STABILITY OF MULTI–COUNTRY UNIONS: GAME THEORETIC ISSUES AND POLICY IMPLICATIONS *

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Abstract

Multicountry unions pose stability problems that can be tackled by dynamic game models of non–conventional form, with payoff functions replaced by subsets of the state–space. A country aspiring to join a multicountry union must guarantee that a set of economic indicators stay within the bounds dictated by the union. Mathematically, this translates into membership of the state vector to a convex set. When such a set–membership holds notwithstanding policy actions of other member countries (under the domestic constraints imposed to each) the overall system exhibits a kind of macroeconomic stability termed in this paper an Invariant Equilibrium. The paper presents this theory in the case of two–person linear discrete–time games and discusses the significance of the results for economic policy.

keywords: *Macroeconomics, Multicountry Models, Games, Controlled Invariance*

1 INTRODUCTION

In response to financial mobility of capital markets, global competition and ensuing threats to economic stability, recent developments in western economies witnessed the formation of multicountry unions of diversified nature - monetary, legislative, commercial, political, etc. The desire of a universal frame within which economic activity is carried out and the commitment to social and political stability as primary values, resulted in a pressure to funnel economic behaviour of enterpreuneurs and policy makers into newly designed and globally shared institutional channels. In macroeconomics, this implied a separation of previously undiversified government liabilities into a national and a supernational level. Classical instruments like monetary and fiscal policy, hitherto centralized and coordinated, were for the first time after WW2 sharply separated. By relegating monetary policy to supernational sovereignty, a key instrument of economic control was subtracted to the destabilizing pressures of contingent and short-sighted political needs and of speculative behaviour in the financial market. Fiscal instruments on the other hand, not only remained in the hands of national goverments but – in the reformers' aspirations – it was hoped they would reconquer the power and effectiveness they had lost under the practice and the restraints of centralized control. It is no surprise that such a doctrinary stand should gain acceptance in Europe after decades of regional market competition based on devaluation of exchange rates of weaker countries as a surrogate to productivity, and increase of fiscal pressure combined to import barriers of richer ones, as a

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defense against inflationary effects of domestic demand. With the net result of an increase of the public debt of fragile countries; export stagnation and suboptimal growth of the stronger ones.

Whatever the degree of success of unions in terms of stability and growth, there is no doubt the new framework poses new problems that only in part can be tackled by traditional methodology. The interplay between a centralized and a decentralized control level has become so pronounced as to make game-theoretic approaches prominent not only for their own merits but also as a complement to more traditional tools like macroeconomic general equilibrium models. On the other hand, traditional game-theoretic models based on a normative principle - maximization of utility, welfare or other aggregate - hardly reflect the spirit and the content of newly created multi-country institutions. Maastricht Treaty, for instance, prescribes tolerance intervals for a selected subset of economic variables. Permanence in the union is better represented by the efforts of member countries to respect set-membership of controlled variables than by abstract maximization of artificially construed aggregates. What modeling implications does this have at a theoretical level?

A theoretical concept relevant to this framework is proposed in this paper - the Doubly Invariant Equilibrium. The formalization starts with a discrete–time linear dynamic game with full–state information and feedback strategies [3]. The novelty is that payoff functions are replaced by subsets of state space. Permanence in a multicountry union requires country variables to stay within a constraining subset, once the union is adhered to. This requires of each member country the adoption of a policy-path divided into two segments. The first leads from the initial state (identified to the moment the country decides to join the union) to a state within the constraints imposed by the union. The second, starting from the formal entry in the union onwards, where the goal is to keep within union bounds. Methodologically, the first policy segment can be dealt with by traditional policy & planning tools (target–instrument analysis, optimal control etc). The post–entry segment poses essentially new and challenging control problems. Namely, the need to maneuvre policy instruments under domestic restraints (like for example limits to fiscal pressure) so as to stay in the union irrespective of policies of other members. Due to the dynamic coupling between country variables, there are spillover effects posing essentially a decentralized stability problem with feedback information. The main finding of the paper is that invariant equilibria dictated by membership in multi-country union indirectly impose domestic restrictions to economic policies. Such restrictions in turn make tolerance margins for the development paths of endogenous variables stricter than originally imposed by the union rules. In other words, there are hidden stability implications that – when brought to the surface – result in narrower margins than explicitly dictated by the union. When an aggregate low-order linear econometric model of a multicountry economy is available, these margins can be calculated and provide normative basis for economic policy as well as deeper insight into the medium–long term consequences and sustainability of union rules.

