, calculated from FANA database).

The intertemporal elasticity of substitution is set to 0.8 (in absolute value) a standard value for French economy following Letournel and Schubert [1991] when labor supply decision is endogenous. It is between the value of 0.5 used by Rios-Rull [1996] and Hairault et al. [2003], for US and French economies respectively, and the value of 1 used by Cooley and Prescott [1995].
The coefficient ξ of leisure vs consumption is set to imply an average, on cohorts between 25 and 54 years, fraction of time devoted of work in the initial

Table 1: Baseline Calibration

Calibrations Targets

$\frac{K}{Y}$=2.95	$\frac{I}{Y}$=24.7%	$\frac{\sum_{a=25}^{54} h_a L_a}{\sum_{a=25}^{54} L_a}$=32%
$\frac{\sum_{a\geq 60}(1-h_a)L_a P_a^c}{\sum_{a\geq 60}(1-h_a)L_a(P_a^c+P_a^b)}$=42%	$\frac{\sum_{a=\underline{r}^a}^{60}(1-h_a)L_a P_a^n}{Y}= 0.5\%$	$\frac{\sum_{a\geq 60}(1-h_a)L_a P_a^n}{\sum_{a\geq 60}(1-h_a)L_a(P_a^n+P_a^b)}$=15%

Deep Parameters

σ= - 0.8	α=0.41	γ=1.7%	a_0=20 yrs.	$\bar{r}^a$=65 yrs.
$\beta^{c=0,..,4}$=0.27	$\beta^{c=5,..,9}$=0.25	$\beta^{c=10,..,14}$=0.41	$\beta^{c=15,..,19}$=0.64	$\underline{r}^a$=55 yrs.
ρ_1^f=1.067	ρ_2^f=-0.177	σ^f=0.0094	ρ^Γ=0.965	σ^Γ=0.0012
ϕ=1	$\frac{\sum_{a\geq 60}(1-h_a)L_a(P_a^c+P_a^b+P_a^n)}{Y}$=14%		$\epsilon_a = e^{0.05(a-20)-0.0006(a-20)^2}$	

Calibrated parameters and steady state values

ρ=0.9605	π=27.4 %	$\Psi^{\underline{r}^a}$=13.6%	Ψ^{60}=3.9%	VP=0.9%
ξ=0.308	η=0.2355	δ=6.8%	r=7.3%	Y/h=7.44
$\frac{\sum_{a=20}^{65} h_a L_a}{\sum_{a=20}^{65} L_a}$=27.9%	θ^c=9.9%	θ^b=14.6%	N/L=0.165	B/Y=1.9%

steady state of 0.32. This fraction of time is obtained by microeconomic evidence from time allocation study Dumontier and Pan Ké Shon [2000]. From this database we have calculated that individuals between 25 and 54 years allocate 31.76 % on average of their discretionary time (time not spent in physiological activities which is 11h37 mn for people) to effective labor (abstracted from formation and travel to work time)[16].
For the life cycle of efficiency units of labour we used the profile estimated by Miles [1999] for U.K. workers : the log of age-specific part of labour productivity is 0.05× age $-0.0006\times$age^2. For calibration purpose we also normalize efficiency units in terms of efficient unit at the age of 20 yrs.

In France the minimum age for be eligible to retirement is actually 55 years, from this age and until the minimum mandatory age of retirement to receive the *basic pension* of 60 years. If the corresponding incomes are not really called retirement benefit the effective status of these individuals is retired : pre-retirement, invalidity, unemployed aged-people exempted of seeking a job, . . . for this reason we choose $\underline{r}^a = 55$ in our model.
The normal age of retirement for new retirees is 60 and the maximum manda-

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[16] This may appear to be a rather strong fraction of time but as a matter of fact the corresponding measure for the age-group 15 to 64 yrs. falls to 25.1 % in France.

tory age is $\bar{r}^a = 65$ (actual legislation).
The share of GDP going to total public pension retirement benefits is equal to 12.6 % in 2000 but following our previous discussion our model seem to be fitted to 2020 demographic structure. According to the COR [2002] projection at this date the share of GDP going to total public pension for person above the age of 60 will be 14%. In 2000, the direct costs of early retirement schemes before the age of 60 roughly amounts to 0.5 % of GDP. In 2000 the share of total pension (here complementary plus basic schemes pension benefits for private wage earners) going to *basic scheme* is roughly 58 % (COR 2002). Among the expenses of the *basic scheme* 15 % are associated to non-contributory benefits (CNAV 2002) [17].
To be consistent with these figures and other steady states values we let some variables of the pension system adjust : the ratio of replacement Ψ^{60} is such that total non-contributory pension after 60 years (11) equals to 15 % of the total basic pension benefits paid to people over 60 yrs. old ; the ratio of replacement $\Psi^{\bar{a}^r}$ (**??**) is such that total non-contributory pension for individual between 55 and 59 equals to 0.5 % of the GDP ; π, the fraction of the *reference wage* used to calculated the contributive basic pension in (9) is such that total pension versed to retiree above 59 yrs are equal to 14 % of GDP ; the contribution rate θ^b is such that (12) is satisfied when $\phi = 1$; the contribution rate θ^c is such that the resources of complementarity system in the steady state ($\theta^c wN$) equals 42 % of total retirement benefits received by retirees older than 59 yrs. ; and at last the steady-state value of the point VP such that (13) is satisfied.

We do not estimate the real process (8) for the Solow residual because in an OLG model we need age specific hours worked to obtain the good measure for Solow's residual. We simply used standard Solow's procedure to obtain the shocks from the residual with aggregate hours worked (still from Insee database). Takes the log values for hours worked, product and capital stock at annual frequencies for 1970-2001 we obtain a value of $\sigma^\Gamma = 0.01139$ for standard deviation and $\rho^\Gamma = 0.938$ for the auto-covariance factor. These

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[17] This figure is rather easy to obtain because since 1993 a special agency called FSV (*fond de solidarité vieilliesse*) has been created especially to finance these non contributory periods to the *general basic scheme* as well as the minimum retirement income. In 2000 the resources from the FSV is equals to 14.9 % of the total of FSV funding plus basic contributions (CNAV 2002). Similar non contributory expenses for the *complementary scheme* are difficult to isolate. But they are relatively small, for instance we know that 2.8 % of total expenses of *complementary scheme* are due to child-rearing with respect to 10 % for the *basic scheme*. It is for this reason that we have assumed that non-contributory expenses are only paid by our basic system.

value are slightly different from Borgy and Hairault [2001] who find 0.009 and 0.95 respectively, the reason certainly lies in the fact that we use the new INSEE database. Notice that this process is less volatile than the one used by Rios-Rull [1996] from US data with $\sigma^{\Gamma} = 0.024$ but more persistent $\rho^{\Gamma} = 0.814$. This is already noted by Hairault [1992] in a paper that compares a standard RBC for both French and US economy.

