

# **Strategic Interactions between Fiscal and Monetary Policies in a Monetary Union**

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**Abstract:** This paper develops a dynamic game model to study strategic interactions between the decision-makers in a monetary union. In such a union, governments of the participation countries pursue national goal when deciding on fiscal policies, whereas the common central bank's monetary policy aims at union-wide objective variables. Considering the example of a negative demand shock, we show how different solution concepts for the dynamic game between the common central bank and the national governments can be used as models of a conflict between national and supra-national institutions (noncooperative Nash equilibrium) and of coordinated policy-making (cooperative Pareto solutions).

## 1. Introduction

Policy-making for a national economy should be supported by a careful analysis of its objectives, its constraints, and the possibilities of achieving an outcome which is in some well-defined sense “better” than other available alternatives. Operations research and economics have produced many theoretical models in order to provide guidelines for arriving at “optimal” solutions for economic decision problems. In the context of fiscal and monetary policies in an international context, especially in a monetary union, where different objectives of policy-makers are nearly inevitable, game theory is an adequate tool to analyze and improve policy-making. Given the intertemporal nature of macroeconomic policy problems, the toolkit of dynamic game theory (see Basar and Olsder, 1999, for instance) recommends itself for obtaining insights and policy recommendations for decision-makers (governments of member countries and the common central bank) in a monetary union (see Petit, 1990).

Those mathematical models for such a macroeconomic system, which give a (largely) realistic picture for the real-world decision problem of concern, are rather soon reaching the limits of analytical tractability. Therefore, in this paper we will use the OPTGAME 2.0 algorithm (Behrens and Neck, 2002) to analyze a simple policy problem in a two-country monetary union. This numerical algorithm is designed for determining solutions of dynamic difference games with a finite planning horizon. In particular, OPTGAME solves discrete-time LQ games, and approximates the solutions of nonlinear-quadratic difference games by iteration. At present, the algorithm calculates the open-loop and the feedback Nash equilibrium solution and the cooperative Pareto-optimal solutions for an arbitrary number of players; extensions to other solution concepts are being implemented. Here we will show that calculating different solution concepts for a dynamic game between the common central bank and national fiscal policy-makers can provide insights into possible conflicts and their solution in this context.

## 2. The Model

We consider a monetary union with two participating countries. *Monetary union* means that national currencies (national central banks) have been entirely replaced by a

common currency (common central bank). Among others, this implies that the exchange rate has disappeared as an instrument of adjustment. In the following description of the model, capital letters indicate nominal values, while lower case letters correspond to real values. The two countries are assumed to be of equivalent size in terms of gross domestic product (GDP). The superscripts <sup>d</sup> and <sup>s</sup> denote demand and supply, respectively. The supply side is mostly exogenous.

The demand side goods market is modeled by a short-run income-expenditure equilibrium relation (IS curve), which is superimposed on an exogenous natural growth path. For  $t = 1, \dots, T$ , real output in country  $i$  ( $i = 1, 2$ ) is given as the sum of the long-run equilibrium level of the real output,  $\bar{y}_{it}$ , and the short-term deviation therefrom,  $\tilde{y}_{it}$ , i.e.

$$y_{it} = \bar{y}_{it} + \tilde{y}_{it} \quad (1)$$

where 
$$\bar{y}_{it} = (1 + \theta)\bar{y}_{i(t-1)}, \quad \bar{y}_{i0} \text{ given}, \quad (2)$$

$$\tilde{y}_{it} = \delta_i \left( \frac{P_{jt}}{P_{it}} - 1 \right) - \gamma_i (r_{it} - \theta) + \rho_i \tilde{y}_{jt} + \eta_i \tilde{f}_{it} + z_{it}, \quad (3)$$

for  $i \neq j$  ( $i, j = 1, 2$ ). The variable  $P_{it}$  ( $i = 1, 2$ ) denotes country  $i$ 's output price (its general price level),  $r_{it}$  ( $i = 1, 2$ ) represents country  $i$ 's current real interest rate, and  $\tilde{f}_{it}$  ( $i = 1, 2$ ) denotes country  $i$ 's short-term (deviation from a zero) real fiscal deficit.  $\tilde{f}_{it}$  ( $i = 1, 2$ ) in (3) is country  $i$ 's fiscal policy instrument, i.e. its control variable. The natural real growth rate,  $\theta$ , is assumed to be equal to the natural real rate of interest (assuming dynamic efficiency, in accordance with neoclassical growth theory). The parameters  $\delta_i, \gamma_i, \rho_i, \eta_i, i = 1, 2$  in (3) are assumed to be positive. The variables  $z_{1t}$  and  $z_{2t}$  are not subject to control and represent exogenous shocks on the demand side goods market.