2 PROBLEM STATEMENT

Consider the linear discrete–time game in state space form

$$x(t+1) = Ax(t) + Bu(t) + Cv(t) \tag{1}$$

where $x(t) \in I\!R^n$, $u(t) \in U, v(t) \in V$ for $t = 0, 1, 2 \ldots$ and $U \subset I\!R^{m_1}, V \subset I\!R^{m_2}$ are compact convex sets containing the origin. Without loss of generality we assume that B, C are full (column) rank matrices. In our context the control variables u, v are policy instruments in the hands of two countries while state components collect endogenous variables. U and V embody domestic policy restraints to which each country is subject. Extension to the n–country case adds nothing but notational clutter and is omitted for ease of discussion.
def 1. *There exists an invariant equilibrium for (1) if two closed convex sets $X_1 \neq I\!R^n$ and $X_2 \neq I\!R^n$ exist such that*

$$\begin{aligned} \forall x \in X_1 \, \exists u \in U \ &: \ Ax + Bu + Cv \in X_1 \quad \forall v \in V \\ \forall x \in X_2 \, \exists v \in V \ &: \ Ax + Bu + Cv \in X_2 \quad \forall u \in U \end{aligned}$$

Evidently, if an invariant equilibrium exists it is possible to find a policy $\hat{u}(t)$ such that once the state is in X_1 country one is able to keep it there no matter what policy $v(t)$ is used by country 2 – and a policy $\hat{v}(t)$ with symmetric properties for country 2. In the definition it was necessary to rule out the entire state space ($X_i \neq I\!R^n$) as this would – trivially and uninterestingly – satisfy the definition. Structure and mutual relationship between the sets X_1 and X_2 can be extremely varied. Examples can be found with X_i coinciding, partially overlapping, orthogonal or even disjoint [4]. In the present context, state variables are typically partitioned into

$$x = \begin{bmatrix} x^1 \\ x^2 \end{bmatrix}$$

with x^i denoting variables of country i. Then X_i reflects a constraint on x^i only and we are in the orthogonal case. An existence condition for invariant equilibrium was proved in [5] for symmetric U, V
Theorem 1 *If X_i are bounded sets a necessary condition for the existence of an invariant equilibrium is that A be a stable matrix. If $X_1 + X_2$ has an interior not containing the origin, then A must be a stable matrix with an eigenvalue equal to 1.*

Symmetry of the sets U, V means that they contain $-u, -v$ whenever they contain u, v. This requisite is harmless when interpreted in terms of policy. It says essentially that instruments cannot deviate either side from their reference value more than a given amount.

Contrary to [5] where sets X_i were parametrized in a class of assigned convex polyhedra we assume here no knowledge of X_i except that they are contained in closed convex sets K^i. We think of K^i as the set of constraints imposed by the union. As remarked in the introduction, our concern is for the second segment of the economic maneuvre, the one originating from the date a point (called feasible) within union bounds K^i has been reached. Therefore we make the following assumption
A1. *the origin of the state space is a feasible point contained in K^i*

There is no loss of generality to assume $0 \in K^i$ as state variables can be re-interpreted as deviations from a feasible point by a suitable coordinate change. The definition of invariant equilibrium entails a double requirement which seen from the viewpoint of a single player (say player 1) takes on the following form. Given K^1, find the largest subset of K^1 which is a V–robust controlled invariant wrt U for (1). We recall that a set X is a V–robust controlled invariant wrt U for (1) if

$$\forall x \in X : \exists u \in U : Ax + Bu + Cv \in X, \quad \forall v \in V.$$

