Testing stationarity of AR(1) process with symmetric stable disturbance

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Abstract

In the paper we propose a modification of the Dickey-Fuller (1979) test for AR(1) process with symmetric stable disturbance. It is an extension of Rachev et al. (1998, 1999ab), and Rachev and Mittnik (2000) researches which anylyse asymptotic properties of DF tests for AR(1) process with symmetric stable disturbance with known characteristic exponent alpha. We consider finite sample properties of the tests, including finding so called response surfaces.

We also propose two modifications of the tests for unknown alpha. Firstly, we replace unknown value of parameter alpha by its estimator and secondly we use linear combination of DF tests. To investigate properties of proposed tests, we use Monte-Carlo simulations. We test if real sizes of our tests differ significantly from assumed values. We also determine the best proposed unit root test in the sense of its power.

Keywords: unit root test; stable distribution

JEL Classification C12, C15, C16

1 Introduction

Stable distribution has become a very popular tool for describing distribution of empirical variables since the Fama (1965ab) papers[1]. A natural question arise about behaviour of econometric tests if variables are not normal, but stable.

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[1]The stable distributions are used to model stock returns (Blume, 1970; Roll, 1970; Teichmoeller, 1971; Officer, 1972; Hagernman, 1978; Akgiray and Booth, 1988; Diebold, 1993); exchange rate (Koedijk and Kool, 1992; Müller et al. (1998); Jorion, 1988; Yang and Brorsen, 1995; Scheicher, 1999); inflation rate (Bidarkota and McCulloch, 1998); real estate prices (Young and Graff, 1995) etc.

In the paper we propose a modification of the Dickey-Fuller (1979) test (DF test) for AR(1) process with symmetric stable disturbance. It is an extension of Rachev et al. (1998, 1999ab), and Rachev and Mittnik (2000) researches which anylyse asymptotic properties of DF tests for AR(1) process with symmetric stable disturbance with known characteristic exponent alpha. We consider finite sample properties of the tests and also propose a modification of the tests for unknown alpha.

The paper is constructed as follows: the second section contains the definition and properties of stable distribution; section three reports classical unit roots testing when applying the Dickey-Fuller (DF) test; the next section discusses asymptotic behaviour of the DF test for stable disturbance with known alpha and section five contains our modification of the DF test in case of unknown alpha and finite sample.

2 Stable distributions

Stable distributions are a rich class of distributions, including (among others) normal, Cauchy and Lévy distributions. According to the generalized central limit theorem, the limit of (scaled) sum of independent, identically distributed random variables is (if it exists) stable. It can be shown that the logarithm of characteristic function of stable distribution is equal to (see DuMouchel, 1975)

$$\log E(\exp(iXt)) = \begin{cases} i\delta t - |ct|^{\alpha}\left[1 - i\beta sign(t)\tan\frac{\pi\alpha}{2}\right] & \text{for } \alpha \neq 1 \\ i\delta t - |ct|\left[1 + i\beta\frac{2}{\pi}sign(t)\log|t|\right] & \text{for } \alpha = 1 \end{cases}, \quad (1)$$

which is denoted $X \sim S(\alpha, \beta, c, \delta)$. For a discussion of other parameterizations, see Zolotarev (1983), Nolan (1999).

The stable distribution is defined by four parameters.

Parameter $\alpha \in (0, 2]$ is a characteristic exponent. For $\alpha = 2$ we have the Gaussian distribution with variance $2c^2$. For $\alpha < 2$, the distribution is "heavy-tailed".

The skewness parameter β determines the shape of the distribution. For $\beta > 0$ the distribution is skewed to the right, for $\beta < 0$ it is skewed to the left, and for $\beta = 0$ it is symmetric (except the normal case which is always symmetric).

Parameter $c \in \mathbf{R}_+$ is the scale parameter and $\delta \in \mathbf{R}$ is the location parameter.

In almost all cases a close formula of density of the stable distribution is not known, so we cannot use directly the maximum likelihood method of parameter estimation. In the paper we use the quantile method of estimation for the symmetric case proposed by McCulloch (1986).

More information about stable distributions are to be found in: Gnedenko and Kolmogorov (1957), Kendall and Stuart (1963-1968), Feller (1966), Zolotarev (1983), McCulloch (1986, 1996), Christoph and Wolf (1993), Samorodnitsky and Taqqu (1994), Nikias and Shao (1995), Nolan (1999).

3 Testing stationarity of normal AR(1) process

In the section we recount the classical Dickey-Fuller (1979) test. Let us consider a normal AR(1) process

$$y_t = \alpha_0 + \alpha_1 \cdot y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.} N(0, \sigma^2), \quad t = 2, \dots, T, \tag{2}$$

The null hypothesis for DF test is the random walk hypothesis[2]. That means that for (2) we assume that:

$$H_0: \quad \alpha_0 = 0, \quad \alpha_1 = 1, \quad y_1 \in \mathbf{R}, \tag{3}$$

and

$$H_1: \quad \alpha_0 \in \mathbf{R}, \quad |\alpha_1| < 1, \quad y_1 \sim N\left(\frac{\alpha_0}{1-\alpha_1}, \frac{\sigma^2}{1-\alpha_1^2}\right). \tag{4}$$

The Dickey-Fuller (DF) statistics is simply a t-Student statistics for $\alpha_1 = 1$ hypothesis, i.e. $DF = \frac{\hat{\alpha}_1 - 1}{s_{\hat{\alpha}_1}}$. It has two versions, denoted $D\tilde{F}_{\alpha_1}$ and DF_{α_0,α_1}:

$$D\tilde{F}_{\alpha_1} = \frac{\tilde{\alpha}_1^- - 1}{s_{\tilde{\alpha}_1^-}}, \tag{5}$$

$$DF_{\alpha_0,\alpha_1} = \frac{\hat{\alpha}_1^+ - 1}{s_{\hat{\alpha}_1^+}}, \tag{6}$$

where:
$\tilde{\alpha}_1^-$ – OLS estimator of the parameter α_1 in the equation $(y_t = \alpha_1 \cdot y_{t-1} + \varepsilon_t)$,
$s_{\tilde{\alpha}_1^-}$ – standard error of $\tilde{\alpha}_1^-$ estimator,
$\hat{\alpha}_1^+$ – OLS estimator of the parameter α_1 in the equation $(y_t = \alpha_0 + \alpha_1 \cdot y_{t-1} + \varepsilon_t)$,
$s_{\hat{\alpha}_1^+}$ – standard error of $\hat{\alpha}_1^+$ estimator.

The critical values of DF statistics are given in Fuller (1976, pp. 371, 373).

Since the distribution of DF_{α_1} statistic depends on y_1 distribution, we subtract the first observation from the whole sample if $y_1 \neq 0$. More precisely, instead of the statistic (5) we use the following statistic (compare Dickey and Fuller, 1979)

$$DF_{\alpha_1} = \frac{\hat{\alpha}_1^- - 1}{s_{\hat{\alpha}_1^-}}, \tag{7}$$

where:
$\hat{\alpha}_1^-$ – OLS estimator of the parameter α_1 in the equation

$$(y_t - y_1) = \alpha_1 \cdot (y_{t-1} - y_1) + \varepsilon_t,$$

$s_{\hat{\alpha}_1^-}$ – standard error of $\hat{\alpha}_1^-$ estimator.

[2] We do not anylyse other unit root tests such as the Phillips-Perron (1988) test, or the KPSS test (Kwiatkowski et al., 1992) etc.