For  $t = 1, \dots, T$ , the current real rate of interest for country  $i$  ( $i = 1, 2$ ) is given by

$$r_{it} = R_{Et} - X_{it}, \quad (4)$$

where  $R_{Et}$  denotes the common nominal rate of interest determined by the common central bank, and  $X_{it}$  ( $i = 1, 2$ ) represents country  $i$ 's rate of inflation. Note that the equilibrium level of the natural long-run interest rate,  $\bar{R}_{Et} = \bar{r}_{it} = \theta$  is "inflation-free", i.e.

$X_{it} = 0$  for  $i=1,2$ . Output prices and inflation rates for  $i=1,2$  and  $t=1,\dots,T$  are determined according to a demand-pull relation:

$$P_{it} = (1 + X_{it})P_{i(t-1)}, \quad P_{i0} \text{ given}, \quad (5)$$

$$X_{it} = \xi_i \tilde{y}_{it}, \quad (6)$$

where  $\xi_1$  and  $\xi_2$  are positive parameters. We also define average variables as

$$y_{Et} = \omega y_{1t} + (1 - \omega)y_{2t}, \quad \omega \in [0,1], \quad (7)$$

$$X_{Et} = \omega X_{1t} + (1 - \omega)X_{2t}, \quad \omega \in [0,1]. \quad (8)$$

Money demand in country  $i$  ( $i=1,2$ ) is the sum of long-run and short-run money demand:

$$m_{it}^d = \bar{m}_{it}^d + \tilde{m}_{it}^d. \quad (9)$$

Short-run money demand is determined by a Keynesian money demand function (LM curve):

$$\tilde{m}_{it}^d = \kappa_i \tilde{y}_{it} - \lambda_i (R_{Et} - \theta). \quad (10)$$

Here  $\kappa_i, \lambda_i$  ( $i=1,2$ ) are positive parameters,  $\theta$  is the natural rate of interest, and  $R_{Et}$  denotes the common nominal interest rate. Due to the long-run equilibrium relations,  $\tilde{y}_{it} = 0$ ,  $\bar{X}_{it} = 0$  and  $\bar{r}_{it} = 0$  ( $i=1,2$ ), long-run equilibrium money demand is given by

$$\bar{m}_{it}^d = \kappa_i \bar{y}_{it}. \quad (11)$$

Hence, there is no money illusion, and the Cambridge equation holds in the long run. This leaves us with the following equilibrium relationship for the long-run quantity of money (both demand and supply) in country  $i$  ( $i=1,2$ ):

$$\bar{M}_{it}^s = \bar{M}_{it}^d = P_{it} \bar{m}_{it}^d = P_{it} \kappa_i (1 + \theta) \bar{y}_{i(t-1)}. \quad (12)$$

This means that in each country, the price level will stay constant in the long run if money supply  $\bar{M}_{it}^s$  grows at the natural rate  $\theta$ . In a monetary union, the sum of the countries' money demands has to be equal to the monetary union's money supply.

In addition, we assume the money market always to clear in the short-run, too, and hence money supply to be equal to the sum of short-run money demands in countries 1 and 2,

$$M_{Et}^s = M_{1t}^d + M_{2t}^d. \quad (13)$$

This leads to

$$M_{Et}^s = \kappa_1 y_{1t} P_{1t} + \kappa_2 y_{2t} P_{2t} - (\lambda_1 P_{1t} + \lambda_2 P_{2t})(R_{Et} - \theta). \quad (14)$$

Note that this implies that the short-run real rates of interest in the two countries can considerably diverge both from each others and from the long-run (natural) real rate of interest.