Given any X, let $\mathcal{R}(X)$ be the set of states that for some $u \in U$ reach in one step a state in X for whatever $v \in V$

$$\mathcal{R}(X) = \{x \in X : \exists\, u \in U : Ax + Bu + Cv \in X, \quad \forall v \in V\}$$

A necessary and sufficient condition for invariance of X is the Nagumo condition [2, 1] which in the case of discrete–time systems takes up the form

$$X \subset \mathcal{R}(X). \tag{2}$$

The set $\mathcal{R}(X)$ can be computed with different techniques depending on the nature of X. We shall denote $\hat{X}$ the maximal subset of K satisfying (2). In the next section it will be shown how to compute $\hat{X}$ in general, and later on particularly when K is a closed convex polyhedron.

3 CONSTRUCTION OF INVARIANT EQUILIBRIA

For given A, B, C the set $\hat{X}$ can be regarded as the image of a mapping U, V into subsets of K. Precisely,
def 2. *The mapping $F_K(U, V) = \hat{X}$ denotes the (possibly empty) largest V–robust invariant set wrt U contained in K for (1).*

Proposition 2. *The mapping $U, V \mapsto F_K(U, V)$ satisfies*

$$F_K(\hat{U}, V) \subset F_K(U, V) \subset F_K(U, \hat{V}) \qquad \forall \hat{U} \subset U, \ \forall \hat{V} \subset V$$

whenever the lhs is non-empty.
Proof. If $X \neq \emptyset$ is V-robust invariant wrt $\hat{U}$

$$\forall x \in X \ \exists u \in \hat{U} : Ax + Bu + CV \subset X$$

which implies for $\hat{U} \subset U$

$$\forall x \in X \; \exists u \in U : Ax + Bu + CV \subset X$$

that is, X is also V-robust invariant wrt U. In particular, for $X = F_K(\hat{U}, V)$ we see that $F_K(\hat{U}, V)$ is also invariant wrt U hence it must be contained in the maximal V-robust invariant wrt U which is precisely $F_K(U, V)$ and this proves the left inclusion. Similarly, if X is V-robust invariant wrt U

$$\forall x \in X \; \exists u \in U : Ax + Bu + CV \subset X$$

which implies for $\hat{V} \subset V$

$$\forall x \in X \; \exists u \in U : Ax + Bu + C\hat{V} \subset X$$

that is $X = F_K(U, V)$ is also $\hat{V}$-robust invariant wrt to U hence it must be contained in the maximal $\hat{V}$-robust invariant wrt to U which is precisely $F_K(U, \hat{V})$ and this proves the right inclusion. ■

Let us now introduce amplitudes $\mu, \nu > 0$ and parametrize U, V as $U = \mu U_0 \quad V = \nu V_0$, with U_0, V_0 given compact convex sets containing the origin. Next define a function $\beta : I\!R \mapsto I\!R$

$$\beta(\mu) = \sup \nu : F_K(\mu U_0, \nu V_0) \neq \emptyset$$

and the set $S = \{\mu, \nu : F_K(\mu U_0, \nu V_0) \neq \emptyset\}$. Think of $\beta(\cdot)$ as the maximal disturbance "tolerated" under control amplitude μ. Notice that if a robust invariant wrt μU_0 exists in K, then the domain of $\beta(\cdot)$ is $\mu \geq \mu_0$ with $\mu_0 = 0$ if $0 \in K$.

Proposition 3. *$\beta(\mu)$ is non-decreasing and S is convex.*
Proof. *(non decr.)* Assume $\mu_1 < \mu_2$ and let $\nu_1 = \beta(\mu_1), \nu_2 = \beta(\mu_2)$. If it were $\nu_1 > \nu_2$, then

$$F_K(\mu_1 U_0, \nu_1 V_0) \neq \emptyset \text{ and } F_K(\mu_2 U_0, \nu_1 V_0) = \emptyset.$$