4 Testing stationarity of stable AR(1) process with known alpha

Rachev et al. (1998, 1999ab) find asymptotic distribution of DF statistics when disturbance ε_t is symmetric stable with known alpha. Asymptotic critical values can be found in Rachev and Mittnik (2000). In the paper we compute critical values for finite samples. Thus, we assume that ε_t is symmetric stable. That means that for

$$y_t = \alpha_0 + \alpha_1 \cdot y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim S(\alpha, 0, c, 0), \quad t = 2, \ldots, T, \tag{8}$$

we assume that:

$$H_0: \quad \alpha_0 = 0, \quad \alpha_1 = 1, \quad y_1 \in \mathbf{R}, \tag{9}$$

$$H_1: \quad \alpha_0 \in \mathbf{R}, \quad |\alpha_1| < 1, \quad y_1 \sim S\left(\alpha, 0, \frac{c}{(1-\alpha_1^{\alpha})^{1/\alpha}}, \frac{\alpha_0}{1-\alpha_1}\right). \tag{10}$$

Assumption $y_1 \sim S\left(\alpha, 0, \frac{c}{(1-\alpha_1^{\alpha})^{1/\alpha}}, \frac{\alpha_0}{1-\alpha_1}\right)$ is necessary for stationarity time series defined by the alternative hypothesis (H_1).

For testing the null hypothesis we use DF_{α_1} and DF_{α_0,α_1} statistics defined by the equations (7) and (6). Distributions of DF statistics will be determined using the Monte-Carlo simulations. Since under null hypothesis the distributions do not depend on parameters y_1 and c, we can assume for example that $y_1 = 0$ and $c = 1$, which leads to

$$H_0': \quad y_t = \sum_{i=2}^{t} \varepsilon_i, \quad \varepsilon_t \sim S(\alpha, 0, 1, 0), \quad t = 2, \ldots, T, \quad y_1 = 0. \tag{11}$$

It has to be stressed that H_0' is not a new null hypothesis – it is only the assumption for Monte-Carlo simulations.

Distributions of DF_{α_1} and DF_{α_0,α_1} statistics depend on parameters α and T. We choose 11 values of the characteristic exponent ($\alpha = \{1.0, 1.1, \ldots, 2.0\}$) and three values of the number of observations ($T = \{100, 500, 1000\}$). For every value of parameters we generate $n = 50\ 000$ times time series $\{y_t\}$ using formula (11). In all simulations we generate stable pseudo-random numbers using McCulloch GAUSS procedure *rndssta* (download from web-page http://www.econ.ohio-state.edu/jhm/jhm.html) which is an application of the Chambers et. al. (1976) method. All procedures are written in GAUSS-386i.

In the figures 1 and 2 we show some empirical density functions for DF statistic without an intercept (DF_{α_1}) and in the figures 3 and 4 – with an intercept (DF_{α_0,α_1}).

The figures show, first of all, that density functions for 100 and 1000 observations are very similar. Secondly, modes of DF_{α_1} distributions are moved to the

Figure 1: Density functions of DF statistic without an intercept for $T = 100$.

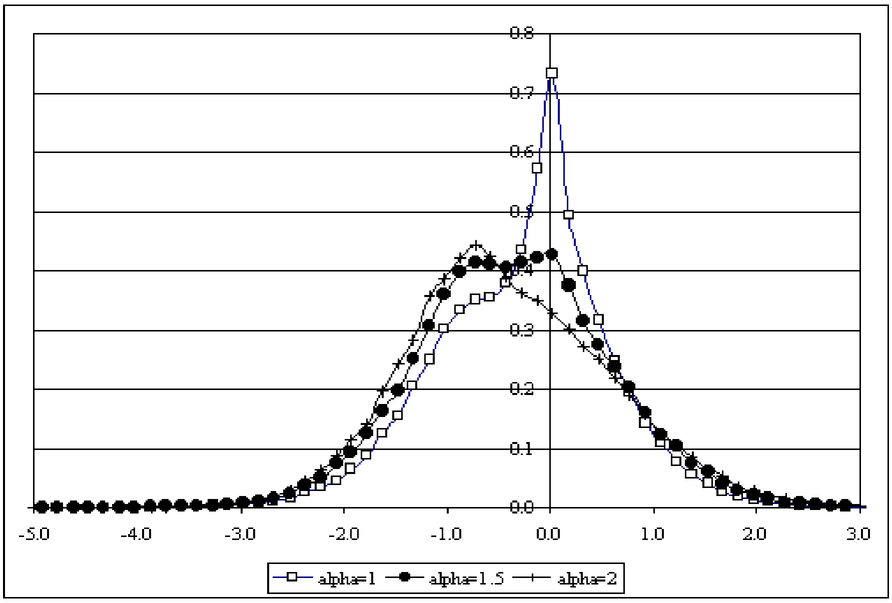


Figure 2: Density functions of DF statistic without an intercept for $T = 1000$.

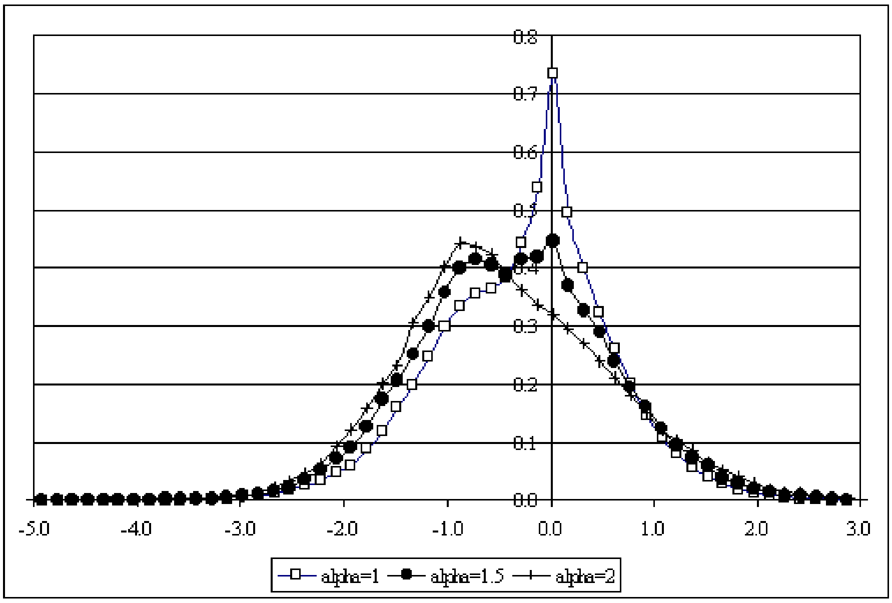


Figure 3: Density functions of DF statistic with an intercept for $T = 100$.

