Competitive Convergence and Divergence: Position and Capability Dynamics

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Abstract

This paper addresses the question of competitive convergence of an industry. When do firms in a given industry make similar strategic choices, compete closely and become more alike over time? I focus on convergence in the equilibrium product portfolio and capability choices. Using the concept of Markov Perfect Nash Equilibrium, I numerically solve, simulate and analyze a dynamic model of competition with multiproduct firms. My main findings are that the industry converges after a few periods when capability depreciation is low and diverges, i.e., firms specialize in different segments, when capability depreciation increases. Other parameters affecting a firm's costs of increasing its capability and the benefits of specializing in the high-end segment, such as: investment efficiency and market size, play a similar role. Also, when firms converge or diverge, they do so in both capability and product dimensions.

1 Introduction

This paper investigates when and why we observe competitive convergence in an industry. When do firms tend to make similar strategic choices (and see their profits shrink)

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and when do they instead specialize in distinct segments? What exogenous factors favor competitive convergence and what endogenous mechanism explains such industry evolution?

I use the term *competitive convergence* to express the idea that strategic distances between firms become smaller over time. Suppose one represented firms as a large vector of important characteristics such as product choice, organizational form, cost structure, distribution channel, and the degree of vertical integration. In this setting, competitive convergence would indicate that the distance between these vectors becomes smaller. Convergence could potentially occur in all of these dimensions, of these I focus on two critical dimensions: product choice and capability. Following Besanko, Dranove and Shanley (2000), I define *capability* as a firm's general ability to perform critical business activities well. The main characteristics of capabilities are: they are valuable across several products, they are embedded in the firm's organizational routines, and they are tacit and therefore cannot be imitated. The critical activities constituting a given firm's capability differ across industries. A firm's capability can also depreciate as technology and management best practices evolve, as consumers become more sophisticated, and better informed of outside opportunities and service benchmarks. Therefore, firms need to invest in order to increase their capability level but also to maintain it. The notion of product position refers to a given product portfolio and the quality of its products.

My main finding is that an industry converges after a few periods when the rate at which capability depreciates is low; it diverges, i.e., firms specialize in different segments, when the rate of depreciation is high. Other factors determining convergence are: investment efficiency and the profitability of the respective product and consumer segments. This paper explains how firms get to contradict the principle of maximal differentiation, get caught in a type of prisoner's dilemma that leads them to competitive convergence. Firms only escape the less profitable equilibrium when it becomes too costly for a firm lagging in capability to catch-up with the leader. The prisoner's dilemma stems from the fact that both firms are better off in a divergent state, but in a convergent state, neither has a unilateral incentive to deviate and scale back its capability and bring the industry to the more profitable divergent state. Also, when firms converge or diverge they do so

¹ "The most critical strategic choices exhibited by a firm are those concerned with the selection of the product areas or segments in which the firm will compete and the basic approaches to those businesses." Rumelt (1981).

in both capability and product dimensions. This comes from the fact that the model incorporates elements of strategic fit, i.e., capability and quality are complements. A related description of the above mechanism leading to competitive convergence is given in the following business strategy interpretation:

"Most companies focus on matching and beating their rivals, and as a result their strategies tend to converge along the same basic dimensions of competition. Such companies share an implicit set of beliefs about "how we compete in our industry or in our strategic group." They share a conventional wisdom about who their customers are and what they value, and about the scope of products and services their industry should be offering. The more that companies share this conventional wisdom about how they compete, the greater the competitive convergence. As rivals try to outdo one another, they end up competing solely on the basis of incremental improvements in cost or quality or both," Chan and Mauborgne (1999).

Although I use a game theoretic approach where firms are far-sighted and rational in the sense that they solve interdependent dynamic programming problems, my model contributes to both competitive strategy and the evolutionary approach to industry dynamics in that it addresses a major strategic question for firms. My research is related to three major research streams: dynamic industrial organization models in the Ericson and Pakes (1995) and Pakes-McGuire (1994) tradition, static industrial organization models and industry evolution models. As my approach and model is close to Ericson and Pakes (1995) (henceforth E-P), I first review their work before describing my model and its specificities.

E-P's dynamic competition model allows for heterogeneity among firms, and idiosyncratic shocks and industry-wide shocks. The profitability of a firm is determined by a single state variable: the efficiency levels of the firms in the industry. Efficiency levels themselves depend on the success of the firm's own investment, on the stochastic outcome of other firms' investments, and on the competitive pressure from outside the industry. If profitability is below a salvation value, the firm exits the industry. These optimal decisions are taken by solving dynamic programming problems that are interdependent across firms through the state and beliefs on industry evolution. At equilibrium, these beliefs are also consistent with the process generated by the optimal decisions of all firms. In a companion paper, Pakes and McGuire (1994) provide an algorithm to compute these types of models, to which I will refer. A good overview of this stream of research can be found in Pakes (2000). This framework has already been used to successfully address issues such as mergers and collusion in a dynamic setting with heterogeneous firms. Welfare results are particularly interesting, as allowing for entry and exit as well as a merger process or collusion process to be endogenous (Gowrisankaran (1999), Fershtman and Pakes (2001)), can actually increase welfare and make consumers better off.

I extend the framework by E-P to address the particular issues in competitive convergence. I then compute the equilibrium investment functions, the Markov transition matrix, and simulate industry evolution for various sets of parameters. The key extensions of the model are: there are two endogenous state variables per firm (capability and product portfolio), investment is three dimensional, firms may sell more than one product, and there are two segments of consumers. Each period, firms compete in prices and dynamically choose how much to invest to increase their capability and to develop and sustain their product offering. I do not directly consider economies of scope; the benefit from multiproducts comes essentially from the demand side.² For tractability I make the following restriction: I consider only two firms that can enter and exit the segments of the industry and can stop producing, but they do not completely exit the industry. Firms that do not produce can still invest and maintain their capability, subject to industry depreciation. No outside firms can enter the industry. This differs from evolutionary models, where entry and exit are the drivers of the evolution process.

Although my model does not incorporate typical elements of industry evolution theories such as entry and exit and therefore does not include a selection process, it does borrow some key elements such as idiosyncratic shocks, heterogeneous capabilities, and costs of expansion. In contrast to the vast majority of models in the evolutionary literature, where decision makers are subject to behavioral continuity, and are myopic profit maximers in a perfectly competitive environment; in my model, firms are rational, forward looking and strategic in an oligopoly setting. (Klepper (2002) and Klepper and Thompson (2002))

My paper contributes to the field of industrial organization in that it is the first dynamic model of competition with heterogeneous and multiproduct firms.³ Little is

²Except that capability is valuable across products, so that investment in capability may increase the quality of the whole product portfolio.

³To my knowledge, with a different focus, only the marketing literature has started addressing a similar type of model (Youn (2002)).

known on this combination of models, although its empirical importance is crucial.

The industrial organization literature has concerned itself with related questions in a static setting, generally with horizontal differentiation and strategic selection of productlines. More specifically, it explores the conditions that lead to a segmentation equilibrium, where individual firms offer products that are close substitutes (comparable to an industry with convergence of product positions), versus an interlacing equilibrium, where individual firms offer products that are distant substitutes (comparable to divergence of product positions). The trade-off is that, when introducing a new product on the market, a potential multiproduct firm has to weigh the benefits of expanding its offer against loss due to increased price competition. These two effects have been described as expansion effect and competition effect by Shaked and Sutton (1990). Static models of product line rivalry such as Brander and Eaton (1984), and Martinez-Giralt and Neven (1988) find that simultaneous scope decisions by both firms favor an interlacing equilibrium, whereas sequential decisions yield a segmentation equilibrium.⁴ In contrast, in my dynamic model, commitment to a certain product position, is neither total, as in a 2-stage sequential-move game, or empty, as in a simultaneous-move game, but rather firms are partially committed to their positions through the necessity of building capability. I further emphasize similarities and differences of the static literature with my model in the discussion section.

In the next section, I present the model. In section three, I first describe my results for a baseline case, then compare them to the case of a social planner and a collusive industry. In section four, I discuss my results and perform some comparative dynamics to examine how the results change as investment efficiency, capability depreciation, market and segment size vary. It is not meant as an exhaustive exercise of robustness but more as an indication that predictions change consistently with economic intuition. In section five, I provide an industry example to illustrate my results. Finally, I conclude and suggest further research. I describe the algorithms in the appendix.

⁴Gilbert and Matutes (1993) study product line rivalry by including both quality and brand name differentiation, where brand name differentiation is modelled as consumers having firm-specific tastes. In a three-stage game, where both firms sequentially commit to their product choice and they then compete in prices, both specialization and product rivalry equilibria can be observed. The basic trade-off is that if firms are not close competitors (i.e., there is significant brand differentiation), they both benefit from discriminating and offering a full product line; otherwise, when brand differentiation is low, they benefit more by specializing to decrease price competition. In equilibrium, firms may crowd the product space when all competitors would be better off specializing.

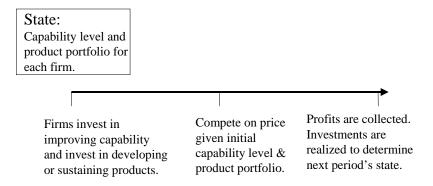


Figure 1: Within-period order of actions

2 The Model

The model focuses on an industry that consists of two firms j = 1, 2, and two segments of consumers k=1,2. There is no industry entry and exit but segment entry and exit. Each firm may produce up to two types of goods $\theta = l$, h, the "low-end" good and the "high-end" good. Firms are distinguished along two dimensions: their capability levels (to be discussed in more detail below) and the composition of their product portfolio (i.e., whether they offer both goods, just the low-end good, just the high-end, or none at all). In each period, static Nash equilibrium prices clear the market. In each period t, firms choose the level of two kinds of investments: y_{jt} to improve their capability levels (and therefore the perceived quality of their goods) and $y_{j\theta t}$ to develop or sustain the product θ . Returns on both investments are stochastic. Figure 1 describes the moves within each period. In each state, investments are chosen so as to maximize the present discounted value of expected future profits, given my rivals optimal investments. I use the concept of Markov Perfect Nash equilibrium (MPNE) to solve for the equilibrium investments. The industry converges competitively if the capability levels and the product portfolios of the two firms become more similar over time. In the following subsections, I elaborate on each of the key elements of the model.