3.1.3 Computational issues

The computational problem is solved by a standard linear quadratic approximation method of the agents policy rule in the spirit of Sims [2002] and Klein [2000]. We did this with the stochastic simulation tools of the program "*Dynare*" developed by Juillard [2003] and (2001). Due to the very large scale of the model[18] it will be extremely cumbersome and very error-prone to write one by one the equations in computer code. But OLG models contains numerous repetitions of the same kind of equations with only slight changes (in age subscripts) so we have developed a companion program written in the programming language "*PYTHON*" to generate automatically the equations codes that are then used as input in "*Dynare*" in order to simulate the model. Before analyse second moment properties of the model we will need to apply the Hodrick-Prescott (HP) filter. Since the method adopted do not resort to simulations but calculate exact policy function we can directly apply the HP filter. Rather, we use the frequency domain technique describe in Uhlig [1999] to obtain filtered moments without the need for any simulations[19].

3.2 Steady State analysis

3.2.1 Baseline Case

As we have already see the model is calibrated to match key ratios and reproduce fairly well the rate of depreciation and the net return to capital. But the model fails to reproduce the pattern of hours worked for the younger people, more precisely it always overstates their participation rates (if the model would take into account time devoted to child rearing it would probably present a better profile for age-specific hours worked). From this concern the value of $\sigma = -0.8$ seems preferable to a lower value because it helps to

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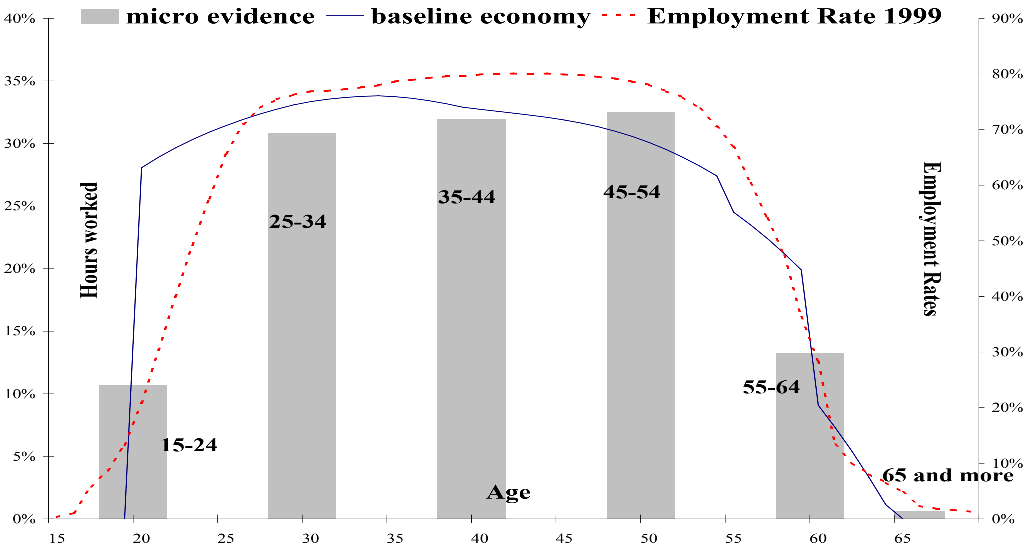
[18] Because of population and descendant variables we have actually 1116 equations with 843 state variables.

[19] To calculate numerically the matrix spectral density of the HP filter and of the policy functions we use a grid of 128 points. For a better precision 512 points would be more satisfactory but it is to computer-resources consuming with regards to the effective gain of precision.

Figure 2: Hours worked in the baseline in the French Data and Employment Rates

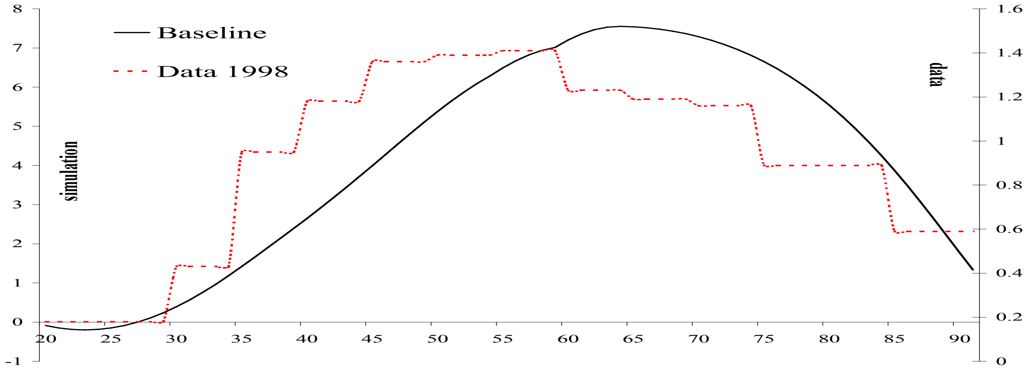


Sources : Insee, 1999's Census for employment rates and Dumontier and Pan Ké Shon [2000] for hours worked (micro-data)

reproduce the growing activity between 20 and 35 yrs observed in the data (see the discussion and Figure 4 below). For the older workers, the model fits very well the profile from the data. This is resulting from our precise description of the whole French PAYG system, indeed Docquier et al. [2002] assume in a model similar to ours that the whole public retirement scheme as the form of the non-contributory basic pension benefit 11, their resulting age-specific labor supply profile then do not presents the decrease in the participation after 55 yrs.

The simulated profile of asset holdings per age presented in figure 3 also differs slightly from the observation in 1998 (the units of account is different). First there appear to be no debt accumulation at the beginning of the active life in the data (as a matter of fact other studies found this). Second if the two profiles are rather similar there is some kind o delay in peak years of accumulation (3 or 4 years) in the simulation. The absence of a bequest motives can not explain this, indeed Docquier et al. [2002] consider bequest motive in a model similar to us for french economy and our corresponding chart presents the same shift between observed age-wealth curve and simulated one. Many things may explain this, among them we consider a an active lifetime beginning in 20 yrs old and lasting in 60 yrs, in France actually the beginning is around 22 yrs and last around 59 yrs ; a second thing

Figure 3: Wealth Profile per age in the baseline and in the French Data (ms en 1998 FF



Sources : Insee, Enquête Patrimoine 1998

is that life-tables are different.

