Uncertainty, Political Preferences, and Stabilization: Stochastic Control Using Dynamic CGE Models

by

Seung-Rae Kim

Woodrow Wilson School
Princeton University
Princeton, NJ 08544-1013
srkim@princeton.edu

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Abstract

Traditional computable general equilibrium (CGE) models have ignored uncertainty – even when applied to fields such as environmental modeling that are replete with economic uncertainty. In contrast, many control theory models have focused on the effects of uncertainty. Thus marrying the tradition of CGE and control modeling can result in price-quantity models with explicit dynamics and careful treatment of uncertainty. This paper is a next step toward the merger of optimal control models with dynamic CGE models. It demonstrates the usefulness of CGE techniques in control theory application and provides a practical guideline to policymakers in this relatively new field. Moreover, it explores the link between economic stabilization and optimal environmental fiscal policy design in a stochastic dynamic general equilibrium framework. Uncertainty, short-term quantity adjustment process, and sector-specific political preferences (e.g., more stabilization priorities on polluting industries) are taken into account in exploring what time paths of adjustments of the economy would be optimal for the government with explicit policy goals. The optimal control solutions could differ not only due to differences in underlying model assumptions or structures, but also depending crucially on uncertainty about the magnitude of various parameters in the economy. In particular, it is also shown that the performance of economic stabilization could vary significantly with asymmetric political preferences/uncertainty across industrial sectors. In such cases, allowing for those components in more general CGE-based economic modeling may identify policies in the inherently stochastic world that may outperform traditional control-theory (macroeconomic) modeling approaches.

Keywords: Optimal control, dynamic CGE models, uncertainty, short-term adjustment process, sector-specific political preferences, macroeconomic stabilization, environmental care

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1. Introduction

In recent years there has been growing research interest in converting static computable general equilibrium (CGE) models into dynamic models.[1] This has also raised the possibility that uncertainty could also be added to these dynamic CGE models and the models solved as stochastic control models.

Nonetheless, unlike aggregate macroeconometric models, CGE modeling has been little used with optimal control formulations for stabilization policy analysis. However, as demonstrated in Smith (1993)'s approach, traditional CGE technique can be integrated with optimal control methods. In other words, adding an explicit objective function to the CGE model and minimizing the weighted deviations of the economy from desired levels, allow one to identify how best to achieve explicit goals for society.

This paper is a next step toward the merger of optimal control models with dynamic CGE models. It addresses the question of what time path of adjustment of an economy would be optimal for a government with explicit policy goals in the face of uncertainty in the economic system.[2]

The main features of the model employed in this paper are as follows. First, it provides rich dynamics by relaxing the standard neoclassical assumptions in CGE modeling, thereby allowing for more realistic adjustment processes towards long-run equilibrium. Economic phenomena can sometimes be best explained by adjustment

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[1] Computable general equilibrium (CGE) models are characterized by their price-endogenous features and the inclusion of resource constraints. CGE models are designed to reflect many markets, many institutions, and their interactions, with prices and quantities determined simultaneously while simulating the results of an external shock or a policy change. Moreover, they can focus on the issues of economic structure with government interventions.

[2] Optimal control theory with CGE approaches would be one of the most appropriate and powerful analytical devices if we can allow for the time path of adjustment towards the new equilibrium in a certain period (even, in response to short-run macroeconomic disturbances due to an unexpected shock) and also incorporate insight with explicit policy weights to reflect the relative importance on various states of the economy.

processes over time ─ especially in the short- and midterm- run period.[3] Thus, instead of immediate market clearing, the model incorporates price-adjusted mechanisms that allow for some quantity-adjusted components together with cross and feedback effects (e.g., unemployment dynamics in labor markets). Second, unlike the usual control theory applications with aggregate macroeconomic models, the model developed here can be applied to deal with sector-specific policy issues (e.g., more stabilization priorities on polluting industries). It can be further used to perform control experiments regarding issues of economic structural reform. Finally, for more realism we incorporate uncertainty and passive learning processes with stochastic components in the dynamic CGE model.[4] Note that the relationships among some particular variables are usually uncertain and the true underlying values for the relevant parameters may be unknown. In such cases the optimality of government policies would be decided with learning about the inherent stochastic world.

Following Johansen (1960)'s linearization method, the dynamic CGE framework is converted to be a stochastic control form amenable to the Duali software (Amman and Kendrick, 1999). Given the variability of an economic system with some stochastic components, this approach can help policy makers determine the timing and extent of government policy intervention.

Under the extended general equilibrium features described above, this model can be used to perform several optimal control experiments for economic stabilization with various sector-specific issues in the face of external shocks. Moreover, in the stochastic control experiments the relationship between uncertainty and the efficiency of government polices can be investigated with explicit declaration of their political preferences. It is thus demonstrated that the inclusion of optimal control formulations

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[3] Dynamic general equilibrium modeling usually deals with long-run effects. Considering the weakness of traditional CGE models, note that it is critical how far one can accommodate non-neoclassical features in the CGE framework without giving up its basic characteristics and internal consistency (For details, see Dervis, de Melo, and Robinson, 1982, pp.169-73)

[4] The stochastic control framework used here is discussed in detail in Kendrick (1981).

into stochastic CGE modeling would allow policymakers to use a wide range of policy experiments with careful treatment of uncertainty.

The rest of the paper is organized as follows. The optimal control CGE model is presented in Section 2, and some control experiments with industry specific emphases on political preferences and on uncertainty are implemented in Section 3. Finally, conclusions are discussed in Section 4.

2. An Optimal Control CGE Model of the U.S. Economy

2.1 General Equilibrium Background of the Model

Building on Smith (1993)'s work, we develop a simple dynamic CGE model that extends the stylized neoclassical CGE structure to allow for the short- or mid-term macroeconomic phenomena such as strains on factor markets. The present model includes decision-making by suppliers, households, government and the foreign sector, and market-clearing conditions, while integrating a traditional CGE modeling technique into a neo-Keynesian macroeconomic framework.