The government budget constraint is given as an equation for government debt of country  $i$  ( $i=1,2$ ),

$$D_{it} = (1 + R_{E(t-1)})D_{i(t-1)} + F_{it} - \beta_i \tilde{B}_{Et}, \quad D_{i0} \text{ given}, \quad (15)$$

where the nominal fiscal deficit of country  $i$  ( $i=1,2$ ) is determined by the identity

$$F_{it} = P_{it} f_{it} = P_{it} \tilde{f}_{it}. \quad (16)$$

$\tilde{B}_{Et}$  denotes the short-term deviations of high-powered money,  $B_{Et}$ , from its long-run equilibrium level,  $\bar{B}_{Et}$ . The equilibrium stock of high-powered money is assumed to grow geometrically at the natural rate  $\theta$ . Hence,

$$B_{Et} = \bar{B}_{Et} + \tilde{B}_{Et} = (1 + \theta)\bar{B}_{E(t-1)} + \tilde{B}_{Et}. \quad (17)$$

$\tilde{B}_{Et}$  represents the control variable of the common central bank. The change in high-powered money is distributed as seigniorage to the two countries according to given positive parameters  $\beta_1 \in [0, 1]$  and  $\beta_2 := 1 - \beta_1$ . Assuming a constant money multiplier,  $\psi$ , the broad money supply of the monetary union is given by

$$M_{Et}^s = \psi B_{Et}. \quad (18)$$

Both national fiscal authorities are assumed to care about stabilization of inflation, output, debt, and fiscal deficits of their own countries. The common central bank is interested in the stabilization of inflation and output in the monetary union and in a low variability of its supply of high-powered money. Hence, the individual objective functions of the national governments and of the common central bank are given by

$$J_i = \frac{1}{2} \sum_{t=1}^T \left( \frac{1}{1+\theta} \right)^t \left( \alpha_{iX} X_{it}^2 + \alpha_{iy} (y_{it} - \bar{y}_{it})^2 + \alpha_{iD} D_{it}^2 + \alpha_{if} \tilde{f}_{it}^2 \right), \quad i = 1, 2 \quad (19)$$

$$J_E = \frac{1}{2} \sum_{t=1}^T \left( \frac{1}{1+\theta} \right)^t \left\{ \alpha_{EX} X_{Et}^2 + \alpha_{Xy} (y_{Et} - \bar{y}_{Et})^2 + \alpha_{EB} \tilde{B}_{Et}^2 \right\}, \quad (20)$$

where all weights are positive numbers  $\in [0,1]$ . The joint objective function for the calculation of the cooperative Pareto-optimal is determined by  $J = \mu_1 J_1 + \mu_2 J_2 + \mu_E J_E$  ( $\mu_1, \mu_2, \mu_E \geq 0, \mu_1 + \mu_2 + \mu_E = 1$ ).

The parameters of the model are specified numerically in the simplest possible way, leaving us with a symmetric monetary union (see Table 1). Lack of space precludes a detailed discussion of the parameter values chosen, the target values assumed for the objective variables of the players (which are basically the long-run equilibrium values of the respective variables), and the initial values of the state variables (see Table 2) of the dynamic game model. Detailed information on this is available from the authors on request.

**Table 1:** Parameter values for a symmetric monetary union for  $i = 1, 2$

Parameter	$T$	$\theta$	$\delta_i$	$\gamma_i$	$\rho_i$	$\eta_i$	$\xi_i$	$\omega$	$\kappa_i$	$\lambda_i$	$\beta_i$	$\psi$	$\alpha$ 's	$\mu_i$
Value	20	0.03	0.5	0.5	0.5	1.0	0.25	0.5	1.0	0.15	0.5	0.5	1.0	0.33

**Table 2:** Initial values for  $i = 1, 2$

Parameter	$\bar{y}_{i0}$	$\tilde{y}_{i0}$	$P_{i0}$	$X_{i0}$	$D_{i0}$	$R_{E0}$	$\bar{B}_{E0}$	$\tilde{f}_{i0}$	$\tilde{B}_{E0}$
Value	1	0	1	0	0	$\theta$	1	0	0

Equations (1)–(20) constitute a nonlinear dynamic game with a finite planning horizon, where the objective functions are quadratic in the deviations of state and control variables from their respective desired values.