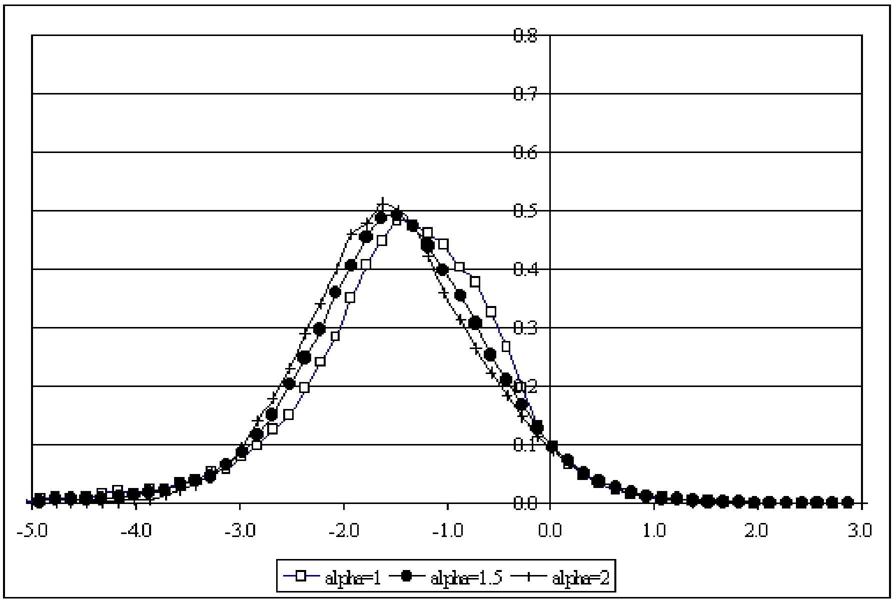
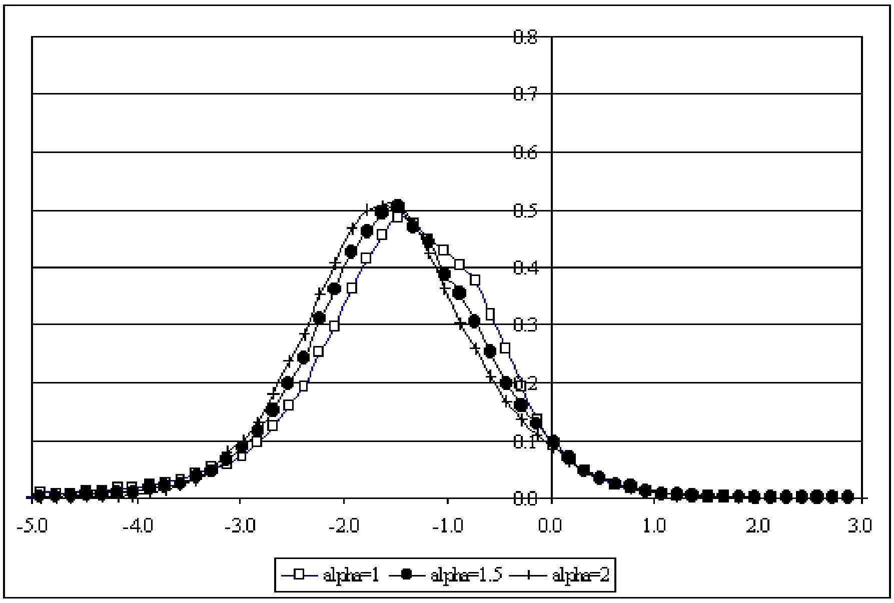


Figure 4: Density functions of DF statistic with an intercept for $T = 1000$.



right, as compared to modes of DF_{α_0,α_1} distributions, especially for "heavy-tailed" case ($\alpha = 1$). Thirdly, if tails of disturbance's distribution become "heavier", modes of DF statistics move to the right.

In table 1 we put critical values of DF tests. For $\alpha = 2$ (normal case, i.e. the classical DF test) critical values for the test without an intercept are similar to the Charemza and Deadman (1997) results (differences are due to a different assumption about y_1), and for the test with an intercept the results are the same as the Fuller (1976) results.

The lower characteristic exponent (i.e. "heavier tails" of the distribution), the smaller absolute values of DF_{α_1} critical values are, and the higher are absolute values of DF_{α_0,α_1} critical values higher (compare Table 1). As a result of non-normality of disturbance of random walk, the classic DF test without an intercept rejects nonstationarity hypothesis too seldom, and the classical DF test with an intercept rejects this hypothesis too often.

To construct confidence intervals for the critical values, we use the following nonparametric method (see e.g. Niemiro, 1999, pp. 158-159). Confidence intervals for a p-th quantile (= critical value at critical level p) of random variable X (= test X) are of form $[X_{\underline{k}:n}, X_{\bar{k}:n}]$, where $X_{k:n}$ – k-th order statistic. If the cumulative distribution function is continuous and strictly increasing in neighbourhood of the p-th quantile, then we can easily prove that for large n the following approximations are valid:

$$\underline{k} \approx np - u\sqrt{np(1-p)},$$

and

$$\bar{k} \approx np + u\sqrt{np(1-p)},$$

where u – appropriate quantile of normal distribution (e.g. 1.96 for confidence level equal to 0.95).

For $n = 50\ 000$ replications of Monte-Carlo simulation and 0.95 level of confidence we use the following order statistics:

- $\underline{k} = 456$ and $\bar{k} = 544$ for $p = 0.01$ (= critical level),
- $\underline{k} = 2404$ and $\bar{k} = 2596$ for $p = 0.05$,
- $\underline{k} = 4868$ and $\bar{k} = 5132$ for $p = 0.10$.

The results of Monte-Carlo simulations are listed in Table 2.

In the next step we approximate critical values by a function $(\mu_0 + \mu_1 \cdot \alpha + \mu_2 \cdot \alpha^2 + \mu_3 \cdot T^{-1})$ (using OLS, parameters with t-values below 1 has been omitted – see Table 3) (we find so called response surfaces).

Increase of characteristic exponent α by 0.1 declines 1% critical values for DF_{α_1} by 0.019, 5% critical values by 0.019-0.031, and 10% critical values by 0.019-0.033. Changing length of time series from 100 to 1000 causes usual unsignificant or not substantial (= 0.02) change in critical values.

For DF_{α_0,α_1} statistic, increase of the characteristic exponent by 0.1 increases 1% critical values by 0.073-0.300, 5% critical values by 0.008-0.057, and 10%

critical values by below 0.012. Changing length of time series from 100 to 1000 increases critical values from 0.016 (10% level of significance) to 0.151 (1%).

Standard errors of estimated response surfaces are not very high; except for 1% DF_{α_0,α_1} critical values they are not higher than 0.02.

We use response functions to estimate asymptotic critical values. They are very similar to Rachev and Mittnik (2000, p. 727) results (see Table 4).

5 Testing stationarity of stable AR(1) process with unknown alpha

The purpose of this section is to propose a modification of DF tests if the characteristic exponent α of process disturbance is unknown. Since in the real life we usually do not know parameters of the DGP, critical values given in Table 1 are not very useful.

We propose two modifications of DF tests, denoted $DF_{\hat{\alpha}}$ and DF_{ϕ}.

Firstly ($DF_{\hat{\alpha}}$ tests), we replace unknown value of parameter α by its estimator $\hat{\alpha}$. Following from null hypothesis (the equations (8)-(9)) we have $\Delta y_t \sim$ i.i.d.$S(\alpha, 0, c, 0)$. So for empirical time series $\{y_t\}$ in the first step we estimate characteristic exponent for the first differences $\{\Delta y_t\}$. To compute critical values we use parameters of response surfaces (Table 3). If we use consistent estimator of characteristic exponent, we can expect that $DF_{\hat{\alpha}}$ tests are asymptotically equivalent to DF_{α_1} and DF_{α_0,α_1} tests.

Secondly, we use combination of DF tests. Consider a following test

$$DF_f = f\left(DF_{\alpha_1} + DF_{\alpha_0,\alpha_1}\right),$$

where $f(\cdot)$ is a function for which critical values of DF_f test do not depend on characteristic exponent of disturbance. It is not easy (or perhaps even impossible) to prove that the function $f(\cdot)$ exists. However, we are able to construct the test with the same critical values for two border α's: $\alpha = 1$ and $\alpha = 2$.