There are two goods, "clean" goods (*i=CLN*) from pollution-non-intensive industries and "dirty" goods (*i=DRT*) from pollution-intensive industries.[5] The technology vector is constructed with a constant-returns-to-scale, perfect competition, and Leontief intermediate input demands (V_{ij}). Sectoral supply (X_i^s) is determined by a Cobb-Douglas production technology with a scale parameter A_i and the two primary factors — labor input (F_{Li}^d) and capital input (F_{Ki}^d).

$$X_i^s = A_i\,(F_{Li}^d)^{\beta_i}\,(F_{Ki}^d)^{(1-\beta_i)} \qquad i = CLN, DRT. \tag{i}$$

Sectoral demands for primary factors (F_{Li}^d and F_{Ki}^d) are derived from the Cobb-Douglas technologies. Factor demand equations assume that the primary factors are

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[5] "Clean" goods and "dirty" goods in the model may be viewed as metaphors for two different groups of industries that are subject to different penalties (or priorities) for stabilization. For example, policymakers can give more stabilization penalties on "dirty" industries due to environmental concerns. For a more detailed description of the model, see Kim and Kendrick (2002).

paid the same average rental rate, P_f, and, for each sector, their remuneration is set equal to the value added price or net producer price (net of both indirect taxes τ_i and domestic intermediate input cost shares $\sum_j \alpha_j^d$) times the partial, β_{fi}, of the production function with respect to each factor.

$$F_{fi}^d = \frac{(1-\tau_i - \sum_j \alpha_j^d)\beta_{fi} P_i^d X_i^s}{P_f} \qquad f = L, K \tag{ii}$$

Pollution (*PE*) is also emitted from the production of dirty industries with emissions coefficient (ε).

$$PE = \varepsilon \; X_{DRT}^s \tag{iii}$$

In the product markets, the potential net production (*GDPP*) is pre-determined in the economy, depending only on the primary factors endowment and technology level. The equilibrium gap (*dr*) is defined as the percentage gap between *GDPP* and the endogenous value of gross domestic product, *GDP*.[6] This is also simultaneously adjusted to the unemployment rate *lur* and the ratio of aggregate price index *CPI* to its reference level, which are all endogenized in this model, taking into account the adjustment costs in factor markets.

$$lur = lur_n + \rho_1 \; CPI / \overline{CPI} + \rho_2 \; dr \, , \tag{iv}$$

where $\rho_1 < 0, \;\; \rho_2 > 0$ and $dr = (GDPP - GDP)/GDPP$ (v)

Endogenizing the short-term adjustment process of prices and wages is critical for analyzing more appropriately the effects of short-term policy changes within the context of optimal control in a certain planning time period. In the model, production and output supply are from profit maximization in accordance with the natural unemployment rate component (lur_n) plus the involuntary unemployment component

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[6] This *dr* is viewed as the ratio of actual deviation from a full-employment neoclassical equilibrium level of the economy. The term 1– *dr* also implies the effective capacity utilization rate of the economy.

under a neo-Keynesian regime. Following the spirit of the Phillps curve, this model assumes that the unemployment rate (*lur*), as a wedge between labor supply and demand, is negatively related to the change in *CPI* and positively to the equilibrium gap, as in eq.(iv).[7] Prices and wages are adjusted towards the model closure, along with some rigidities constrained by this "equilibrium gap augmented" Phillips surface.

Domestic composite prices (P_i) are the weighted averages of the domestic prices (P_i^d) and the world prices (P_i^w), with weights based on imports (M_i) and domestic consumption of domestic goods (*Di*).

$$P_i = (P_i^w)^{\vartheta_i} (P_i^d)^{(1-\vartheta_i)} \qquad \text{(vi)}$$

The consumer price index (*CPI*) is an average of aggregate prices weighted by private household consumption for each commodity (C_i).

$$CPI = \prod_i (P_i)^{\lambda_i} \qquad \text{(vii)}$$

Labor supply potential is a function of initial labor supply ($\overline{F}_L^s$) times the real wage rate (P_L / CPI) with a real wage elasticity of θ, which is, in turn, adjusted by the unemployment rate.

$$F_L^s = (1 - lur)\overline{F_L^s}\left[\frac{P_L}{CPI}\right]^{\theta} \qquad \text{(viii)}$$

Factor incomes (R_f) are simply the sum of factor demand by domestic production and by the government. Household income (R_Q) is obtained as labor income plus capital income from the value-added (or net) production less direct government tax. Private households allocate their expenditures according to their Cobb-Douglas preferences.

$$C_i = \alpha_{ci} R_Q / P_i \qquad \text{(ix)}$$

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[7] For a similar treatment and discussion on general equilibrium with unemployment, see Patinkin (1966), Hansen (1970) pp.141-47, and Dervis, de Melo, and Robinson (1982).

Sectoral investment (I_i) is given as a fixed share of total investment.

$$I_i = \alpha_{Ii} I \tag{x}$$

Sectoral export (E_i) and import (M_i) depend on the relative price of domestic and world goods with their own price elasticity of demand parameters.

$$E_i = \overline{E}_i (P_i^w / P_i^d)^{\eta_i} \tag{xi}$$

$$M_i = \overline{M}_i (P_i^d / P_i^w)^{\mu_i} \tag{xii}$$

Capital inflows are the balancing term of the value of imports less the value of exports.

Government revenue (R_G) is the sum of factor taxes, indirect taxes on domestic production and direct taxes from household income. Private household and government saving are the residual of their income less the expenditure on good and services. The government makes purchases of goods and services for each sector, G_i, in real terms.

