

# The Beveridge Curve, Job Creation and the Propagation of Shocks

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## Abstract

This paper proposes modifications to the popular model of equilibrium unemployment by Mortensen and Pissarides [30]. I augment the model by introducing (1) costly planning for brand-new jobs, (2) the option to mothball preexisting jobs, and (3) a non-trivial job rejection decision for new employment relationships. These modifications greatly improve the model's ability to replicate the Beveridge curve as well as observed correlations between vacancies and job creation. It is also shown that persistent behavior of vacancies in my model serves to enhance the model's propagation mechanism.

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# 1 Introduction

Over the last decade, the job-matching models introduced by Mortensen and Pissarides [30] has become a popular framework for analyzing the behavior of job flows and unemployment. Reflecting this popularity, there have been numerous attempts to assess the quantitative performance of this class of models. For example, Cole and Rogerson [11] examine the model’s ability to deliver plausible cyclical properties of job flows and employment in the U.S. in a reduced form framework. Collard et al. [12] estimate the structural parameters and undertake formal statistical tests of the model.<sup>1</sup> Overall, the conclusion from these studies is that the framework does a good job in explaining important empirical regularities regarding job flows and employment in the U.S.

There is, however, an important dimension along which the model performs poorly, i.e., the model is unable to generate realistic dynamics in vacancies and unemployment, known as the Beveridge curve. The U.S. data show that the cyclical components of vacancies and unemployment are highly negatively correlated, and that those two series exhibit considerable persistence.<sup>2</sup> The simulation results in existing papers (see e.g. Andolfatto [4], Merz [29], Mortensen and Pissarides [30]), however, indicate that cyclical components of the two series exhibit virtually no correlation, and that vacancies are not persistent in the model.

The sources of the inability of the model to generate a realistic Beveridge curve lie in a free entry condition — a widely-used assumption in this class of models — and an associated “echo effect.” The condition states that firms immediately enter the matching market by simply posting a vacancy when doing so is expected to yield positive returns. Consequently, the expected returns to opening a vacancy are equalized to the posting cost: with vacancies being constant, a negative aggregate productivity shock would decrease the expected returns, and thus the number of vacancies would drop immediately to ensure that the free-entry condition holds. Moreover, the firms’ incentive to post vacancies quickly rises as the adverse shock increases unemployment, since this raises the chance that the firm successfully finds

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<sup>1</sup>Other recent studies include Costain and Reiter [13].

<sup>2</sup>See Shimer [42] for more details of the characteristics of the vacancy series.

a worker from the pool of unemployed. Because of this echo effect, vacancies recover soon after the adverse shock, and thereby can not be persistent. Given this, the model cannot generate the quantitatively meaningful Beveridge curve.

The behavior of vacancies in the model also has a significant implication for the behavior of job creation. In the Mortensen and Pissarides model, an aggregate matching function determines the number of jobs created.<sup>3</sup> Given that unemployment is sluggish in the model, the matching function implies that the behavior of vacancies dominates job creation in the short-run. Thus, the model generates job creation series that are not persistent, which accords with observed data, but does so at the cost of generating vacancy series that are not persistent either, which is counterfactual.

This paper modifies the Mortensen and Pissarides model in the following sensible ways. First, I augment the model with costly *planning*. Specifically, the firms are allowed to post vacancies only after completing the planning process, which occurs with some probability each period. Clearly, such planning lags can play an important role in generating persistent vacancies.

The second modification is that firms with currently operating jobs are given an option to *mothball* their jobs, rendering them temporarily inactive. The firms that are mothballing their jobs then decide whether to repost vacancies at the margin of one-time *retooling* costs. Firms are allowed to repost mothballed jobs as vacancies with certainty as soon as doing so is profitable. This feature is sensible in that preexisting jobs have already incurred their planning costs. It implies that the flows of preexisting jobs respond to shocks promptly, in contrast to the flows of brand-new jobs that involve time-consuming planning process.

The third modification to the Mortensen and Pissarides model is to introduce a non-trivial job rejection decision for new employment relationships.<sup>4</sup> Recall that in the original Mortensen and Pissarides model, all “meetings” result in job creation under the assumption that new matches enjoy the highest level of idiosyncratic

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<sup>3</sup>More precisely, the assumption that new matches start with the highest idiosyncratic productivity level yields the outcome that “meetings” always result in “matching”.

<sup>4</sup>This type of model is discussed by Pissarides [33], [35] and is called a stochastic job matching model.

productivity. Instead, we adopt the specification where even newly-formed meetings draw idiosyncratic productivity shocks, and decide whether to create new jobs (i.e., start producing) or not. In particular, the model imposes an additional cost for new employment relationships, often called a job creation cost. This additional cost lowers the return of new matches, thereby making the job rejection rate for new matches higher than the job destruction rate for ongoing jobs. Under this specification, the number of jobs created is influenced not only by the aggregate meeting function, but also by the rate at which job rejection takes place.<sup>5</sup>

The quantitative evaluations of the model show that these modifications greatly improve the model's ability to replicate the Beveridge curve, and observed correlations between vacancies and job creation.<sup>6</sup> It is also shown that persistent behavior of vacancies in my model serves to enhance the propagation mechanism through the channel of job creation. There is a group of papers that stress the role of higher job destruction as a channel through which a recession shock propagates over the business cycle.<sup>7</sup> My claim is not necessarily inconsistent with the view. Rather, this paper provides a new insight that persistent declines in vacancies after a recessionary shock, which accords with an empirical fact, imply that the propagation mechanism through a job creation channel could possibly be more pervasive than existing models claim.

This paper is organized as follows. Section 2 lays out the benchmark Mortensen and Pissarides matching model. By solving the model under reasonable param-

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<sup>5</sup>Thus, the relevant definition of the number of jobs created is  $(1 - \rho^n)m(u, v)$  in our model, where  $m$  is the number of meetings that depend positively on unemployment ( $u$ ) and vacancies ( $v$ ), and where  $\rho^n$  is the job rejection rate, whereas it was simply defined as  $m(u, v)$  in the benchmark Mortensen-Pissarides model.

<sup>6</sup>A recent paper by Shimer [42] argues that the productivity shock necessary to generate realistic vacancy-unemployment dynamics is implausibly large. However, our model successfully matches the empirical regularities regarding vacancies and unemployment under standard assumptions about the productivity process.

<sup>7</sup>Theoretical papers in this group include den Haan et al. [19] Gomes et al. [22], and Ramey and Watson [36]. Davis and Haltiwanger [14] and [17] empirically show that job destruction is more important in driving employment fluctuations. Caballero and Hammour [9], on the other hand, argue through their econometric exercises that the most important effect of a recessionary shock operates through lower job creation.

terizations, I will show that the model fails to generate a realistic Beveridge curve, and that the model counterfactually predicts strong positive correlation between job creation and vacancies. Section 3 extends the benchmark model by introducing the three modifications mentioned above. Section 4 presents the parameterizations of the extended model. Section 5 quantitatively evaluates the extended model in reference to the benchmark model. Section 6 concludes the paper.

## 2 Benchmark Model

This section lays out a discrete time version of the standard Mortensen and Pissarides [30] matching model that is characterized by matching frictions and endogenous job destruction. There is a continuum of identical consumer-workers with total mass equal to one in this economy, along with a continuum of potential firms, having infinite mass. Further, each firm consists of only one job to which only one worker is attached. Workers are assumed to be risk neutral, with discount factor  $\beta$  lying between zero and one. Time spent working is restricted to be either zero or one, meaning that workers provide one unit of labor when employed and zero when unemployed. Labor is the only input for production.<sup>8</sup> To hire a worker, a firm first must open a vacancy that incurs a cost  $c_v$  per period. Other important assumptions are that workers search for their jobs only when unemployed,<sup>9</sup> and that the workers' decision about labor force participation is ignored.

### 2.1 Employment Relationship

Each worker-firm pair that engages in production produces output according to the production function:

$$z_{it}y_t,$$

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<sup>8</sup>Andolfatto [4], Merz [29] and den Haan et al. [19] embed the labor market matching into dynamic stochastic general equilibrium models with capital and a risk-averse household. Since these models with capital encounter the same problems addressed in this paper, I focus on the problems by ignoring capital. In principle, adding capital should not alter our results.

<sup>9</sup>In other words, there is no on-the-job-search. See Mortensen [31], Pissarides [34], Chapter 4 in Pissarides [35] and the references therein for the models with on-the-job search.

where  $z_{it}$  gives a random disturbance that is specific to  $i$  th pair in period  $t$ , and  $y_t$  indicates a random aggregate productivity disturbance in period  $t$ , which follows a first order Markov process. The idiosyncratic productivity shock is assumed to be i.i.d. across jobs and time. The distribution of  $z_{it}$  is described by a cumulative distribution function  $H(z_{it})$  whose support is assumed to be  $[0, \infty)$ . The firm that produces output incurs a constant operating cost  $\kappa$ . Worker-firm pairs can be destroyed either by exogenous or endogenous reasons as in den Haan et al. [19], i.e., matched pairs are destroyed with constant probability  $\rho^{ex}$  per period, and those that are not subject to exogenous separation may choose to separate endogenously.

The worker who is separated from a job, whether exogenously or endogenously, obtains  $b+U_{it}$  where  $b$  is the current-period unemployment benefit,<sup>10</sup> and  $U_{it}$  denotes the expected present discounted value of the unemployed worker net of the current-period unemployment benefit. The firm's outside option alternative to production is zero in the benchmark model, as we will see shortly.