2.1 States

Given each period's Nash equilibrium in prices, a firm's investment depends on the state of the industry: the capability level and the products it already produces and the capability levels and product portfolio of its competitors. An industry's state is a vector $s \in \Omega^2 \times \Theta^2$ comprised of two variables:

- the vector of capability levels of the active firms: $\boldsymbol{\omega} = (\omega_1, \omega_2) \in \Omega^2$, $\omega_j \in \Omega = \{1, ..., \overline{\omega}\}$, for j = 1, 2.
- the vector of product portfolios of these same firms: $\boldsymbol{\theta} = (\theta_1, \theta_2) \in \Theta^2$, $\theta_j = (l, h) \in \Theta = \{(l, h) | l \in \{0, 1\}, h \in \{0, 1\}\}$ for j = 1, 2. That is, $\theta_j = (1, 0)$ means that firm j offers the low-end product, but not the high-end product.

A complete state will be denoted as: $s = (\omega_1, \omega_2; \theta_1, \theta_2)$, or more compactly as $s = (\omega, \theta)$. I extend the notation of θ to denote a particular product, when bold it refers to the full vector of product portfolios in the current state.

2.2 Utility and Demand

Consumer behavior follows a standard discrete choice framework. Consumer preferences over product attributes (quality and price) are segment specific. In addition, each consumer has idiosyncratic preferences over the set of products offered in each period. In particular, consumer h = 1, ..., H of segment k, derives a utility from a good of type θ , produced by firm j, of:

$$u_{j\theta}^{kh} = u_0 + \beta^k f_{\theta}(\omega_j) - \alpha^k p_{j\theta} + \varepsilon_{j\theta}^h,$$

where $f_{\theta}(\omega_{j})$ is interpreted as the quality of product θ offered by firm j (it is discussed below), $p_{j\theta}$ is the price that firm j charges for product θ , $\varepsilon_{j\theta}^{h}$ is consumer h's idiosyncratic preference for product θ produced by firm j, and α^{k} and β^{k} are preference parameters. The exogenous idiosyncratic preference terms $\varepsilon_{j\theta}^{h}$ have a Type I extreme value distribution and are i.i.d. across j's, θ 's and h's. The preference parameters α^{k} and β^{k} are segment specific. I assume that $\beta^{2} > \beta^{1} > 0$ and that $\alpha^{1} > \alpha^{2}$, which implies that consumers in segment 1 are less sensitive to quality and more sensitive to prices than consumers in segment 2.⁵ The price of a firm product $p_{j\theta}$ is the same across segments, so there is no second-degree price discrimination. The function f_{θ} translates the capability

⁵This type of utility function is more common in the marketing literature, it follows from Berry, Carnall, and Spiller (1997). The choice of a utility function with segments is important in this question of convergence or divergence because it creates two differentiated masses of consumers. A unique segment of consumers would strongly favor industry convergence.

of the firm in exploiting its production technology, assets, resources and knowledge into quality attributes for good of type θ . In order to retain symmetry, f is product specific but not firm specific. I will sometimes refer to f as the quality function. Product l is the low-end product, while h is the high-end product.

I assume: $f_l(\omega_j) < f_h(\omega_j)$, for all $\omega \in \Omega$, which implies that for a given capability of firm j, the quality of its l product is lower than that of its h product. It is also assumed that there is more potential for increasing the quality of h than l: $0 < f'_l(\omega) < f'_h(\omega)$. Finally, there are decreasing returns to increasing its capability and $f_h^{0}(\omega) < f_l^{0}(\omega) < 0$.

Maximizing this utility yields the probability that a consumer from segment k buys θ from firm j:

$$x_{j\theta}^{k}(s) = \frac{\exp\left(\beta^{k} f_{\theta}(\omega_{j}) - \alpha^{k} p_{j\theta}\right)}{1 + \sum_{(i,l) \in I_{s}} \exp\left(\beta^{k} f_{l}(\omega_{i}) - \alpha^{k} p_{il}\right)},\tag{1}$$

where I_s is the set of index couples of unique firm-product combinations in state s. If $s = (\omega_1, \omega_2; (1, 1), (1, 0))$, then $I_s = \{(1, l_1 = 1), (1, h_1 = 1), (2, l_2 = 1)\}$. Note that there is an outside good that gives a utility normalized to u_0 for consumers in either segment. The probability that good θ from firm j is being bought across segments is: $x_{j\theta} = \lambda^1 x_{j\theta}^1 + \lambda^2 x_{j\theta}^2$, where λ^k is the relative size of segment k, $\lambda^1 + \lambda^2 = 1$.

2.3 Nash Equilibrium in Prices

The per-period profit maximization problem for firm j is:

$$\Pi_{j}(s, p_{-j}) = \max_{p_{jl}, p_{jh}} \Pi_{j}(s, p_{jl}, p_{jh}, p_{-j})
= \max_{p_{jl}, p_{jh}} M \left[(p_{jl}(s) - c_{l}) x_{jl} (p_{jl}, p_{jh}, p_{-j}, s) l_{j} + (p_{jh}(s) - c_{h}) x_{jh} (p_{jl}, p_{jh}, p_{-j}, s) h_{j} \right].$$
(2)

M is the total size of the market in this industry. To focus on issues of interest, the marginal cost of production is assumed to be the same for all firms, in all states, and thus differs only across product types.

For given levels of ω_1 and ω_2 , there are 16 possible states They are described by: $s = (\omega_1, \omega_2; (l_1, h_1), (l_2, h_2))$, where $l_j = 0, 1$ and $h_j = 0, 1$. I solve for the symmetric Nash equilibrium in prices for each of these states, conditional on the capability levels ω_1 and ω_2 .

The system of first-order conditions for a given state s and a given firm j is:

$$\mathbf{X}_i(s) + \mathbf{\Phi}_i(s) [\mathbf{p}_i(s) - \mathbf{c}] = 0,$$

where the matrices of demand, the derivative of demand and the markup are defined as below. I introduce an index that lists the products sold by firm j in state s: $n_j(s) = 1$, $|\theta_j(s)|$, in my model though this is an index for just l, just h or both l and h.

$$\mathbf{X}_{j}(s) = \begin{bmatrix} x_{j1}(s) \\ x_{j|\theta_{j}(s)|}(s) \end{bmatrix},$$

$$\mathbf{\Phi}_{j}(s) = \begin{bmatrix} \sum_{k} \lambda^{k} \frac{\partial x_{j1}^{k}(s)}{\partial p_{j1}} \dots & \sum_{k} \lambda^{k} \frac{\partial x_{j}^{k}(s)}{\partial p_{j1}} \\ \sum_{k} \lambda^{k} \frac{\partial x_{j1}^{k}(s)}{\partial p_{j|\theta_{j}(s)|}} \dots & \sum_{k} \lambda^{k} \frac{\partial x_{j}^{k}(s)}{\partial p_{j|\theta_{j}(s)|}} \end{bmatrix},$$
and $[\mathbf{p}_{j}(s) - \mathbf{c}] = \begin{bmatrix} p_{j1}(s) - c_{1} \\ p_{j|\theta_{j}(s)|}(s) - c_{|\theta_{j}(s)|} \end{bmatrix}.$

Further:

$$\frac{\partial x_{jn_{j}(s)}^{k}(s)}{\partial p_{jn_{j}(s)}(s)} = -\alpha^{k} x_{jn_{j}(s)}^{k} \left(1 - x_{jn_{j}(s)}^{k}\right), \qquad (3)$$

$$\frac{\partial x_{jn_{j}(s)}^{k}(s)}{\partial p_{j-n_{j}(s)}(s)} = \alpha^{k} x_{j-n_{j}(s)}^{k} x_{jn_{j}(s)}^{k},$$

for
$$n_{j}(s) = 1, ..., |\theta_{j}(s)|$$
 and $-n_{j}(s) = 1, ..., |\theta_{j}(s)|$ with $-n_{j}(s) \neq n_{j}(s)$.

Solving the $|\theta_j(s)|$ F.O.C. equations for j = 1, 2 does not yield a closed form for the Nash equilibrium prices. I solve the optimization program numerically. (See Appendix B for the algorithm.)

I compute equilibrium prices in all states although there is no theoretical proof of the existence of this equilibrium in the states where either firm has more than one product.⁶ Further, one cannot rule out the possibility that several equilibria exist; however the computations for the parameters used in this analysis mostly lead to unique outcomes. In the set of states where four products are offered, in a very limited number of cases, I select the equilibrium closest to the ones in neighboring states.

⁶This is because there is no existence result for multiproduct firms and logit demand with discrete segments. Andersen, de Palma and Thisse (1992) show that a unique Nash equilibrium in prices exists in the case of a multiproduct oligopoly and nested logit demand with no outside alternative, identical quality and equal marginal production costs across variants. Caplin and Nalebuff (1991), provide conditions under which there exists a Nash equilibrium in prices in a multiproduct model of product differentiation, where consumer types can be Normal, Pareto, Weibull and Beta distributed. They find existence results in the case of logit demand with random coefficients and duopoly competition, however their result does not apply to this case where the support of the preference parameters is not convex (segments are discrete).

2.4 The dynamic maximization program

Laws of motion of states. The investment technology for incumbent firms is such that if firm j has a capability of $\tau \omega_{jt}$ at time t, then its capability at time t+1 is $\tau \omega_{jt+1}$, where τ is a constant exogenous scaling factor, whose purpose is to make the state space coarser. Capability states evolve according to the following law of motion:

$$\tilde{\omega}_{it+1} = \min\{\max\{\omega_{jt} + \tilde{\nu}_{jt} - \tilde{\nu}_{0t}, 1\}, \overline{\omega}\}, \text{ for } \omega_{jt} \in \{1, 2, ..., \overline{\omega}\}, \ t = 1, ..., \infty$$
 (4)

where the random variable $\tilde{\nu}_{jt} \in \{0, 1\}$ is endogenous. It is the increment in capability resulting from the firm's investment y_{jt} , it is assumed to be independent across firms and stochastically increasing in y. The random variable $\tilde{\nu}_0 \in \{0, 1\}$ is exogenous, it is the increment in capability of the consumer's outside alternative. $\tilde{\nu}_{0t}$ represents industry-wide capability depreciation, and its realization is common to all firms. Industry depreciation can be thought of as improvement in outside available technologies, best practices, or increased customer expectations due to these factors. When a firm's capability depreciates, the utility derived by a given good decreases, and firms need to increase their capability to maintain the same utility level. Therefore, capability depreciation can be thought of as consumers becoming more sophisticated as technological improvements or service improvements affect overall appreciation for products in this industry.

The min and max functions are only introduced to bound the set of capabilities; they are only active when $\omega_{jt} \in \{1, \overline{\omega}\}$. Consequently, a firm with a capability of 1 is not affected by a positive realization of the outside alternative, similarly, a firm with a capability of $\overline{\omega}$ cannot increase its capability even by investing. In the numerical computation, the upper bound is sufficiently large so that it does not bind and the firms stop investing at $\omega_{jt} \leq \overline{\omega} - 1$.