But $\mu_1 U_0 \subset \mu_2 U_0$ (since $0 \in U_0$) and, by Prop. 2, we would get

$$\emptyset \neq F_K(\mu_1 U_0, \nu_1 V_0) \subset F_K(\mu_2 U_0, \nu_1 V_0) = \emptyset$$

hence $\mu_1 < \mu_2 \Rightarrow \beta(\mu_1) \leq \beta(\mu_2)$.
(convex.) If $X_1 = F_K(\mu_1 U_0, \nu_1 V_0)$ and $X_2 = F_K(\mu_2 U_0, \nu_2 V_0)$ are non-empty

$$\begin{aligned} AX_1 + C\nu_1 V_0 &\subset X_1 - B\mu_1 U_0 \\ AX_2 + C\nu_2 V_0 &\subset X_2 - B\mu_2 U_0 \end{aligned}$$

hence for $\alpha \in [0\,1]$

$$(1-\alpha)[AX_1 + C\nu_1 V_0] + \alpha[AX_2 + C\nu_2 V_0] \subset (1-\alpha)[X_1 - B\mu_1 U_0] + \alpha[X_2 - B\mu_2 U_0]$$

or

$$AX + C\nu V \subset X - B\mu U_0$$

with

$$\mu = (1-\alpha)\mu_1 + \alpha\mu_2, \quad \nu = (1-\alpha)\nu_1 + \alpha\nu_2$$

and $X = (1-\alpha)X_1 + \alpha X_2$ is non-empty. ■

We are now in a position to rephrase the existence of an invariant equilibrium as per def. 1 in terms of the sets $F_k(U, V)$ by stating that an invariant equilibrium exists if and only if

$$\begin{aligned} F_{K^1}(\mu U_0, \nu V_0) &\neq \emptyset \qquad (3) \\ F_{K^2}(\nu V_0, \mu U_0) &\neq \emptyset. \qquad (4) \end{aligned}$$

Notice that a trivial equilibrium $\{0\} = F_{K^i}(\{0\}, \{0\})$ always exists ($0 \in K^i$) so we need only be concerned with non-trivial equilibria. A pair μ, ν satisfying (3–4) will be called a non-trivial invariant equilibrium if $\mu U_0 \neq \{0\}, \nu V_0 \neq \{0\}$. Denote by

$$E = \{\mu, \nu : F_{K^1}(\mu U_0, \nu V_0) \neq \emptyset, \; F_{K^2}(\mu V_0, \nu U_0) \neq \emptyset\}$$

the set of all invariant equilibria for fixed K^i. Since E is the intersection of convex sets (Prop.3) it is either empty or convex. Let

$$\beta_1(\mu) = \sup \nu : F_{K^1}(\mu U_0, \nu V_0) \neq \emptyset$$
$$\beta_2(\nu) = \sup \mu : F_{K^2}(\nu V_0, \mu U_0) \neq \emptyset$$

and let $\mu \geq \mu_0, \nu \geq \nu_0$ be the domains of $\beta_1(\cdot)$ and $\beta_2(\cdot)$. Assuming that K^i contain robust invariants, a typical graph of β_1, β_2 is shown in Fig. 1

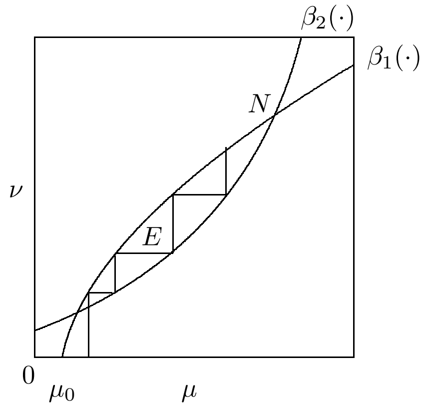

Fig. 1 Tolerated amplitudes and the tatonnement process.

with invariant equilibria in the cross-hatched region E.