Sectoral final demand is evaluated as the sum of private and public consumption, investment, and net exports. Thus, gross domestic product (*GDP*) is defined as

$$GDP = \sum_i \left(C_i + I_i + G_i + E_i - M_i \right) \tag{xiii}$$

Gross investment (I) is determined by initial investment ($\overline{I}$) times a function of real capital income change with an investment return elasticity of γ. Capital stock (F_K^s) is the depreciated prior period supply ($(1-\delta)\overline{F_{K,-1}^s}$) plus investment from the prior period (I_{-1}) (assuming undifferentiated by source), which is the only inter-temporal linkage in our simple CGE model.

$$F_K^s = (1-\delta)\overline{F_{K,-1}^s} + I_{-1}, \quad \text{where} \quad I = \overline{I}[(R_K / CPI)/\overline{R_K}]^{\gamma} \tag{xiv}$$

To summarize, the requirements of general equilibrium closure are that in each period the demand for each factor and for each product equal supply:

Labor market clearing

$$\sum_i \frac{(1-\tau_i - \sum_j \alpha_j^d)\beta_{Li} P_i^d X_i^s}{P_L} = (1-lur)\overline{F_L^s}\left[\frac{P_L}{CPI}\right]^\theta \qquad \text{(xv)}$$

Capital market clearing

$$\sum_i \frac{(1-\tau_i - \sum_j \alpha_j^d)\beta_{Ki} P_i^d X_i^s}{P_K} = (1-\delta)\overline{F_{K,-1}^s} + I_{-1} \qquad \text{(xvi)}$$

Product markets clearing

$$X_i^d = X_i^s, \quad \text{where } X_i^d = \sum_j V_{ij} + C_i + I_i + G_i + E_i - M_i \qquad \text{(xvii)}$$

To develop this CGE model with simultaneous states, controls and exogenous variables into system equations for a stochastic control version, one can convert the original nonlinear equations into a form amenable to the Duali software (Amman and Kendrick, 1999) which requires a state-space form with all linear relationships expressed in lagged terms. Three main state variables are introduced to use in calculating values from existing endogenous state variables: x = {gross domestic product (*GDP*), employment (*L*), pollution emissions (*PE*)}. This allows us to exploit more realistic policy objectives in the framework of optimal control. The policy or control variables, freeing up in the model, are government expenditures on goods and services: u = {government purchases of commodities (G_i)}. In this open economy, we also have exogenous variables: z = {world prices of commodities (P_i^w)}.

2.2 Optimal Control CGE Framework

Based on the CGE model described above, we have developed a small optimal-control CGE model of the U.S. economy. To do this, first, the nonlinear CGE framework is converted to classic system equations for an optimal control model, which will be used to perform various computational experiments. Then, if we add an explicit objective function (or social loss function) such as a quadratic "tracking" criterion function to the stochastic CGE model, it allows one to determine the level of policy (or control) variables to minimize the penalty-weighted squared deviation of the economy from the desired levels. In this case, as demonstrated in Smith (1993), the traditional CGE modeling is a special case of the optimal control problem – one in which the control variables are forced on the model and the resulting states are merely calculated as the CGE simulation. Specifically, for handling the standard CGE simulation within control theory applications such as the Duali, we can set to zero the weights (w_n) on the states variables and set to maximum possible value the weight (λ_m) on the controls.

The underlying CGE model in Section 2.1 can be implemented in percentage rates of change using the Johansen (1960)'s method. The principle of this method is a Taylor approximation around the base value to replace all non-linear equations by linear approximations, which are linear function of the log-deviations of variables from their base values. Without loss of the basic general equilibrium properties of the model in section 2.1 we can obtain a reduced linearized CGE model [eq.(1) thru (19)] in Table 1 by taking the total derivative of each function and dividing by the base period value. In this case, all variables with the superscript "*" in the equations represent the percentage deviations from the base period values such as the steady-states, means or secular trends. The lagged states variables are indicated by the subscript "-1".

Table 1. The Optimal Control CGE Framework

Objective function

$$\min\ J = \sum_t \left[\sum_n w_n (x_{nt}{}^* - \tilde{x}_{nt}{}^*)^2 + \sum_m \lambda_m (u_{mt}{}^* - \tilde{u}_{nt}{}^*)^2 \right] \qquad (1)$$

Gross Domestic Product

$$GDP^* = \sum_i \Phi_{iQ}(R_Q^* - P_i^*) + \sum_i \Phi_{iG} G_i^* + \sum_i \Phi_{iE} E_i^* - \sum_i \Phi_{iQ} M_i{}^* + \Phi_I I^* \qquad (2)$$

Employment

$$L^* = \sum_i \Psi_{iL} F_{Li}^{d\,*} \qquad (3)$$

Pollution emissions

$$PE^{*} = {X^{s}_{DRT}}^{*} \tag{4}$$

Domestic Commodity Prices

$$P_i^{d^*} = 1/(1-\vartheta_i)P_i^{*} - \vartheta_i/(1-\vartheta_i)P_i^{w^*} \tag{5}$$

Gross Domestic Supply

$$X_i^{s^*} = \sum_f \beta_{fi} {F_{fi}^{d}}^{*} \tag{6}$$

Factor Demand

$$F_{fi}^{d^*} = P_i^{d^*} + X_i^{s^*} - {P_f}^{*} \tag{7}$$

Consumer Price Index

$$CPI^{*} = \sum_i \lambda_i {P_i}^{*} \tag{8}$$

Unemployment Rate

$$lur^{*} = \rho_1\ CPI^{*} - \rho_2\ GDP^{*} \tag{9}$$

Labor Equilibrium

$$P_L^{\ *} = \sum_i (\Psi_{iL}/\theta){F_{Li}^{d}}^{*} + CPI^{*} - \rho_1/[\theta(1+\rho_1)]\ lur^{*} \tag{10}$$

Capital Equilibrium

$$P_K^{\ *} = \frac{\Psi_{CLN,K}}{\delta + \Psi_{CLN,K}}\left\{{P_{CLN}^{d}}^{*} + {X_{CLN}^{s}}^{*}\right\} + \frac{\delta}{\delta + \Psi_{CLN,K}}\left\{{P_{DRT}^{d}}^{*} + {X_{DRT}^{s}}^{*}\right\} - \frac{\gamma\Psi_{CLN,K}}{\delta + \Psi_{CLN,K}}\ I_{-1}^{\ *} \tag{11}$$