3.2.2 Alternative Calibration

The Table 2 presents the key steady-state variables under various alternative choice of deep parameters. With a lesser intertemporal elasticity of consumption demand in absolute value ($\sigma = -0.5$ as in Rios-Rull [1996] or Hairault et al. [2003]), the model generates a huge decrease in the capital to output ratio (16.5 % lower than in the baseline case) and a corresponding increase in the net rate of return to assets holdings (38 % greater). The stock of capital is lower because households are more willing to smooth their consumption over their lifetime. As a matter of fact it acts like a decrease in the effective discount rate faced by individuals. Indeed when one considers a higher value of the subjective discount factor (with the value of ρ=1.011 estimated by Hurd [1989]) the agents are more patient and the economy generates a dramatic increase in the capital and investment to output ratios.
To isolate the role of the intertemporal elasticity of consumption demand we now consider (column 4 of Table 2 the same lower elasticity but with ρ fixed in a way such initial K/Y remains at its baseline calibration value. Now the effects on the key steady state ratios are nil. The main change is to lower the average fraction of time devoted to working and particulary among people between 30 and 54 yrs. (as indicated also on figure 2).
Other parameters have an important influence on activity behaviour (and then on initial contribution rates) like replacement ratios or point value but more of all like ξ the parameter that shifts the relative weight of consumption versus leisure. Adopting a smaller share of output going to capital income also affects wage and capital to output ratio. On the contrary a lower de-

Table 2: Alternative Calibration : Steady - State Analysis

	baseline	σ	ρ	σ:-0.5	α	δ	π	VP	Ψ^{60}
		0.5	1.011	ρ:0.984	0.36	0.05	0.26	0.855%	0.03
N	0.164	0.154	0.197	0.161	0.165	0.163	0.165	0.165	0.165
Y / h	7.446	6.516	10.28	7.38	5.55	8.15	7.45	7.45	7.45
θ	0.246	0.264	0.205	0.245	0.269	0.238	0.239	0.24	0.24
r	0.073	0.101	0.020	0.073	0.072	0.074	0.073	0.073	0.073
w	3.587	3.164	4.975	3.587	2.899	3.926	3.592	3.591	3.592
$\bar{h}^{\star}$	0.279	0.263	0.336	0.276	0.279	0.276	0.28	0.28	0.28
K/Y	2.95	2.462	4.722	2.95	2.604	3.359	2.956	2.955	2.955
I/Y	0.247	0.206	0.395	0.247	0.218	0.221	0.247	0.247	0.247
C/Y	0.753	0.794	0.605	0.753	0.782	0.779	0.753	0.753	0.753

Note : Except the parameters explicitly defined the others parameters are those of the baseline economy presented in Table 1. * here $\bar{h}$ is the fraction of time spent working among the people of working age which is different from h the total hours worked.

preciation rate exclusively influences capital to output ratio and allocation between consumption and investment with very few impacts on hours.

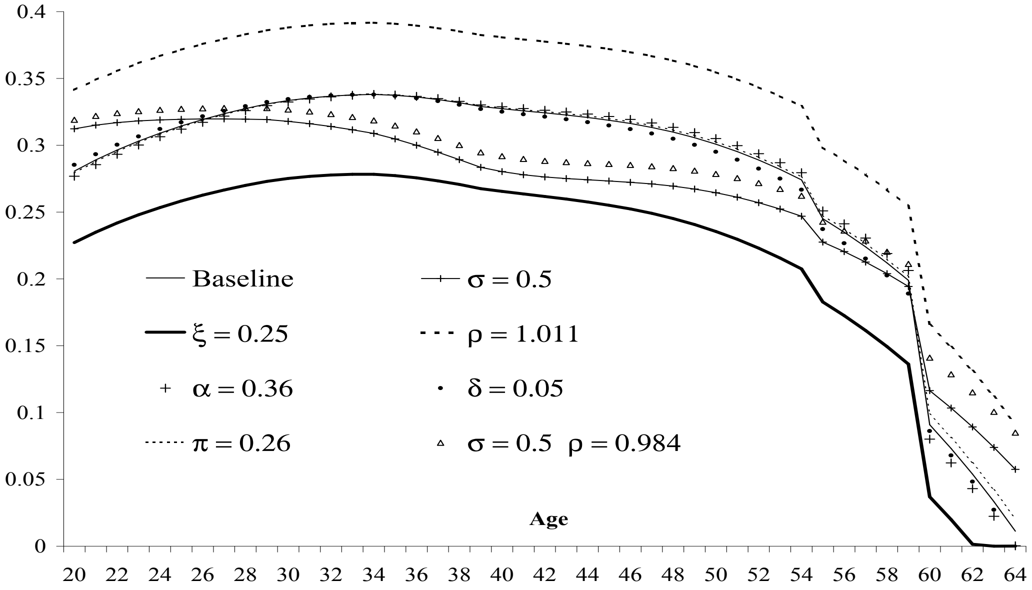
3.3 Dynamics analysis

It is important to see that fertility and productivity shocks are very different in terms of uncertainty across time. Because individuals are born before they enter to labor force the agents are aware of most the economic changes implied by unexpected births at date t (abstracted from the current extra cost of rearing these unexpected new children which is relatively weak). In other words once the fertility shock occurs, its consequences on the size of future working-age population, potential future ratios of dependency, . . . are well appreciated by agents with rational expectations (at least when survival probabilities are known). The technological shock on the contrary implies a lot of surprises in the current period.

3.3.1 Responses to the technological shock

Figures 5 report the dynamic of aggregate (growth adjusted) real variables per capita, hours worked and factor prices consecutive to a 1 percent productivity shock ϵ^{Γ}. At the beginning all aggregate quantities are increasing. After roughly more than 100 years they return to their initial value, this highly persistent positive effect is explained by the high estimated value of the autocorrelation parameter of the technological shocks.

Figure 4: Hours worked by age under various alternative calibration



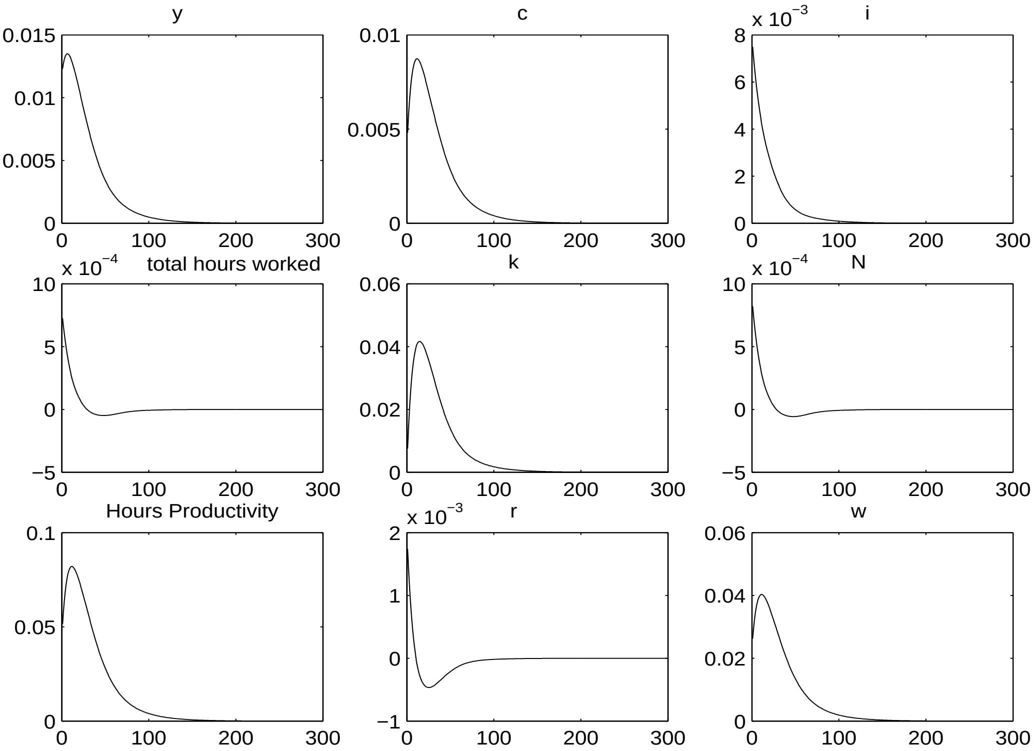
The whole dynamic of this economy is very closed to the dynamic in standard representative agent economy as presented for instance in King, Plosser and Rebelo [1988a] for US economy and Hairault [1992] for french economy.