I further specify:

$$\gamma\left(\tilde{\nu}_{jt} = 1 | y_{jt}\right) = \frac{ay_{jt}}{1 + ay_{jt}} \text{ and } \gamma\left(\tilde{\nu}_{jt} = 0 | y_{jt}\right) = \frac{1}{1 + ay_{jt}},$$
$$p\left(\tilde{\nu}_{t0} = 1\right) = \delta \text{ and } p\left(\tilde{\nu}_{t0} = 0\right) = 1 - \delta.$$

Thus, the parameter δ indicates the rate at which firms' capabilities stochastically depreciate. The functional form of the investment function was chosen to satisfy continuity in y and strict concavity, in order to bound the equilibrium investment choices.

For product introduction, the laws of motion are:

$$\tilde{l}_{jt+1} = l_{jt} + \tilde{\mu}_{jt} \tag{5}$$

$$\tilde{h}_{jt+1} = h_{jt} + \tilde{\eta}_{jt}, \tag{6}$$

where $\tilde{\mu}_{jt}$ and $\tilde{\eta}_{jt}$ are the stochastic investment outcomes. They are independent across firms, stochastically increasing in y_{ljt} and y_{hjt} respectively, and take the values: $\{-1,0,1\}$. The random variables $\tilde{\mu}_{jt}$ and $\tilde{\eta}_{jt}$ govern how a firm's product portfolio evolves over time. Note that investment is required both to create new products and to sustain existing ones.

The probability distributions governing $\tilde{\mu}_{jt}$ and $\tilde{\eta}_{jt}$ are as follows:

If
$$l_{jt} = 0$$
: $\tilde{\mu}_{jt} = \{ \begin{cases} 1 \\ 0 \end{cases}$, else $l_{jt} = 1$: $\tilde{\mu}_{jt} = \{ \begin{cases} 0 \\ -1 \end{cases}$.

$$\rho_l \left(\tilde{\mu}_{jt} = 1 | y_{ljt}, l_{jt} = 0 \right) = \frac{b_l y_{ljt}}{1 + b_l y_{ljt}} \text{ and } \rho_l \left(\tilde{\mu}_{jt} = 0 | y_{ljt}, l_{jt} = 0 \right) = \frac{1}{1 + b_l y_{ljt}}.$$

$$\rho_l \left(\tilde{\mu}_{jt} = 0 | y_{ljt}, l_{jt} = 1 \right) = \frac{b_{ll} y_{ljt} + \delta_l}{1 + b_{ll} y_{ljt}} \text{ and } \rho_l \left(\tilde{\mu}_{jt} = -1 | y_{ljt}, l_{jt} = 1 \right) = \frac{1 - \delta_l}{1 + b_{ll} y_{ljt}}.$$

If
$$h_{jt} = 0$$
: $\tilde{\eta}_{jt} = \{ \begin{cases} 1 \\ 0 \end{cases}$, else $h_{jt} = 1$: $\tilde{\eta}_{jt} = \{ \begin{cases} 0 \\ -1 \end{cases}$.

$$\begin{split} \rho_h \left(\tilde{\eta}_{jt} = 1 | y_{hjt}, h_{jt} = 0 \right) &= \frac{b_h y_{hjt}}{1 + b_h y_{hjt}} \text{ and } \rho_h \left(\tilde{\eta}_{jt} = 0 | y_{hjt}, h_{jt} = 0 \right) = \frac{1}{1 + b_h y_{hjt}}. \\ \rho_h \left(\tilde{\eta}_{jt} = 0 | y_{hjt}, h_{jt} = 1 \right) &= \frac{b_{hh} y_{hjt} + \delta_h}{1 + b_{hh} y_{hjt}} \text{ and } \rho_h \left(\tilde{\eta}_{jt} = -1 | y_{hjt}, h_{jt} = 0 \right) = \frac{1 - \delta_h}{1 + b_{hh} y_{hjt}}. \end{split}$$

For a given amount of investment, the chances of successfully introducing a high-end product is assumed to be lower than for a low-end product: $b_h < b_l$. Also: $b_h < b_{hh}$, $b_l < b_{ll}$, which implies that it requires more investment to develop a product than to maintain one. Generally, in the numerical analysis below, $\delta_l = \delta_h$ and is a small number. Firms have the possibility of not offering any product at all, while keeping their capability level, subject to industry depreciation. However, they do not have the option of exiting and collecting a salvation value.

Bellman Equation Both firms choose their optimal investments by maximizing expected future discounted cash flows, given the state-to-state transitions and given the other firm's investments. Each firm's discount factor is denoted by β , with $\beta \in (0,1)$. Let $V_1(s)$, with $s = (\omega_1, \omega_2, (l_1, h_1), (l_2, h_2))$, denote Firm 1's expected net present value of being in the industry, given that it behaves optimally, that it has a capability of ω_1 , a product portfolio (l_1, h_1) and that Firm 2 has a capability of ω_2 and a product portfolio (l_2, h_2) :⁷

$$= \max_{y \geq 0, y_{l} \geq 0, y_{h} \geq 0} \left[\beta \sum_{\nu_{1}, \mu_{1}, \eta_{1}} W_{1}(\omega_{1} + \nu_{1}, (l'_{1}, h'_{1})) \rho_{l}(\mu_{1} | y_{l1}, l_{1}) \rho_{h}(\eta_{1} | y_{h1}, h_{1}) \gamma(\nu_{1} | y_{1}) \right]$$

$$g_{1}(s, y_{1}, y_{l1}, y_{h1}, V_{1})$$

$$s.t. (4), (5), (6).$$

In the above expression, $W_1(\omega_1 + \nu_1, (l'_1, h'_1))$ is the expectation over industry capability depreciation and Firm 2's capability and product portfolio states of the discounted future cash flow, given that Firm 2 invests y_2 , y_{l2} , y_{h2} . Thus:

$$W_{1}(\omega_{1} + \nu_{1}, (l'_{1}, h'_{1})) = \sum_{\nu_{2}, \nu_{0}, \mu_{2}, \eta_{2}} V_{1}((\omega'_{1}, \omega'_{2}, (l'_{1}, h'_{1}), (l'_{2}, h'_{2})) \rho_{l}(\mu_{2}|y_{l2}, l_{2}) \rho_{h}(\eta_{2}|y_{h2}, h_{2}) \gamma(\nu_{2}|y_{2}) p(\nu_{0}), (7)$$

given that the states follow (4), (5), (6) and $g_1(s, y_1, y_{l1}, y_{h1}, V_1)$, is Firm 1's dynamic objective function.

Investment Functions and Dynamic Equilibrium. The optimal investment functions must solve the Bellman equation and satisfy the first-order condition with respect to the firm's choice variables in each state (See Appendix C for a statement of those first-order conditions). The static model results in a symmetric profit matrix, $\Pi(s) = \Pi_1(\omega_1, \omega_2, (l_1, h_1), (l_2, h_2)) = \Pi_2(\omega_2, \omega_1, (l_2, h_2), (l_1, h_1))$. Agents solve value functions that are interdependent only through the state and beliefs about rival's actions. The

⁷Note that I allow firms to invest more in a given period than the current profit flow. This implicitly assumes the existence of a perfect capital market, where firms can borrow from their expected discounted profits at an interest rate of $\frac{1}{3} - 1$.

⁸For simplicity, in the expressions in this section, time subscripts have been dropped from all variables and we use primes to denote next-period capabilities and product-line decisions.

investment strategies for each player, at all states, are chosen to be optimal given all other agents' optimal decisions. I therefore restrict myself to symmetric Markov Perfect Nash Equilibria (MPNE) in pure strategies. The optimal strategies and transition probabilities are functions only of states, independent of the history of how they were reached.⁹

Computation and Parameterization. To compute the symmetric MPNE, I adapt the algorithm of Pakes and McGuire (1994). The algorithm that I used can be found in Appendix B. All programs have been written in Fortran 90. 10

Parameter	Explanation	Value
u_o	Utility of outside good	5
β	Marginal utility of quality	(1, 2.8)
α	Marginal disutility of price	(1.8, 0.67)
λ	Segment sizes (1 and 2 respectively)	(.9, .1)
c	Marginal costs (l and h respectively)	(0,19)
$f_h(\tau\omega)$	h quality function for $\omega \in \{1,, 12\}$	$4.5\sqrt{\tau\omega}$
	h quality function for $\omega \in \{13,, 20\}$	$4.5\sqrt{\tau 13} + \ln\left(2 - e^{-4.5(\sqrt{\tau \omega} - \sqrt{\tau 13})}\right)$
	h quality function for $\omega \in \{21,, 25\}$	$4.5\sqrt{\tau 13} + \ln\left(2 - e^{-4.5(\sqrt{\tau 21} - \sqrt{\tau 13})}\right)$
$f_l(\tau\omega)$	l quality function for $\omega \in \{0,, 12\}$	$\sqrt{ au\omega}$
	l quality function for $\omega \in \{13,, 20\}$	$\sqrt{\tau 13} + \ln\left(2 - e^{-(\sqrt{\tau\omega} - \sqrt{\tau 13})}\right)$
	l quality function for $\omega \in \{21,, 25\}$	$\sqrt{\tau 13} + \ln \left(2 - e^{-(\sqrt{\tau 21} - \sqrt{\tau 13})} \right)$
τ	Scaling factor	2

Static Reference Parameters

The parameters of both the static and dynamic problems are of the same order of magnitude as in Pakes and McGuire (1994). They were chosen to reflect key features of the

⁹Following Doraszelski and Satterthwaite (2002): given that profits are bounded in all states, that firms discount with a factor $0 < \beta < 1$, that the number of states is bounded, and that $g(s, y, y_l, y_h, V_1)$ is a continuous function of (y, y_l, y_h) for all V_1 and s, existence of a symmetric MPNE in pure strategies is guaranteed if $(y_1(s), y_{l1}(s), y_{h1}(s))$ is bounded (in equilibrium it always is) and if g_1 is quasiconcave in (y_1, y_{l1}, y_{h1}) for all (y_2, y_{l2}, y_{h2}) , V_1 and s. This last condition, which is sufficient for existence, may not be satisfied for certain subsets of my parameters. However, I computed equilibria for a whole range of parameters.

¹⁰Initially, the programs were written in Matlab 6.1. As the routines were becoming very computationally intensive, I switched to Fortran90. Fortran90 computed results close to 100 times faster than Matlab.

model and to minimize computational time.