Given the outside options for the worker and firm, the separation decision of the matched-pair can be described as follows. Let  $G_{it}$  denote the expected present discounted value of joint returns of the worker-firm relationship in period  $t$ . The surplus of the matched pair over the outside options in period  $t$  is then written as:

$$S_{it}^e = z_{it}y_t - \kappa + G_{it} - (U_{it} + b). \quad (1)$$

The worker and firm bargain over this joint surplus. The negotiation is resolved according to the Nash bargaining solution, where the firm and worker take fixed proportion of  $S_{it}$ ,  $\pi$  and  $1 - \pi$ , respectively. Since the current-period return becomes lower as  $z_{it}$  declines, there exists a level  $\hat{z}_{it}$  such that  $S_{it} < 0$  for  $z_{it} < \hat{z}_{it}$ , where both parties agree to abandon their relationship, while  $S_{it} \geq 0$  for  $z_{it} \geq \hat{z}_{it}$ , where both parties agree to maintain their relationship, and engage in production in this period.  $\hat{z}_{it}$  is referred to as the job destruction margin. Associated with  $\hat{z}_{it}$  is the

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<sup>10</sup>The symbol  $b$  is referred to as “unemployment benefit” for convenience even though it is not a transfer from the government. More precisely,  $b$  should be considered as home production or utility from leisure that unemployed workers enjoy.

endogenous separation rate  $\rho_{it}^e$  :

$$\rho_{it}^e = \int_0^{\hat{z}_{it}} dH(z_{it}).$$

## 2.2 Matching Market

Unemployed workers and firms with vacant jobs engage in search activity in a matching market, which is characterized by a constant-returns-to-scale<sup>11</sup> aggregate matching function:

$$m_t = m(u_t, v_t), \quad (2)$$

where  $m_t$  denotes the number of matches formed in the period- $t$  matching market,  $u_t$  denotes unemployment,  $v_t$  denotes vacancies. The matching function  $m(\cdot)$  is increasing in both arguments. On average, an unemployed worker finds a firm each period with probability:

$$\frac{m(u_t, v_t)}{u_t} \equiv \lambda_t^w. \quad (3)$$

Similarly, a vacant job is filled with probability:

$$\frac{m(u_t, v_t)}{v_t} \equiv \lambda_t^f. \quad (4)$$

Specifically, the matching function takes the following form proposed by den Haan et al. [19]:

$$m_t = \frac{u_t v_t}{(u_t^l + v_t^l)^{1/l}}.$$

A major advantage of this functional form over the widely used Cobb-Douglas specification is that the matching probabilities defined in (3) and (4) take on values between zero and one for all  $u_t \in [0, 1]$  and  $v_t \in [0, \infty)$ .

## 2.3 Equilibrium Conditions

Consider now the situation facing a firm in the matching market, having a vacant job. Assume that new matches start with some known high level of idiosyncratic

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<sup>11</sup>This assumption is supported by numerous empirical studies. Previous work for the US includes Blanchard and Diamond [6] and Bleakly and Fuhrer [8]. See Petrongolo and Pissarides [32] for an extensive survey on this issue.

productivity,  $\bar{z}$ , such that job rejection never occurs for all possible states of the model economy.<sup>12</sup> Under this specification, if the firm finds a worker with probability  $\lambda_t^f$ , then production takes place, and it obtains a share  $\pi$  of total surplus. Also, free entry restricts the value of a vacant job to be zero every period. The Bellman value-transition equation for a vacant job is then written as:<sup>13</sup>

$$0 = -c^v + \beta \lambda_t^f E_t \pi S_{t+1}^n, \quad (5)$$

where  $S_{t+1}^n$  denotes the total surplus for the newly matched pair, defined as:

$$S_{t+1}^n \equiv \bar{z} y_{t+1} - \kappa + G_{t+1} - (U_{t+1} + b). \quad (6)$$

Equation (5) is called the “free entry condition,” meaning that in equilibrium, the posting cost equals the expected returns from posting a vacancy. This is the key equation in the benchmark model that determines the level of vacancies each period.

The job creation rate is defined consistently with the specification that meetings are equivalent to matches:

$$cre_t = \frac{m(u_t, v_t)}{n_t},$$

where  $n_t \equiv 1 - u_t$ .<sup>14</sup>

Equilibrium values of  $U_t$  and  $G_t$  are determined as follows. Consider first the situation facing an unemployed worker in the period- $t$  matching market. If the unemployed worker does not match with a firm in period  $t$ , then his continuation value is  $U_{t+1} + b$  in the next period. Alternatively, if the worker finds the firm with probability  $\lambda_t^w$ , then he engages in production and receives a share  $1 - \pi$  of total surplus  $S_{t+1}^n$  on top of  $U_{t+1} + b$ . The Bellman equation for the unemployed worker is thus written as:

$$U_t = \beta E_t [\lambda_t^w (1 - \pi) S_{t+1}^n + U_{t+1} + b]. \quad (7)$$

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<sup>12</sup>This slight modification to the original Mortensen and Pissarides model is made because the support of the idiosyncratic productivity shock is assumed to be unbounded above in this paper. This modification does not have any important implications.

<sup>13</sup>In what follows, the  $i$  subscripts are dropped because the idiosyncratic productivity shocks are assumed to be independent across time, and the decision problems are identical for each worker and each firm.

<sup>14</sup>The term job creation is usually used to indicate the job creation rate, but it should not cause any confusion to the reader. The same attention should be paid to the term job destruction.



Next, consider the expected joint returns of the worker-firm pair that produces in period  $t$ . Given that the firm's outside option is zero, the joint outside option for the relationship equals  $U_{t+1} + b$  when the relationship is severed at the beginning of period  $t + 1$ . If the relationship survives the separation process in period  $t + 1$ , it receives  $S_{t+1}^e$  in addition to  $U_{t+1} + b$ . Thus, the following Bellman equation holds:

$$G_t = \beta E_t \left[ (1 - \rho^x) \int_{\hat{z}_{t+1}}^{\infty} S_{t+1}^e dH(z_{t+1}) + U_{t+1} + b \right]. \quad (8)$$

Finally, the law of motion for unemployment closes the model:

$$u_t = u_{t-1} + [\rho^{ex} + (1 - \rho^{ex})\rho_t^e] (1 - u_{t-1}) - m(u_{t-1}, v_{t-1}). \quad (9)$$

Figure 1 summarizes the recursive structure of the benchmark model. It should be clear that the period  $t$  aggregate state variables of the economy consist of aggregate productivity, the number of unemployed at the beginning of period  $t$ ,  $u_{t-1} - m_{t-1}$ , and the number of matches  $m_{t-1}$ .<sup>15</sup> Given that the last two variables contain the equivalent information to  $u_{t-1}$  and  $m_{t-1}$ , a set of period- $t$  aggregate state variables  $\mathbf{s}_t$  is written as:

$$\mathbf{s}_t = \{y_t, u_{t-1}, m_{t-1}\}.$$

The recursive equilibrium is then defined by a list of functions,  $G(\mathbf{s}_t)$ ,  $U(\mathbf{s}_t)$ ,  $v(\mathbf{s}_t)$  and  $\hat{z}(\mathbf{s}_t)$  such that (a) the Bellman equations (7) and (8), and the free entry condition (5) hold, (b) the job destruction condition  $S^e(z_t; \mathbf{s}_t) = z_t y_t - \kappa + G(\mathbf{s}_t) - U_t(\mathbf{s}_t) - b = 0$  determines  $\hat{z}(\mathbf{s}_t)$  and (c) these conditions are satisfied under the evolution of aggregate productivity  $y_t$  (specified below), unemployment (9) and the matching technology (2). Appendix B.1 gives more detailed description of the recursive equilibrium of the benchmark model and its solution algorithm.

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<sup>15</sup>The fact that the list includes the number of matches may seem odd. This is because new matches and existing pairs are severed at a different rate (existing pairs are destroyed at rate,  $\rho^{ex} + (1 - \rho^{ex})\rho_t^e$  while new matches are never severed before production).

## 2.4 Cyclical Behavior of Vacancies and Job Creation in the Benchmark Model

This subsection demonstrates the counterfactual properties of the benchmark model. The model is numerically solved under the following commonly-used aggregate productivity process:

$$\ln y_{t+1} = \xi \ln y_t + \varepsilon_{t+1} \quad (10)$$

where  $\varepsilon_t$  is taken to be independently and identically distributed (i.i.d.) normal with zero mean and standard deviation  $\sigma_\varepsilon$ . The idiosyncratic productivity shocks are assumed to be i.i.d. lognormal with mean zero, following den Haan et al. [19]. The existing literature enables me to parameterize the quarterly model fairly easily, so the discussion is put together in Tables 1 through 3.

Figure 2 presents the impulse responses to a one-standard deviation negative aggregate shock.<sup>16</sup> The upper right panel shows that vacancies respond to shocks immediately. As mentioned before, this immediate response comes from the free entry condition (Equation (5)), i.e, the negative shock lowers the expected returns for the firm with the constant posting cost ( $c^v$ ), and thus vacancies drop, eventually raising the matching probability up to the point where the free entry condition is restored. Furthermore, vacancies quickly move back to the pre-shock level because of the echo effect: as unemployment rises, the matching probability for the firm becomes higher, eliminating the firms' incentive to keep cutting vacancies any further. As the lower right panel of the figure illustrates, this behavior of vacancies initially dominates the behavior of job creation. But soon after the initial drop, job creation exhibits the strong echo effect, being greatly influenced by the behavior of unemployment. An important thing to note here is that the behavior of vacancies and job creation is consistent with the claim in the literature that higher unemployment caused by the adverse shock is mainly driven by higher job destruction.<sup>17</sup>

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<sup>16</sup>The second moment properties are examined in the later section in comparison to the extended model.

<sup>17</sup>See papers cited in Introduction.

### 3 The Extended Model

This section extends the benchmark model in the following three ways. The other basic environment is otherwise unchanged.

First, firms must engage in costly *planning* to introduce brand-new jobs into the economy. Vacancies for those jobs are posted only after completing a planning stage. In turn, new projects enter the planning stage if they are of sufficiently high quality. Second, firms with currently operating jobs are given an option to *mothball* their jobs after *temporary* job destruction. The firms that are mothballing their jobs can then reactivate them, by incurring one-time *retooling* costs. The mothballing is different from the planning in the sense that firms with preexisting jobs can choose to repost vacancies immediately if doing so is profitable. Finally, newly-formed meetings draw their initial idiosyncratic productivity level from stochastic productivity distribution, and decide whether to start producing or break up.

Having introduced the notion of the planning and mothballing, the model has a meaningful distinction between “job” and “match.” A “new job” means that it embodies the brand-new features, for example, reflecting newly discovered technologies or innovations. On the other hand, the notion of a “match” has nothing to do with the characteristic of the job, whether preexisting or new. Rather, it simply refers to an employment relationship between a worker and firm. Therefore, “new matches” do not necessarily mean that the relationships are associated with brand-new jobs. Similarly, “job creation” does not necessarily mean that the jobs created are new ones, but it simply means that new employment relationships are formed.