Export Quantities

$$E_i^{*} = \eta_i(P_i^{w^*} - P_i^{d^*}) \tag{12}$$

Import Quantities

$$M_i^{*} = \mu_i(P_i^{d^*} - P_i^{w^*}) \tag{13}$$

Factor Incomes

$$R_f^{*} = P_f^{*} + \sum_i \Psi_{if} F_{fi}^{d^*} \tag{14}$$

Household Income

$$R_Q^{*} = \sum_i \Gamma_{Qf} R_f^{*} + \Gamma_{QG} R_G^{*} \tag{15}$$

Government Income

$$R_G^{*} = \sum_f \Lambda_f R_f^{*} + \Lambda_G R_G^{*} + \sum_i \Lambda_i (P_i^{d^*} + X_i^{s^*}) + \Lambda_Q R_Q^{*} + \Lambda_{KA} I^{*} \tag{16}$$

Gross Investment

$$I^{*} = \gamma(R_K^{*} - CPI^{*}) \tag{17}$$

Intermediate Demand

$$V_i^* = \sum_j \upsilon_{ij} X_i^{s^*} \tag{18}$$

Output Equilibrium

$$P_i^* = -\frac{1}{\Theta_{iQ}} X_i^{s^*} + \frac{\Theta_{iV}}{\Theta_{iQ}} V_i^* + R_Q^* + \frac{\Theta_{iG}}{\Theta_{iQ}} G_i^* + \frac{\Theta_{iE}}{\Theta_{iQ}} E_i^* - \frac{\Theta_{iM}}{\Theta_{iQ}} M_i^* \tag{19}$$

where share parameters evaluated at the base period values are

$$\Phi_{iQ} = \frac{C_i}{GDP}, \Phi_{iG} = \frac{G_i}{GDP}, \Phi_{iE} = \frac{E_i}{GDP}, \Phi_{iM} = \frac{M_i}{GDP}, \Phi_I = \frac{I}{GDP} \; ;$$

$$\Psi_{iL} = \frac{F_{Li}^d}{L} \; ; \; \Psi_{iK} = \frac{F_{Ki}^d}{K} \; ; \; \Gamma_{Qf} = \frac{P_f \sum_i F_{fi}}{R_Q} \; ; \; \Lambda_i = \frac{\tau_i P_i^d X_i^s}{R_G} \; ; \; \upsilon_{ij} = \frac{V_{ij}}{V_i} \; ;$$

$$\Theta_{iQ} = \frac{C_i}{X_i^s}, \Theta_{iV} = \frac{V_i}{X_i^s}, \Theta_{iG} = \frac{G_i}{X_i^s}, \Theta_{iE} = \frac{E_i}{X_i^s}, \Theta_{iM} = \frac{M_i}{X_i^s}.$$

As is typical in the control theory literature, the dynamic evolution of the states variables in the above model can be represented by the state-space form of eq. (2) thru eq. (19) in matrix notation. In this case, following Kendrick (1981, Chapter 4), sectoral government expenditures, $G_i^* = u_{i,-1}$, are set equal to government obligations of the previous period and the exogenous sectoral world prices, $P_i^{w^*} = z_{i,-1}$, to those of the previous period, so as to write the model in the usual format of control theory. After stacking these new states into the augmented state vector x and doing simple matrix manipulation, we finally get the system equation for our optimal control CGE model:

$$x_{k+1} = A(\theta_k)x_k + B(\theta_k)u_k + C(\theta_k)z_k + \xi_k , \; k = 0, 1, \cdots , \text{T-1}. \tag{20}$$

where x_0 is given, $\xi_k \sim N(0,Q)$, and $\theta_k \sim N(\hat{\theta}_k, \Sigma_{0!0}^{\theta\theta})$. Here ξ_k and θ_k are normally distributed with means and covariances as shown above. System equation (20) is a system of first-order difference equations in which the current 26 states are functions of their previous period states, 2 previous period controls and 2 previous period exogenous variables. The additive error term, ξ_k, is normally distributed with mean zero and covariance Q. The evolution of uncertain parameters, θ_k, is specified as

$$\theta_{k+1} = D\theta_k + \zeta_k, \qquad k = 0, 1, \cdots, T\text{-}1 \tag{21}$$

which permits time-varying random parameters.[8] In general it is difficult to know the relative uncertainty across parameters since there is no general criteria yet to select which of the parameters in *A*, *B* and *C* matrix should be treated as uncertain. However, as will be seen in next section, it would be reasonable to assume that all of the non-zero parameters in the system equations are uncertain.

2.3 Data Sources, Calibration, and Stochastic Elements

Considering the exploratory nature of joint stochastic control-CGE models, this paper employs the simplest approach to data sets and model calibration. We used a social accounting matrix (SAM) for the 1989 U.S. economy. This data was basically recompiled from the original SAM database constructed by the U.S. International Trade Commission for analysis of U.S. trade policy. For our purpose, the original 31 industrial sectors in the U.S. SAM, benchmarked to 1989, were amalgamated into two broad sectors: pollution-non-intensive industries (or "clean" industries) and pollution-intensive industries (or "dirty" industries).[9] Two commodities from the two broad industries are the base for CGE modeling. The model for stochastic control CGE experiments described in Section 2.2 was calibrated directly from the U.S. SAM. Thus, for our model, the U.S economy has been divided into a SAM composed of two broad industrial sectors, two factor sectors (labor, capital), one household sector, one

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[8] For a detailed discussion of the structure of the optimal control model, see Kim and Kendrick (2002).