Let $f(\cdot)$ be a linear function:

$$DF_{\phi} = \phi \cdot DF_{\alpha_1} + (1 - \phi) \cdot DF_{\alpha_0,\alpha_1}, \tag{12}$$

where $0 \leq \phi \leq 1$. From Monte-Carlo simulation (Table 1) we know that the difference between critical values for $\alpha = 1$ and $\alpha = 2$ is positive for $\phi = 1$ (i.e. DF_{α_1} test) and negative for $\phi = 0$ (DF_{α_0,α_1} test). Since critical values of the DF_{ϕ} test are continuous functions of ϕ (see Appendix 1 for proof), following from the Bolzano-Cauchy theorem, there exists $\phi \in (0, 1)$, for which the difference between critical values for $\alpha = 1$ and $\alpha = 2$ is equal to 0. We should note, however, that for $\alpha \in (1, 2)$ critical values of the test can differ, even in an asymptotic case.

We find ϕ's with precision 0.01 by Monte-Carlo simulations (50 000 replications). Simulation is repeated for $\phi = \{0.00, 0.01, \ldots, 1.00\}$. The criterion is minimum (square) difference between critical values for $\alpha = 1$ and $\alpha = 2$. All simulations are made under the same conditions: parameter *seed* (starting point in GAUSS random number generating procedure) is set at the level 28031970. The results of the simulation are in Table 5 (critical values are averages for $\alpha = 1$ snd $\alpha = 2$). As we see, for different sizes and numbers of observations optimal ϕ's are different.

To investigate properties of proposed tests, we use Monte-Carlo simulations. Firstly, we find real small-sample sizes of the proposed tests.

In the first step of Monte-Carlo simulation for $DF_{\hat{\alpha}}$ tests we simulate $\{y_t\}$ series using modified null hypothesis (11) (for assumed α and T values). Secondly, we estimate characteristic exponent α for the first differences $\{\Delta y_t\}$ using McCulloch GAUSS procedure (download from web-page http://www.econ.ohio-state.edu/jhm/jhm.html), which is an application of McCulloch (1986) quantile method. Next for $\hat{\alpha}$ we compute critical values of the tests, using response surfaces (Table 3). Null hypothesis is rejected if DF statistics are smaller than the critical values. The results of simulations are in table 6.

Standard error of estimator of probability p (= size of the test) is equal to $\sqrt{\frac{p(1-p)}{n}}$ (n – number of observations). Therefore standard error of estimator of significance level is equal to 0.04% for assumed size equal to 1%, 0.10% for assumed size equal to 5% and 0.13% for assumed size equal to 10%. Estimators of sizes differ significantly from assumed values in 26 cases of total 198 (= 13%) which is not acceptable fraction for 5% level of significance.

For DF_{ϕ} tests results are much worse. Using the results from Table 5 we compute real sizes of the test (see Table 7). Real sizes of the test differ significantly from assumed sizes in too many cases (47 of total 99 = 47%).

In comparison, powers of above tests are computed. We also take into account DF tests for known alpha – so three types of tests are considered. In every case null and alternative hypotheses are given by the equations (9) and (10). In addition autoregressive parameter α_1 in alternative hypothesis is chosen to be equal 0.95. Since distributions of DF statistics do not depend on parameters α_0 and c in alternative hypothesis (10), we can set them arbitrarily, e.g. $\alpha_0 = 1$ and $c = 1$:

$$H_1' : \begin{cases} y_t = 1 + 0.95 \cdot y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.} S(\alpha, 0, 1, 0), \\ t = 2, \ldots, T, \quad y_1 \sim S\left(\alpha, 0, \frac{1}{(1-0.95^{\alpha})^{1/\alpha}}, \frac{1}{1-0.95}\right). \end{cases} \tag{13}$$

We should distinguish between assumption $\alpha_1 = 0.95$ which is essential for the results and assumptions $\alpha_0 = 1$ and $c = 1$ which have no impact on results.

Powers of DF tests are found in the Monte-Carlo simulations. We generate $n = 50\ 000$ time series for which DGP is the alternative hypothesis. Power of the tests is a fraction of rejecting null hypothesis, if:

(i) We use critical values depending on known characteristic exponent α (table 1),

(ii) We use critical values computed as a function of estimator of characteristic exponent $\hat{\alpha}$. The function is estimated response surface (Table 3). Estimator $\hat{\alpha}$ is estimated for time series $\{\Delta y_t\}$,

(iii) We use DF_ϕ test, where ϕ's and critical values are given in table 5.

We can compare directly powers of the tests (ii) and (iii). Simulations for tests (i) inform us about lack of power due to lack of information about disturbance distribution. The results of simulations are in Tables 8, 9 and 10.

If alpha is known, power of DF test without an intercept is higher for 100 observations, and lower for 1000 observations, except 1% critical level with $\alpha < 1.25$ (see Table 8). There is no rule for behaviour of power as α increases; it can increase or decrease with α or have the maximum for certain alpha value.

Powers of DF tests using response surfaces (see Table 9) are similar to the previous case. It is probably the result of the length of time series – for 1000 observations RMSE of α estimator is about 0.004.

Comparing Tables 9 and 10 we can indicate which modification of DF tests is better in sense of power. For 100 observations we should use DF test without an intercept and for 1000 observations and large α's – DF test with an intercept (among tests for unknown α). On the other hand, for the intermediate case (500 observations) and for 1000 observations with small α's, the DF_ϕ test has the highest power. Let us note, that this test does not take into account information about observed estimator of characteristic exponent.

A Proof that critical values of DF_ϕ test are continuous functions of ϕ

Full proof is available from the author upon request. For $\lim_{n\to\infty} \phi_n = \phi$ there is $DF_{\phi_n} \xrightarrow{\text{d}} DF_\phi$. Therefore cumulative distributions and quantile functions converge. There is a little problem if the distribution of DF_ϕ is not strictly increasing.

References

[1] Akgiray V., G. Booth (1988), The Stable law model of stock returns, *Journal of Business & Economic Statistics*, **6**, 51-57

[2] Bidarkota P.V., J.H. McCulloch (1998), Optimal univariate inflation forecasting with symmetric stable shocks, *Journal of Applied Econometrics*, **13**, 659-670

[3] Blume M.E. (1970), Portfolio theory: a step towards its practical application, *Journal of Business*, **43**, 152-173

[4] Chambers J.M., C. Mallows, B.W. Stuck (1976), A method for simulating stable random variables, *Journal of the American Statistical Association*, **71**, 340-344

[5] Charemza W.W., D.F. Deadman (1997), *New Directions in Econometric Practice*, Edward Elgar, Cheltenham

[6] Christoph G., W. Wolf (1993), *Convergence Theorems with Stable Limit Law*, Academie Verlag, Berlin

[7] Cootner P. (1964), *The Random Character of Stock Market Prices*, Cambridge, Mass, M.I.T. Press

[8] Dickey D.A., W.A. Fuller (1979), Distributions of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association*, **74**, 427-431

[9] Dickey D.A., W.A. Fuller (1981), Likelihood ratio statistics for autoregressive time series with a unit root, *Econometrica*, **49**, 1057-1072

[10] Diebold F.X. (1993), Comment on "Modeling asset returns with alternative stable distributions," by S. Mittnik and S.T. Rachev, *Econometric Reviews*, **12**, 339-342

[11] DuMouchel W.H. (1975), Stable distributions in statistical inference: 2. Information from stably distributed samples, *Journal of the American Statistical Association*, **70**, 386-393

[12] Fama E.F. (1965a), The behavior of stock market prices, *Journal of Business*, **37**, 34-105