3.3.2 Responses to the fertility shock

Figures 6 report the dynamic of economic variables following a 1 percent fertility shock ϵ^f. These responses appear to be very more complexes than after a technological shock. The main reason lies in the dynamic of population structure which display complex roots as an echo of the complex dynamic of birth rates l_0 as indicated in Figs 7. As in France recent history (since the 2^{nd} world war) the baby-boom (Γ^f) almost vanished after 20 years. But because of the fertility dynamic (2) the initial baby boom is followed after 20 periods by another but smaller baby boom, associated to the echo of the initial greater number of children, like what happened for France in the mid 1960's. After births oscillate like a in a vanishing wave until it moves to the initial births number. As a result the whole structure of population admit cyclical changes (but with delay regarding to ageing of extra born people). Then the total population oscillates : growing for roughly 65 years then decreasing after (when the initial baby boomers will begin to massively deceased).
The corresponding dependency ratio (population under 20 adjusted from relative cost plus population over 65 over total population between 20 and

Figure 5: Baseline Economy : impulse response functions to a technological shock

(Deviations from initial steady state)



65 yrs) then fluctuates a lot across time : it is increasing in first periods, because there is more children, then it decreases when these people become adults and increases again after 65 periods as these individuals progressively enter into retirement age.

As a consequence output and consumption follow the waving profile of the potential working age population. There is almost no jump at initial time, birth rate shocks only affect slightly cost of children and then allocation of consumption as would done a very small increase in a consumption tax.

Figure 6: Baseline Economy : impulse response functions to a fertility shock

(Deviations from initial steady state - Economic variables)

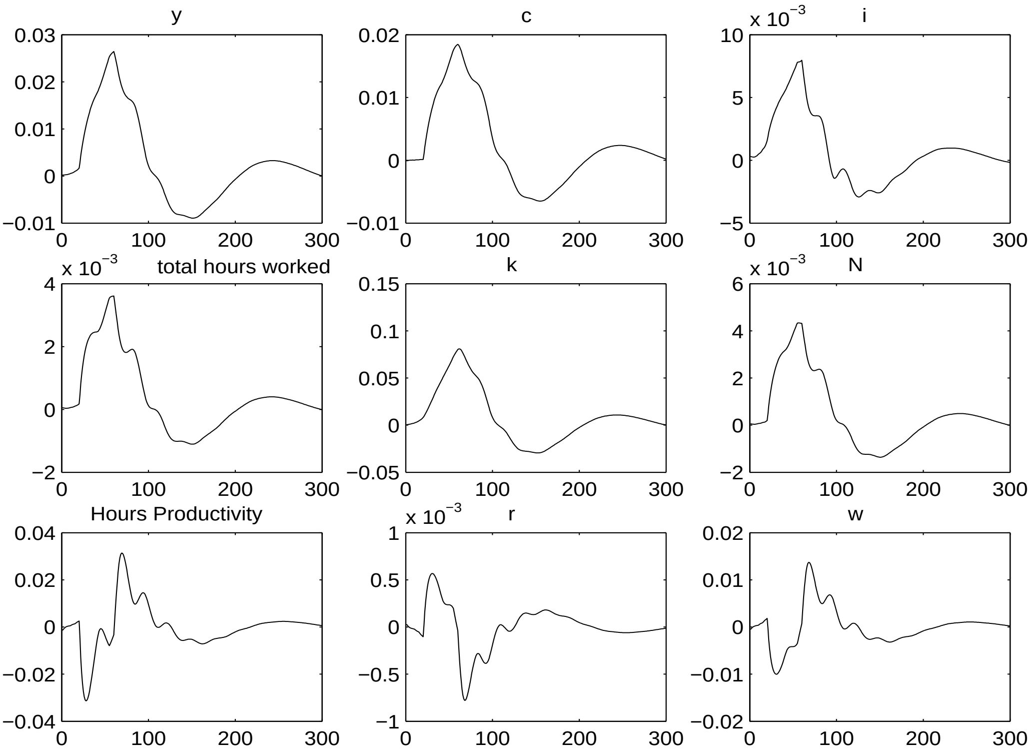
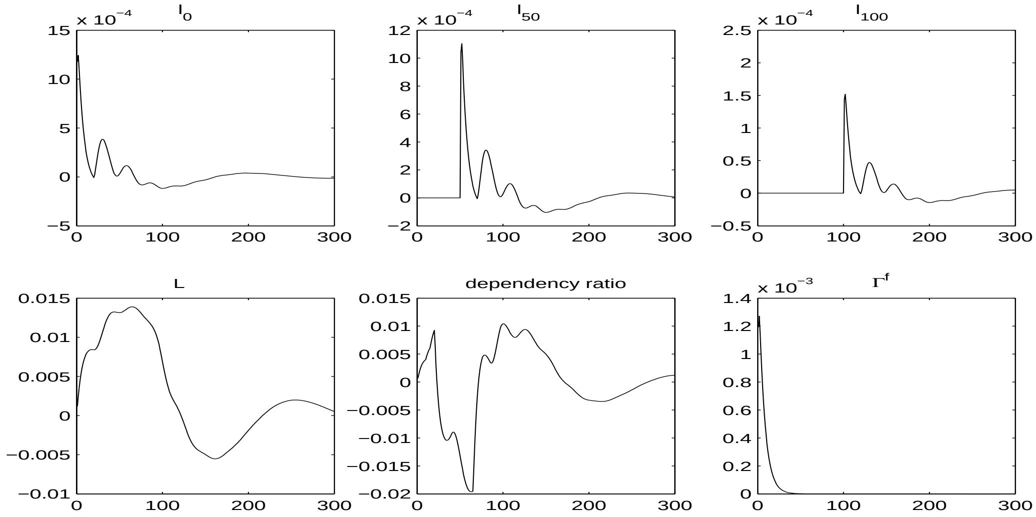


Figure 7: Baseline Economy : impulse response functions to a fertility shock

(Deviations from initial steady state- Demographics)



4 Cyclical Properties

As long as the different cohorts have the same relative risk aversion the relative volatility of the consumption for each cohort at date t would be perfectly correlated in an perfect *Arrow-Debreu* environment (Bohn 1998). Here the discrepancy then traduce inefficiencies in intergenerational risk-sharing induced by incomplete financial markets.