#### 3.1 Planning

Every period, a *fixed flow of new projects* is born into the economy.<sup>18</sup> Each project is endowed with a parameter  $\alpha$ , which represents the “quality” of the project in the sense that it gives the maximum number of jobs that can be created by the

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<sup>18</sup>The specification in this subsection is inspired by Fonseca et al. [20], but I modify it for convenience.

project.<sup>19</sup> The quality of projects is distributed as some known distribution function  $F(\alpha)$ , whose support is  $[0, A]$ . Note here that each project can create multiple jobs, so the notion of a project is distinguished from that of a job in this model. Letting  $P_t$  be a continuation value of a job in the planning pool in period  $t$ , the expected returns to each project is written as  $\alpha P_t$ . Similarly to Fonseca et al. [20], assume that starting a new project incurs fixed sunk costs  $K$ , called start-up costs. Given this structure, only the projects that satisfy the equality:

$$K \leq \alpha_t P_t,$$

are actually initiated. In other words, projects whose quality is better than a *reservation project quality*  $\hat{\alpha}_t$  deserve to be started. The projects disappear from the market if the costs  $K$  are not paid, i.e., those projects that are of poorer quality than the reservation quality are lost every period. The reservation quality satisfies:

$$K = \hat{\alpha}_t P_t. \tag{11}$$

If  $P_t \leq K/A$ , then  $\hat{\alpha}_t = A$ , meaning that there will be no entry in the period. The mean quality of the new projects conditional on acceptance is written as:

$$E(\alpha \mid \alpha \geq \hat{\alpha}_t) = \frac{\int_{\hat{\alpha}_t}^A \alpha dF(\alpha)}{1 - F(\hat{\alpha}_t)}.$$

Denoting the total mass of projects introduced each period as  $\eta$ , the entry of new jobs into the economy, denoted as  $\theta_t$ , is written as:

$$\theta_t = \eta [1 - F(\hat{\alpha}_t)] E(\alpha \mid \alpha \geq \hat{\alpha}_t) = \eta \int_{\hat{\alpha}_t}^A \alpha dF(\alpha). \tag{12}$$

Firms post vacancies after completing the planning process. It is also assumed that the transition from the planning stage to vacancy pool occurs with fixed probability  $\lambda^p$ .<sup>20</sup> The introduction of planning captures the idea that to initiate new projects

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<sup>19</sup>Since all jobs are ex ante identical in the model, projects are differentiated only by the maximum number of jobs that are created by each project.

<sup>20</sup>In the R&D literature, it is typically the case that the rate of innovation is proportional to the research effort (e.g. Aghion and Howitt [2], [3], Grossman and Helpman [23], [24] and Segerstrom, Anant and Dinopoulos [40] and Segerstrom [41]). As we abstract from the “planning effort”, the fixed probability seems to be the most uncontroversial specification.

is costly, for instance, because it requires research, internal approval, applying for licenses/permits, etc..

Here I show an important result that is exploited when the model is calibrated. Combining Equations (11) and (12), we have:

$$\theta_t = \eta \int_{K/P_t}^A \alpha dF(\alpha). \quad (13)$$

We then see from this that:

$$\frac{d\theta_t}{dP_t} = \eta \frac{K^2}{P_t^3} dF(K/P_t) > 0, \quad (14)$$

meaning that a higher (lower) value of  $P_t$  leads to more (less) entry. Given this result, I use the following reduced-form relationship between the entry ( $\theta_t$ ) and the value of  $P_t$  rather than fully utilizing the structure represented by Equation (11) and (12). Taking the first order approximation of Equation (13) around the steady-state gives:

$$\theta_t = \theta_s + \phi(P_t - P_s), \quad \phi > 0 \quad (15)$$

where the subscript  $s$  stands for the steady-state values of the corresponding variables, and  $\phi$  is the above derivative evaluated at the steady-state, namely:

$$\eta \frac{K^2}{P_s^3} dF(K/P_s). \quad (16)$$

As we will see later, using the reduced-form relationship dramatically simplifies the calibration process of the model.

The Bellman equation of a job under the planning process can be written as:

$$P_t = -c^p + \beta E_t [\lambda^p V_{t+1} + (1 - \lambda^p) P_{t+1}], \quad (17)$$

where  $c^p$  denotes a flow cost of planning. In writing the above equation, we implicitly assume that the parameter values of the model are such that the condition  $V_t > P_t > 0$  is satisfied for all possible states of the economy. This condition is maintained throughout this paper.

## 3.2 Mothballing

The firms with active jobs are allowed to make their jobs temporarily inactive, by mothballing the jobs with no cost. The firms can reactivate the temporarily inactive jobs by *retooling* their jobs. The mothballing/retooling decision is described as follows. At the beginning of each period, the firm in the mothballing stage draws an idiosyncratic retooling cost  $a_t c^r$ , where  $a_t$  is distributed as i.i.d. with mean one, so that  $E(a_t c_v) = c^r$ . The distribution of  $a_t$  is described by a cumulative distribution function  $M(a_t)$  whose support is taken to be  $[0, \infty)$ . The firm then decides whether to make the payment  $a_t c^r$  and retool the job, or to mothball the job another period, avoiding the payment. In contrast to the planning process, it is assumed that the payment of the retooling cost entitles the firm to repost its vacancy with certainty. The mothballing/retooling decision is thus based upon the cost  $a_t c^r$  and the surplus of posting a vacancy over the value of the mothballing. If the idiosyncratic cost drawn is low enough so that the equality:

$$V_t - Q_t \geq a_t c^r,$$

holds, then the firm retools the job and reposts a vacancy, where  $Q_t$  denotes the value of the job being mothballed. Note here that the left hand side is ensured to be positive and does not depend on  $a_t$ . Then, there exists a level  $\hat{a}_t$  such that  $V_t - Q_t \geq a_t c^r$  for  $a_t \leq \hat{a}_t$ , where reposting the vacancy is profitable, while  $V_t - Q_t < a_t c^r$  for  $a_t > \hat{a}_t$ , where the firm mothballs another period. The retooling/mothballing margin  $\hat{a}_t$  satisfies:

$$V_t - Q_t = \hat{a}_t c^r, \tag{18}$$

and the associated transition rate is written as:

$$\rho_t^q = \int_0^{\hat{a}_t} dM(a_t).$$

Finally, assuming that exogenous job destruction occurs at rate  $\rho^{qx}$  before the firm draws its cost, the Bellman equation for the mothballed job is written as:

$$Q_t = \beta(1 - \rho^{qx}) E_t \left[ \int_0^{\hat{a}_{t+1}} (V_{t+1} - a_{t+1} c^r) dM(a_{t+1}) + \int_{\hat{a}_{t+1}}^{\infty} Q_{t+1} dM(a_{t+1}) \right]. \tag{19}$$

### 3.3 Job Destruction

Endogenous job destruction is similar to the benchmark model except that the firm's outside option is not zero in the extended model. That is, the firm obtains  $Q_t$  after temporary job destruction. Taking this into consideration, the surplus ( $S_t^e$ ) for a matched pair is written as:

$$S_t^e = z_t y_t - \kappa + G_t - (U_t + b) - Q_t.$$

It is again assumed that the worker and firm bargain over this joint surplus and the negotiation is resolved according to the Nash bargaining solution, where the firm and worker take fixed proportion of  $S_t^e$ , given by  $\pi$  and  $1 - \pi$ , respectively. Recall here that each firm consists of multiple jobs in the extended model. But to avoid the complications caused by this specification, it is assumed that wage bargains take place at the individual level. That is, the firm engages in Nash bargains with each employee separately by taking the wages of all other employees as given.

Since the current-period return becomes lower as  $z_t$  declines, there exists a level  $\hat{z}_t^e$  such that  $S_t^e < 0$  for  $z_t < \hat{z}_t^e$ , where both parties agree to abandon their relationship, while  $S_t^e \geq 0$  for  $z_t \geq \hat{z}_t^e$ , where both parties agree to maintain their relationship, and engage in production in this period.  $\hat{z}_t^e$  is referred to as the job destruction margin. Associated with  $\hat{z}_t^e$  is the endogenous separation rate  $\rho_t^e$ :

$$\rho_t^e = \int_0^{\hat{z}_t^e} dH(z_t).$$

### 3.4 Job Rejection

The new employment relationships that meet in period  $t - 1$  also draw i.i.d. idiosyncratic productivity shocks at the beginning of period  $t$ . The aggregate productivity shock is also realized at the same time. The decision as to whether to start producing or not is again based on the surplus over the outside options for the worker and the firm, which are as above  $U_t + b$  and  $Q_t$ , respectively. Furthermore, this job rejection decision is differentiated from the job destruction decision in the sense that the firm must pay an additional job creation cost,  $\iota$ . This type of cost is often used in the literature and may be thought of as a training cost for a new worker. The

cost obviously makes the current-period returns lower, thus the job rejection rate higher.

We now define the surplus for a new relationship, and the associated job rejection margin:

$$S_t^n \equiv z_t y_t - \kappa - \iota + G_t - (U_t + b) - Q_t. \quad (20)$$

The negotiation of the division of the surplus is again resolved according to the Nash bargaining solution. Similarly to the case of job destruction, since the current-period payoff becomes low as  $z_t$  declines, there exists a level  $\hat{z}_t^n$  such that  $S_t^n < 0$  for  $z_t < \hat{z}_t^n$ , where both parties agree to reject the new relationship, while  $S_t^n \geq 0$  for  $z_t \geq \hat{z}_t^n$ , where both parties agree to create a new job, and engage in production in the period.  $\hat{z}_t^n$  is referred to as the *job rejection margin*. Associated with  $\hat{z}_t^n$  is the endogenous job rejection rate  $\rho_t^n$ :

$$\rho_t^n = \int_0^{\hat{z}_t^n} dH^n(z_t^n).$$

### 3.5 Interpretation

As mentioned at the beginning of this section, the firms have an option to mothball their jobs. Endogenous job separations thus do not completely destroy the jobs, but simply make them temporarily inactive. Importantly, the idea of the temporary job destruction is consistent with i.i.d. idiosyncratic productivity shocks: given the fact that i.i.d. shocks do not contain any information about the future development of jobs, it may be inappropriate to assume that idiosyncratic shocks induce permanent destruction of jobs. Instead, the model stands in the position where permanent destruction of jobs occurs due to the long-term reasons such as obsolescence of technology, and such permanent job destruction occurs at a constant rate.