### 3. Optimal Fiscal and Monetary Policies

Several experiments were performed with the model, using different assumptions about the paths of the exogenous non-controlled variables. For lack of space, we report only the results of one of them. This is a symmetric shock acting on both economies. In particular, we assume that autonomous real output (GDP) in both economies falls by 1.5% below the long-run equilibrium path for the first four periods and less for the next three periods:  $z_{i0} = 0$ ,  $z_{i1} = z_{i2} = z_{i3} = z_{i4} = -0.015$ ,  $z_{i5} = -0.01$ ,  $z_{i6} = -0.005$ ,  $z_{i7} = -0.0025$ , and  $z_{it} = 0$  for  $t \geq 8$  and  $i = 1, 2$ .

Without policy intervention, this demand side shock leads to lower output and inflation (compared to the long-run equilibrium path) during the first five periods, but higher output and inflation afterwards (see Fig.3). That is, the uncontrolled dynamic system adjusts in dampened oscillations, getting close to the long-run path after twenty periods. The maximum deviation of output from its equilibrium path is approximately 0.33% (in the first period). That is, even without policy intervention there are sufficiently strong negative feedbacks in the system to reduce the impact of the shock on output to about one fifth of the original shock at most in the case of a temporary symmetrical shock. This is mainly due to a strong reaction of the rate of interest, which falls to values near zero in the first four periods (but rises above the long-run value afterwards). Due to the symmetry of the economies and of the shock, the reactions of all variables are identical in both economies.

When policy-makers are assumed to react on this shock according to their preferences as expressed in their objective functions, several outcomes are possible, depending on the assumptions made about the respective other policy-makers. Here we consider two noncooperative equilibrium solutions of the resulting dynamic game, the

open-loop and the feedback Nash equilibrium solution, and one cooperative solution, the Pareto-optimal collusive solution (all players get the same weight  $\mu_i = 1/3$ ,  $i = 1, 2, E$ ). The feedback Nash equilibrium solution is more interesting than the open-loop one because the former is subgame perfect or Markov perfect, while the latter is only valid if it is assumed that all policy-makers commit themselves unilaterally and decide upon trajectories of their instrument variables once for all at  $t = 0$ .

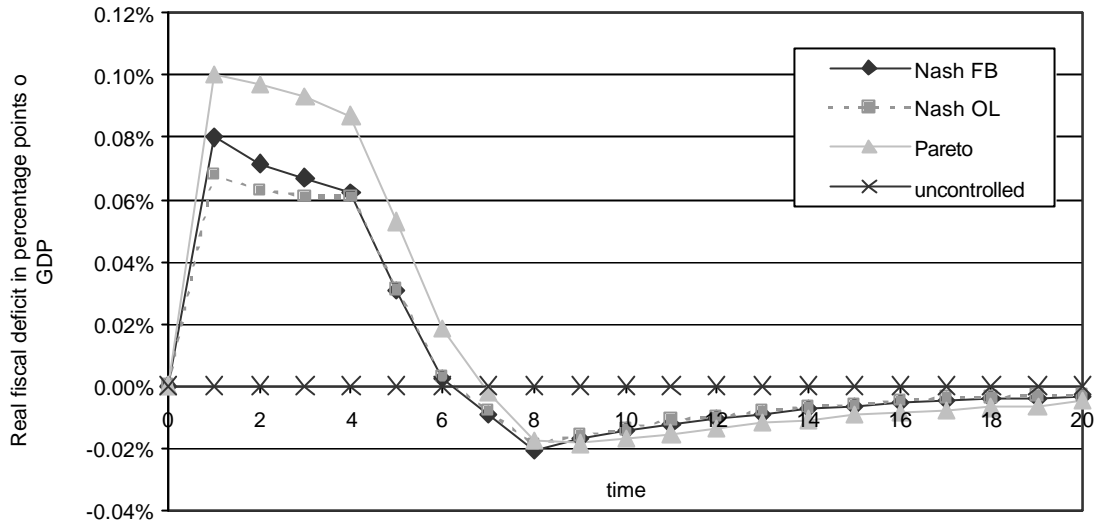
The time paths of the control variables — real fiscal deficit (for either country) and additional high-powered money — under the three solution concepts considered are shown in Figs.1 and 4, respectively, those of the state (and objective) variables — deviations from long-run equilibrium output and government debt — in Figs.3 and 4, respectively. Inflation rates show the same qualitative pattern as outputs, price levels remain below the equilibrium value of one for all periods, and the common nominal rate of interest exhibits a behavior very similar to the uncontrolled case (falling to low values in periods one to four, rising up to about 3% later on). All country-specific variables show exactly the same time paths for both countries. More detailed results are available from the authors on request.