Table 3: Cyclical Behaviour of the real French Economy : Current moments

(Deviations of variables from trend 1970-2001)

	GDP	Cons	Durables	Non Dur	Inv.	Hrs	Hrs Prod.	Cap
St dev %	1.55	1.06	5.5	0.82	5.22	1.12	0.94	0.98
St Rel to GDP	1	0.69	3.54	0.52	3.36	0.72	0.6	0.63

Current cross-correlation

	GDP	Cons	Non Dur.	Durables	Inv.	Hrs	Hrs Prod.	Cap
GDP	1	0.8	0.75	0.78	0.91	0.8	0.7	0.4
Total consumption	0.8	1	0.98	0.82	0.74	0.65	0.55	0.38
Non-durable Cons.	0.75	0.98	1	0.71	0.68	0.56	0.57	0.44
Durable Cons.	0.78	0.82	0.71	1	0.77	0.71	0.44	0.14
Tot. Investment	0.91	0.74	0.68	0.77	1	0.81	0.55	0.25
Total Hours	0.8	0.65	0.56	0.71	0.81	1	0.14	0.31
Hours Prod.	0.7	0.55	0.57	0.44	0.55	0.14	1	0.3
Capital Stock	0.4	0.38	0.44	0.14	0.25	0.31	0.3	1

Source : Insee, French Annual National Accounts, ESA-95 basis. “Hrs.” = total hours worked, “Hrs Prod.” = average hours productivity, “Cap.” = Net Capital Stock included Administration.

Table 3 reports main business cycle statistics for the French Economy over the period 1970-2001. These includes standard deviations and cross-correlations of GDP, total consumption including its durables, total consumption of durables, total consumption of non durables plus services, total investment (excluding durables but including government) and the corresponding capital stock and total hours worked (all variables comme from Insee National Accounts on ESA-95 basis). The data, in annual and real terms, have been filtered, after taking logs, by using the Hodrick-Prescott filter (with a value of 100 for the smoothing parameter).

The table shows that all these variables are pro-cyclical. Investment is more volatile than output, which is itself more volatile than consumption ; total hours worked are slightly more volatile than the average productivity of hours worked. A standard and important feature in RBC studies is that the correlation between hours worked and hours productivity is very low. Here contrary to the quarterly business cycle features describe in Hairault [1992] the value remains positive this is due to differences in frequency of the two studies. Borgy and Hairault [2001] also find positive and very low correlation with annual data but with a different database (OECD). We also state that the magnitude of fluctuations in output hours worked and capital are closed compared to investment of both producer and consumer'durables that are very volatile. Consumption of nondurables and services fluctuates two times less than output.

Table 4: Cyclical Behaviour of the real French Economy : Across-Time moments

(Deviations of variables from trend 1970-2001)

Auto-correlation

lags	-4	-3	-2	-1
GDP	-0.4	-0.16	0.17	0.61
Total consumption	-0.25	-0.1	0.15	0.56
Total Investment	-0.48	-0.22	0.23	0.7
Total Hours	-0.39	-0.21	0.1	0.61
Hours Productivity	-0.31	-0.16	0.11	0.52
Capital Stock	-0.17	0	0.32	0.47

Cross-correlations of GDP with

time	-3	-2	-1	0	1	2	3
Total consumption	-0.3	0.08	0.58	0.8	0.52	0.31	0.05
Non-Durables	-0.35	0.01	0.5	0.75	0.55	0.41	0.16
Durables	-0.1	0.3	0.67	0.78	0.29	-0.04	-0.24
Total Investment	-0.01	0.32	0.69	0.91	0.54	0.08	-0.31
Total Hours	-0.04	0.3	0.64	0.8	0.38	-0.04	-0.25
Hours Productivity	-0.22	-0.07	0.25	0.7	0.56	0.33	0.04
Capital Stock	-0.39	-0.3	-0.12	0.4	0.54	0.53	0.36

Source : INSEE, Annual National Accounts, ESA-95 basis.

The table 4 reports auto-correlations across periods of the same variables as well as the correlations of aggregate variables with real output, including three periods of leads and lags. It shows that investment as well as hours lead the cycle while capital stock follows it.

4.1 Fluctuations caused by shocks to productivity

Table 5 shows the results of simulating our artificial economy with baseline parameters when economy is only hit by technology shocks. The output fluctuates less than the French economy : 73 % of the variance of output can be explained by variations in Solow residuals. As already remarked by Rios-Rull [1996] this is less than can do a representative-agent model (see for instance Borgy and Hairault [2001] for this kind of model for French economy). But it is nevertheless an interesting feature for French Economy, because Hairault [1992] shows that, on the contrary to US studies, simulating a standard King et al. [1988a] model for France overstates the volatility of output from data. So from this perspective an OLG with a large social security department model seems more adequate to represent french business cycle than a representative-agent model.

Table 5: Technological shocks : Current moments

(deviation of variables from trend - quantities per capita and growth adjusted)

	y	c	i	h	N	y/h	k	w	r
St. Dev.	1.131	0.617	2.808	0.507	0.469	0.649	0.44	0.685	2.305
Rel. to y	1	0.546	2.482	0.448	0.415	0.574	0.389	0.605	2.038

Current cross-correlation

	y	c	i	h	N	y/h	k	w	r
y	1	0.965	0.984	0.973	0.971	0.983	0.571	0.987	0.93
c	0.965	1	0.903	0.877	0.875	0.997	0.767	0.995	0.8
i	0.984	0.903	1	0.998	0.998	0.936	0.417	0.943	0.98
h	0.973	0.877	0.998	1	1	0.914	0.364	0.922	0.99
N	0.971	0.875	0.998	1	1	0.912	0.36	0.92	0.991
y/h	0.983	0.997	0.936	0.914	0.912	1	0.711	1	0.847
k	0.571	0.767	0.417	0.364	0.36	0.711	1	0.697	0.229
w	0.987	0.995	0.943	0.922	0.92	1	0.697	1	0.858
r	0.93	0.8	0.98	0.99	0.991	0.847	0.229	0.858	1

The table 6 reports auto-correlations. The autocorrelograms are well reproduced both in timing and in magnitudes. On the contrary we state that consumption lead more the cycle than investment, it is the opposite in the data. Here also the phase shift between hours and output from the model is not satisfactory.

We find the standard RBC result that Solow residual shocks implemented in our OLG economy can not explained the weak correlation between hours worked and their productivity. But as with a representative agent model the variance of quantities are correctly ordered. The relative volatility of investment to consumption is around 5 like in the data. But every thing being equal all variables are less volatile than in the data and relative to the output and especially hours worked. All variables are pro-cyclical with strongest correlation than in the data. It is logical since the coefficient of autocorrelation of the Solow residual is very large.