[13] Fama E.F. (1965b), Mandelbrot and the stable paretian hypothesis, *Journal of Business*, **38**, 34-105

[14] Fama E.F., R. Roll (1971), Parameter estimates for symmetric Stsble distributions, *Journal of the American Statistical Association*, **66**, 331-338

[15] Feller W. (1966), *An Introduction to Probability Theory and its Applications*, tom 2, John Wiley & Sons Inc., New York

[16] Gnedenko B.W., A.N. Kolmogorow (1957), *Rozklady graniczne sum zmiennych losowych niezaleznych*, tranlation from Russian, Krakowska Drukarnia Naukowa, Kraków

[17] Gribbin D.W., R.W. Harris, H. Lau (1992), Futures prices are not stable-paretian distributed, *Journal of Futures Market*, **12**, 475-487

[18] Hagernman R. (1978), More evidence on the distribution of security returns, *Journal of Finance*, **33**, 1213-1220

[19] Kendall M.G., A. Stuart (1963-1968), *The Advanced Theory of Statistics*, Hafner Publishing Company, New York

[20] Koedijk K.G., C.J.M. Kool (1992), Tail estimates of East European exchange rates, *Journal of Business and Economic Statistics*, **10**, 83-96

[21] Kwiatkowski D., P.C.B. Phillips, P. Schmidt, Y. Shin (1992), Testing the null hypothesis of stationarity against the alternative of a unit root, *Journal of Econometrics*, **54**, 159-178

[22] Mandelbrot B. (1963), The variation of certain speculative prices, *Journal of Business*, **36**, 394-419

[23] McCulloch J.C. (1986), Simple consistent estimators of stable distribution parameters, *Communications in Statistics: Simulation & Computation*, **15**, 1109-1136

[24] McCulloch J.H. (1996), Financial Applications of Stable Distributions, w: G.S. Maddala, C. Rao (red.), *Statistical Methods in Finance* (Handbook of Statistics, 14), Elsevier, Amsterdam, 395-425

[25] McCulloch J.H. (1998), Numerical approximation of the symmetric stable distribution and density, w: R. Alder, R. Feldman, M. Taqqu (red.), *A Practical Guide to Heavy Tailed Data*, Birkhäuser, Boston

[26] Müller U.A., M.M. Dacorogna, O.V. Picet (1998), *Heavy Tails in High-Frequency Financial Data*, w: R. Adler, R. Feldman, M. Taqqu (red.), *A Practical Guide to Heavy Tailed Data*, Birkhäuser, Boston

[27] Niemiro W. (1999), *Rachunek prawdopodobienstwa i statystyka matematyczna*, Szkola Nauk Scislych, Warszawa

[28] Nikias C.L., M. Shao (1995), *Signal Processing with Alpha-Stable Distributions and Applications*, John Wiley & Sons, New York

[29] Nolan J.P. (1999), *Maximum Likelihood Estimation and Diagnostics for Stable Distributions*, (web page http://academic2.american.edu/ jpnolan/stable/stable.html)

[30] Officer R. (1972), The distribution of stock returns, *Journal of the American Statistical Association*, **67**, 807-812

[31] Phillips P.C.B, P. Perron (1988), Testing for a unit root in time series regresson, *Biometrika*, **75**, 335-346

[32] Rachev S.T., S. Mittnik, J.-R. Kim (1998), Time series with unit roots and infinite-variance disturbances, *Applied Mathematical Letters*, **11**, 69-74

[33] Rachev S.T., J.-R. Kim, S. Mittnik (1999a), Stable paretian models in econometrics: Part 1, *Mathematical Scientists*, **24**, 24-55

[34] Rachev S.T., J.-R. Kim, S. Mittnik (1999b), Stable paretian models in econometrics: Part 2, *Mathematical Scientists*, **24**, 113-127

[35] Rachev S., J-S. Mittnik (2000), *Stable Paretian Models in Finance* (2000), Series in Financial and Quantitative Analysis, Wiley & Sons

[36] ROLL R. (1970), *The Bahavior of Interest Rates: The Application of the Efficient Market Model to U.S. Treasury Bills*, Basic Books, New York

[37] SAMORODNITSKY G., M.S. TAQQU (1994), *Stable Non-Gaussian Processes: Stochastic Models with Infinite Variance*, Chapman&Hall, New York

[38] SCHEICHER M. (1999), Nonlinear dynamics: evidence for a small stock exchange, *Empirical Economics*, **24**, 45-59

[39] TEICHMOELLER J. (1971), A note on the distribution of stock price changes, *Journal of the American Statistical Association*, **66**, 282-284

[40] YANG S.R., B.W. BRORSEN (1995), Nonlinear dynamics of daily foreign exchange rates, *Advanced in Quantitative Finance and Accounting*

[41] YOUNG M.S., R.A. GRAFF (1995), Real estate is not normal: A fresh look at real estate return distributions, *Journal of Real Estate Finance and Economics*, **10**, 225-259

[42] ZOLOTAREV V.M. (1983), *Odnomernye ustojcivye raspredelenija*, Izdatelstvo "Nauka", Moskva

Table 1: Critical values of DF statistics, depending on characteristic exponent and number of observations (50 000 Monte-Carlo replications).

– for 1% level of significance

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	-2.38	-2.36	-2.40	-5.47	-5.10	-5.20
1.1	-2.40	-2.39	-2.41	-5.20	-5.03	-5.01
1.2	-2.45	-2.38	-2.43	-4.95	-4.75	-4.80
1.3	-2.45	-2.40	-2.41	-4.67	-4.46	-4.42
1.4	-2.48	-2.42	-2.44	-4.37	-4.19	-4.22
1.5	-2.50	-2.47	-2.47	-4.24	-4.03	-4.06
1.6	-2.52	-2.48	-2.49	-3.98	-3.91	-3.84
1.7	-2.53	-2.48	-2.53	-3.83	-3.75	-3.72
1.8	-2.52	-2.51	-2.53	-3.68	-3.61	-3.65
1.9	-2.54	-2.56	-2.53	-3.55	-3.53	-3.53
2.0	-2.59	-2.59	-2.56	-3.52	-3.45	-3.45

– for 5% level of significance

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	-1.68	-1.69	-1.69	-3.22	-3.20	-3.21
1.1	-1.70	-1.73	-1.72	-3.18	-3.14	-3.13
1.2	-1.76	-1.75	-1.76	-3.13	-3.06	-3.12
1.3	-1.79	-1.77	-1.76	-3.07	-3.04	-3.03
1.4	-1.81	-1.79	-1.79	-3.04	-2.98	-3.01
1.5	-1.84	-1.83	-1.83	-3.01	-2.96	-2.96
1.6	-1.86	-1.84	-1.86	-2.98	-2.95	-2.94
1.7	-1.87	-1.87	-1.89	-2.96	-2.92	-2.93
1.8	-1.90	-1.90	-1.91	-2.93	-2.90	-2.91
1.9	-1.91	-1.92	-1.91	-2.90	-2.89	-2.88
2.0	-1.93	-1.94	-1.94	-2.89	-2.87	-2.87

– for 10% level of significance

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	-1.35	-1.36	-1.36	-2.64	-2.62	-2.64
1.1	-1.38	-1.39	-1.39	-2.63	-2.62	-2.61
1.2	-1.43	-1.42	-1.42	-2.63	-2.60	-2.62
1.3	-1.45	-1.44	-1.44	-2.60	-2.60	-2.59
1.4	-1.47	-1.47	-1.47	-2.59	-2.58	-2.59
1.5	-1.50	-1.50	-1.50	-2.60	-2.57	-2.58
1.6	-1.53	-1.53	-1.53	-2.59	-2.58	-2.57
1.7	-1.55	-1.55	-1.56	-2.60	-2.57	-2.57
1.8	-1.57	-1.57	-1.58	-2.59	-2.59	-2.58
1.9	-1.59	-1.60	-1.60	-2.58	-2.57	-2.57
2.0	-1.61	-1.61	-1.61	-2.58	-2.57	-2.57

Table 2: Confidence intervals for critical values of DF statistics (confidence level 0.95) depending on characteristic exponent and number of observations (50 000 Monte-Carlo replications).