The static parameters where chosen to sufficiently differentiate the segments and products so that both multiproduct duopolists with close capabilities choose to target the high quality product h to the quality-sensitive segment 2 and the low quality product l to the price-sensitive segment and make positive sales in both products, at least for intermediate and high capability levels. Given that segment 2 is more profitable than segment 1, the difference in marginal costs and segment sizes compensates for the differences in marginal utilities with respect to prices and qualities. The marginal cost c_h is sufficiently high so that almost no consumers of segment 1 would buy it if it were priced at marginal cost, even for a high quality. If only the high-end good is available, consumers of segment 1 prefer the outside good.

Both products are differentiated by their marginal costs and by how the firm's capability translates into quality. Recall that $f_h(\omega) > f_l(\omega)$ so that a given capability level translates into higher quality for product h than for product l. One interpretation of this assumption is that capability can also be thought of as a component of brand capital, and strong brands bring more value to high-end goods than low-end goods.

The functions f_h and f_l have three sections to decrease the returns to investment for high capability values. I chose $\overline{\omega}$ and f so that, given the other parameter values, firms stop investing when they are close to the upper-bound of the capability set. I adjusted these so that the ergodic set of states is in the interior of the upper bound of the capability set, in order to eliminate the possibility that my results are driven by a boundary effect. The parameter τ allowed me to decrease the size of the state space and save computational time while retaining the key features of the model.

Parameter	Explanation	Value
a	Investment in capability efficiency parameter	1
(b_l, b_h)	Investment in l and h efficiency parameters	(3,1)
(b_{ll},b_{hh})	Investment in maintaining l and h efficiency parameters	(5,5)
$\delta_l = \delta_h$	Parameter limiting the probability of segment exit	0.05
β	Discount factor, or discount rate of 10%	0.9091
δ	Industry capability depreciation rate	$\{0.1,, 0.5\}$

Dynamic reference parameters

These parameters were chosen to generate reasonable investment incentives, i.e., to make h a more expensive product to develop and sustain than l. Firms are differentiated

by their state, portfolio and capability, while in all other respects they are the same. I therefore focus on symmetric equilibria to reduce the computational burden.

Using the above set of parameters, I first solve this model computationally for a duopoly, then for a perfectly colluding duopoly in both prices and investments, and finally for a welfare-maximizing social planner -one that chooses investments to maximize social welfare given duopoly prices. To appreciate industry evolution and how it depends on the parameters, I simulate the model 10,000 times and collect some key statistics and the distribution of states at different points in time. In the baseline case, I keep all parameters fixed but δ , the probability that industry capability depreciates (I will also refer to this as the rate of capability depreciation). I then provide some comparative dynamics where I compare the baseline case to industries where segment 2 is larger, where investment in capability is more efficient, and where the market is larger.

3 Results

In this section, I describe the equilibrium outcome of the model and the simulations. My main finding is that the industry converges in both the capability and product dimensions after several periods when the rate of capability depreciation is low and that it diverges when the rate is higher. This result is also true when other exogenous parameters are affected, but for exposition purposes, I focus mainly on the probability of capability depreciation. I report other comparative dynamics in the discussion section. The discussion subsection explains and provides intuition for the results described below.

Transitory dynamics. For $\delta=0.1$ and up to $\delta=0.3$, firms make similar choices and tend to stay close in terms of capabilities and product portfolio. The upper-part of Figure 2, illustrates a sample industry simulation with $\delta=0.2$. Figure 3 shows the distribution of product states at three different points in time (t=5,10,50). For example, when $\delta=0.1$, after 50 periods, the modal outcome for an industry starting in a nascent state is for each firm to offer a high-end product and a low-end product. The steady state, subject to some randomness, is that both firms have a capability level of $\tau\omega=2\times11=\widehat{\tau\omega}$ and a product portfolio of (l,h). This can be seen also in Figure 4, which shows the distribution of capability states for each product state after 50 periods for the case where $\delta=0.1$. It is clear that for these parameter values, firms converge in both dimensions over time.

Typically, in a nascent industry, firms will initially invest heavily in their capabilities. If a firm takes the lead in terms of capability, it will substantially increase its investment in order to solidify its position. This is more important for low capability values because it guarantees a greater revenue stream for several future periods. A laggard is only most likely to catch-up when the leader approaches the capability level where the f function becomes relatively flat $(\widehat{\tau\omega})$. Also, the concavity of the quality function makes initial capability differences translate into larger quality and profit differences. 11 The largard may eventually lose the ability to offer its h product and may re-introduce it later when its capability increases and eventually catches up the leader's capability around $\widehat{\tau \omega}$. This is most clearly illustrated by the capability investment function and quiver plot in Figure 6. The quiver plot examines the expected movement of the capability state by computing the probability-weighted average of $(\omega'_1 - \omega_1, \omega'_2 - \omega_2)$ and represents this as a vector for each product portfolio configuration in the capabilities space. The capability investment function is only shown for Firm 1 in the $((l_1, h_1), (l_2, h_2))$ product state, as it is very similar across other product states. 12 Note that because capability depreciation is industry-wide, δ does not directly affect the probability of a change in the difference between capabilities; instead δ can be thought of as a negative drift on the whole industry's capability.

The general shape of the investment in h function exhibits complementarity with capability. The complementarity originates from the profit function, as the marginal profit of selling h increases with capability and the marginal profit of capability increases with the offering of the h product. Complementarity between l and capability is state-dependent in equilibrium. Generally, for a leading firm, capability and the low-end product are substitutes, whereas for the laggard they are complements.¹³ This relation stems from costly cannibalization between the high-end and low-end products when a leader, say Firm 1, has a relatively high capability. When capability is high, the quality of both h_1 and l_1 is relatively high, but cannibalization of sales by l_1 is costly as h_1 is the

¹¹Computations with a quality function f that is more concave yielded a steeper investment function and a tougher race along the diagonal that also increases more as δ increases.

 $^{^{12}}$ To be consistent with previous notation I should write this state as: ((1,1),(1,1)). However, to be clearer I replace the 1's with the character representing the product in the results section.

¹³Athey and Schmutzler (2000) investigate circumstances under which a firm with a quality lead (or cost lead) will invest more to extend its lead. Unfortunately, these results do not apply because investments across firms are not strategic substitutes in all states, and the capability and product investments are not always strategic complements.

high-margin product. However, if the rival (laggard) offers l_2 and has a capability that is sufficiently close to Firm 1's, then Firm 2 already siphons sales from h_1 . As a result, cannibalization of Firm 1's high-end product has already occurred, and the introduction of l_1 results only in a positive expansion effect.

For $\delta > 0.2$, and as δ increases, firms have an incentive to invest more in order to catch-up on the industry-wide negative capability drift. At the same time, the returns to investment are lower. The direct effect is that increasing a firm's capability becomes more expensive. Although the investments in products do not change much as δ increases, the capability investment function exhibits an even greater increase in investment along the diagonal as in the low δ case. When δ is high, it becomes very costly for the laggard to catch-up with the leader when the leader reduces its capability investment as it reaches the flat part of the f function. Therefore, the laggard does not invest at all when it drops too far behind, so that in contrast to the low δ case, beyond a certain capability gap, the leader secures itself a considerable advantage, as its rival is stuck in capability-absorbing states. The possibility of deterring rivals from investing in capability gives firms an even greater strategic incentive to lead in capability. This yields industry dynamics that resemble a preemption race. To illustrate this, one can see in Figure 6 how the quiver plots and investment functions evolve as δ increases. Although, the quiver plots tend to suggest that capabilities evolve in a deterministic way, one should keep in mind that these are just expectations and therefore are very different from realizations. The plots confirm that firms race in capability along the diagonal and that the leader is able to promptly exploit its advantage by increasing the capability gap. Correspondingly, the simulations show that the industry evolves towards states where firms diverge both in terms of capability and product offerings, as in the sample simulation of Figure 2, rows three and four of Figure 3, and Figure 5.

Industry Performance. In this section, I compare the performance of the competitive duopoly to that of a welfare maximizing social planner and a perfectly collusive industry. Table 8 shows some statistics based on 10,000 simulations of the respective equilibrium policy functions.

Interestingly, accounting for the endogeneity of the market structure implies that the duopoly expected discounted profit is not monotonically decreasing with δ . This is easily explained by the mode of the distribution of states in the bottom half of the

table. As δ increases from 0.2 to 0.3, the mode of the distribution of states switches from the convergence state $((l_1, h_1), (l_2, h_2))$ where firms compete head-to-head and make low profits to the divergence state $(((l_1, 0), (0, h_2)))$ or $((0, h_1), (l_2, 0))$ where firms are specialized in their own niches and are local monopolies.

Industry evolution under a social planner is very focused around the convergence states. This can be explained by the logit consumer surplus that greatly rewards variety. The social planner chooses welfare maximizing investments under the constraint that firms compete in prices. The equilibrium prices are therefore different from marginal cost. This implies that a competitive duopoly's evolution will be most efficient when the depreciation rate is low. Similarly, consumer surplus will be highest when the depreciation rate is low.

Industry evolution of a perfectly collusive industry is focused on offering the highend product. For low depreciation rates, the mode of the long-run distribution of the industry is that each firm offers a high-end product, whereas as the depreciation rate increases the mode becomes the state where only one firm offers the high-end product.¹⁴

Summary. As the high-end product is both the most profitable, and a complement to capability, both firms invest heavily in capability and race to develop the high-end product. The leader gets to develop the high-end product first and to make substantial profits before the laggard develops it, if the latter is not 'too far behind' in terms of capability and cost of capability.

Industries with a low rate of capability depreciation tend to converge competitively, whereas firms in industries with a large capability depreciation rate diverge and specialize in their own segment. The low capability firm specializes in the low-end product whereas the firm with the high capability specializes in the high-end product. I find that convergence or divergence occurs simultaneously in the capability and the product dimensions. Also, in essence, the industry will either converge or diverge. Some other

¹⁴Industry performance measure would be more interesting if I allowed for more firms and entry and exit in the industry. In particular, it would be interesting to know whether this could change the ordering of welfare or consumer surplus between the competitive duopoly and the perfect cartel. Allowing for more entry and exit is likely to increase consumer surplus and to decrease profits. It is difficult to predict what the effect on total welfare and on the market structure would be. Fershtman and Pakes (2000) find in the context of price collusion and competitive investments, that consumer surplus is greater with collusion. The positive effect of collusion on variety and quality more than compensates consumers for the negative effect of collusive prices. It is not clear whether their results would hold in this framework of multiproducts.

states have also a significant probability of being reached in the steady state. In Table 8, the distribution of states for the competitive duopoly shows, for instance that for small depreciation rates, the states of type ((h,0),(l,h)) and ((l,h),(l,0)) are also very probable. This is because they are actually convergence states ((l,h),(l,h)) where one firm has had a negative realization of $\tilde{\mu}_{jt}$ or $\tilde{\eta}_{jt}$ in that given period (here t=100). The latter firm is very likely to offer the product again in the next period. This is also visible on the simulation paths from Figure 2.