### 3.6 Other Bellman Equations

Equilibrium values of  $U_t$ ,  $G_t$ , and  $V_t$  are determined as follows. First, a vacant job that has successfully completed the planning or mothballing processes searches for a worker in the matching market. The firm finds a worker with probability  $\lambda_t^f$ . If



the job is destroyed by an exogenous reason with probability  $\rho^{nx}$ , then the firm obtains zero return. If the job survives the exogenous destruction but is endogenously rejected, the firm goes to the mothballing stage, obtaining the continuation value  $Q_{t+1}$ . Finally, if the firm successfully accepts the worker, the firm receives a share  $\pi$  of total surplus  $S_{t+1}^n$  in addition to the outside option  $Q_{t+1}$ . Also, I assume zero vacancy-posting cost in the extended model.<sup>21</sup>

The Bellman value-transition equation for a vacant job is thus written as:

$$V_t = \beta E_t \left[ \lambda_t^f (1 - \rho^{nx}) \int_{z_{t+1}^n}^{\infty} \pi S_{t+1}^n dH(z_{t+1}) + \lambda_t^f (1 - \rho^{nx}) Q_{t+1} + (1 - \lambda_t^f) V_{t+1} \right]. \quad (21)$$

Consider next the expected joint returns of a worker-firm pair that produced in period  $t$ . For the worker, the separation, whether exogenous or endogenous, yields the value  $U_{t+1} + b$ . On the other hand, the firm obtains zero returns in the case of permanent destruction of the job (i.e., exogenous destruction) while it receives a continuation value  $Q_{t+1}$  in the case of temporary job destruction (i.e., endogenous destruction). Thus, the following Bellman value-transition equation holds:

$$G_t = \beta E_t \left[ (1 - \rho^{ex}) \int_{z_{t+1}^e}^{\infty} S_{t+1}^e dH(z_{t+1}) + U_{t+1} + b + (1 - \rho^{ex}) Q_{t+1} \right] \quad (22)$$

Finally, consider the situation facing an unemployed worker in period  $t$ . If the unemployed worker fails to match with the firm, then he again obtains  $U_{t+1} + b$  in period  $t + 1$ . Alternatively, if the worker finds the firm with probability  $\lambda_t^w$ , and survives the separation process at the beginning of period  $t + 1$ , then he engages in production and receives a share  $1 - \pi$  of total surplus  $S_{t+1}^n$  on top of  $U_{t+1} + b$ . The continuation value of the unemployed worker is thus written as:

$$U_t = \beta E_t \left[ \lambda_t^w (1 - \rho^{nx}) \int_{z_{t+1}^n}^{\infty} (1 - \pi) S_{t+1}^n dH(z_{t+1}) + U_{t+1} + b \right] \quad (23)$$

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<sup>21</sup>This assumption is made simply for convenience, and thus does not cause any important consequences.

### 3.7 Timing Summary and Evolution of State Variables

Figure 3 summarizes the structure of the extended model. The number of jobs in the planning pool ( $p_t$ ) evolves according to:

$$p_t = (1 - \lambda^p)p_{t-1} + \theta_t. \quad (24)$$

Note that as the first column of the figure illustrates, new entry in period  $t$  must spend at least one period in planning pool before moving on to vacancy pool, i.e., entry into planning pool comes after the outflow from planning pool into vacancy pool. This assumption implies that it takes at least two periods until new entrants in period  $t$  start producing.

Next, the law of motion for mothballed jobs ( $q_t$ ) is written as:

$$q_t = (1 - \rho_t^q)(1 - \rho^{qx})q_{t-1} + \rho_t^e(1 - \rho^{ex})(1 - u_{t-1}) + \rho_t^n(1 - \rho^{nx})m(u_{t-1}, v_{t-1}) \quad (25)$$

As in the case of planning process, outflow from mothballing pool into vacancy pool takes place before flows into the mothballing pool (from the pool of active jobs). One important difference from outflow from the planning pool, however, is that the transition rate from mothballing pool to vacancy pool,  $\rho_t^q$  can respond to an aggregate shock within the same period whereas the corresponding rate from the planning pool  $\lambda^p$  is treated as a fixed parameter in the model. A shock therefore will induce an immediate change of flows of previously mothballed jobs into vacancy pool. This can be seen from the below equation (26) for evolution of stock of vacancies. Notice further that this implies that job creation from mothballed jobs responds to the shock only with a one-period lag.

Given the flows into and out of vacancy pool, the law of motion for vacancies can be written as:

$$v_t = v_{t-1} + \lambda^p p_{t-1} + \rho_t^q(1 - \rho^{qx})q_{t-1} - m(u_{t-1}, v_{t-1}). \quad (26)$$

The second and third terms of the right-hand side are flows into vacancy pool from planning and mothballing pools, respectively, and the fourth term is the outflow from vacancy pool.

Finally, the evolution of unemployment is written as:

$$u_t = u_{t-1} + [\rho^{ex} + (1 - \rho^{ex})\rho_t^e](1 - u_{t-1}) - (1 - \rho^{xn})(1 - \rho_t^n)m(u_{t-1}, v_{t-1}). \quad (27)$$

The second term is flow into unemployment pool and the third term is flow out of unemployment pool. The only difference from equation (9) is that the last term takes into account the possibility of job rejection.

### 3.8 Recursive Equilibrium

The model's aggregate state is now of much greater dimensionality. It is characterized by aggregate productivity  $y_t$ , unemployment  $u_{t-1} - m_{t-1}$ , vacancies  $v_{t-1} - m_{t-1}$ , new meetings  $m_{t-1}$ , jobs in planning pool  $p_{t-1}$  and jobs being mothballed  $q_{t-1}$ .<sup>22</sup> These are equivalently summarized by a set of the following five variables:

$$\mathbf{s}_t = \{y_t, u_{t-1}, v_{t-1}, p_{t-1}, q_{t-1}\}$$

Definition of the model's equilibrium becomes more complicated than the preceding case. First, instead of vacancy-posting decision (Equation (5)), the model here features an entry decision into planning pool (Equation (13)). As mentioned before I use the "reduced form" specification that governs the entry of new jobs. Equation (13) is thus replaced by Equation (15). The other extensions of the benchmark model also include a non-trivial job rejection decision, and a vacancy-reposting (mothballing/retooling) decision (Equation (18)).

The recursive equilibrium is defined by a list of functions,  $P(\mathbf{s}_t)$ ,  $Q(\mathbf{s}_t)$ ,  $V(\mathbf{s}_t)$ ,  $G(\mathbf{s}_t)$ ,  $U(\mathbf{s}_t)$ ,  $\hat{z}^n(\mathbf{s}_t)$ ,  $\hat{z}^e(\mathbf{s}_t)$ ,  $\theta(\mathbf{s}_t)$  and  $\hat{a}(\mathbf{s}_t)$  such that (a) the Bellman equations (17), (19), (21), (22) and (23) hold; (b) the job rejection and destruction margins  $\hat{z}^e(\mathbf{s}_t)$  and  $\hat{z}^n(\mathbf{s}_t)$  are determined by  $\hat{z}^e y_t - \kappa + G(\mathbf{s}_t) - U(\mathbf{s}_t) - b - Q(\mathbf{s}_t) = 0$  and  $\hat{z}^n y_t - \kappa - \iota + G(\mathbf{s}_t) - U(\mathbf{s}_t) - b - Q(\mathbf{s}_t) = 0$ , respectively; (c) entry of new jobs into planning pool  $\theta(\mathbf{s}_t)$  is determined by Equation (15); (d) the mothballing/retooling margin  $\hat{a}(\mathbf{s}_t)$  is determined by Equation (18); (e) (a) through (d) are satisfied under evolution of aggregate productivity and other state variables, (10), (24), (25), (26) and (27). See Appendix B.2 for more detailed description of the recursive equilibrium of the extended model and its solution algorithm.

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<sup>22</sup>All of these variables should be thought of as their values at the beginning of period  $t$ .

## 4 Parameterization

I take the same aggregate productivity process and idiosyncratic productivity process as in the benchmark model. For the present model, I need to make additional assumptions about the distribution of the retooling cost,  $M(a)$ . It is again taken to be i.i.d. lognormal. Notice, however, that the reduced-form specification for entry of new jobs allows me to avoid making a detailed specification of the quality distribution  $F(\alpha)$ .

The symbols for the parameters and the corresponding concepts are put together in Table 4. The determination of those parameter values are summarized in Table 5. Table 6 presents the steady-state values in the model.

I start with the following steady-state relationships of job and employment flows (steady-state version of equations (24) through (27))

$$\lambda^p p_s = \theta_s, \quad (28)$$

$$[\rho^{qx} + \rho_s^q(1 - \rho^{qx})] q_s = \rho_s^e(1 - \rho^{ex})(1 - u_s) + \rho_s^n(1 - \rho^{nx})u_s\lambda_s^w, \quad (29)$$

$$\lambda^p p_s + \rho_s^q(1 - \rho^{qx})q_s = u_s\lambda_s^w, \quad (30)$$

$$[\rho^{ex} + (1 - \rho^{ex})\rho_s^e](1 - u_s) = (1 - \rho^{xn})(1 - \rho_s^n)u_s\lambda_s^w \quad (31)$$

where the subscript  $s$  denotes the steady state values of corresponding variables. Note also that  $m_s = u_s/\lambda_s^w$  is used in equations (29) through (31). Consider first equation (31). The main challenge here is to obtain the endogenous and exogenous job rejection rates,  $\rho_s^n$  and  $\rho^{xn}$ <sup>23</sup> Because no direct observations for these values are available for the U.S. labor market, I use one piece of evidence provided by Berman [5], in which he estimates the overall rejection rate conditional on meeting using the Israeli data. According to the study, the job acceptance rate fluctuates around 0.9 through 0.6 for the period of 1979 through 1990. Based on this, I have chosen the overall acceptance rate being 0.8:

$$\rho^{xn} + (1 - \rho^{xn})\rho_s^n = 0.2.$$

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<sup>23</sup>Since it is straightforward to obtain the steady-state unemployment, endogenous job destruction rate, and exogenous job destruction rate, I do not bother to discuss it here (see Table 5 and 6).