– for 1% level of significance

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	(-2.42, -2.34)	(-2.41, -2.34)	(-2.44, -2.36)	(-5.57, -5.32)	(-5.24, -4.99)	(-5.35, -5.04)
1.1	(-2.44, -2.37)	(-2.43, -2.36)	(-2.45, -2.38)	(-5.41, -5.04)	(-5.22, -4.89)	(-5.18, -4.84)
1.2	(-2.50, -2.41)	(-2.41, -2.35)	(-2.47, -2.39)	(-5.08, -4.79)	(-4.89, -4.64)	(-4.94, -4.68)
1.3	(-2.48, -2.41)	(-2.44, -2.36)	(-2.46, -2.39)	(-4.82, -4.55)	(-4.58, -4.39)	(-4.52, -4.30)
1.4	(-2.50, -2.43)	(-2.45, -2.39)	(-2.48, -2.41)	(-4.48, -4.30)	(-4.29, -4.11)	(-4.36, -4.13)
1.5	(-2.53, -2.47)	(-2.51, -2.44)	(-2.51, -2.44)	(-4.32, -4.18)	(-4.10, -3.97)	(-4.13, -3.99)
1.6	(-2.56, -2.49)	(-2.51, -2.45)	(-2.53, -2.47)	(-4.05, -3.90)	(-3.98, -3.85)	(-3.91, -3.77)
1.7	(-2.56, -2.50)	(-2.52, -2.46)	(-2.57, -2.51)	(-3.89, -3.79)	(-3.80, -3.68)	(-3.76, -3.68)
1.8	(-2.55, -2.49)	(-2.53, -2.48)	(-2.56, -2.50)	(-3.73, -3.64)	(-3.65, -3.56)	(-3.69, -3.62)
1.9	(-2.56, -2.50)	(-2.59, -2.53)	(-2.55, -2.50)	(-3.58, -3.52)	(-3.56, -3.49)	(-3.56, -3.49)
2.0	(-2.62, -2.55)	(-2.62, -2.56)	(-2.59, -2.53)	(-3.56, -3.49)	(-3.48, -3.42)	(-3.48, -3.43)

– for 5% level of significance

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	(-1.70, -1.66)	(-1.71, -1.68)	(-1.71, -1.67)	(-3.26, -3.19)	(-3.23, -3.17)	(-3.24, -3.17)
1.1	(-1.73, -1.69)	(-1.75, -1.71)	(-1.74, -1.71)	(-3.22, -3.15)	(-3.17, -3.11)	(-3.16, -3.09)
1.2	(-1.78, -1.74)	(-1.77, -1.73)	(-1.78, -1.74)	(-3.15, -3.10)	(-3.10, -3.04)	(-3.15, -3.09)
1.3	(-1.80, -1.77)	(-1.79, -1.75)	(-1.77, -1.74)	(-3.10, -3.05)	(-3.07, -3.01)	(-3.06, -3.01)
1.4	(-1.82, -1.79)	(-1.81, -1.77)	(-1.80, -1.77)	(-3.06, -3.01)	(-3.01, -2.96)	(-3.03, -2.99)
1.5	(-1.85, -1.82)	(-1.84, -1.81)	(-1.84, -1.81)	(-3.03, -2.99)	(-2.98, -2.94)	(-2.99, -2.94)
1.6	(-1.88, -1.84)	(-1.86, -1.83)	(-1.87, -1.84)	(-3.00, -2.95)	(-2.97, -2.93)	(-2.96, -2.92)
1.7	(-1.89, -1.86)	(-1.89, -1.85)	(-1.90, -1.87)	(-2.98, -2.94)	(-2.94, -2.90)	(-2.94, -2.91)
1.8	(-1.91, -1.88)	(-1.92, -1.89)	(-1.92, -1.89)	(-2.95, -2.91)	(-2.92, -2.89)	(-2.92, -2.89)
1.9	(-1.93, -1.90)	(-1.94, -1.90)	(-1.93, -1.90)	(-2.91, -2.88)	(-2.91, -2.87)	(-2.89, -2.86)
2.0	(-1.95, -1.91)	(-1.96, -1.92)	(-1.95, -1.92)	(-2.90, -2.87)	(-2.88, -2.85)	(-2.88, -2.85)

– for 10% level of significance

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	(-1.36, -1.33)	(-1.37, -1.35)	(-1.37, -1.34)	(-2.66, -2.62)	(-2.64, -2.60)	(-2.66, -2.62)
1.1	(-1.39, -1.37)	(-1.41, -1.38)	(-1.40, -1.37)	(-2.65, -2.61)	(-2.64, -2.61)	(-2.63, -2.59)
1.2	(-1.44, -1.41)	(-1.43, -1.41)	(-1.43, -1.40)	(-2.65, -2.62)	(-2.61, -2.58)	(-2.63, -2.60)
1.3	(-1.46, -1.44)	(-1.45, -1.43)	(-1.46, -1.43)	(-2.62, -2.59)	(-2.61, -2.58)	(-2.61, -2.58)
1.4	(-1.49, -1.46)	(-1.48, -1.46)	(-1.48, -1.45)	(-2.61, -2.58)	(-2.59, -2.57)	(-2.61, -2.57)
1.5	(-1.52, -1.49)	(-1.51, -1.49)	(-1.52, -1.49)	(-2.62, -2.59)	(-2.59, -2.56)	(-2.59, -2.56)
1.6	(-1.54, -1.52)	(-1.55, -1.52)	(-1.54, -1.52)	(-2.61, -2.58)	(-2.60, -2.57)	(-2.59, -2.56)
1.7	(-1.57, -1.54)	(-1.56, -1.54)	(-1.57, -1.55)	(-2.61, -2.58)	(-2.58, -2.56)	(-2.59, -2.56)
1.8	(-1.58, -1.56)	(-1.58, -1.56)	(-1.59, -1.56)	(-2.60, -2.58)	(-2.60, -2.57)	(-2.59, -2.56)
1.9	(-1.60, -1.58)	(-1.61, -1.59)	(-1.61, -1.59)	(-2.59, -2.57)	(-2.59, -2.56)	(-2.59, -2.56)
2.0	(-1.62, -1.60)	(-1.62, -1.60)	(-1.62, -1.60)	(-2.59, -2.57)	(-2.58, -2.55)	(-2.58, -2.55)

Table 3: Response surfaces for critical values (standard errors in brackets).