[9] In this paper, "clean" industries are those with a relatively low portion of pollution intensities in their production such as services and agriculture, whereas "dirty" industries are those with a relatively large proportion of pollution intensities such as energy and metal processing, petrochemicals, transportation and the other manufacturing products.

investment sector, one government sector, and one sector which represents the rest of the world. The key elasticities of the model not available from the SAM were uncertain but approximately chosen from the relevant literature. The real wage elasticity of θ we adopt is 0.3 for the U.S economy.[10] The investment return elasticity of γ adopted here is 1.1 which is higher than the real wage elascitities.[11] The parameter values associated with the "equilibrium gap augmented" Phillips surface in eq. (9) are $lur_n = 0.05$, $\rho_1 = -0.05$, and $\rho_2 = 0.66$. Thus, in the base case, the model can collapse into the fully neoclassical CGE framework. The elasticities used for the U.S import demand and export supply are chosen to represent the middle ground of published estimates obtained from the most recent studies. For the dirty industries, import and export elasticities are set at 1.3 and 1.65, respectively, while the clean industries have import and export elasticities set at 0.5 and 0.65.[12] Here, trade flows of dirty industry product are more price-responsive than those of clean industries.

Table 2 represents the coefficients of the estimated matrices *A*, *B* and *C*. In the state-space form, matrix *A* has non-zero values only in a column vector associated with total investment of previous period (*TINV*), since this variable is the only inter-temporal linkage in our simple CGE model. However, any uncertainty contained in each of the above key parameters in the original structural CGE framework would be distributed throughout the final (state-space form) system equations (20) during the matrix transformation. Thus the systems equations would also have some estimates of variance and covariance of coefficients as well as the residuals from equations.

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[10] See Stuart (1984), Browning (1987), Russek (1996), and Fuchs, Krueger and Poterba (1997).

[11] Reliable estimates of the elasticity of investment with respect to rates of return are relatively difficult to find because of the data problems. However, Engle (1974)'s estimates are useful for our model. The elasticity estimates by Engle (1974) for the four major durable and nondurable industries are 0.64, 1.57, 0.80, and 1.5. The value adopted here is the simple average of these values and it seems somewhat plausible compared to the other estimates reported by Jorgenson and Stephenson (1969).

[12] These are the industry-wide average values from the estimates by Reinert and Roland-Holst (1992) and Shiells and Reinert (1993).

Table 2. Estimated Model and Parameter Values : 26 States, 2 Controls and 2 Exogenous Variables

26 States

Parameter Matrix of System Equations

X=	A(TINV)=	B=		C=	
GDP	-0.02104	0.80050	0.16544	0.14473	0.86883
L	0.03708	0.10117	0.02166	0.01160	0.10649
PE	0.24541	-0.19883	0.06273	-0.03625	0.32229
PD1	-0.15482	0.47960	0.09889	0.07132	0.50277
PD2	-0.14643	0.59096	0.12255	0.08819	0.62348
XS1	0.15991	0.24528	0.02902	0.03521	0.14396
XS2	0.24541	-0.19883	0.06273	-0.03625	0.32229
FDL1	0.01845	0.16718	0.01028	0.02243	0.04716
FDL2	0.11235	-0.16557	0.06765	-0.03216	0.34620
CPI	-0.14692	0.47456	0.09795	0.09150	0.52052
LUR	0.02167	-0.55348	-0.11438	-0.10037	-0.60101
PL	-0.01337	0.55770	0.11763	0.08410	0.59957
PK	-0.50158	0.67973	0.13569	0.09913	0.68731
E1	0.10063	-0.31174	-0.06428	0.60364	-0.32680
E2	0.24160	-0.97508	-0.20221	-0.14552	0.62126
M1	-0.07741	0.23980	0.04944	-0.46434	0.25139
M2	-0.19035	0.76824	0.15931	0.11465	-0.48948
RL	0.02371	0.65887	0.13928	0.09570	0.70606
RK	0.01650	0.62910	0.12558	0.09174	0.63611
RQ	0.02209	0.64484	0.13335	0.09362	0.67560
RG	0.02988	0.63143	0.13044	0.09053	0.65959
TINV	0.17976	0.17000	0.03040	0.00027	0.12714
V1	0.18149	0.13317	0.03753	0.01717	0.18898
V2	0.21138	-0.02209	0.04931	-0.00781	0.25132
P1	-0.15095	0.46762	0.09642	0.09452	0.49022
P2	-0.12635	0.50994	0.10575	0.07610	0.67510

2 Controls

$$U=\begin{bmatrix} G1 \\ G2 \end{bmatrix}$$

2 Exogenous

$$Z=\begin{bmatrix} PW1 \\ PW2 \end{bmatrix}$$

3. Control Experiments

The optimal control CGE model developed in Section 2 provides a full economic specification with both price and quantity equations and it incorporates the speed of evolution of the economic system over time. In addition, it allows for more realistic adjustment processes towards long-run equilibrium, which is important especially for capturing the short- and midterm- run effects of temporary external shocks.

In what follows, we will use Duali to perform some control experiments with the U.S. control-CGE model, focusing on the roles of asymmetric political preferences and uncertainty across industrial sectors. Duali is a specialized software that can receive as inputs the desired paths for target and control variables, weighting penalty matrices, and the state-space representation of the economic model with or without its stochastic specifications. With the help of this software, we can easily compute the optimal feedback rule and the solution paths for the states and controls.

How does the structure of sectoral political preferences matter for macroeconomic stabilization?

How do changes in the structure of policymaker's political preferences affect the macroeconomic performance of stabilization policy? For instance, the macroeconomic performance can vary with the degree of policymaker's environmental preferences. This section performs some experiments with alternative penalty weight schemes associated with issue of pollution emissions. Specifically, as an illustration, the case of macroeconomic stabilization with sector-specific political preferences (i.e., dirty or clean industries) is compared to the case with no sector-specific political preferences.