In order to break this down into exogenous and endogenous rejection rates, I impose the restriction that the proportion of endogenous rejection to exogenous rejection equals to the corresponding ratio of endogenous job destruction to exogenous job destruction:

$$\frac{\rho_s^n}{\rho^{xn}} = \frac{0.018}{0.038}.$$
<sup>24</sup>

This restriction together with the estimate of the overall acceptance rate enables me to obtain  $\rho^{xn} = 0.142$  and  $\rho_s^n = 0.067$ . Next, consider the transition probability  $\lambda^p$  from planning pool to the vacancy pool. One useful piece of evidence about the parameter is provided by Reynolds and White [37] who present detailed information regarding the start-up activities of new firms. According to the survey results in the book, it takes about a year or so to start up new firms. So  $\lambda^p = 0.25$  is selected as the estimate. In order to obtain the steady-state values for the number of jobs under the planning and mothballing processes, I impose the following two restrictions: (1) the exogenous job destruction of the mothballed jobs takes place at the same rate as the exogenous destruction of active jobs,  $\rho^q x = \rho^e x$ , and (2) the steady-state transition rate from mothballing pool to vacancy pool is set equal to the one from planning pool to vacancy pool,  $\lambda^p = \rho_s^q$ . These two restrictions in addition to the estimates obtained so far allow one to pin down  $p_s = 0.134$  and  $q_s = 0.068$  from equations (28) through (30). Given the estimates for  $p_s$  and  $\lambda^p$ , the entry of the new jobs, i.e.,  $\theta_s$  is identified as 0.045.

The AR(1) coefficient of aggregate productivity  $\xi$  is chosen to be 0.933 following Mortensen and Pissarides [30]. The standard deviation of the innovation  $\sigma_\varepsilon$  is selected so that the volatility of employment is close to the observed US data.<sup>25</sup> Importantly, the target level of the employment volatility is achieved using a smaller standard deviation than the one typically assumed in the RBC literature.<sup>26</sup>

The seven parameters,  $\kappa$ ,  $b$ ,  $i$ ,  $\pi$ ,  $\beta$ ,  $l$  and  $\sigma_z$  are set in a relatively straightforward way, so I skip the discussion (see Table 5 and 6).

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<sup>24</sup>The numbers 0.018 and 0.038 correspond to the rates of endogenous job destruction and exogenous job destruction, respectively.

<sup>25</sup>The same values for  $\xi$  and  $\sigma_\varepsilon$  are used for the benchmark model in order to make the comparison between the two models meaningful.

<sup>26</sup>In this paper,  $\sigma_\varepsilon = 0.0051$  is chosen whereas the RBC literature typically uses 0.007.

The parameters  $\phi$ ,  $\sigma_a$  and  $c^r$  play important roles in determining the behavior of vacancies. Apparently, the first parameter  $\phi$  governs the responsiveness of entry of brand-new jobs (see Equation 15), and thus vacancy-flow for the new jobs. The other two parameters  $\sigma_a$  and  $c^r$  are associated with the marginal condition of the vacancy-reposting decision (Equation (18)), and therefore control behavior of vacancy-flow of preexisting jobs. These three parameters are selected so that the former flows dominate the latter in the long-run, which is a key to generate persistent vacancies.<sup>27</sup> Recall here that the parameter  $\phi$  is a function of the underlying structural parameters. The present approach therefore allows me to sidestep the task of calibrating the underlying structural parameters. Given that it is difficult to pin down the flow of new projects using available data, the present approach has the advantage of making the calibration much more transparent.

Finally, the planning cost per period  $c^p$  is selected to ensure the condition that  $V_t > P_t > 0$  holds for all states, given the other parameter values.

## 5 Performance

The model is solved numerically with the similar method to the preceding model. Appendix B.2 gives a succinct description of the algorithm. Using the numerical solution, this section quantitatively evaluates the two models based on their second moment properties. I use the Conference Board's help-wanted index for the proxy for vacancies, which has been used by a number of authors.<sup>28</sup> On the other hand, the most widely-used job flow series are the ones constructed by Davis et al. [15]. The data set, however, covers only the manufacturing sector whereas the help-wanted index is available only for the aggregate U.S. economy. Therefore I also use the job flow series constructed by Ritter [38] (see also Ritter [39]) which cover

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<sup>27</sup>The meaning of this approach will be clearer in the next section where the performance of the model is examined.

<sup>28</sup>The series is the index of counts of help-wanted advertisements in 51 major newspapers in the U.S. Abraham [1] compares the index with actual vacancies in Minnesota where both series are available through two business cycles from 1972 to 1981, and has shown that the index closely track the actual vacancies.

non-manufacturing industries as well.<sup>29</sup> The sample periods are 1972.Q2 – 1993.Q4 for Davis et al.’s series and 1972.Q2 – 1993.Q2 for Ritter’s series.

The statistics of the model economies are based on 100 simulated samples. Each sample has the same number of periods (87) as Davis et al.’s [15] job flow series, where initial conditions are randomized by ignoring the first 200 observations. All the results below are based on HP filtered data.

## 5.1 Persistence

Table 7 shows the first order autocorrelation coefficients of variables of interest. The notable feature of the extended model is that it can successfully generate persistent vacancies, consistent with the data, while the benchmark model performs very poorly along this dimension. This can also be seen from the upper right panel of Figure 4, which presents the response of vacancies to a one-standard deviation negative aggregate shock. The figure displays a clear contrast with the response in the benchmark model.

Obviously, the persistence of vacancies is made possible by the planning lags. In order to clarify the mechanism behind the persistence of vacancies, it is helpful to see the responses of the two job flows into the vacancy pool from the planning and mothballing pools (see Figure 5). As expected, the flow from the planning stage responds to the shock only slowly, and is persistent. On the other hand, the flow of preexisting jobs into vacancy pool exhibits a considerably different pattern: on impact, there is a sharp drop of the flow caused by the lower transition rate  $\rho_t^j$ . But following the initial response, it quickly bounces back. This is because the adverse shock causes a spike in temporary job destruction, producing large flows into the mothballing stage. Given the different behavior of the two job flows, it is easy to see that in order for the model to yield persistent declines of job flows into vacancy pool, the former flow must dominate the latter flow in the long-run. Note, however, that the flow of preexisting jobs plays an important role in determining the short-run responses of vacancies and job creation.

Now, examine the persistence of job creation. As is seen from the impulse re-

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<sup>29</sup>Details of the Ritter’s series are documented in Appendix.

sponse (the lower right panel of Figure 2), job creation in the extended model is not very persistent even though vacancies become persistent. There are mainly two reasons for the result that job creation responds to the shock sharply in the short-run. The first reason is the decline in vacancies caused by the lower the transition rate from mothballing pool into vacancy pool. This further contributes to produce the sharp reduction in job creation. Secondly, the adverse shock immediately lowers the job acceptance rate, decreasing the job creation rate independently of behavior of vacancies and unemployment.<sup>30</sup> After the initial declines, job creation moves back to pre-shock level promptly. This is because increases in unemployment contribute to the recovery of job creation as in the case of the benchmark model. Observe, however, that the persistent declines in vacancies substantially reduce the extent of the echo effect compared to the benchmark model. The weaker echo effect has an important implication for the propagation mechanism, which we will see shortly.

Finally, consider persistence of job destruction. The clear message from the impulse responses and autocorrelations in the two models is that job destruction is less persistent in the extended model. Although we cannot draw a definitive conclusion as to which model is more consistent with the data along this dimension, the less persistent behavior of job destruction provides an interesting insight about the propagation mechanism of the extended model. The discussion regarding this point is also left for a later subsection (see subsection 5.6).

## 5.2 Beveridge Curve

Table 8 presents dynamic correlations between unemployment and vacancies. Not surprisingly from the patterns in impulse responses of unemployment and vacancies, the extended model performs well along this dimension: it successfully generates a realistic Beveridge curve, in contrast to the benchmark model which exhibits virtually no correlation between the two. Moreover, the extended model is also able to mimic the observed correlation pattern that vacancies tend to lead unemployment.

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<sup>30</sup>Notice that given the timing of events and our definition of job creation, a decline in the job acceptance rate shows up in the response of job creation in the same period, whereas changes in vacancies appear in job creation with a one-period lag.



This can also be observed in the impulse responses, showing that initial declines in vacancies is faster than initial increases in unemployment. Recall here the discussion in the previous subsection: the key feature behind this successful result is that the firms that are mothballing preexisting jobs can respond relatively quickly to aggregate shocks.

### **5.3 Correlations between Job Flows and Vacancies**

Tables 9 and 10 present correlations between vacancies and job flows. The third row of Table 9 confirms the prediction presented in Subsection 2.4, i.e., in the benchmark model, job creation and vacancies are highly positively correlated contemporaneously. Since there is no close link between job creation and vacancies in the extended model, the strong contemporaneous correlation is no longer observed. The observed data clearly more consistent with the extended model.

The dynamic correlations of vacancies with job destruction also highlight the realistic behavior of vacancies in the extended model: the extended model achieves good data match along this dimension, whereas the benchmark model appears grossly inconsistent with the data.

### **5.4 Correlations between Job Flows and Employment**

Tables 11 through 13 give dynamic relationships between job flows and employment, the statistics conventionally used in the literature for assessing the quantitative ability of the Mortensen and Pissarides model. Recall that the literature typically concludes that the model is able to deliver satisfactory quantitative results along these dimensions. It is therefore important to make sure that the extended model performs as well as the benchmark model.