Table 6: Technological shocks : Across-Time moments

(deviation of variables from trend - quantities per capita and growth adjusted)

	Auto-correlation					Cross-correlations of y with				
lags	-1	-2	-3	-4	-5	-1	-2	-3	-4	-5
y	0.505	0.148	-0.083	-0.213	-0.265	0.505	0.148	-0.083	-0.213	-0.265
c	0.593	0.253	-0.001	-0.17	-0.264	0.631	0.335	0.101	-0.066	-0.17
i	0.47	0.107	-0.115	-0.229	-0.265	0.400	0.017	-0.203	-0.303	-0.318
h	0.468	0.105	-0.117	-0.231	-0.266	0.365	-0.024	-0.241	-0.331	-0.336
N	0.468	0.105	-0.117	-0.231	-0.266	0.362	-0.027	-0.244	-0.333	-0.337
y/h	0.559	0.213	-0.032	-0.186	-0.263	0.594	0.277	0.043	-0.112	-0.199
k	0.848	0.556	0.234	-0.049	-0.262	0.737	0.684	0.518	0.313	0.117
w	0.552	0.205	-0.039	-0.189	-0.263	0.585	0.264	0.029	-0.123	-0.207
r	0.462	0.098	-0.123	-0.233	-0.265	0.268	-0.131	-0.331	-0.393	-0.366

4.2 Fluctuations caused by changes in population structure after fertility shocks

Fertility shocks considered alone explain less than one quarter of the volatility of output from data (Table 7). Now the consumption is relatively more volatile relative to output than in the data, the opposite is true for investment. This tend to indicate that investment is less a buffer against population structure changes than for technology shocks. The main insight is about hours volatility. Here the relative volatility of hours to output is more than the double of than in the data. Then hours volatility is greater than productivity of hours worked, a stylized fact that cannot be reproduce with Solow residual shocks.

A very interesting features is that hours and productivity are negatively and almost perfectly correlated each other. As a matter of fact productivity (as well as wage rate) is counter-cyclical when fertility shocks occur. The reason is obvious, output is not very influenced by changes in population structure whereas effective hours worked are and both of them are pro-cyclical.

Another feature is important although logical is the rate of return and the wage rate are strictly opposite in the correlogram.

Table 7: Fertility shocks : Current moments

(deviation of variables from trend - quantities per capita and growth adjusted)

	y	c	i	h	N	y/h	k	w	r
St. Dev.	0.356	0.308	0.754	0.651	0.552	0.32	0.227	0.227	0.631
Rel. to y	1	0.865	2.117	1.826	1.55	0.897	0.638	0.638	1.772

Current cross-correlation

	y	c	i	h	N	y/h	k	w	r
y	1	0.883	0.812	0.966	0.967	-0.852	0.632	-0.782	0.782
c	0.883	1	0.444	0.830	0.809	-0.704	0.627	-0.581	0.581
i	0.812	0.444	1	0.814	0.841	-0.752	0.428	-0.772	0.772
h	0.966	0.83	0.814	1	0.996	-0.958	0.422	-0.905	0.905
N	0.967	0.809	0.841	0.996	1	-0.948	0.416	-0.916	0.916
y/h	-0.852	-0.704	-0.752	-0.958	-0.948	1	-0.154	0.97	-0.97
k	0.632	0.627	0.428	0.422	0.416	-0.154	1	-0.020	0.02
w	-0.782	-0.581	-0.772	-0.905	-0.916	0.97	-0.02	1	-1
r	0.782	0.581	0.772	0.905	0.916	-0.97	0.02	-1	1

Table 8: Fertility shocks : Across-Time moments

(deviation of variables from trend - quantities per capita and growth adjusted)

	Auto-correlation					Cross-correlations of y with				
lags	-1	-2	-3	-4	-5	-1	-2	-3	-4	-5
y	0.918	0.734	0.506	0.275	0.062	0.918	0.734	0.506	0.275	0.062
c	0.907	0.7	0.45	0.206	-0.006	0.877	0.762	0.59	0.401	0.221
i	0.914	0.741	0.533	0.320	0.11	0.664	0.455	0.234	0.027	-0.158
h	0.906	0.699	0.452	0.210	-0.004	0.822	0.581	0.313	0.061	-0.155
N	0.907	0.703	0.459	0.219	0.006	0.823	0.584	0.317	0.066	-0.15
y/h	0.9	0.683	0.425	0.177	-0.037	-0.649	-0.365	-0.073	0.183	0.384
k	0.962	0.854	0.69	0.486	0.259	0.763	0.826	0.822	0.76	0.653
w	0.903	0.692	0.443	0.2	-0.016	-0.562	-0.27	0.022	0.271	0.461
r	0.903	0.692	0.443	0.2	-0.016	0.562	0.27	-0.022	-0.271	-0.461

4.3 Fluctuations with both sources of uncertainty

When both sources of shocks are considered the model economy now accounts for 76.5 % of the volatility of the French output (Table 9). This slight improvement is not the main point of the paper. The last part of the Table reports the variance decompositions for key variables in order to precise the forces that drive the business cycle.

Table 9: Two shocks : Current moments and variance decomposition

(deviation of variables from trend - quantities per capita and growth adjusted)

	y	c	i	h	N	y/h	k	w	r
St. Dev.	1.186	0.690	2.907	0.825	0.725	0.723	0.495	0.721	2.389
Rel. to y	1	0.582	2.452	0.695	0.611	0.610	0.417	0.608	2.015

Current cross-correlation

	y	c	i	h	N	y/h	k	w	r
y	1	0.942	0.97	0.799	0.821	0.728	0.571	0.819	0.917
c	0.942	1	0.832	0.775	0.782	0.661	0.738	0.763	0.759
i	0.970	0.832	1	0.759	0.790	0.725	0.409	0.801	0.966
h	0.799	0.775	0.759	1	0.996	0.17	0.352	0.313	0.776
N	0.821	0.782	0.79	0.996	1	0.210	0.352	0.345	0.803
y/h	0.728	0.661	0.725	0.17	0.210	1	0.535	0.986	0.62
k	0.571	0.738	0.409	0.352	0.352	0.535	1	0.585	0.198
w	0.819	0.763	0.801	0.313	0.345	0.986	0.585	1	0.702
r	0.917	0.759	0.966	0.776	0.803	0.620	0.198	0.702	1

Variance decomposition (in percentage)

	y	c	i	h	N	y/h	k	w	r
ϵ^{Γ}	90.97	80.04	93.27	37.79	41.87	80.47	78.93	90.09	93.02
ϵ^{f}	9.03	19.96	6.73	62.21	58.13	19.53	21.07	9.91	6.98

The main fact of our analysis is that the model now successfully reproduces the actual correlation between hours worked and productivity of hours worked reported in Table 3. Not surprisingly, 90 % of the output volatility is due to Solow residual fluctuations. What is new here is that changes in population structure are the a more dominant source of hours worked volatility than technology shocks. As a result the first ones induces counter-cyclical changes and the second pro-cyclical changes then the simulated 0.17 correlation coefficient between hours worked and their productivity is almost equal to what the data shows (0.14).