– for 1% level of significance

parameter	**DF test without an intercept**	**DF test with an intercept**
intercept	-2.18 (0.02)	-9.39 (0.26)
α	-0.189 (0.011)	5.279 (0.364)
α^2	0 (-)	-1.138 (0.121)
T^{-1}	-2.3 (0.8)	-16.8 (2.6)
R^2	0.915	0.991
σ_ϵ	0.019	0.061

– for 5% level of significance

parameter	**DF test without an intercept**	**DF test with an intercept**
intercept	-1.32 (0.04)	-4,00 (0,05)
α	-0.433 (0.052)	1,054 (0,073)
α^2	0.062 (0.017)	-0,243 (0,024)
T^{-1}	0 (-)	-3,7 (0,5)
R^2	0.989	0,988
σ_ϵ	0.009	0,012

– for 10% level of significance

parameter	**DF test without an intercept**	**DF test with an intercept**
intercept	-0.96 (0.02)	-2,81 (0,03)
α	-0.470 (0.029)	0,255 (0,044)
α^2	0.070 (0.010)	-0,066 (0,015)
T^{-1}	0 (-)	-1,7 (0,3)
R^2	0.997	0,893
σ_ϵ	0.005	0,007

Table 4: Asymptotic critical values of DF statistics: empirical (simulated values in brackets) [computed from response function in square brackets].

	DF test without an intercept			**DF test with an intercept**		
alpha	1%	5%	10%	1%	5%	10%
1.1	-2.43 (-2.42) [-2.39]	-1.72 (-1.72) [-1.72]	-1.38 (-1.38) [-1.39]	-4.82 (-5.01) [-4.96]	-3.12 (-3.13) [-3.13]	-2.61 (-2.62) [-2.61]
1.5	-2.46 (-2.49) [-2.46]	-1.83 (-1.84) [-1.83]	-1.5 (-1.51) [-1.50]	-3.99 (-4.05) [-4.03]	-2.94 (-2.96) [-2.96]	-2.56 (-2.58) [-2.58]
1.9	-2.54 (-2.56) [-2.54]	-1.92 (-1.93) [-1.92]	-1.6 (-1.60) [-1.59]	-3.5 (-3.53) [-3.47]	-2.86 (-2.87) [-2.87]	-2.56 (-2.57) [-2.57]

Note: Empirical and simulated values are from Rachev and Mittnik (2000, p. 727).

Table 5: ϕ's and critical values for DF_ϕ tests (50 000 Monte-Carlo replications).

– ϕ's

level of significance	$T = 100$	$T = 500$	$T = 1000$
1%	0.71	0.63	0.67
5%	0.42	0.40	0.41
10%	0.14	0.18	0.18

– critical values

level of significance	$T = 100$	$T = 500$	$T = 1000$
1%	-2.67	-2.70	-2.65
5%	-2.27	-2.28	-2.26
10%	-2.37	-2.30	-2.29

Table 6: True sizes of $DF_{\hat{\alpha}}$ tests (50 000 Monte-Carlo replications).

– for assumed 1% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	0.010	0.011	0.010	0.011	0.010	0.011
1.1	0.010	0.010	0.011	0.010	0.009	0.010
1.2	0.010	0.010	0.010	0.010	0.010	0.010
1.3	0.010	0.010	0.010	0.010	0.010	0.010
1.4	0.010	0.010	0.010	0.010	0.010	0.010
1.5	0.010	0.011	0.009	0.010	0.010	0.011
1.6	0.010	0.010	0.010	0.010	0.010	0.011
1.7	0.010	0.010	0.010	0.009	0.011	0.011
1.8	0.011	0.010	0.010	0.009	0.010	0.011
1.9	0.010	0.011	0.011	0.008	0.010	0.011
2.0	0.011	0.011	0.010	0.006	0.010	0.010

– for assumed 5% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	0.050	0.049	0.049	0.053	0.051	0.050
1.1	0.050	0.049	0.049	0.050	0.050	0.050
1.2	0.050	0.049	0.050	0.049	0.050	0.049
1.3	0.049	0.050	0.048	0.052	0.050	0.050
1.4	0.050	0.048	0.049	0.049	0.050	0.050
1.5	0.051	0.052	0.048	0.050	0.049	0.050
1.6	0.050	0.050	0.051	0.049	0.051	0.050
1.7	0.052	0.051	0.048	0.048	0.050	0.050
1.8	0.052	0.049	0.050	0.049	0.049	0.050
1.9	0.050	0.051	0.052	0.047	0.049	0.051
2.0	0.053	0.053	0.052	0.047	0.048	0.050

– for assumed 10% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	0.099	0.098	0.097	0.101	0.103	0.100
1.1	0.098	0.099	0.100	0.102	0.101	0.100
1.2	0.098	0.099	0.100	0.098	0.100	0.100
1.3	0.100	0.098	0.098	0.101	0.102	0.102
1.4	0.098	0.097	0.099	0.100	0.100	0.101
1.5	0.101	0.102	0.097	0.101	0.100	0.100
1.6	0.099	0.099	0.101	0.098	0.101	0.100
1.7	0.101	0.100	0.097	0.099	0.101	0.100
1.8	0.100	0.100	0.100	0.099	0.098	0.100
1.9	0.102	0.101	0.101	0.098	0.100	0.103
2.0	0.103	0.103	0.101	0.099	0.101	0.103

Table 7: True sizes for DF_ϕ tests (50 000 Monte-Carlo replications).

– for assumed 1% critical value

alpha	$T = 100, \phi = 0.71$	$T = 500, \phi = 0.63$	$T = 1000, \phi = 0.67$
1.0	0.009	0.009	0.009
1.1	0.010	0.009	0.009
1.2	0.008	0.009	0.008
1.3	0.008	0.009	0.008
1.4	0.009	0.008	0.009
1.5	0.009	0.008	0.009
1.6	0.008	0.008	0.009
1.7	0.009	0.008	0.009
1.8	0.009	0.008	0.010
1.9	0.009	0.008	0.009
2.0	0.010	0.010	0.010

– for assumed 5% critical value

alpha	$T = 100, \phi = 0.42$	$T = 500, \phi = 0.40$	$T = 1000, \phi = 0.41$
1.0	0.051	0.049	0.050
1.1	0.050	0.050	0.048
1.2	0.047	0.050	0.048
1.3	0.049	0.049	0.048
1.4	0.047	0.048	0.047
1.5	0.047	0.048	0.047
1.6	0.047	0.048	0.049
1.7	0.049	0.048	0.047
1.8	0.047	0.046	0.048
1.9	0.048	0.047	0.049
2.0	0.048	0.049	0.051

– for assumed 10% critical value

alpha	$T = 100, \phi = 0.14$	$T = 500, \phi = 0.18$	$T = 1000, \phi = 0.18$
1.0	0.102	0.097	0.102
1.1	0.101	0.098	0.098
1.2	0.099	0.098	0.099
1.3	0.100	0.098	0.097
1.4	0.097	0.099	0.099
1.5	0.096	0.098	0.098
1.6	0.098	0.097	0.098
1.7	0.099	0.095	0.098
1.8	0.097	0.095	0.099
1.9	0.099	0.095	0.098
2.0	0.096	0.098	0.102

Table 8: Power of DF tests, for which critical values depend on known α (autoregressive parameter is equal to 0.95, 50 000 Monte-Carlo replications) (results for the tests with higher power are **bold**.)