The extended model actually does a better job than the benchmark model. In particular, the benchmark model has trouble generating the negative contemporaneous correlation between job destruction and job creation (see Table 13). With regard to dynamic correlations between job flows, my model gives a closer data match than the benchmark model. The relatively poor performance of the benchmark model

along this dimension is associated with fact that the echo effect in job creation is too strong.<sup>31</sup>

## 5.5 Volatilities

The upper panel of Table 14 compares standard deviations of job flows and the relative standard deviation of vacancies to that of unemployment. According to the table, job creation and vacancies in the extended model exhibit smaller variability than the observed series. Part of the smaller variability of vacancies is due to the properties of the matching function proposed by den Haan et al. [19]. Remember that the matching probabilities of the matching function take on values between 0 and 1 for all  $u_t \in [0, 1]$  and  $v_t \in [0, \infty)$ . This property is essential to avoid the truncation problem that the typical Cobb-Douglas matching function suffers from. But avoidance of the truncation problem necessarily implies variable elasticities of the matching function, and this property makes it impossible to obtain a large standard deviation of vacancies without deteriorating other dimensions. To see this, first note that the elasticities of the matching function are written as:

$$\varepsilon_{mv} \equiv \frac{\partial m}{\partial v} \frac{v}{m} = \frac{u^l}{u^l + v^l}, \quad \varepsilon_{mu} \equiv \frac{\partial m}{\partial u} \frac{u}{m} = \frac{v^l}{u^l + v^l}.$$

Evaluating these expressions at the steady-state values of unemployment and vacancies, we obtain  $\varepsilon_{mv} = 0.75$  and  $\varepsilon_{mu} = 0.25$ . Given that estimates of the elasticities typically used in the literature are around 0.5, our model gives much higher elasticity with respect to vacancies and much lower elasticity with respect to unemployment. Suppose that we were able to obtain enough variability of vacancies, say, roughly the same standard deviation as unemployment, without changing the elasticities. In this case, however, vacancies would have had a much larger impact on the number of meetings than unemployment does, because of the larger elasticity, deteriorating the cyclical properties of job creation.

The true problem of the extended model may lie in the low standard deviation of job creation. One possible explanation for this problem would be that fluctuations

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<sup>31</sup>In fact, den Haan et al. [19] point out that the echo effect in their model is too strong, although their model delivers the negative correlation.

of the job rejection (or acceptance) rate could be more important in reality than supposed in the model. Recall the assumption that new relationships and ongoing relationships draw idiosyncratic productivity shocks from the same distribution. But as Pissarides [35] writes, the distribution function in the two cases may not be the same in general. In particular, the assumption that new matches face idiosyncratic productivity shocks having a *smaller* variance may be an useful specification that generates a larger variability in the job rejection rate, and thereby in job creation.<sup>32</sup>

## 5.6 Propagation of Shocks

The last row of Table 14 shows the standard deviations of aggregate output in the two models.<sup>33</sup> Figure 6 compares impulse responses of aggregate output in the two models. They clearly illustrate a stronger propagation mechanism embedded in the extended model. The weaker echo effect caused by persistent behavior of vacancies is at the center of the propagation mechanism in our model. That is, after a recessionary shock, vacancies persistently decline in the extended model, pushing down job creation and thereby employment. In the benchmark model, however, the contribution from low job creation after the shock quickly dies out because of the very strong echo effect.

The implication of the persistent behavior of vacancies carries over to the behavior of job destruction also. That is, in the extended model, since both unemployment and vacancies are persistent, workers who separate from firms experience persistently lower job-finding probabilities in recessions. Figure 7, which compares the impulse responses of  $\lambda_t^w$  in the two models, clearly illustrates this point. This effect, in turn, creates the incentive for ongoing matched pairs to remain in their current employment relationships, making job destruction series in the extended model less persistent.<sup>34</sup>

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<sup>32</sup>Note that the smaller the variance is, the greater the mass of rejected workers is for the same change in the job rejection margin.

<sup>33</sup>Note again that I have chosen the standard deviation of the aggregate productivity innovation so that the extended model can match the standard deviation of employment. I use the same value for the benchmark model.

<sup>34</sup>As is mentioned in Introduction, my results in this subsection is not necessarily inconsistent

## 6 Conclusion

This paper points out the counterfactual properties of the widely-used model of equilibrium unemployment by Mortensen and Pissarides [30], and introduces sensible modifications to the model. In particular, the model developed in the paper features meaningful heterogeneity between new jobs and preexisting jobs: to develop new jobs requires costly planning, whereas firms with preexisting jobs are allowed to mothball their jobs, and to reactivate them with no planning lags. The calibration exercises have shown that these modifications dramatically improve the quantitative performance of the model. First, the planning lags serve to generate persistent behavior of vacancies, thereby generating an empirically plausible Beveridge curve. Second, the feature that the preexisting jobs can be reactivated relatively easily plays an important role in replicating the short-run behavior of vacancies and job creation. The first point also highlights an interesting insight about the model's propagation mechanism: the persistent declines of vacancies caused by an adverse shock put downward pressures on job creation, and thus employment, enhancing the propagation mechanism of our model.

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with the existing literature that emphasizes job destruction channel in propagating shocks.

# Appendix

## A Job Flow Series by Ritter

Ritter’s approach to measuring gross job creation and destruction is similar to Davis et al.’s approach, but he uses a breakdown of employment by industry taken from the BLS Current Employment Statistics (CES) survey. Using the finest possible industries as counting units, the empirical measures of the gross job creation and job destruction rates are defined analogously to Davis et al.’s measures:<sup>35</sup>

$$\begin{aligned} cre_t &= \frac{1}{E_t} \sum_{i=1}^{N_t} \delta_{it}^{(+)} \Delta E_{it}, \\ des_t &= \frac{1}{E_t} \sum_{i=1}^{N_t} \delta_{it}^{(-)} |\Delta E_{it}|. \end{aligned}$$

where  $\delta_{it}^{(+)} \left( \delta_{it}^{(-)} \right)$  is equal to 1 if employment is increasing (decreasing) in industry  $i$  and 0 otherwise,  $E_{it}$  denotes employment in industry  $i$  in period  $t$ ,  $N_t$  denotes the number of industries, and  $E_t$  denotes total employment in period  $t$ . As mentioned by Ritter [38], the most significant problem with this approach is the netting of job creation and destruction within industries: as a measurement unit gets larger, we are likely to measure net rather than gross flows. This netting problem, however, may not be severe when a detailed industry breakdown is available, as is the case for the Ritter’s series. For example, Haltiwanger and Schuh [26] construct a similar industry-based series of job creation and destruction rates for the U.S. manufacturing sector using the LRD (Longitudinal Research Database), which is the original data source of the Davis et al.’s plant level job flow data, and find that the plant-based and industry-based job flows exhibit strikingly similar cyclical patterns. Because of this evidence (though indirect), I use Ritter’s series as proxies of job creation and destruction for the aggregate U.S. economy.

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<sup>35</sup>The data are constructed using 573 industries in the nonfarm private industries since 1972.

## B Recursive Equilibrium and Solution Algorithm

This section gives the full description of recursive equilibria of the two models together with their solution algorithms. The solution algorithm of each model is a straightforward application of the method called Projection Parameterized Expectation Algorithm (PEA). The method is a variant of a projection method discussed by Judd [27] and [28]. See also Gasper and Judd [21] and Christiano and Fisher [10].

In what follows, I follow a convention of using an unprimed variable to denote a current-period value, and a primed variable to denote a next-period value.

### B.1 Benchmark Model

First recall that the current period state variables consist of  $\mathbf{s} = \{y, u, m\}$ . The recursive equilibrium is a list of functions  $G(\mathbf{s}), U(\mathbf{s}), v(\mathbf{s})$  and  $\hat{z}(\mathbf{s})$  such that (a) the Bellman equations:

$$G(\mathbf{s}) = \beta E \left[ (1 - \rho^x) \int_{\hat{z}'(\mathbf{s}')} S^e(z'; \mathbf{s}') dH(z') + U(\mathbf{s}') + b \mid y \right] \quad (\text{B.1.1})$$

$$U(\mathbf{s}) = \beta E [\lambda^w (1 - \pi) S^n(\mathbf{s}') + U(\mathbf{s}') + b \mid y] \quad (\text{B.1.2})$$

hold where  $S^e(z'; \mathbf{s}') = z'y' - \kappa + G(\mathbf{s}') - U(\mathbf{s}') - b$ ,  $S^n(\mathbf{s}') = \bar{z}y' - \kappa + G(\mathbf{s}') - U(\mathbf{s}') - b$ , and  $\lambda^w = m'/u'$ ; (b) the free entry condition:

$$\frac{c^v}{\beta\pi\lambda^f} = E [S^n(\mathbf{s}') \mid y] \quad (\text{B.1.3})$$

holds where  $\lambda^f = m'/v(\mathbf{s})$ ; (c) the job destruction condition  $\hat{z}y - \kappa + G(\mathbf{s}) - U(\mathbf{s}) - b = 0$  defines  $\hat{z}(\mathbf{s})$ ; (d) these conditions are satisfied under the evolution of aggregate productivity  $y$ , unemployment and matching technology:

$$\ln y' = \xi \ln y + \varepsilon' \quad (\text{B.1.4})$$

$$u' = u + [\rho^{ex} + (1 - \rho^{ex})\rho^e(\hat{z}(\mathbf{s}))](1 - u) - m \quad (\text{B.1.5})$$

$$m' = m(u', v(\mathbf{s})) \quad (\text{B.1.6})$$

The first step to numerically solve the model is to approximate the right hand side of Equations (B.1.1), (B.1.2), and (B.1.3) by a tensor product of second-degree Chebyshev polynomials of each state variable. Note that each function have

$3 \times 3 = 27$  unknown coefficients, so there are  $27 \times 3 = 81$  unknown coefficients in total. I use fixed-point iteration to solve for these coefficients. The iteration proceeds as follows. Using some initial guess for the 81 unknown coefficients, the current period job destruction margin can be computed by the job destruction condition, which also gives us the job destruction rate from  $\rho^e(\hat{z}(\mathbf{s})) = \int_{\hat{z}(\mathbf{s})}^{\infty} dH(z)$ . The integral to obtain the job destruction rate is computed by Simpson's rule with 15 nodes. We can then compute  $u'$  from the evolution of unemployment (B.1.5). Next, using the approximating function for  $E[S^n(\mathbf{s}') | y]$  in the free entry condition (B.1.3) allows one to obtain the equilibrium level of vacancies  $v(\mathbf{s})$ . The matching technology (B.1.6) then reveals the outcome of the matching market  $m'$  given the levels of vacancies and unemployment. Given the next-period values of state variables,  $u'$  and  $m'$ , and the distribution of the aggregate productivity shock  $\varepsilon'$ , we are able to actually compute the conditional expectations appeared on the right hand side of Equations (B.1.1), (B.1.2), and (B.1.3). The integral inside the bracket of (B.1.1) is again computed by Simpson's rule with 15 nodes. The conditional expectations associated with the aggregate productivity shock are numerically computed by Gauss-Hermite quadrature with 5 nodes. These conditional expectations are evaluated at twenty seven points that are chosen by finding three zeros of Chebyshev polynomial sequence for each state variable, and taking all possible combinations of the roots. The new set of coefficients of the approximating functions are obtained by equating the values of right hand side of Equations (B.1.1), (B.1.2) and (B.1.3) to values of approximating functions at twenty seven grid points. Since there are twenty seven coefficients in each approximating functions, this uniquely pins down the new set of coefficients.