For our economy, the policy goal of the economy (in aggregated terms) is to stabilize *GDP*, employment (*L*) and pollution emissions (*PE*) around the base case values, that is around zero, in the face of unexpected shocks. Assume that at period 0, the U.S economy is initially shocked (below its base case values) due to a temporary 5% deterioration in the international price-competitiveness of the dirty industries. Then, the optimal solutions for macroeconomic stabilization with three alternative penalty weight schemes are compared. Figure 1 provides the comparison of the optimal control solutions for the cases of (i) more of the political preferences on clean industries (sector1-focused scheme), (ii) more of the political preferences on dirty industries (sector2-focused scheme), and (iii) no sector-specific political preferences (equal scheme).[13] The graphs in Figure 1 show that the optimal control paths for all three schemes outperform the autonomous responses of the economic system. However, the stabilization performance of "sector2-focused scheme" in our experiment is somewhat worse than that of "equal scheme," since more controls on economic activities of the U.S. dirty industries tend to slow down directly the speed of recovery from the initial recession that was centered on the dirty industrial sectors. On the contrary, "sector1-focused scheme" performs better than "equal scheme." We can also see that in the case of "sector1-focused scheme" the fiscal policy of spending on clean goods (*G1*) plays a major role compared to that of dirty goods (*G2*), and *vice versa* in the case of "sector1-focused scheme."

The results above imply that the macroeconomic performance of stabilization could vary significantly with sector-specific political preferences (i.e., dirty or clean industries). This consideration would be especially important when policymakers address the issues of industrial, environmental, macroeconomic stabilization concerns simultaneously in an open economy framework.

On the other hand, for a substantial gain in realism, policymakers might need to take uncertainty into account. Indeed, economic models are biased due to the true underlying values of parameters being unknown (multiplicative uncertainty) and all the variances in states is not fully explained by the equations defining these states (additive uncertainty). Note that there are random shocks frequently hitting the

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[13] The optimal control case with "equal scheme" is undertaken with the equal penalty weight structure on the states (*GDP*, employment and pollution emissions). For the optimal control case "sector1-focused (sector 2-focused) scheme," the penalties on all sectoral activities of the dirty (clean) industries are now set to zero. Also, the model in Duali can be easily implemented to simulate the autonomous response of the model to a change in initial conditions. To do so, change the value of the nonzero elements in the W vector back to zero and and the value of the two elements in the Λ matrix to large numbers such as 9999999. For all cases, check to be sure that all desired paths are set to zero.

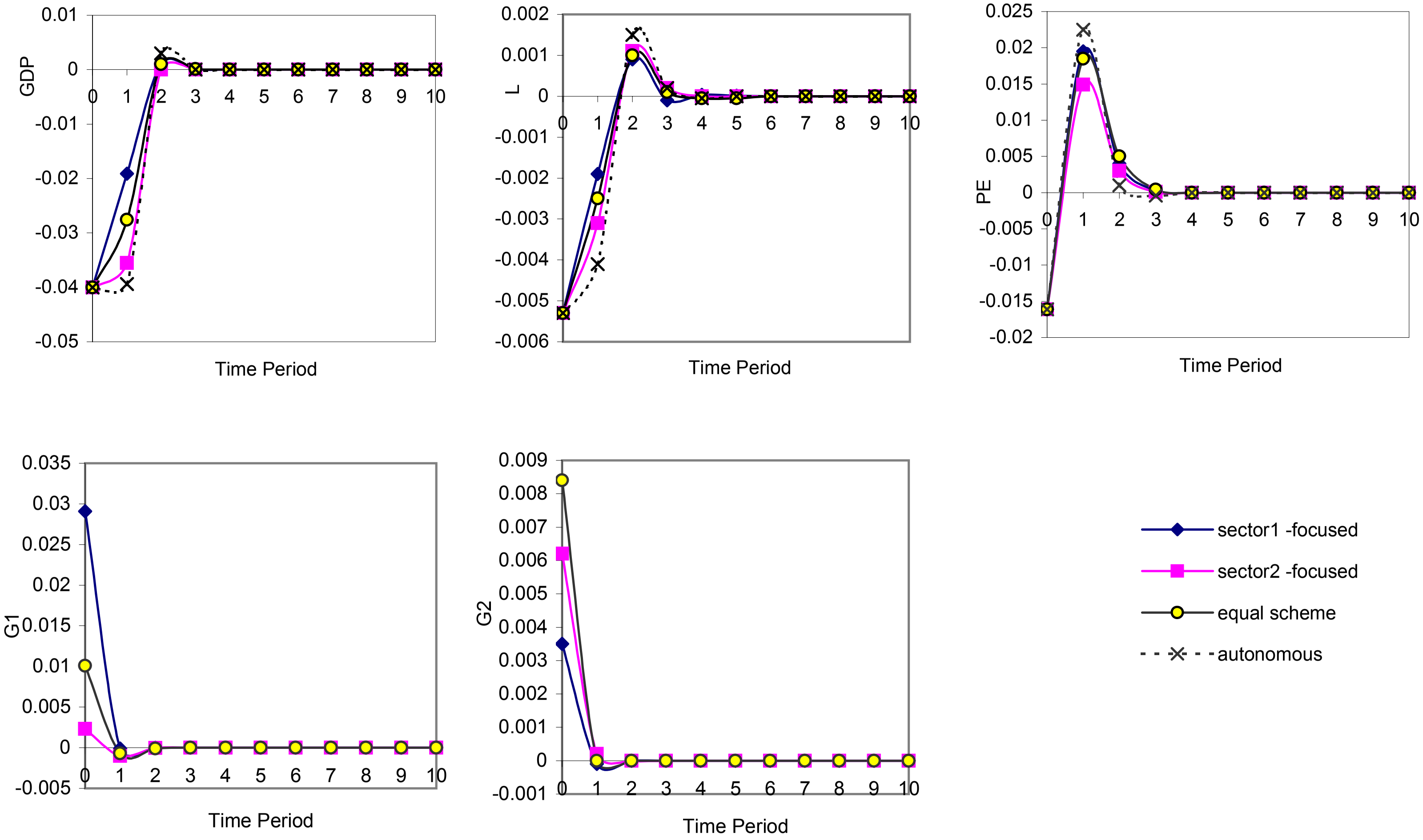

Figure 1. Optimal Control Solutions under Alternative Political Preferences Schemes in the CGE Model

economy, and the actual values of the model parameters, variables and initial conditions are never known with certainty.