4 Discussion

To provide further insight regarding the effect of δ on the likelihood of convergence or divergence, consider an industry in state $(8,8,(l_1,h_1),(l_2,h_2))$. The profit functions of both firms, as a function of Firm 1's capability, are represented in Figure 7. If Firm 1 has one or two unproductive investments, the state may become $(6,8,(l_1,h_1),(l_2,h_2))$, and it is likely to stop investing in the high-end product h as the marginal profit from selling becomes smaller than the marginal cost of sustaining it. This comes from the fact that h_1 is targeted at consumers from segment 2, and therefore Firm 1 has to compensate the quality difference with h_2 by reducing p_{h1} considerably (recall that segment 2 consumers have a low marginal disutility from price and a high marginal utility from quality). Therefore, Firm 1 makes slightly less profit in $(6,8,(l_1,h_1),(l_2,h_2))$ compared to state $p_{l2}((6,8,(l_1,0),(l_2,h_2)))$. The difference in Firm 1's profit between the two product states increases as ω_1 decreases and as $(\omega_2 - \omega_1)$ increases. As Firm 1 stops offering h, the profit of Firm 2 increases substantially, as can be seen in the lower graph in Figure 7. This increases the profit from winning the race and, therefore, encourages capability investment even more. The kink at point B in the upper graph corresponds to Firm 2 pricing l_2 above the choke price.¹⁵ This increase in Firm 1's profit gives it an incentive

¹⁵In product state $s = (2, 8, (l_1, h_1), (l_2, h_2))$, the equilibrium prices are: $((p_{l1}, p_{h1}), (p_{l2}, p_{h2})) = ((0.7503, 20.5049), (1.4779, 50.8398))$ and profits are: $(\pi_1, \pi_2) = (0.1765, 3.7838)$. However, note that there is another Nash equilibrium where both have higher profits, which corresponds to the profits in the specialized state where the low-capability firm prices h_1 out of the market, and the high-capability firm prices l_2 out of the market: $((p_{l1}, p_{h1}), (p_{l2}, p_{h2})) = ((1.0408, \infty), (\infty, 58.9527))$ and $(\pi_1, \pi_2) = (0.4391, 3.8460)$. The computational routine converged to the former equilibrium. There is a unique equilibrium in state $(1, 8, (l_1, h_1), (l_2, h_2))$, as the marginal cost of h is already part of the set of choke prices for $\omega_1 = 1$. This explains point A in Figure 7. I generally keep the equilibrium selected by the

to scale back its capability if it is not able to catch-up with the leader. The sooner Firm 1 scales back, the sooner Firm 1 increases its profit substantially, and the more incentive they have to race along the diagonal.

As the probability δ of capability depreciation increases, for given investments, the likelihood that a capability gap increases is the same. However, increasing one's capability is costlier, so that above a certain level of δ , a laggard will find it too costly to invest in order to catch-up, even when the leader decreases its investment as it approaches the flat part of the f function. Therefore, as it becomes more likely that a laggard will drop out of the race and not return, the expected gain from winning the race increases; in return, this not only increases both firms' incentive to race, but especially the firm that achieves a small lead on the capability diagonal. The fact that firms invest more along the diagonal as δ increases is clear in Figure 6.

Finally, as the laggard's decision to remain in the race or drop out will essentially depend on the expected gain from winning the race as well as the cost of racing, the likelihood that an industry will converge or diverge will depend on parameters affecting this trade-off. I discuss this further in the comparative dynamics section.

The existence of a comparable investment race has also been described in Besanko and Doraszelski (2002) in the context of capacity races. In their model, firms invest to increase their capacity and they compete in each period in either prices or quantities. Interestingly, they similarly find that a higher probability that capacity depreciates results in an asymmetric market structure whereas a lower rate of depreciation results in symmetric market structure in the case of price competition.

An interesting property of my equilibrium is that not only are joint profits better in the specialized state than in the convergent state, but individual profits are also better in a specialized state.¹⁶ Convergence is therefore not driven by where joint profits are greatest. Firms find themselves stuck into a type of prisoner's dilemma where firms focus on beating their rival in terms of capability but where neither has a unilateral incentive to deviate to specialize in its own niche. This describes in a game-theoretic framework what Chan and Mauborgne's earlier quote suggests.¹⁷

computational routine, except in 3 grid points I forced it to select the equilibrium closest in terms of prices and profits to the neighboring states.

 $^{^{16}\}pi_1(5, 15, (l_1, 0), (0, h_2)) > \pi_1(15, 15, (l_1, h_1), (l_2, h_2)).$

¹⁷Budd, Harris and Vickers (1993), in attempting to understand the factors that favor a process of increasing dominance by the leader as opposed to a process of catch-up by the laggard, find that several

4.1 Comparative Dynamics

In this section I examine the responses of the prediction of the model to a change in the segment size, to a change in investment efficiency, and a change in segment size, for different values of the probability of capability depreciation δ . Again, I compute the equilibrium investment functions and simulate the model for 100 periods, 10,000 times. The results of the comparative dynamics exercise are summarized in Table 9. I report the distribution of states at t = 100 for $\delta \in \{0.2, 0.3, 0.4, 0.6\}$.

In Case 1 the size of segment 2 is increased from $\lambda^2 = 0.1$ in the baseline case to $\lambda^2 = 0.3$. In Case 2, the total size of the market is increased for M = 5 to M = 6. In Case 3, investment efficiency is increased from a = 1 to a = 3. Actually, all three cases show analogous results as they have the effect of making segment 2 more attractive. In the case of the change in segment size, selling the high-end product targeted at segment 2 now increases profits from that segment threefold. Therefore, a lagging firm will invest more in order to catch-up with the leader and get a piece of that very profitable market. As a result, the industry will continue converging for larger values of δ than in the base case. In Case 2, the absolute number of high-end consumers has increased, so that the mechanism is similar as above. Finally, in the case of an increase in investment efficiency a, it becomes cheaper for the laggard to keep up with the leader, or stated differently, capability depreciation has less impact on industry evolution as firms can counter its effect with fewer resources devoted to investment. Hence, the industry will continue converging for larger values of δ than in the baseline case.

My result relates to those of the static industrial organization literature. In static models of product line competition with simultaneous moves, (that is, when firms are unable to commit to a certain product choice), the outcome is the interlacing equilibrium, or full product line competition. In contrast, in the sequential-move versions of the model, (that is, when firms are able to commit), the segmentation equilibrium (or specialization) can be obtained. In my model, commitment is not determined by the modeler's decision of the order of moves; rather, it underlines a more strategic approach to positioning, where complementarity (i.e., strategic fit) between capability and products is emphasized rather than an approach where quality and product choices are both effects are at work. They identify joint-profit effects and joint-cost effects. Some of these effects are coming into play in the equilibrium evolution of my model, however, because their setting is abstract and simplified, it is not clear when and why they affect the results.

chosen within a same decision period, as in a more marketing approach of the question. The value of parameters like δ and a determines whether it is profitable for a laggard to continue investing to catch-up with the leader. A high δ (or low a or low M) serves as a commitment device for the laggard to scale-back its capability and bring the industry to diverge, rather than race with the leader. Following this interpretation, my prediction that low rates of capability depreciation favor convergence and that high rates favor divergence are consistent with the static literature's predictions. My approach, however, is unique in that it highlights the dynamics that lead firms, in a given state, to either converge or diverge. The outcomes of industry convergence or divergence or interlacing and segmentation may not be reached in the real-world, therefore, as the exogenous parameters of the model will vary, it is critical to understand how the firms will react and adjust and what are the dynamics that shape the industry. This is unique to MPNE dynamic models of competition where firms are heterogeneous.

5 Industry Example and Implications

The U.S. car rental industry provides an interesting example of the dynamics highlighted in the previous section. In particular, the industry exhibits some evidence of competitive convergence. In this section, I first describe the industry and then interpret its evolution through the lens of my model.

Since 1990, revenues have grown by almost 100 percent and the industry has become more consolidated. Today, seven players account for 85 percent of total industry revenues (Enterprise (21 percent), Hertz (19.7 percent), Avis (13.1 percent), Budget (9.8 percent), National (9.3 percent), Alamo (7.7 percent), and Dollar (4.6 percent)). Ownership of the main players has changed significantly in this period of time. The trend is toward private car rental companies that are independent from large automobile manufacturers and focused on their own profitability.

The market for car rentals can be divided in two main segments: airport rentals and

¹⁸In Gilbert and Matutes' (1998) static model consumers have a firm-specific taste (in contrast to the above large idiosyncratic preference) which softens price competition and therefore favors an equilibrium where firms discriminate by offering the full product line. It would be interesting to include a firm-specific taste to account for this kind of effect, and in the case where it favors convergence, compare the value firms get with the value without the brand effect. If a brand effect softens competition but favors convergence as opposed to divergence, firms may be worse off with the brand effect.

local market rentals. Back in 1996, 64 percent of the market was from airport transactions and 36 percent from local transactions. In 1999, the ratio is 59 percent (airport) versus 41 percent. Despite the increase in air traffic and business travel, the local transactions segment has grown more rapidly. Several trends have been affecting industry growth: shorter and more frequent vacations and increased customer sophistication. Consumers tend to more often rent vehicles tailored for special lifestyle occasions. Consumers also expect their insurance companies to provide a replacement vehicle when their own vehicle is being repaired.

In addition to the above trends, most of the individual firms' actions show that they are no longer focusing on their core segments but are now trying to attract new types of customers. The segment of local renters is no longer a niche dominated by small independent players; rather, it is a growing segment in the car rental industry that has been attracting the large players previously specialized in other markets. As Figures 10 and 11 suggest, there seems to be a slow movement towards convergence.¹⁹ It is not clear whether convergence will go all the way through for the main firms or whether they will remain somewhat specialized.

In light of the results of my model, the evolution of the car-rental industry can be interpreted as follows. Capability of rental car firms is related to a firm's ability to provide the set of services that airport customers especially value: airport locations (as opposed to off-site locations), nationwide reservation systems, frequent traveler miles, wide variety of new cars in the fleet, and prompt service. Thus, capability ω can be thought of as influenced both by location and by IT capabilities. In the early 1990's, the industry has attained an equilibrium in a specialized state: Hertz, Avis and National serve the airport customers; smaller local firms, including Enterprise, serve local markets. Airport firms have a high capability ω , while local firms have a low ω . Without any further changes, the model predicts that this pattern of specialization will tend to persist.