<sup>36</sup> The iteration continues until convergence of the 81 Chebyshev coefficients is achieved.

## B.2 Extended Model

As mentioned in the main text, aggregate state of the model is summarize by a set of following five variables  $\mathbf{s} = \{y, u, v, p, q\}$ . The definition of the model's equilibrium becomes more complicated than the preceding case: the recursive equilibrium is a

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<sup>36</sup>This method is called collocation.

list of functions  $P(\mathbf{s}), Q(\mathbf{s}), V(\mathbf{s}), G(\mathbf{s}), U(\mathbf{s}), \hat{z}^n(\mathbf{s}), \hat{z}^e(\mathbf{s}), \theta(\mathbf{s})$  and  $\hat{a}(\mathbf{s})$  such that:

(a) Bellman equations

$$P(\mathbf{s}) = -c^p + \beta E[\lambda^p V(\mathbf{s}') + (1 - \lambda^p)P(\mathbf{s}') | y], \quad (\text{B.2.1})$$

$$Q(\mathbf{s}) = \beta(1 - \rho^{qx})E\left[\int_0^{\hat{a}(\mathbf{s}')} (V(\mathbf{s}') - a(\mathbf{s}')c^r)dM(a') + \int_{\hat{a}(\mathbf{s}')}^{\infty} Q(\mathbf{s}')dM(a') | y\right], \quad (\text{B.2.2})$$

$$V(\mathbf{s}) = \beta E\lambda^f(1 - \rho^{nx})\left[\int_{\hat{z}^n(\mathbf{s}')}^{\infty} \pi S^n(z'; \mathbf{s}')dH(z') + \lambda^f(1 - \rho^{nx})Q(\mathbf{s}') + (1 - \lambda^f)V(\mathbf{s}') | y\right] \quad (\text{B.2.3})$$

$$G(\mathbf{s}) = \beta E\left[(1 - \rho^{ex})\int_{\hat{z}^e(\mathbf{s}')}^{\infty} S^e(z'; \mathbf{s}')dH(z') + U(\mathbf{s}') + b + (1 - \rho^{ex})Q(\mathbf{s}') | y\right], \quad (\text{B.2.4})$$

$$U(\mathbf{s}') = \beta E\left[\lambda^w(1 - \rho^{nx})\int_{\hat{z}^n(\mathbf{s}')}^{\infty} (1 - \pi)S^n(z'; \mathbf{s}')dH(z') + U(\mathbf{s}') + b | y\right], \quad (\text{B.2.5})$$

hold where  $S^e(z'; \mathbf{s}') = z'y' - \kappa + G(\mathbf{s}') - U(\mathbf{s}') - b - Q(\mathbf{s}')$  and  $S^n(z'; \mathbf{s}') = z'y' - \kappa - \iota + G(\mathbf{s}') - U(\mathbf{s}') - b - Q(\mathbf{s}')$ ; (b) the job rejection and destruction margins  $\hat{z}^e(\mathbf{s})$  and  $\hat{z}^n(\mathbf{s})$  are determined by

$$\hat{z}^e y - \kappa + G(\mathbf{s}) - U(\mathbf{s}) - b - Q(\mathbf{s}) = 0, \quad (\text{B.2.6})$$

and

$$\hat{z}^n y - \kappa - \iota + G(\mathbf{s}) - U(\mathbf{s}) - b - Q(\mathbf{s}) = 0, \quad (\text{B.2.7})$$

respectively; (c) entry of new jobs  $\theta(\mathbf{s})$  into planning pool is determined by

$$\theta(\mathbf{s}) = \theta_s + \phi [P(\mathbf{s}) - P_s]; \quad (\text{B.2.8})$$

(d) the mothballing/retooling margin  $\hat{a}(\mathbf{s})$  is determined by

$$V(\mathbf{s}) - Q(\mathbf{s}) = \hat{a}(\mathbf{s})c^r; \quad (\text{B.2.9})$$



(e) these conditions are satisfied under the evolution of the state variables:

$$\ln y' = \xi \ln y + \varepsilon' \quad (\text{B.2.10})$$

$$p' = (1 - \lambda^p)p + \theta(\mathbf{s}), \quad (\text{B.2.11})$$

$$q' = (1 - \rho^q(\hat{a}(\mathbf{s}))(1 - \rho^{qx})q + \rho^e(\hat{z}^e(\mathbf{s}))(1 - \rho^{ex})(1 - u) + \rho^n(\hat{z}^n(\mathbf{s}))(1 - \rho^{nx})m(u, v), \quad (\text{B.2.12})$$

$$v' = v + \lambda^p p + \rho^q(\hat{a}(\mathbf{s}))(1 - \rho^{qx})q' - m(u, v), \quad (\text{B.2.13})$$

$$u' = u + [\rho^{ex} + (1 - \rho^{ex})\rho^e(\hat{z}^e(\mathbf{s}))](1 - u) - (1 - \rho^{xn})[1 - \rho^n(\hat{z}^n(\mathbf{s}))]m(u, v). \quad (\text{B.2.14})$$

In the extended model, I solve for the five functions,  $P(\mathbf{s})$ ,  $Q(\mathbf{s})$ ,  $V(\mathbf{s})$ ,  $G(\mathbf{s})$  and  $U(\mathbf{s})$  above. The problem here is that given that  $\mathbf{s}$  contains five variables, it is computationally very expensive to use tensor product methods.<sup>37</sup> Under this type of situation, Judd [27] suggests to use a complete set of polynomials instead of tensor product. He shows that the complete polynomials give us nearly as good an approximation as the tensor product with far fewer elements. More specifically, I first form five-fold tensor product bases of second-degree Chebyshev polynomials of each state variables. This set includes  $3^5 = 243$  bases. The higher terms than third degree are then dropped. This reduces the number of bases to 51. The iteration of determining the unknown coefficients ( $51 * 5 = 255$ ) proceed as follows. Using some initial guess for the 255 unknown coefficients, we obtain the next-period values of state variables in the following way. First, the current period job destruction and job rejection margins can be computed by the conditions, (B.2.6) and (B.2.7) respectively. We can numerically compute the rates of job destruction and rejection,  $\rho^e(\hat{z}^e(\mathbf{s}))$  and  $\rho^n(\hat{z}^n(\mathbf{s}))$ , by using Simpson's rule as before, which further allows one to obtain  $u'$  from the law of motion for unemployment (B.2.14). Next, entry into planning pool  $\theta(\mathbf{s})$  can be computed from (B.2.8). Using this in (B.2.11) yields  $p'$ . Finally, the marginal condition (B.2.9) gives us the mothballing/retooling margin  $\hat{a}(\mathbf{s})$ ,

<sup>37</sup>For example, if I were to use a tensor product of second-order Chebyshev polynomial bases, each approximating function would include  $3^5 = 243$  unknown coefficients, so in total  $243 * 5 = 1215$  unknown coefficients would have to be determined.

from which I numerically compute the transition rate  $\rho^q(\hat{a}(\mathbf{s}))$  (again by Simpson's rule). Using the transition rate  $\rho^q(\hat{a}(\mathbf{s}))$  together with the rates of job destruction and rejection,  $\rho^e(\hat{z}^e(\mathbf{s}))$  and  $\rho^n(\hat{z}^n(\mathbf{s}))$  in Equations (B.2.12) and (B.2.13) makes it possible to compute  $q'$  and  $v'$ . Given the next-period values of the state variables,  $p', q', v'$  and  $u'$ , and the distribution of the aggregate productivity shock  $\varepsilon'$ , we can actually compute the right hand side of Equations (B.2.1), (B.2.2), (B.2.3), (B.2.4), and (B.2.5). Here the integrals associated with idiosyncratic productivity and an idiosyncratic retooling cost are computed by Simpson's rule. The conditional expectations associated with the aggregate productivity shock by Gauss-Hermite quadrature with 5 nodes. The conditional expectations are evaluated at 243 grid points. They are as usual chosen by finding three zeros of Chebyshev polynomial sequence of each state variable, and taking all possible combinations of the roots. The difference from the preceding case arise here because the number of unknown coefficients (51 for each approximating function) is smaller than the number of the grid points (243). The new set of coefficients of the approximating functions are thus obtained by least squares, i.e., regressing the computed values of the right hand side of (B.2.1), (B.2.2), (B.2.3), (B.2.4), and (B.2.5) on the values of 51 complete polynomial bases at the 243 grid points. The iteration continues until convergence of the unknown coefficients is achieved.