Theorem 4. *A non-trivial invariant equilibrium exists if and only if* $\beta_2(\beta_1(\mu)) - \mu \geq 0$, *for some* $\mu > \mu_0$.
Proof. Assume

$$0 < \mu \leq \beta_2(\nu), \tag{5}$$

for $\nu = \beta_1(\mu)$. Then $F_{K^1}(\mu U_0, \beta_1(\mu)V_0) \neq \emptyset$, hence

$$F_{K^1}(\mu U_0, \nu V_0) \neq \emptyset$$

and (2) holds. On the other hand (5) implies $\mu U_0 \subset \beta_2(\nu)U_0$ hence, by Prop.2

$$\emptyset \neq F_{K^2}(\nu V_0, \beta_2(\nu)U_0) \subset F_{K^2}(\nu V_0, \mu U_0) \quad \mu, \nu > 0$$

and (3) holds, so a non-trivial invariant equilibrium exists. If, on the contrary $\beta_2(\beta_1(\mu)) - \mu < 0 \quad \forall \mu > \mu_0$, then $\beta_2(\nu) < \mu$ for $\nu = \beta_1(\mu)$ hence $F_{K^1}(\mu U_0, \beta_1(\mu)V_0) = F_{K^1}(\mu U_0, \nu V_0) \neq \emptyset$ but, due to Prop.2, $F_{K^2}(\nu V_0, \mu U_0) = \emptyset$ so (3) is violated and no invariant equilibrium exists. ■

When E is non-empty, we say an invariant equilibrium $\hat{\mu}, \hat{\nu}$ is Nash if it satisfies

$$\hat{\nu} \geq \nu \qquad \forall (\hat{\mu}, \nu) \in E \tag{6}$$
$$\hat{\mu} \geq \mu \qquad \forall (\mu, \hat{\nu}) \in E \tag{7}$$

(see point N in Fig. 1). Notice if an invariant Nash equilibrium exists, the condition of Thm 4. must hold in some interval $\mu_0 < \mu \leq \mu_1 < \infty$.

Proposition 5. *An invariant Nash equilibrium exists if and only if an invariant equilibrium exists.*
Proof. The "only if" part is obvious. To prove the "if" part notice that if no $\hat{\nu}$ satisfies (6) then $V = \nu V_0$ is unbounded and

$$AX_1 + CV \subset X_1 - BU$$

with U, X_1 bounded. But this is impossible if C is full rank. Similarly if no $\hat{\mu}$ satisfies (7). ■
We close this section with a brief note on computational aspects. An invariant Nash equilibrium can be approximated by tatonnement, as sketched in Fig. 1. This requires computation of $\beta_i(\cdot)$. These functions are computable if their value at each point of the domain can be obtained in a finite number of steps. This entails to check non-emptyness of $F_{K^1}(\mu U_0, \nu V_0), F_{K^2}(\nu V_0, \mu U_0)$. Two algorithms are available for this task, an external and an internal algorithm. For reasons of space we only recall the former and we

address the reader to [6] for more details. The external algorithm, known as the **Invariance Kernel Algorithm** [7] is based on the recursion for $t = 0, 1, 2, \ldots$

$$\begin{array}{rcl} K_t = K & \quad if \quad & t = 0 \\ K_t = \mathcal{R}(K_{t-1}) \cap K & \quad if \quad & t > 0. \end{array}$$

The algorithm generates a sequence

$$K \supset K_1 \supset K_2 \supset \ldots$$

A property of K_t is that it contains nothing but the initial states for which it is possible to stay in K at least t times with some control in U for all disturbances in V. Thus the sequence either converges to $\hat{X}$ if $\hat{X} \neq \emptyset$ or, it yields $K_t = \hat{X} = \emptyset$ at some $t < \infty$. To guarantee finite time termination when $\hat{X} \neq \emptyset$ a modification has been proposed in [7] and address to that ref for details.