As can be seen from the graphs, both fiscal and monetary policies react on the negative demand shock in an expansionary and hence counter-cyclical way: both countries create fiscal deficits during the first five to six periods and surpluses afterwards, and the central bank raises its supply of high-powered money during the first six years and reduces it afterwards. This results in less output loss and lower deflation than in the uncontrolled solution. What is remarkable is the small magnitude of the (absolute) values of the instruments involved: the highest value of the fiscal deficit created is one tenth of one percentage point of GDP (in period one), for example, which would be nearly invisible in terms of the Maastricht criteria if applied in the European Economic and Monetary Union. This is due to the strong self-stabilizing forces in the model used, acting especially through the interest rate channel, as noted already for the uncontrolled solution. As there is not much need for counter-cyclical action, it is not surprising that optimal (equilibrium) policies entail only cautious activities.

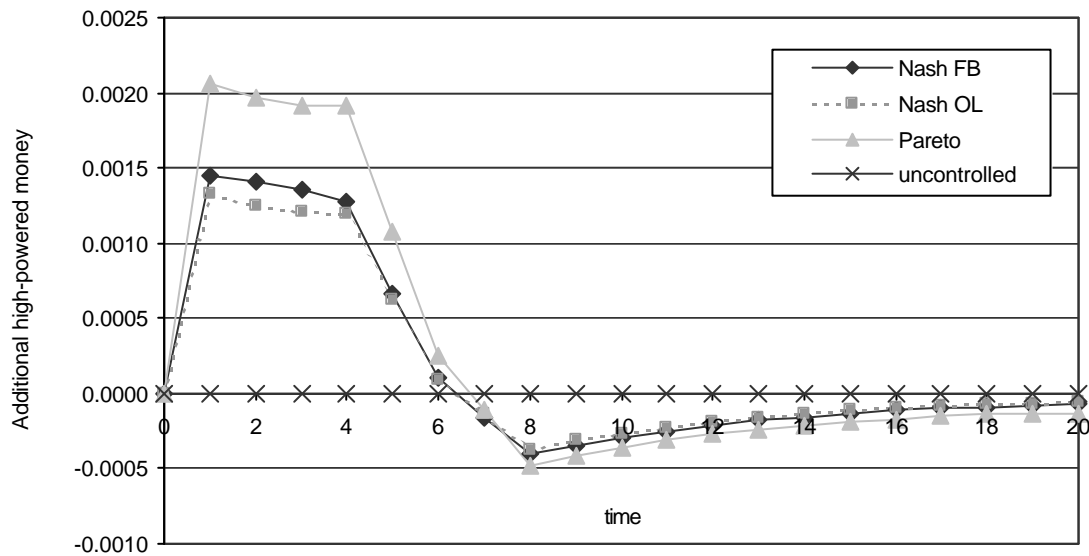
Comparing the noncooperative equilibrium solutions and the cooperative solution yields another interesting observation. All show qualitatively the same behavior, and the two noncooperative Nash equilibrium solutions are very close together in terms of all



control and state variables. The collusive solution, although not too distant from the other two, exhibits more active policy-making (higher fiscal deficits and money creation in the first periods). This different policy-mix does not change the path of the rate of interest



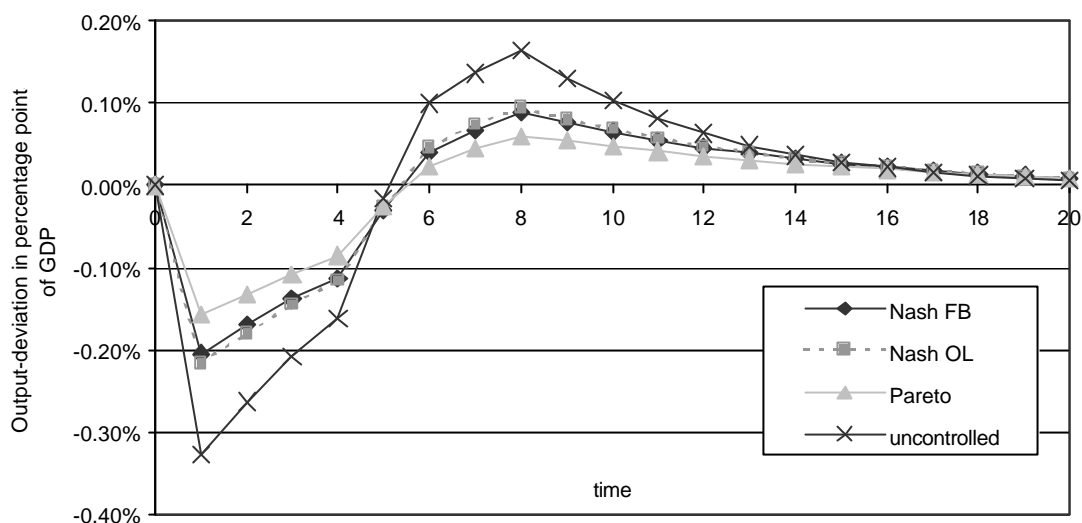
**Fig.1.** Country  $i$ 's real fiscal deficit