Two events upset the equilibrium. First, although capability investments are very low in the low-end segment of a specialized industry, perhaps by good luck, Enterprise achieves favorable IT investments.²⁰ This enhanced capability enables it to improve its fleet management skills and reservation network: the capability gap between Enterprise

¹⁹The contrast between the specialization of car-rental firms may be clearer when I collect data for the early 1990s.

²⁰ "And it [Enterprise] employs a highly sophisticated computer network to track the whereabouts and service history of each of its 315,000 cars, keeping inventory lean and cars on the road an average of six months longer than Hertz and Avis do. At the same time, the company is known for its extra level of

and the airport firms narrows, simultaneously inducing some local firms to exit. Second, as airports expand in the 1990s, the airport rental companies are moved further off-site at airports, reducing an inherent advantage over local firms. This exogenous event, which could be interpreted as a firm-specific positive realization of ν_0 further reduces the capability gap between the airport and local firms.²¹

As capabilities become very close, Enterprise has a greater incentive to develop a product aimed at airport customers, plus a greater incentive to invest more in improving its capability. Simultaneously, airport firms enter the local segment. High capability firms now have an incentive to set up local agencies as this expands their customer base (expansion effect) while limiting cannibalization. The resulting increase in competition (competition effect) from setting a local agency is now small since Enterprise already provides similar quality than a local agency of Hertz would and thereby draws customers away from Hertz's airport agency. For example, if a traveler is flying to Chicago to do business in the Oakbrook area, and is willing to take a cab to rent a car from the Enterprise in Oakbrook, this could provide a similar quality than renting at Hertz's airport agency but at a cheaper price. It would then become more profitable for Hertz to set up a local agency rather than loose its customer altogether. Basically, companies specialized in the high-end segment incurred loss of market share due to Enterprise's increased capability and, therefore, it became profitable for them to expand in the lowend segment.

Although my model can interpret the evolution of this time period of the car-rental industry, it cannot provide further insight into the evolution of the industry as it is composed of several other firms. In particular, it is not clear whether convergence of a few firms favors convergence of more firms.

6 Conclusion and Further Research

Solving for the equilibrium investment functions and simulating the model, I have found that industries with a low rate of capability depreciation (or large market, or high investment efficiency) tend to converge competitively, whereas industries with a large rate service: it often delivers cars to customers' homes or takes customers to the cars." from *The New York Times*, Jan. 23 1997: "Enterprise's Unconventional Path; Rental Car Giant Successfully Shuns Industry Shakeout".

²¹ A firm-specific capability depreciation is not formalized in the model, but could easily be included.

of capability depreciation (or small market, or low investment efficiency) tend to diverge, i.e., firms specialize in their own segment. I find that, in equilibrium, convergence or divergence occurs in both the capability and the product dimensions.

The main intuition behind this result is that both firms invest to improve their option of specializing in the most profitable high-end product. As capability is complementary to the high-end product, both firms will invest and race in order to achieve a capability lead over its rival. When firms are close in terms of capability they will tend to both offer the full product-line as the expansion effect of offering an extra-good dominates the competitive effect that occurs from the rival's close presence. Whereas when firms become more distant in terms of capability, the cannibalization within the product-line becomes more important (as competition from the other firms' product-lines decreases) so that the expansion effect is not worth the cannibalization anymore. Therefore, as firms move closer together in the capability space they will invest in both products, and when they move apart, the leader will tend to drop the low-end product whereas the laggard will drop the high-end product. Depending on the attractiveness of ultimately specializing in the high-end product, the laggard will try to catch-up in capability when the leader's marginal returns to investment decrease, so that the industry converges competitively. Otherwise, if investment is too costly the laggard will rapidly let its capability depreciate and specialize in the low-end product, so that the industry diverges.

An interesting property is that not only are joint profits better in the specialized state than in the convergent state, but individual profits are also better in a specialized state. Firms find themselves stuck into a type of prisoner's dilemma where they race to beat their rival in terms of capability but neither has a unilateral incentive to deviate to specialize in its own niche. This result is a game-theoretic description of Chan and Mauborgne's approach to competitive convergence. The relative values of the exogenous dynamic parameters virtually serve to commit the laggard to scale back its activity and let the industry diverge.

However, exogenous parameters are not stable in the real world, so that the state distributions in 100 periods from now are unlikely to be reached as such. Although, I focused on the nascent industry as a benchmark, the dynamics that I have described relate to an industry at any state of its evolution. As the parameters are endogenous in the real world, this leads to several essential questions for empirical applications, whether case-based or data-based: how could I identify the parameters? How could managers affect these dynamic parameters like δ and a and affect them in their favor? How do the

parameters correlate?

A key to finding applications of this model would be to identify levels of δ . A low δ could perhaps be associated to more mature industries with less innovation, and a high δ to younger industries or industries where the innovation rate is high. It is also likely that similar industries may be subject to the same variations of parameters such as δ , as capability depreciation can stem from common factors such as a trend of increased consumer sophistication. There may be both economy-wide time variations of δ as well as across country economy-wide variations in δ that could help identify my results in real-world applications.

This model and research could be interestingly extended to allow for a more complete entry and exit decision, including the possibility of having more than two firms. This would include selection effects and more representative welfare effects. Another interesting factor that could be extended in the model and reinforce competitive convergence could come from demand growth and uncertainty related to demand growth. Indeed, the formation of informational cascades (Bikhchandani, Hirschleifer, Welch, (1992)) could generate herd behavior as firms seek to update their beliefs on the future state of demand.

This area of research is potentially very fruitful for better understanding the fundamental area of competitive strategy or strategic industry dynamics and evolutions of industries. The Ericson and Pakes (1995) framework is particularly well suited as a convincing economic and game-theoretic bridge between rigorous economic foundations and applied research and competitive strategy, more specifically. Some theory advances, such as an existence proof of pure strategy symmetric Markov Perfect Nash Equilibrium with multidimensional investments and states, and computational advances, such as the computation of mixed Markov strategies, would greatly help develop this area of research.

A Appendix: Algorithm for Equilibrium Profits

Profit functions may be multi-peaked because of the existence of multiproducts and consumer segments. This raises the issue of numerically finding local best responses rather than optimal global best responses. To guard against local best responses, the algorithm uses an iterative procedure on best response functions and checks that each

Nash equilibrium candidate is a mutual global best response using a grid search. This has been more effective than solving for the first-order conditions. Using this approach, the results are not sensitive to starting values. To compute the matrix of profits, for each state, I computed the Nash equilibrium prices using the above iterative procedure and replaced them in the profit function:

- 1. Set parameter values.
- 2. Set value of state s and initial prices: $(\mathbf{p}_0^1, \mathbf{p}_0^2)$. ²²
- 3. For i = 1, $\mathbf{p}_1^i = \arg\max_{\mathbf{p} \in \mathbb{R}^{l_1 + h_1}} \Pi_i(\mathbf{p}, \mathbf{p}_0^2)$. ²³
- 4. Update the initial values $(\mathbf{p}_0^1, \mathbf{p}_0^2)$ with $(\mathbf{p}_1^1, \mathbf{p}_0^2)$.
- 5. For i = 2, repeat step 3, using the updated initial values.²⁴
- 6. Repeat steps 3, 4 and 5 until: $|\mathbf{p}_n \mathbf{p}_{n-1}| < 10^{-5}$ or n > 100. If n > 100 then exit the algorithm. Otherwise, set the variable: Nash=1 in this state.
- 7. For the previous price candidates $(\mathbf{p}_*^1, \mathbf{p}_*^2)$: check for i = 1, 2, that $\Pi_i(\mathbf{p}_*^i, \mathbf{p}_*^{-i}) \geq \Pi_i(\mathbf{p}^i, \mathbf{p}_*^{-i})$, for all $\mathbf{p}^i \in G$,where $G \subset \mathbb{N}$, is a grid of prices from a vector of zeros to a vector close to choke prices. If there is a profitable deviation, i.e. there exists $(\mathbf{p}_{**}^i, \mathbf{p}_*^{-i})$, such that $\Pi_i(\mathbf{p}_{**}^i, \mathbf{p}_*^{-i}) > \Pi_i(\mathbf{p}_*^i, \mathbf{p}_*^{-i})$ then set Nash=0 and repeat steps 2 to ,6 until Nash=1 (and limit the number of cycles), otherwise proceed to step 8.
- 8. Repeat steps 2 to 7 until the Nash equilibrium prices have been found for the entire state space.

For 13 capability levels (or $(13+1) \times 13/2 \times 3 \times 3 = 819$ unique duopoly states) this routine took over 12 hours to converge on a standard computer, using Matlab 6.1 or 8 min on the same computer using Fortran 90.

²²The initial prices where as often as possible, the equilibrium prices from the same optimization problem with nearby parameters. I also computed the results with marginal costs and joint profit maximizing prices as initial values.

²³The optimization routine that I used is DBCOAH from the Fortran 90 IMSL library. It uses a Newton method and line search and active set strategy to handle the bounds on the prices. A very helpful explanation of different optimization routines can be found in Judd (1998). I provided the analytical gradient and Hessian of the profit function.

²⁴Gauss-Seidel updating was somewhat faster than Gauss-Jacobi.

B Appendix: Algorithm for Dynamic Duopoly

- 1. Choose parameters specific to the dynamic problem: $a, b_l, b_h, b_{ll}, b_{hh}, \delta, \beta, c$.
- 2. Set the upper bound for capability sufficiently large so that in equilibrium firms do not visit states where $\omega_j = \overline{\omega}$.
- 3. Load the equilibrium matrix of duopoly profits: Π . Π has been computed independently of the dynamic problem by the previous algorithm. The matrix of profits is of size: $(\overline{\omega}+1)/2\times4\times(\overline{\omega}+1)/2\times4$. There are $(\overline{\omega}+1)/2$ capability levels, as capability increases in increments of 2 and there are 4 possible portfolio choices per firm.
- 4. Set initial values for matrices: $\mathbf{V}^0, \mathbf{y}_l^0, \mathbf{y}_h^0, \mathbf{y}^0$. Again, as often as possible I used the equilibrium values of these functions for the model with neighboring parameters.
- 5. For each iteration k, cycle through the state space in a deterministic order to update the values of these matrices. By symmetry, I compute only the value function and policy functions for Firm 1. For a given state s, I compute $W_1^{k-1}(\omega_1 + \nu_1, (l'_1, h'_1))$, the expected discounted value of future net cash flows, conditional on Firm 2's current year investments using the value function and investments at iteration k-1 in equation 7. The expectation is taken over the Firm 2's possible realizations of its investments, and capability depreciation. For this same state, I then maximize the value function using:

$$V_{1}^{k}\left(s\right) = \max_{y^{k} \geq 0, y_{\theta}^{k} \geq 0} \left[\begin{array}{c} \Pi_{1}\left(s\right) - y_{1}^{k} - y_{1l}^{k} - y_{1h}^{k} + \\ +\beta \sum_{\nu_{1}, l_{1}^{l}, h_{1}^{0}} W_{1}^{k-1}(\omega_{1} + \nu_{1}, (l_{1}^{\prime}, h_{1}^{\prime})) prob(\nu_{1}|y^{k}) prob\left(l_{1}^{\prime}, h_{1}^{\prime}|y_{l}^{k}, y_{h}^{k}\right) \end{array} \right].$$

This yields the updated values: $V^k(s), y_l^k(s), y_h^k(s), y^k(s)$. For the optimization problem I used the IMSL routine: DBCONG, with analytical gradient. This routine uses a quasi-Newton method with BFGS updating and an active set strategy for the simple bounds on the variables. For further information on this optimization method, see Judd (1998).