4 THE CASE OF POLYHEDRAL SETS

When the sets K, U, V are closed convex polyhedra, the computation $\mathcal{R}(K_t)$ and the testing of the invariance condition (2) can be reduced to a sequence of LP programs as briefly sketched below. Let us now define

$$K = \{x : Gx \leq g\} \quad U = \{u : H_1 u \leq h_1\} \quad V = \{v : H_2 v \leq h_2\}.$$

Then

$$\mathcal{R}(K) = \{x : \exists u : G(Ax + Bu + Cv) \leq g, \quad H_1 u \leq h_1 \quad \forall v : H_2 v \leq h_2\}$$

If the inequality is to hold for all $v \in V$ it must hold with GCv replaced by

$$\hat{w} = \left\{ \max_{H_2 v \leq h_2} [GC]_i \, v \right\}.$$

Notice that the computation of this vector requires solving as many LP problems as there are rows in G. Then

$$\mathcal{R}(K) = \{x : \exists u : G(Ax + Bu) \leq g - \hat{w} \quad H_1 u \leq h_1\}$$

which can be expressed as

$$\mathcal{R}(K) = \{x : \exists u : \begin{bmatrix} GB \\ H_1 \end{bmatrix} u \leq \begin{bmatrix} g - GAx - \hat{w} \\ h_1 \end{bmatrix}\}.$$

It is known from convex analysys [5] that the set of $b \in I\!R^p$ such that the system $Mx \leq b$ has a solution can be expressed as $Qb \geq 0$ where the rows of Q are the generators of the cone $\mathcal{N}(M') \cap I\!R^p_+$. Thus if M is identified to $\begin{bmatrix} GB \\ H_1 \end{bmatrix}$ we get

$$\mathcal{R}(K) = \{x : Q \begin{bmatrix} g - GAx - \hat{w} \\ h_1 \end{bmatrix} \geq 0\} = \{x : Q_1 GAx \leq Q_1(g - \hat{w}) + Q_2 h_1\}$$

where $Q = [Q_1 | Q_2]$. The external and internal algorithms require intersection of convex polyhedra and these can be determined by simply appending inequalities. Finally, invariance condition (2) requires polyhedral inclusion and this can be checked up by solving, again, linear programs.

5 NUMERICAL RESULTS

In this section we test the theory with data from Italy and the rest of the European Monetary Union (REMU). A least–squares regression over a sample of 10 yrs data led to the state-space model $x(t+1) =$

$Ax(t) + Bu(t) + Cv(t)$ with

$$A = \begin{bmatrix} 1.5162 & -0.0218 & -0.5858 & -0.2202 \\ 0.3090 & -0.1839 & 0.5385 & -0.1352 \\ 1.3716 & 0.5631 & -0.4503 & -0.0449 \\ 2.9261 & -0.0664 & -0.6754 & -0.8782 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.3285 \\ -0.8157 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 \\ 0 \\ 0.1423 \\ -1.3108 \end{bmatrix}$$

$$y^1(t) = \begin{bmatrix} -2.3833 & -2.0444 & 0 & 0 \end{bmatrix} x(t)$$
$$y^2(t) = \begin{bmatrix} 0 & 0 & -4.9845 & 1.5999 \end{bmatrix} x(t)$$

where x_1, x_2 are current debt and unemployment, u fiscal pressure and y^1 inflation indexes for Italy (and correspondingly x_3, x_4, v, y^2 for REMU). The state matrix is asymptotically stable, with eigenvalues in modulus

$$|\lambda| \in \{0.8246, \quad 0.8246, \quad 0.4686, \quad 0.7912\}.$$

Maastricht treaty imposes bounds to inflation rate ($\leq 2.7\%$) and public debt ($\leq 60\%$ of GDP). The indexes are set up so that each variable in the model represents deviations from a feasible point which is chosen centrally within the Maastricht bounds. This is 1.35% (2.7/2) for inflation rate; 30% (60/2) for the ratio of debt to GDP. Hence a nominal margin of 1.35 points for the inflation rate and 30 points for debt to GDP ratio are left either side of the feasible point. When projected in state space the bounds define two convex regions as shown in fig 2 (large polygons, Italy on the left). It is assumed that the first segment of the economic maneuvre has been completed. Therefore u, v reflect variables concerning post-entry maneuvres only. At $u = v = 0$ the state is identically zero implying no deviations from the origin. Computing the invariant Nash equilibrium with the Invariance Kernel Algorithm we get the results shown in Figs. 3 (small polygons). The invariant set for Italy (small polygon on the left) turns out to be 4.74% of the nominal margin whereas the invariant set for REMU is about 42% of the nominal margin.