Thus, the structure of political preferences can also affect the macroeconomic consequences of policy procedures in the face of uncertainty. We performed an experiment of 100 Monte Carlo runs with two different political preferences across industrial sectors, assuming that there is uncertainty associated with all parameters in the B matrix with the 20% standard deviations of each of these parameter mean values.[14] For each run, we compute the values of the quadratic tracking function in eq. (1). Then, the simulation results are encapsulated in a plot of pairs of criterion values from certainty equivalence procedure (CE) and open loop feedback procedure with parameter uncertainty (OLF) across Monte Carlo runs.[15]

Figure 2 summarizes the simulation results. Here the 45 degree line indicates when the criterion values of OLF policies are equal to CE policies. Thus a greater number of points above the 45 degree line imply that OLF policy performs better than CE policy. In the first panel of Figure 2, for a majority of the Monte Carlo runs the criterion values of OLF is smaller, and thus better, than the criterion value of CE.[16] However, the second panel indicates a similar performance for both control procedures. Therefore, we can see that the macroeconomic performance comparison of policy procedures towards uncertainty could be conditioned on the structure of policymaker's political preferences (i.e., relative penalty weights in criterion functions). However, note that there are no general theoretical results yet regarding the

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[14] Following Amman and Kendrick (1999)'s approach, for each of the two cases the Monte Carlo runs are done using Duali. In Duali all the random variables regarding additive noise, uncertain parameters, measurement errors and uncertain initial states are generated by Monte Carlo routines using the covariance matrices and the probability distributions.

[15] Stochastic control experiments generate a dynamic stochastic environment through random shock generation. These experiments use specific solution procedures: Certainty Equivalence (CE) and Open Loop Feedback with parameter uncertainty (OLF). CE considers only the additive uncertainty and ignores parameter uncertainty, while OLF uses both the mean and covariance values of the parameter estimates with passive learning. For this solution procedure in Duali, see Amman and Kendrick (1999).

[16] Using a simple macroeconomic model, Amman and Kendrick (1999) also found a similar result.

relative performance of CE and OLF, and the results may also depend on differences in the sizes and specifications in a wide variety of models.[17]

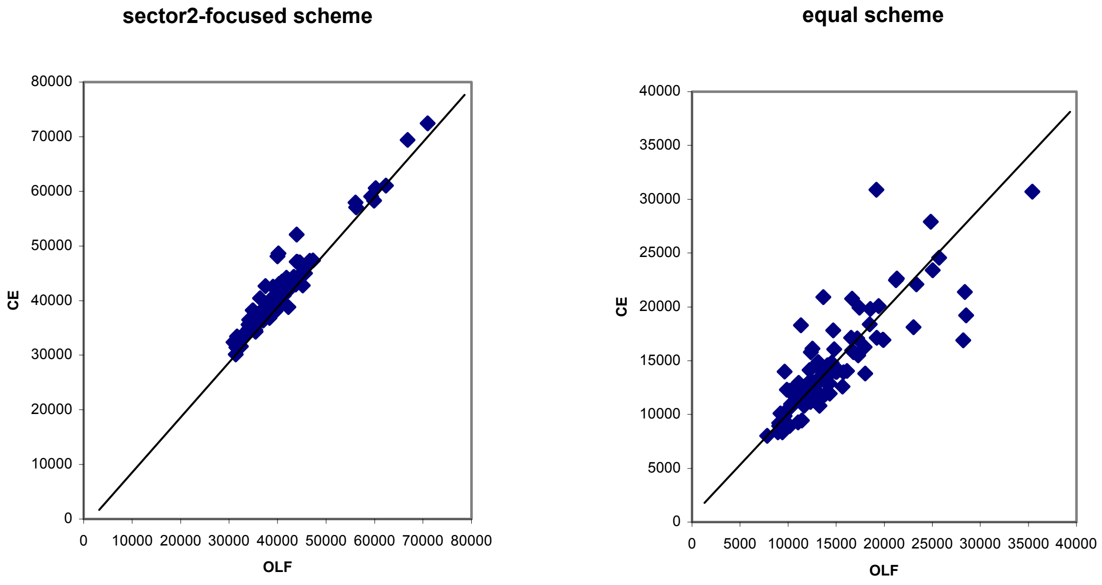

Figure 2. Comparison of Criterion Values Across Monte Carlo Runs

How does the structure of uncertainty matter for sectoral government expenditures?

How does the existence of parameter uncertainty cause policymakers to use their control instruments as compared to the case of no parameter uncertainty in a dynamic setting. In this section we will investigate the question of whether or not the existence of parameter uncertainty causes policymakers to use their controls in a more conservative fashion as compared to the case of no parameter uncertainty in a dynamic setting. For the cases of models with several states and controls, we cannot obtain the general answer to the above question. The result depends on the unknown covariance matrices associated with all of the states and controls in a complex way.

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[17] For a detailed discussion, see Kendrick (2002).

The policymaker's goal is to stabilize the economy in the face of unexpected temporary shocks. The desired levels of *GDP*, employment (*L*) and total pollution emissions (*PE*) were set to their initial levels in 1989 with equal policy weights for each. As before, we assume that the U.S economy is initially shocked below its base-case value due to the terms-of-trade shocks at period 0 and also that there is uncertainty in connection with six out of the parameters in the *B* matrix with the 20% standard deviations of each of these parameters. The vector of the base case mean values of these uncertain parameters (θ_0) and the variance-covariance matrix of uncertain parameters ($\sum_0$) are

$$\theta_0 = \begin{bmatrix} b_{11} = .80051 \\ b_{12} = .16544 \\ b_{21} = .10117 \\ b_{22} = .02166 \\ b_{31} = -.19883 \\ b_{32} = .06273 \end{bmatrix}, \ \Sigma_0 = \begin{bmatrix} .002563 & & & & & \\ & .00109 & & & & \\ & & .00041 & & & \\ & & & .00002 & & \\ & & & & .00158 & \\ & & & & & .00016 \end{bmatrix}.$$

Here, b_{11}, b_{21} and b_{31} are *GDP*, *L* and *PE* parameters associated with government expenditure on clean goods (*G1*), respectively, and b_{12}, b_{22} and b_{32} are the parameters associated with government expenditure on dirty goods (*G2*).