Other observed moments are also fairly well reproduce by the model like the relative volatility of consumption to output to be compared with the relative consumption of non durables with output reported in Table 3 ; the relative volatility of total hours worked to the output ; the correlation between hours and output. But the investment and then the stock of capital still appear to be too few volatiles and the correlation between output and consumption too strong.

Table 10: Two shocks : Across-Time moments

(deviation of variables from trend - quantities per capita and growth adjusted)

	Auto-correlation					Cross-correlations of y with				
lags	-1	-2	-3	-4	-5	-1	-2	-3	-4	-5
y	0.542	0.201	-0.03	-0.169	-0.235	0.542	0.2	-0.03	-0.169	-0.235
c	0.656	0.343	0.089	-0.095	-0.212	0.656	0.388	0.165	-0.003	-0.115
i	0.5	0.15	-0.072	-0.192	-0.24	0.42	0.051	-0.169	-0.277	-0.305
h	0.74	0.475	0.237	0.043	-0.103	0.409	0.124	-0.067	-0.18	-0.234
N	0.723	0.452	0.217	0.031	-0.108	0.412	0.117	-0.078	-0.19	-0.242
y/h	0.626	0.305	0.057	-0.115	-0.219	0.422	0.189	0.027	-0.072	-0.119
k	0.872	0.619	0.33	0.063	-0.152	0.73	0.694	0.553	0.370	0.189
w	0.587	0.253	0.01	-0.151	-0.239	0.477	0.213	0.029	-0.086	-0.143
r	0.493	0.139	-0.083	-0.203	-0.248	0.291	-0.099	-0.306	-0.383	-0.374

5 Alternative Rules of the pension system for retirement

5.1 Age specific variables volatility

In this section we analyse how different rules of adjustment in social security system modify the relative volatility of variables. In this experiment the variables of interest are the relative volatility of age non-assets income (y_a) between cohort and the relative volatility of consumption per age. The table 11 reports such figures. It shows that under the baseline simulation the consumption of mature people (40 yrs old here) is third time more volatile that consumption of 80 yrs old people. So the baseline public pension system seem to report risk from retirees to active people, indeed at 60 yrs the consumption volatility decreases. On the other side consumption variance of new adult is also lower than for 40 years old people, this is because there are not subject to capital income risk and that because their age-specific productivity is lower than 40 yrs old people there are less subject to wage income risk, but there are still support much risk than retirees because contribution rates are volatile whereas most part of pension benefits are not. As a matter of fact we can observe that capital income and smoothing behaviour plays an important role, because non-assets income is more volatile for the high savers population (40 and 60 yrs old) than for young adult and old retirees. But asset income volatility is growing with age.

Table 11: Other Variables of interest : Current moments

	y	c	c_{20}	c_{40}	c_{60}	c_{80}	y_{20}	y_{40}	y_{60}	y_{80}	θ
St. Dev.	1.186	0.69	1.011	1.197	0.744	0.306	1.224	1.23	2.055	0.172	1.395
Rel. to y	1	0.582	0.853	1.01	0.628	0.258	1.032	1.038	1.733	0.145	1.176

Current cross-correlation

	y	c	c_{20}	c_{40}	c_{60}	c_{80}	y_{20}	y_{40}	y_{60}	y_{80}	θ
y	1	0.942	0.94	0.878	0.932	0.434	0.92	0.971	0.95	0.816	-0.901
c	0.942	1	0.89	0.922	0.947	0.581	0.81	0.883	0.819	0.693	-0.821
c_{20}	0.94	0.89	1	0.785	0.948	0.497	0.902	0.97	0.949	0.809	-0.786
c_{40}	0.878	0.922	0.785	1	0.832	0.458	0.669	0.785	0.728	0.593	-0.753
c_{60}	0.932	0.947	0.948	0.832	1	0.629	0.886	0.932	0.879	0.771	-0.794
c_{80}	0.434	0.581	0.497	0.458	0.629	1	0.438	0.447	0.348	0.437	-0.346
y_{20}	0.92	0.81	0.902	0.669	0.886	0.438	1	0.966	0.96	0.851	-0.82
y_{40}	0.971	0.883	0.97	0.785	0.932	0.447	0.966	1	0.981	0.862	-0.86
y_{60}	0.95	0.819	0.949	0.728	0.879	0.348	0.96	0.981	1	0.847	-0.817
y_{80}	0.816	0.693	0.809	0.593	0.771	0.437	0.851	0.862	0.847	1	-0.824
θ	-0.901	-0.821	-0.786	-0.753	-0.794	-0.346	-0.82	-0.86	-0.817	-0.824	1
w	0.819	0.763	0.921	0.632	0.875	0.494	0.889	0.894	0.90	0.702	
r	0.917	0.759	0.827	0.769	0.746	0.127	0.826	0.881	0.921	0.774	-0.836

Variance decomposition (in percentage)

	y	c	c_{20}	c_{40}	c_{60}	c_{80}	y_{20}	y_{40}	y_{60}	y_{80}	θ
ϵ^{Γ}	90.97	80.04	94.49	63.05	92.51	85.99	89.17	95.13	97.49	76.6	59.5
ϵ^{f}	9.03	19.96	5.51	36.95	7.49	14.01	10.83	4.87	2.51	23.4	40.5

As a matter at a time t the incomes of the different cohorts are not perfectly correlated, so there are potential trade-off between risk sharing across cohorts than can be altered when the rules of the social security are modified. Let now study this issue.

5.2 Alternatives rules of the social security system

Let consider three alternative fashion to insure the time-to-time balance of social security constraint.

Rule1 : the rule on complementarity scheme is such that the contribution rate θ^c stay constant, at its baseline level, and that the value of point VP is the instrument that adjust in order to meet the constraint (13), the rules of the basic pension scheme stay those of the baseline.

Rule2 : the rule on basic scheme is such that the contribution rate θ^b stay constant, at its baseline level, the coefficient π in the calculus of contributory basic pension benefit adjusts in order to meet the constraint

(12), the rules of the complementary pension scheme and of the non-contributory basic pension benefit stay those of the baseline.

Rule3 : the rule on basic scheme is such that the contribution rate θ^b stay constant, at its baseline level, the ratio of replacement $\Psi^6 0$ in the calculus of the non-contributory basic pension benefit adjusts in order to meet the constraint (12), the rules of the complementary pension scheme and of the contributory basic pension benefit stay those of the baseline.