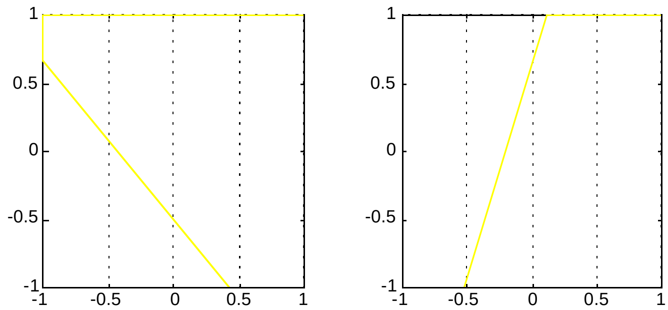

Fig. 2 Maastricht feasible regions.

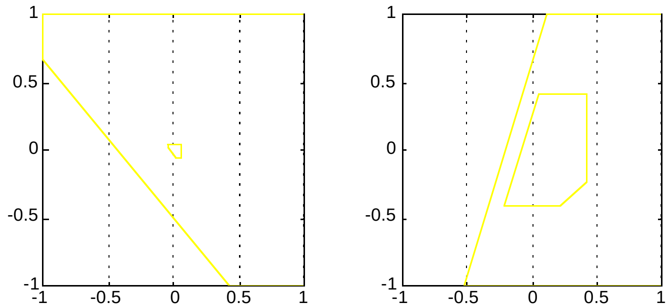

Fig. 3 Invariant Nash equilibrium sets.

In terms of the original variables, this means for Italy a tolerance for inflation $1.35 \pm .062$, that is between 1.29% and 1.41% and for REMU of $1.35 \pm .567$, that is between 0.79% and 1.92%. Similarly, for the ratio of debt to GDP: this parameter for Italy must be comprised between $.3 \pm .014$, that is between 28.6% and 31.4% of GDP and for REMU of $.3 \pm .126$, that is between 17.4% and 42.6% of GDP. Neglecting the lower bounds (that clearly reflect a purely theoretical value) the original Maastricht bound on inflation rate (2.7%) is lowered to 1.41% for Italy and to 1.92% for REMU. Likewise, the original Maastricht bound on debt to GDP ratio (60%) is lowered to 31.4% for Italy and to 42.6% for REMU.

6 DISCUSSION AND CONCLUSION

The main finding of our analysis is that when dynamic interactions among member countries are taken into account the theoretical constraints fixed by a supernational economic authority, monetary unions being a case in point, are considerably reduced. This is so because union dictated constraints are – in general – not invariant: one country may comply and yet, due to dynamic cross–coupling, push another one out. To preclude this destabilizing occurrence one should adopt the notion of invariant equilibria, as amply discussed in the paper. The end effect is that original bounds must be duly restrained. This has a three–fold implication

1. Although non-economic criteria (historical, political, etc) concur in the choice of union rules, the choice should be supplemented by adequate analysis. Given a set of candidate countries wishing to form a union, invariant equilibria should be studied in order to assess the degree of confidence or realism that union bounds be sustainable in the long run.

2. The question of the enlargement of the union to new members should be discussed, along with politically inspired criteria, with economic – indeed game theoretic – analysis aimed to assess the likelyhood that the prospective member will be able to comply with union rules. In this respect traditional tools may fail to recognize limits that only emerge with invariant equilibrium analysis.

3. The bounds emerging from invariant equilibrium analysis may be strongly asymmetric, as the numerical case we discussed clearly shows. Some member countries typically have larger margins of tolerance than others*. What determines the stability of the union is the country with narrower bounds. If union stability is a shared and primary concern, it may not be inappropriate that union efforts be directed to sustain the member country which is closer to critical conditions, that is the one with smaller equilibrium set.

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*it is significant that at the time of this writing one of the prominent issues in the agenda of EC planners should concern revision of rules in the so called stability and growth pact