- 6. Repeating 6 for all s yields the updates: $\mathbf{V}^k, \mathbf{y}_l^k, \mathbf{y}_h^k, \mathbf{y}^k$.
- 7. Repeat steps 6 and 7 until the value function and all policy functions converge relatively in the sup norm, or k > M.

For 13 capability levels this routine took about 12 hours to converge on a standard computer, in Matlab 6.1 or about 8 min on the same computer using Fortran90, and sometimes much less time if the initial values where carefully chosen.

C Appendix: Investment Functions

The FOC of the Bellman equation are:

$$\begin{bmatrix}
-1 + \beta \frac{a}{(1+ay)^2} \sum_{\mu_1, \eta_1} \begin{bmatrix} W_1(\omega_1 + 1, (l'_1, h'_1)) - \\ W_1(\omega_1, (l'_1, h'_1)) \end{bmatrix} \rho(\mu_1 | y_{l1}, l_1) \rho(\eta_1 | y_{h1}, h_1) \end{bmatrix} y_1 = 0, \ y_1 \ge 0$$

$$\begin{bmatrix}
-1 + \beta \frac{b_l}{(1+b_l y_l)^2} \sum_{\nu_1, \eta_1} \begin{bmatrix} W_1(\omega_1 + \nu_1, (1, h'_1)) - \\ W_1(\omega_1, (0, h'_1)) \end{bmatrix} \gamma(\nu_1 | y_1) \rho(\eta_1 | y_{h1}, h_1) \end{bmatrix} y_{l1} = 0, \ y_{l1} \ge 0,$$

$$\begin{bmatrix}
-1 + \beta \frac{b_h}{(1+b_h y_h)^2} \sum_{\nu_1, \mu_1} \begin{bmatrix} W_1(\omega_1 + \nu_1, (l'_1, 1)) - \\ W_1(\omega_1, (l'_1, 0)) \end{bmatrix} \gamma(\nu_1 | y_1) \rho(\mu_1 | y_{l1}, l_1) \end{bmatrix} y_{h1} = 0, \ y_{h1} \ge 0.$$

An interior solution exists if the investment functions solve:

$$y_{1}(s) \ = \ \frac{-1 + \sqrt{\beta a \sum_{\mu_{1},\eta_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+1,(l'_{1},h'_{1})\right) \\ -W_{1}\left(\omega_{1},(l'_{1},h'_{1})\right) \end{array} \right] \rho\left(\mu_{1}|y_{l1},l_{1}\right) \rho\left(\eta_{1}|y_{h1},h_{1}\right)}{a}}, \\ y_{1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{l} \sum_{\nu_{1},\eta_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(1,h'_{1})\right) \\ -W_{1}\left(\omega_{1},(0,h'_{1})\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\eta_{1}|y_{h1},h_{1}\right)}{b_{l}}, \text{ if } l_{1} = 0,} \\ y_{1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{l} \left(1 - \delta_{l}\right) \sum_{\nu_{1},\eta_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(1,h'_{1})\right) \\ -W_{1}\left(\omega_{1},(0,h'_{1})\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\eta_{1}|y_{h1},h_{1}\right)}{b_{l}}, \text{ if } l_{1} = 1,} \\ y_{h1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{h} \sum_{\nu_{1},\mu_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(l'_{1},1)\right) \\ -W_{1}\left(\omega_{1},(l'_{1},0)\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\mu_{1}|y_{l1},l_{1}\right)}{b_{h}}, \text{ if } h_{1} = 0,} \\ y_{h1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{h} \sum_{\nu_{1},\mu_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(l'_{1},1)\right) \\ -W_{1}\left(\omega_{1},(l'_{1},0)\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\mu_{1}|y_{l1},l_{1}\right)}{b_{h}}, \text{ if } h_{1} = 1.} \\ y_{h1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{h} \left(1 - \delta_{h}\right) \sum_{\nu_{1},\mu_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(l'_{1},1)\right) \\ -W_{1}\left(\omega_{1},(l'_{1},0)\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\mu_{1}|y_{l1},l_{1}\right)}{b_{l}}} \\ y_{h1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{h} \left(1 - \delta_{h}\right) \sum_{\nu_{1},\mu_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(l'_{1},1)\right) \\ -W_{1}\left(\omega_{1},(l'_{1},0)\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\mu_{1}|y_{l1},l_{1}\right)}{b_{l}}} \\ y_{h1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{h} \left(1 - \delta_{h}\right) \sum_{\nu_{1},\mu_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(l'_{1},1)\right) \\ -W_{1}\left(\omega_{1},(l'_{1},0)\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\mu_{1}|y_{l1},l_{1}\right)}{b_{l}}} \\ y_{h1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{h} \left(1 - \delta_{h}\right) \sum_{\nu_{1},\mu_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(l'_{1},1)\right) \\ -W_{1}\left(\omega_{1},(l'_{1},0)\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\mu_{1}|y_{l1},l_{1}\right)}{b_{l}}} \\ y_{h1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{h} \left(1 - \delta_{h}\right) \sum_{\nu_{1},\mu_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(l'_{1},1)\right) \\ -W_{1}\left(\omega_{1},(l'_{1},0)\right) \end{array} \right] \gamma\left(\nu_{1}|y_{1}\right) \rho\left(\mu_{1}|y_{1},l_{1}\right)}{b_{l}}} \\ y_{h1}(s) \ = \ \frac{-1 + \sqrt{\beta b_{h} \left(1 - \delta_{h}\right) \sum_{\nu_{1},\mu_{1}} \left[\begin{array}{c} W_{1}\left(\omega_{1}+\nu_{1},(l'_{1},1)\right) \\ -W_{1}\left(\omega_{1},(l'_{1},0$$

Note that real solutions exist only if the term under the square root is nonnegative. The second-order sufficient condition for a maximum is that the bordered Hessian of g be negative definite. The second-order necessary condition is that the bordered Hessian of g be negative semi-definite²⁵. It is not possible to show analytically that these conditions are satisfied. However, I have found numerical solutions that satisfy the Bellman equation.

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²⁵The sufficient condition is also sufficient for quasiconcavity of g on its domain, if it holds for all nonnegative investments. The necessary condition is also necessary for quasiconcavity of g.

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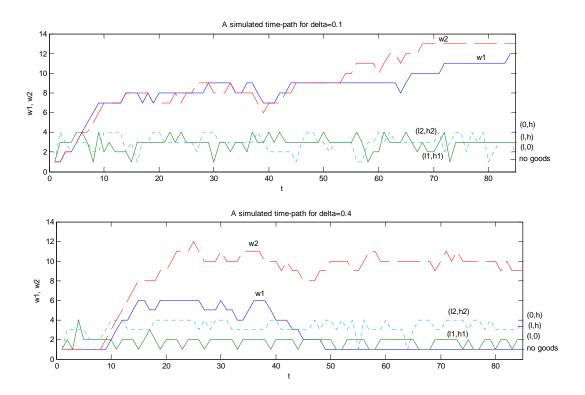


Figure 2: Two industry simulation time-paths for $\delta = 0.1$ and $\delta = 0.4$, respectively.

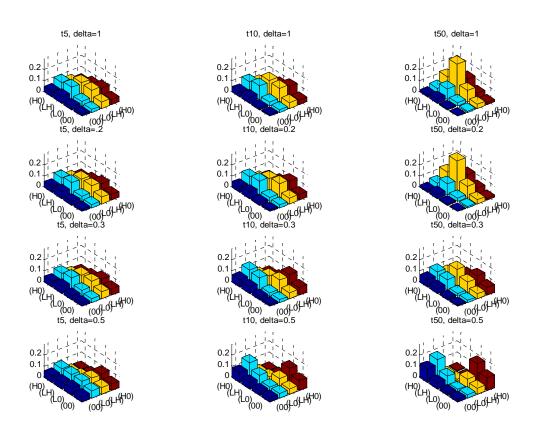


Figure 3: Distribution of product states of a nascent industry at 3 points in time, for $\delta \in \{0.1, 0.2, 0.3, 0.5\}.$

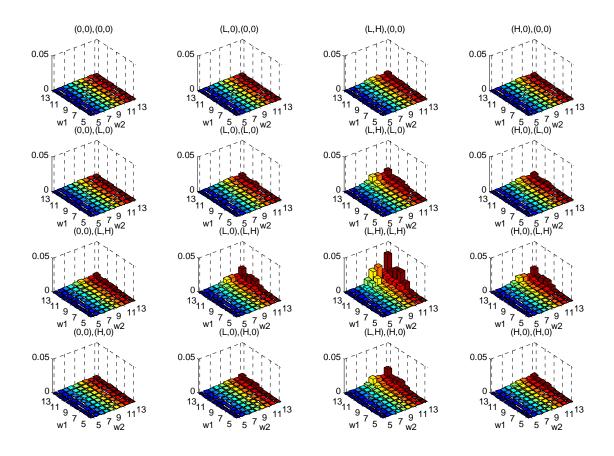


Figure 4: The distribution of states of a nascent industry after 50 periods, when δ =0.1. The distribution is represented for each product state as a function of capability states.