– for 1% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	**0.039**	**0.827**	**0.929**	0.032	0.037	0.686
1.1	**0.036**	**0.819**	**0.930**	0.028	0.038	0.682
1.2	**0.041**	**0.815**	**0.928**	0.022	0.054	0.908
1.3	**0.040**	**0.803**	0.925	0.018	0.105	**0.952**
1.4	**0.043**	**0.782**	0.925	0.016	0.182	**0.986**
1.5	**0.044**	**0.761**	0.918	0.013	0.270	**0.993**
1.6	**0.046**	**0.741**	0.921	0.012	0.446	**0.997**
1.7	**0.044**	**0.717**	0.917	0.017	0.524	**0.998**
1.8	**0.047**	**0.696**	0.908	0.021	0.628	**0.999**
1.9	**0.047**	0.656	0.901	0.023	**0.709**	**1.000**
2.0	**0.045**	0.635	0.888	0.027	**0.759**	**1.000**

– for 5% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	**0.237**	0.894	0.951	0.093	**0.936**	**0.998**
1.1	**0.231**	0.892	0.953	0.092	**0.935**	**0.998**
1.2	**0.229**	0.891	0.952	0.087	**0.950**	**0.999**
1.3	**0.214**	0.892	0.955	0.087	**0.957**	**0.999**
1.4	**0.214**	0.889	0.957	0.095	**0.959**	**0.999**
1.5	**0.210**	0.889	0.957	0.093	**0.963**	**0.999**
1.6	**0.204**	0.880	0.961	0.102	**0.965**	**0.999**
1.7	**0.200**	0.872	0.963	0.106	**0.968**	**1.000**
1.8	**0.190**	0.865	0.965	0.112	**0.968**	**1.000**
1.9	**0.195**	0.848	0.969	0.125	**0.970**	**1.000**
2.0	**0.184**	0.839	0.970	0.121	**0.971**	**1.000**

– for 10% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	**0.488**	0.916	0.959	0.188	**0.991**	**0.999**
1.1	**0.455**	0.917	0.963	0.200	**0.992**	**0.999**
1.2	**0.449**	0.917	0.963	0.198	**0.991**	**0.999**
1.3	**0.412**	0.921	0.966	0.196	**0.992**	**0.999**
1.4	**0.400**	0.922	0.967	0.205	**0.993**	**0.999**
1.5	**0.390**	0.927	0.969	0.202	**0.994**	**1.000**
1.6	**0.373**	0.922	0.973	0.209	**0.994**	**1.000**
1.7	**0.358**	0.921	0.976	0.217	**0.994**	**1.000**
1.8	**0.346**	0.921	0.980	0.224	**0.994**	**1.000**
1.9	**0.338**	0.919	0.985	0.237	**0.995**	**1.000**
2.0	**0.324**	0.919	0.991	0.229	**0.996**	**1.000**

Table 9: Powers of $DF_{\hat{\alpha}}$ tests (autoregressive parameter is equal to 0.95, 50 000 Monte-Carlo replications).

– for assumed 1% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	0.034	0.825	0.928	**0.047**	0.056	0.472
1.1	**0.038**	0.821	0.927	0.034	0.065	0.660
1.2	**0.039**	0.806	0.928	0.027	0.089	0.818
1.3	**0.041**	0.798	0.926	0.021	0.134	0.919
1.4	**0.041**	0.781	0.922	0.018	0.200	**0.968**
1.5	**0.043**	0.760	0.924	0.016	0.302	**0.988**
1.6	**0.045**	0.741	0.920	0.016	0.414	**0.994**
1.7	**0.045**	0.718	0.916	0.014	0.526	**0.998**
1.8	**0.046**	0.692	0.910	0.015	0.616	**0.999**
1.9	**0.047**	0.661	0.903	0.017	0.681	**1.000**
2.0	**0.048**	0.642	0.894	0.018	0.726	**1.000**

– for assumed 5% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	**0.193**	0.889	0.950	0.105	0.945	0.998
1.1	**0.200**	0.891	0.951	0.095	0.950	0.999
1.2	**0.206**	0.889	0.954	0.093	0.955	0.999
1.3	**0.207**	0.890	0.953	0.089	0.958	0.999
1.4	**0.206**	0.886	0.956	0.092	0.961	0.999
1.5	**0.201**	0.882	0.959	0.094	0.962	0.999
1.6	**0.202**	0.878	0.960	0.098	0.966	1.000
1.7	**0.205**	0.874	0.962	0.104	0.967	1.000
1.8	**0.197**	0.865	0.966	0.108	0.968	**1.000**
1.9	**0.196**	0.854	0.969	0.113	0.968	**1.000**
2.0	**0.195**	0.844	0.973	0.116	0.968	**1.000**

– for assumed 10% critical value

	DF test without an intercept			**DF test with an intercept**		
alpha	$T = 100$	$T = 500$	$T = 1000$	$T = 100$	$T = 500$	$T = 1000$
1.0	**0.416**	0.911	0.959	0.199	0.991	0.999
1.1	**0.415**	0.915	0.961	0.195	0.992	0.999
1.2	**0.414**	0.916	0.963	0.198	0.992	0.999
1.3	**0.397**	0.920	0.965	0.199	0.993	0.999
1.4	**0.389**	0.919	0.967	0.202	0.994	1.000
1.5	**0.375**	0.921	0.971	0.207	0.993	1.000
1.6	**0.366**	0.921	0.972	0.212	0.993	**1.000**
1.7	**0.362**	0.925	0.976	0.218	0.994	**1.000**
1.8	**0.347**	0.923	0.981	0.224	0.995	**1.000**
1.9	**0.340**	0.920	0.986	0.230	0.995	**1.000**
2.0	**0.336**	0.918	0.992	0.230	0.995	**1.000**

Note: Real sizes of the tests can differ from assumed values. Results for tests with higher power (including DF_{ϕ} tests are **bold**.

Table 10: Power of DF_{ϕ} tests (autoregressive parameter is equal to 0.95, 50 000 Monte-Carlo replications).

– for assumed 1% critical value

alpha	$T = 100, \phi = 0.71$	$T = 500, \phi = 0.63$	$T = 1000, \phi = 0.67$
1.0	0.029	**0.864**	**0.960**
1.1	0.029	**0.857**	**0.960**
1.2	0.030	**0.853**	**0.963**
1.3	0.029	**0.846**	**0.964**
1.4	0.033	**0.840**	0.965
1.5	0.034	**0.829**	0.969
1.6	0.036	**0.819**	0.970
1.7	0.037	**0.817**	0.975
1.8	0.039	**0.805**	0.978
1.9	0.043	**0.793**	0.984
2.0	0.045	**0.780**	0.989

– for assumed 5% critical value

alpha	$T = 100, \phi = 0.42$	$T = 500, \phi = 0.40$	$T = 1000, \phi = 0.41$
1.0	0.135	**0.992**	**0.999**
1.1	0.136	**0.992**	**0.999**
1.2	0.138	**0.993**	**0.999**
1.3	0.141	**0.992**	**1.000**
1.4	0.149	**0.993**	**1.000**
1.5	0.152	**0.993**	**1.000**
1.6	0.160	**0.992**	**1.000**
1.7	0.164	**0.993**	**1.000**
1.8	0.173	**0.993**	1.000
1.9	0.180	**0.994**	1.000
2.0	0.185	**0.994**	1.000

– for assumed 10% critical value

alpha	$T = 100, \phi = 0.14$	$T = 500, \phi = 0.18$	$T = 1000, \phi = 0.18$
1.0	0.224	**0.995**	**0.999**
1.1	0.224	**0.995**	**0.999**
1.2	0.222	**0.996**	**0.999**
1.3	0.225	**0.996**	**0.999**
1.4	0.231	**0.996**	**1.000**
1.5	0.236	**0.996**	**1.000**
1.6	0.243	**0.997**	1.000
1.7	0.246	**0.997**	1.000
1.8	0.255	**0.998**	1.000
1.9	0.260	**0.998**	1.000
2.0	0.266	**0.999**	1.000

Note: Real sizes of the tests can differ from assumed values. Results for tests with higher power (including tests using response surfaces) are **bold**.