So all these new rules of adjustment describes some kind of defined contribution reinforcement of the whole social security system but they differs in the kind of pension benefit that is affected.
First of all, Table 12 indicates that, whatever be the source of the surrounding uncertainties, under the defined contribution rules Rule1, Rule2 and Rule3 the volatility of the output becomes smaller than in the baseline defined benefits economy. This reduction in the variance of y is higher with Rule2 and to a lesser extent with the Rule3 than with the Rule2, meaning that non-contributive defined contribution systems smooth more the fluctuations of GDP than contributive ones. In all these case investment appears to be less a buffer against consumption risk than in the baseline : the relative volatility of consumption to output increase under the three alternatives rules whereas it is the opposite for investment. The volatility of hours worked decreases under alternative rules when technology shocks occur whereas it increases when the uncertainty is on birth shocks.
In the case of technology shocks the Rule1 tends to re-allocate consumption risks from older people (60 and 80) to younger ones (20 and 40). In the case of demographic shock it is the opposite, active people people bearing more risk than in the baseline case. For the Rule2 most of the extra volatility of consumption, in comparison to baseline, is bear by older people under technology shocks. With surroundings uncertainties in population structure the accrued volatility of consumption is diluted in all cohorts and a little more to new retirees population. The case of the Rule3 is less ambiguous because whatever be the source of the shock the consumption risk of younger people always increase relative to those of the other cohorts.

Table 12: Volatility under various rules for PAYG system

	Std. Dev.	Standard Deviation relative to y										
	y	c	i	h	c_{20}	c_{40}	c_{60}	c_{80}	y_{20}	y_{40}	y_{60}	y_{80}
Solow Residual shocks alone												
baseline	1.131	0.546	2.482	0.448	0.869	0.841	0.633	0.251	1.022	1.061	1.794	0.133
Rule1	1.073	0.567	2.407	0.369	0.769	0.760	0.609	0.321	1.007	0.995	1.544	0.554
Rule2	0.959	0.556	2.449	0.247	0.869	0.823	1.878	3.641	1.168	1.224	3.916	12.14
Rule3	1.073	0.573	2.384	0.369	0.760	0.736	0.585	0.357	0.949	0.975	1.526	0.662
Birth rate shocks alone												
baseline	0.356	0.865	2.117	1.826	0.666	2.043	0.572	0.321	1.130	0.762	0.914	0.233
Rule1	0.28	1.001	1.864	1.887	0.872	2.552	0.567	0.315	1.390	0.504	0.894	1.661
Rule2	0.248	1.067	1.872	1.916	1.022	2.858	1.191	0.553	1.727	1.054	2.706	15.7
Rule3	0.27	1.016	1.924	1.905	0.956	2.623	0.576	0.339	1.652	0.654	1.064	1.791
Both shocks together												
baseline	1.186	0.582	2.452	0.695	0.853	1.01	0.628	0.258	1.032	1.038	1.733	0.145
Rule1	1.109	0.604	2.377	0.595	0.776	0.977	0.607	0.32	1.035	0.972	1.511	0.681
Rule2	0.99	0.602	2.417	0.537	0.879	1.072	1.842	3.528	1.21	1.214	3.851	12.4
Rule3	1.106	0.608	2.359	0.586	0.773	0.958	0.584	0.356	1	0.959	1.5	0.776
Contribution of ϵ^{Γ} to the variance												
baseline	90.97	80.04	93.27	37.79	94.49	63.05	92.51	85.99	89.17	95.13	97.49	76.6
Rule1	93.63	82.48	96.08	35.95	91.97	56.59	94.44	93.83	88.53	98.29	97.77	62.07
Rule2	93.71	80.21	96.23	19.90	91.51	55.27	97.37	99.85	87.20	95.26	96.90	89.91
Rule3	94.06	83.43	96.05	37.25	90.92	55.48	94.24	94.64	83.94	97.24	97.02	68.37

6 Conclusion

This paper is still a work in progress. A lot of things remain to do. First we will present more sensitivity analysis of the cyclical properties of the model to various parameter value but also to others structural assumption like : the "cost" of children rearing, the unintended bequests profile, the assumption of imperfect annuities market, . . . The analysis of risk-sharing consequences of alternative rules of the PAYG system is also preliminary and need to be completed.

Appendix : Baseline Economy : Policy Functions (Truncated)

		y	c	i	h	N	y/h	k	w	r
Constant		1.000	0.753	0.247	0.134	0.164	7.446	2.950	3.587	0.073
Γ	(-1)	1.268	0.496	0.773	0.075	0.085	5.295	0.773	2.703	0.179
Γ^f	(-1)	-0.032	0.016	-0.048	-0.008	-0.009	0.226	-0.048	0.080	-0.005
k	(-1)	0.255	0.029	-0.829	0.027	0.032	0.390	0.087	0.210	-0.012
l_0	(-1)	0.008	-0.005	0.013	0.002	0.002	-0.053	0.013	-0.019	0.001
l_1	(-1)	0.010	-0.007	0.016	0.002	0.003	-0.063	0.016	-0.024	0.001
l_{10}	(-1)	0.062	-0.047	0.108	0.014	0.017	-0.338	0.108	-0.154	0.009
l_{100}	(-1)	-0.166	-0.028	-0.731	-0.039	-0.046	0.948	-0.731	0.413	-0.023
l_{20}	(-1)	1.126	0.607	0.426	0.302	0.314	-8.355	0.426	-2.807	0.159
l_{30}	(-1)	1.337	0.754	0.997	0.298	0.373	-6.587	0.997	-3.333	0.189
l_{40}	(-1)	1.111	1.009	2.890	0.224	0.310	-4.121	2.890	-2.769	0.157
l_{50}	(-1)	0.705	1.189	5.176	0.150	0.197	-3.052	5.176	-1.758	0.100
l_{60}	(-1)	-0.638	0.929	6.011	-0.141	-0.178	3.042	6.011	1.591	-0.090
l_{70}	(-1)	-0.817	0.937	5.904	-0.191	-0.228	4.491	5.904	2.036	-0.115
l_{80}	(-1)	-0.560	0.886	4.361	-0.131	-0.156	3.123	4.361	1.395	-0.079
l_{90}	(-1)	-0.273	0.606	0.881	-0.065	-0.076	1.577	0.881	0.681	-0.039
a_{20}	(-1)	-0.002	0.001	0.009	0.000	0.000	0.010	0.009	0.004	0.000
a_{40}	(-1)	-0.002	0.001	0.009	0.000	0.000	0.008	0.009	0.004	0.000
a_{60}	(-1)	-0.002	0.001	0.008	0.000	-0.001	0.013	0.008	0.005	0.000
a_{80}	(-1)	-0.001	0.001	0.008	0.000	0.000	0.004	0.008	0.002	0.000
a_{100}	(-1)	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000
ε^Γ		1.315	0.514	0.801	0.078	0.088	5.490	0.801	2.802	0.186
ε^f		0.172	-0.088	0.260	0.045	0.048	-1.201	0.260	-0.429	0.024

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