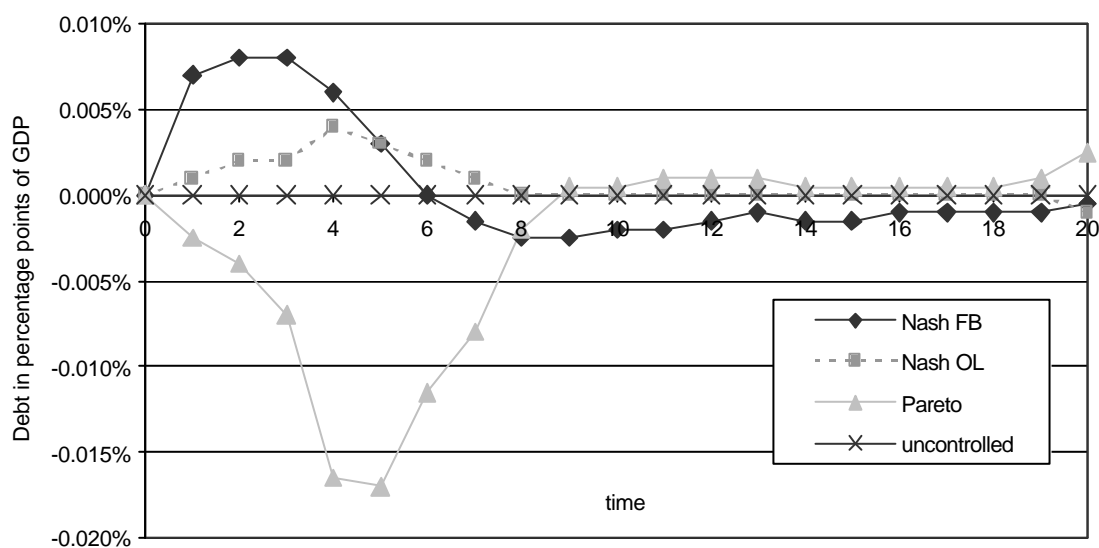
**Fig.2.** Additional high-powered money

(higher deficits increase, higher money supply decreases the interest rate, ceteris paribus), but does so for the public debt trajectory: in the noncooperative solutions, government debt is increased, in the cooperative solution it is increased, reflecting the

relatively higher increase of fiscal deficits as compared to monetary supply increases in the noncooperative solutions. It remains to be seen whether these policy patterns remain under alternative assumptions about the economic model or the shock.



**Fig.3.** Country  $i$ 's output-deviation from its long-run equilibrium level



**Fig.4.** Country  $i$ 's debt

## 4. Concluding Remarks

Applying dynamic game theory and the OPTGAME 2.0 algorithm to a simple macroeconomic model of fiscal and monetary policies in a monetary union, we obtained several insights into the design of economic policies facing a symmetric negative demand shock. In particular, optimal policies of both the governments and the common central bank are counter-cyclical but not very active, at least for the model under consideration. The outcomes of the different solution concepts of dynamic game theory are rather close to each other. In particular, a periodic update of information and related reduction of commitment (a change from an open-loop to a feedback Nash equilibrium solution) does not cause benefits or costs to either decision-maker. Cooperative economic policies (both fiscal and monetary ones) are more active or “aggressive” than noncooperative ones, resulting in a somewhat different policy-mix with higher stabilization effects. Further research will have to show how sensitive these results are with respect to the assumptions about the model and the shock.

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