The general equilibrium effects of changing the degree of model uncertainty can be traced to investigate the question of where optimal policies should lie. To investigate the consequences of changing the level of relative uncertainty of the model parameters corresponding to one of the policy variables, the standard deviation of the *G2* parameters, b_{12}, b_{22} and b_{32}, were doubled from 20% to 40%.

Figure 3 compares the alternative optimal paths of the policy variables in the case of a doubling in the relative uncertainty of the *G2* parameters. As we can see in the graphs, the new path of *G2* is flatter than before, which implies that a relatively higher uncertainty in the *G2* parameters induces a more cautious use of that policy as a control instrument. On the contrary, government expenditure *G1* with a relatively certain parameters fluctuates more than before and is used more vigorously. This seems plausible but the results would be conditioned on the structure of penalty weights in the criterion functions and differ according to the structure of model assumptions. However, this experiment clearly suggests that policymakers need to consider the relative degree of associated parameter uncertainty among policy variables when choosing levels of policy intervention.

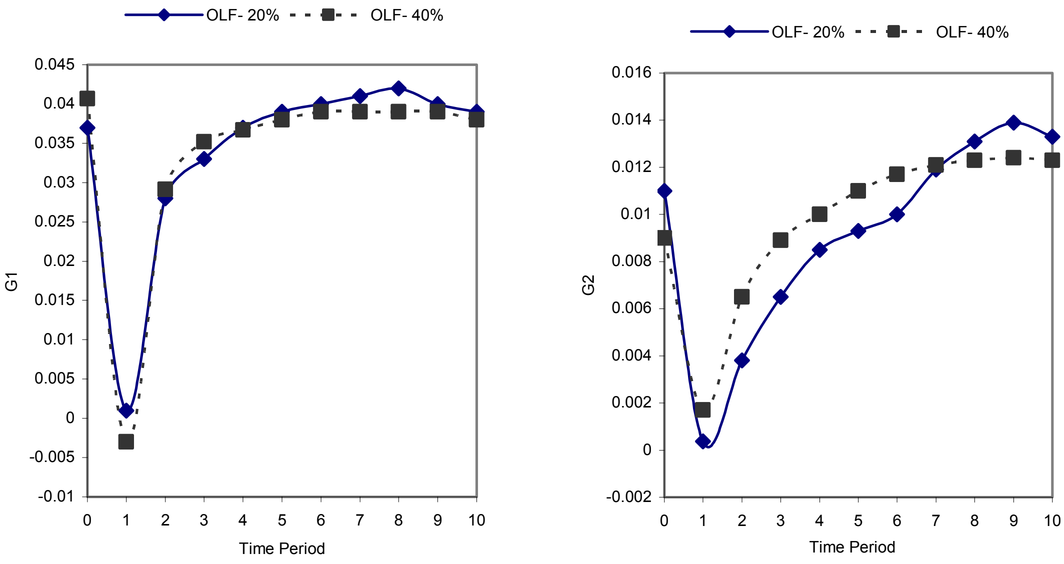

Figure 3. Effects of Relative Parameter Uncertainty of Policy Variables

4. Conclusions

Traditional CGE models have ignored uncertainty ─ even when applied to fields such as environmental modeling that are replete with economic uncertainty. In contrast, many control theory models have focused on the effects of uncertainty. Thus marrying the tradition of CGE and control modeling can result in price-quantity models with explicit dynamics and careful treatment of uncertainty.

In this perspective our paper explores an operational optimal control model of the U.S economy with traditional CGE approaches. It is intended to demonstrate the usefulness of CGE techniques in control theory application and also to provide a practical guideline to policymakers in this relatively new field. First, we develop a small dynamic SAM-based CGE model for the U.S economy. Then we compute the optimal control paths for the policy and state variables to guide the economy toward its desired goals as compared to the autonomous responses of the system in the face of unexpected shocks. Specifically, as an example, the U.S. control CGE model is used to explore the links between uncertainty, environmental care, and optimal government expenditure policy in a dynamic general equilibrium framework.

Instead of immediate market clearing, the model incorporates price-adjusted mechanisms that allow for some quantity-adjusted components (such as unemployment dynamics in labor markets) together with cross and feedback effects. This consideration is of great importance for short- or mid-term economic stabilization policies against unexpected external shocks. The results indicate that the optimal control solutions could differ not only due to differences in underlying model assumptions or structures, but also depending crucially on uncertainty about the magnitude of various parameters in the economy. In particular, it is also demonstrated that the performance of economic stabilization could vary significantly with asymmetric political preferences and uncertainty across industrial sectors. In such cases, allowing for all these components in more general CGE-based economic modeling may identify policies in the inherently stochastic world that may outperform traditional control-theory modeling approaches.

An interesting extension of the paper would be to consider the industry classifications of exportable, importable and non-tradable sectors, and then to analyze some (sector-specific or strategic) international trade policy issues. Another possible extension might include the consideration of monetary variables in the CGE framework, so that we could examine both fiscal and monetary policy in relation to industrial and international trade policy issues. Incorporation of additional inter-temporal linkages such as investment accelerator, durable consumption behavior, or the dynamic Phillips curve would enrich the model dynamics and possible control-theory CGE applications as well.

To mention some limitations of the model in this paper, it relies on the parameter values artificially drawn from the relevant literature rather than consistently estimated in a unified framework of the model. Thus, for more practical implementation of the model, we will need a fairly disaggregated econometric model of the U.S. economy (in reduced form). Further, it depends on the standard simplifying assumptions on the model economy and thus could be extended to consider the other effects of imperfect competition, monetary and financial behaviors, and distributional consequences.

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