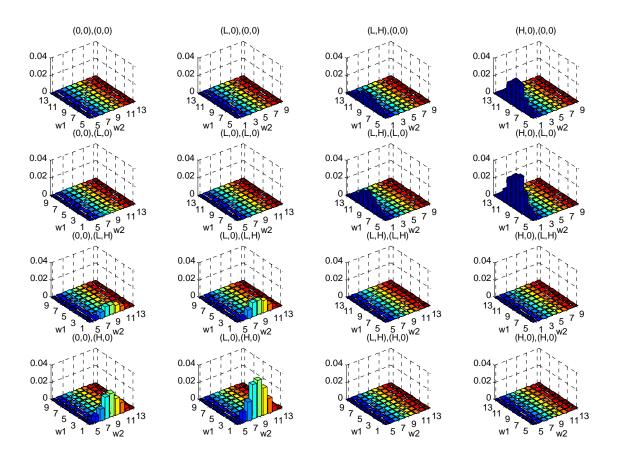


Figure 5: The distribution of states of a nascent industry after 50 periods, when $\delta = 0.5$. The distribution is represented for each product state as a function of capability states. Divergence in both capability and product dimensions can be clearly seen.

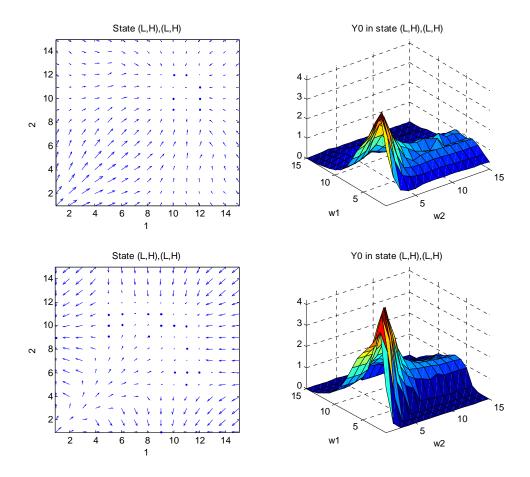


Figure 6: Quiverplot and Firm 1's investment in capability, as a function of both capabilities, from top to bottom, respectively for $\delta \in \{0.2, 0.5\}$. I do not represent these plots for states that are not part of the ergodic set.

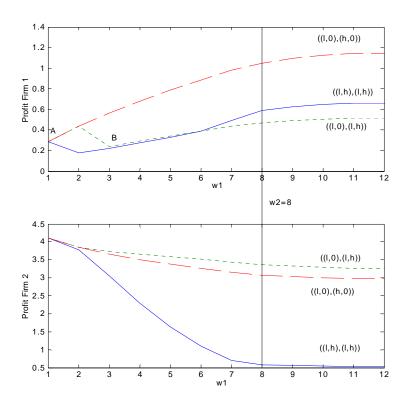


Figure 7: Firm 1's profit in product states ((l,h),(l,h)),((l,0),(l,h)),((l,0),(0,h)), as it lags in capability from $\omega_1=\omega_2=8$ to $\omega_1=1$.

Ná	ascent ir	ndustry i	nitial stat	e is: no i	firm has	develop	ped any	y produc	t, both ha	ve a cap	ability o	f 1		
		delta=0.1	1	delta=0.2			delta=0.3			delta=0.4	delta=0.5			
Mean over 10,000 simulations	Comp. Duop.	Social Planner	Collusion	Comp. Duop.	Social Planner	Collusion	Comp. Duop.	Social Planner	Collusion	Comp. Duop.	Comp. Duop.	Social Planner	Collusion	
Expected Profits	43.18	-80.35	93.55	39.80	-84.70	81.16	40.60	-88.99	66.94	38.20	31.86	-92.06	37.61	
Expected Welfare	338.44	352.55	218.03	321.59	336.21	196.22	298.86	317.29	172.57	275.85	256.28	270.92	128.82	
Expected Consumer Surplus	295.26	432.91	124.48	281.78	420.91	115.06	258.26	406.29	105.62	237.65	224.43	362.98	91.22	
Expected Invt. in Capability	35.99	40.57	38.35	40.35	45.93	43.49	43.83	52.11	48.10	45.40	44.53	59.02	50.32	
Expected Invt. in	9.75	16.63	0.93	9.33	16.39	1.11	8.30	16.08	1.40	7.36	6.83	15.27	2.40	
Expected Invt. in H	19.70	23.16	22.93	19.03	22.38	22.13	18.25	20.81	21.05	17.17	15.24	17.77	17.16	
						t=100								
No products	0.00	0.00	0.04	0.00	0.00	0.05	0.01	0.00	0.05	0.02	0.03	0.00	0.07	
Monop L	0.02	0.01	0.00	0.02	0.01	0.01	0.04	0.01	0.02	0.05	0.06	0.01	0.04	
Monop (L,H)	0.07	0.05	0.01	0.08	0.05	0.03	0.06	0.05	0.13	0.07	0.08	0.05	0.30	
Monop H	0.03	0.01	0.35	0.03	0.01	0.37	0.13	0.01	0.45	0.21	0.25	0.01	0.59	
((L,0),(L,0))	0.03	0.03	0.00	0.03	0.03	0.00	0.03	0.03	0.00	0.03	0.02	0.03	0.00	
((L,H),(L,0)) or ((L,0),(L,H))	0.21	0.23	0.00	0.21	0.23	0.00	0.18	0.23	0.00	0.17	0.16	0.23	0.00	
Divergence	0.08	0.06	0.01	0.09	0.06	0.01	0.22	0.06	0.01	0.31	0.36	0.06	0.00	
Convergence	0.28	0.38	0.00	0.27	0.38	0.00	0.15	0.38	0.00	0.06	0.01	0.37	0.00	
((H,0),(L,H)) or ((L,H),(H,0))	0.23	0.20	0.02	0.23	0.20	0.02	0.12	0.20	0.01	0.05	0.01	0.20	0.00	
((H,0),(H,0))	0.04	0.03	0.55	0.04	0.03	0.51	0.02	0.03	0.33	0.01	0.00	0.03	0.00	
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

Expected amounts, are the means over the 10,000 simulations of the discounted sums of the corresponding amount in each realized state.

Divergence: ((H,0),(L,0)) or ((L,0),(H,0))

Convergence: ((L,H),(L,H))
Comp. Duop.: Competitive Duopoly

Parameters of base case

Figure 8: Industry Performance, Base Case

Nascent industry initial state. Results are based on 10,000 simulations.																
	delta=0.2 delta=0.3						delta=0.4				delta=0.6					
Average	Base Case	Case 1	Case 2	Case 3	Base Case	Case 1	Case 2	Case 3	Base Case	Case 1	Case 2	Case 3	Base Case	Case 1	Case 2	Case 3
Exp. Profits	40	106	52	49	41	102	48	46	38	101	49	41	22	108	31	33
Exp. Disc. Welfare	322	705	401	355	299	665	377	340	276	612	345	322	238	485	296	275
Exp. Disc. CS	282	599	349	306	258	563	329	294	238	510	296	281	215	377	265	242
Exp. Total Invt. in Capability	40	61	45.64	23	44	71	50.50	25	45	83	52.58	28	39	100	48.14	32
Exp. Disc. Invt. in L	9	10	10.71	10	8	10	10.10	10	7	9	8.66	9	7	8	7.82	8
Exp. Disc. Invt. in H	19	31	20.45	19	18	31	19.61	19	17	30	18.73	18	12	27	14.10	15
							t	=100								
No products	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.02	0.00	0.02	0.00	0.03	0.01	0.02	0.02
Monop L	0.02	0.01	0.02	0.02	0.04	0.01	0.02	0.02	0.05	0.01	0.04	0.02	0.06	0.03	0.05	0.04
Monop (L,H)	0.08	0.06	0.07	0.08	0.06	0.06	0.08	0.08	0.07	0.06	0.10	0.08	0.10	0.18	0.11	0.10
Monop H	0.03	0.03	0.03	0.03	0.13	0.03	0.05	0.03	0.21	0.03	0.16	0.03	0.26	0.18	0.22	0.15
((L,0),(L,0))	0.03	0.01	0.03	0.03	0.03	0.01	0.03	0.03	0.03	0.01	0.02	0.03	0.03	0.02	0.02	0.03
((L,H),(L,0)) or ((L.0).(L.H))	0.21	0.13	0.21	0.20	0.18	0.13	0.22	0.21	0.17	0.13	0.21	0.20	0.15	0.23	0.19	0.19
Divergence:	0.09	0.06	0.08	0.08	0.22	0.06	0.12	0.08	0.31	0.06	0.28	0.09	0.37	0.22	0.38	0.24
Convergence	0.27	0.33	0.30	0.28	0.15	0.33	0.26	0.27	0.06	0.33	0.10	0.27	0.00	0.06	0.01	0.12
((H,0),(L,H)) or ((L.H).(H.0))	0.23	0.31	0.22	0.23	0.12	0.31	0.18	0.23	0.05	0.31	0.07	0.23	0.00	0.06	0.01	0.10
((H,0),(H,0))	0.04	0.07	0.04	0.04	0.02	0.07	0.03	0.04	0.01	0.07	0.01	0.04	0.00	0.01	0.00	0.02
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Base Case: Competitive duopoly with parameters as described as base case.

Case 1: Competitive duopoly with base case parameters, except for the size of segment: lambda=(0.7, 0.3), compared to (0.9, 0.1)

Case 2: Competitive duopoly with base case parameters, except for market size: M=6 compared to M=5.

Case 3: Competitive duopoly with base case parameters, except for investment efficiency: a=3 (compared to 1)

Divergence: ((H,0),(L,0)) or ((L,0),(H,0))

Convergence: ((L,H),(L,H))

Figure 9: Industry Performance, Comparative Dynamics

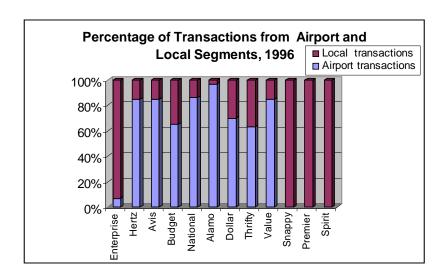


Figure 10: Percentage of transactions from airport and local market segments, 1996, for firms with a revenue>\$50mil. Source: Auto Rental News

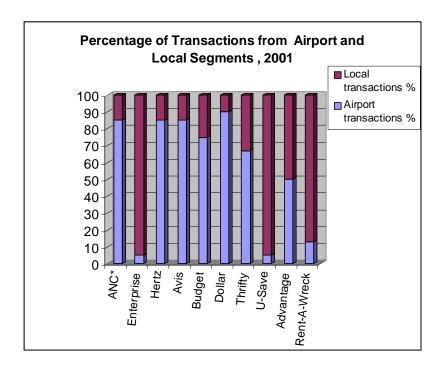


Figure 11: Percentage of transactions from airport and local market segments, 2001 for firms with a revenue>\$50mil. Source: Auto Rental News