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Table 1: **Parameters in the benchmark model**

Symbol	Concept
$\xi$	$AR(1)$ coefficient of the aggregate productivity process
$\sigma_\varepsilon$	s.d. of the innovation of the aggregate productivity process $\varepsilon_t$
$\kappa$	operating cost for production
$b$	unemployment benefit
$\sigma_z$	s.d. of the idiosyncratic productivity shock for ongoing pairs
$\rho^{ex}$	exogenous job destruction rate.
$\pi$	bargaining power of the firm
$l$	parameter in the matching function proposed by den Haan et al. [19]
$\beta$	discount factor
$c^v$	vacancy posting cost per period

Table 2: **Parameter values in the benchmark model**

Values	Description
$\xi = 0.933$	Follow Mortensen and Pissarides [30].
$\sigma_\varepsilon = 0.0051$	Set at the same value as the one for the extended model (see Section 4).
$\kappa + b = 0.921$	Assigned so that the model yields the target rates of endogenous job destruction rate
$\sigma_z = 0.25$	Selected to match the standard deviation of job destruction.
$\pi = 0.5$	No evidence. Commonly used in the literature.
$\rho^{ex} = 0.038$	Computed from the overall job destruction rate reported by Davis et al. [15] and the endogenous job destruction rate (see the following table)
$l = 1.29$	Follow den Haan et al. [19].
$\beta = 0.99$	Implies a real interest rate of 1% per quarter.
$c^v = 0.14$	Implied value in the steady-state version of the model given the other parameter values

Table 3: **Steady state values in the benchmark model**

Symbol	Concept	Value	Source
$\lambda^f$	matching probability for firms	0.71	den Haan et al. [19]
$\lambda^w$	matching probability for workers	0.45	den Haan et al. [19]
$\rho^e$	endogenous job destruction rate	0.018	Hall [25].
$u$	unemployment rate	0.11	Blanchard and Diamond [7]



Table 4: **Parameters in the extended model**

Symbol	Concept
$\xi$	$AR(1)$ coefficient of the aggregate productivity process
$\sigma_\varepsilon$	standard deviation of aggregate productivity innovation $\varepsilon_t$
$\kappa$	operating cost for production
$b$	unemployment benefit
$i$	job creation costs
$\pi$	bargaining power of the firm
$\beta$	discount factor
$l$	parameter in the matching function proposed by den Haan et al. [19]
$\sigma_z$	standard deviation of the idiosyncratic productivity shock
$\rho^{ex}$	exogenous job destruction rate for operating jobs
$\rho^{nx}$	exogenous job rejection rate
$\rho^q$	exogenous job destruction rate for mothballed jobs
$\lambda^p$	transition probability from planning stage to vacancy pool
$c^p$	planning costs per period
$c^r$	mean of the retooling costs
$\sigma_a$	standard deviation of the distribution of retooling costs $M(a)$

Table 5: **Parameter values in the extended model**

Values	Description
$\beta = 0.99$	Implies a real interest rate of 1% per quarter.
$\pi = 0.5$	No available evidence. Commonly used value in the literature.
$\xi = 0.933$	Follow Mortensen and Pissarides [30].
$\sigma_\varepsilon = 0.0051$	Selected to match the standard deviation of employment.
$\kappa + b = 0.950$ and $i = 0.06$	Assigned so that the model yields the target rates of endogenous job destruction and rejection rates
$\sigma_z = 0.13$	Selected to match the standard deviation of the job destruction rate.
$l = 2.41$	Implied by the target steady-state meeting probabilities.
$\lambda^p = 0.25$	Based on the survey results by Reynolds and White [37]
$\rho^{ex} = 0.038$	Overall job destruction rate reported by Davis et al. [15] and the endogenous job destruction rate.
$\rho^{ex} = 0.142$	Berman [5] and a restriction mentioned in the text
$\phi = 4.55, c^r = 0.09$ $\sigma_a = 0.23$	Selected to match persistence of vacancies
$c^p = 0.005$	Selected to ensure $V_t > P_t > 0$ .

Table 6: **Steady state values in the extended model**

Symbol	Concept	Values	Source
$\lambda_s^f$	meeting probability for firms	0.89	den Haan et al. [19], Berman [5]
$\lambda_s^w$	meeting probability for workers	0.56	den Haan et al. [19], Berman [5]
$\rho_s^e$	endogenous job destruction rate	0.018	Hall [25]
$\rho_s^n$	endogenous job rejection rate	0.067	Berman [5] and a restriction mentioned in the text.
$p_s$	jobs in the planning stage	0.18	See the text.
$q_s$	mothballed jobs	0.07	See the text.
$\lambda_s^q$	transition rate of mothballed jobs	0.25	Restriction.
$u_s$	unemployment rate	0.11	Blanchard and Diamond [7]

Table 7: **Autocorrelations**

	U.S. data		Model	
	DHS	Ritter	Benchmark	Extended
$Corr(cre_t, cre_{t-1})$	0.42 (0.131)	0.55 (0.107)	0.61	0.68
$Corr(des_t, des_{t-1})$	0.64 (0.063)	0.56 (0.071)	0.66	0.55
$Corr(n_t, n_{t-1})$	0.91 (0.016)		0.89	0.92
$Corr(u_t, u_{t-1})$	0.91 (0.024)		0.89	0.91
$Corr(v_t, v_{t-1})$	0.92 (0.017)		0.35	0.80

All series are logged and HP filtered. Standard errors presented in parenthesis are computed by the den Haan and Levin's [18] GMM-VARHAC procedure. DHS denotes the results using the series constructed by Davis, Haltiwanger and Schuh [16]. Ritter denotes the results using the series constructed by Ritter [38].  $n = LHEM/PO16$ ,  $u = LHUR \cdot LHPAR/10000$ ,  $v = LHELX \cdot LHUR \cdot LHPAR/10000$  where  $LHEM$  : total employment,  $PO16$  : population,  $LHUR$  : unemployment rate,  $LHPAR$  : labor force participation rate,  $LHELX$  : help-wanted ads as percentage of unemployed. These data are taken from DRI-Webstract (former CITIBASE). Sample period for DHS series is 1972Q2-1993Q4. Sample period for other series is 1972Q1-1993Q1.

Table 8: **Beveridge curve**

$Corr(v_{t+k}, u_t)$	-3	-2	-1	0	1	2	3
U.S. data	-0.60 (0.068)	-0.80 (0.054)	-0.94 (0.014)	-0.95 (0.014)	-0.81 (0.046)	-0.59 (0.058)	-0.35 (0.109)
Benchmark model	-0.43	-0.43	-0.30	0.10	0.45	0.52	0.48
Extended model	-0.55	-0.71	-0.79	-0.74	-0.54	-0.33	-0.14

All series are logged and HP filtered. Standard errors presented in parenthesis are computed by the den Haan and Levin's [18] GMM-VARHAC procedure.  $u = LHUR \cdot LHPAR/10000$ ,  $v = LHELX \cdot LHUR \cdot LHPAR/10000$ , where  $LHUR$  : unemployment rate,  $LHPAR$  : labor force participation rate,  $LHELX$  : help-wanted ads as percentage of unemployed. These data are taken from DRI-Webstract (former CITIBASE). Sample period is 1972Q1-1993Q1.

Table 9: **Dynamic correlations between job creation and vacancies**

$Corr(cre_{t+k}, v_t)$	-3	-2	-1	0	1	2	3
U.S. data	0.62 (0.101)	0.59 (0.083)	0.50 (0.094)	0.26 (0.116)	-0.06 (0.120)	-0.32 (0.126)	-0.41 (0.130)
Benchmark model	0.25	0.36	0.53	0.76	0.05	-0.25	-0.34
Extended model	0.12	0.20	0.30	0.41	0.39	0.08	-0.14

All series are logged and HP filtered. Standard errors presented in parenthesis are computed by the den Haan and Levin's [18] GMM-VARHAC procedure. Job creation series is constructed by Ritter [38].  $v = LHELX \cdot LHUR \cdot LHPAR/10000$  where  $LHELX$ : help-wanted ads as percentage of unemployed,  $LHUR$ : unemployment rate,  $LHPAR$ : labor force participation rate. These data are taken from DRI-Webstract (former CITIBASE). Sample period for is 1972Q1-1993Q1.

Table 10: **Dynamic Correlations between job destruction and vacancies**

$Corr(des_{t+k}, v_t)$	-3	-2	-1	0	1	2	3
U.S. data	-0.60 (0.099)	-0.59 (0.083)	-0.53 (0.068)	-0.32 (0.077)	0.01 (0.115)	0.28 (0.101)	0.41 (0.097)
Benchmark model	0.46	0.40	0.14	-0.57	-0.47	-0.37	-0.29
Extended model	-0.41	-0.50	-0.56	-0.56	-0.08	0.19	0.36

All series are logged and HP filtered. Standard errors presented in parenthesis are computed by the den Haan and Levin's [18] GMM-VARHAC procedure. Job destruction series is constructed by Ritter [38].  $v = LHELX \cdot LHUR \cdot LHPAR/10000$  where  $LHELX$ : help-wanted ads as percentage of unemployed,  $LHUR$ : unemployment rate,  $LHPAR$ : labor force participation rate. These data are taken from DRI-Webstract (former CITIBASE). Sample period for is 1972Q1-1993Q1.

Table 11: **Dynamic Correlations between job creation and employment**

$Corr(cre_{t+k}, n_t)$	-3	-2	-1	0	1	2	3
U.S. data (DHS)	0.20 (0.096)	0.07 (0.155)	-0.10 (0.166)	-0.40 (0.103)	-0.64 (0.081)	-0.64 (0.060)	-0.57 (0.046)
U.S. data (Ritter)	0.54 (0.130)	0.45 (0.095)	0.30 (0.113)	0.02 (0.115)	-0.29 (0.113)	-0.47 (0.117)	-0.54 (0.109)
Benchmark model	0.06	-0.11	-0.36	-0.71	-0.89	-0.78	-0.58
Extended model	0.46	0.46	0.38	0.19	-0.13	-0.40	-0.52

All series are logged and HP filtered. Standard errors presented in parenthesis are computed by the den Haan and Levin's [18] GMM-VARHAC procedure. DHS: job creation series is constructed by Davis, Haltiwanger and Schuh [16], and corresponding employment series ( $n$ ) is obtained by  $LPM6/PO16$  where  $LPM6$  : employees in manufacturing sector,  $PO16$  : population. The data are taken from DRI-Webstract (former CITIBASE). Sample period is 1972Q2-1993Q4. Ritter: job creation series and corresponding employment series ( $n$ ) are constructed by Ritter [38]. Sample period is 1972Q1-1993Q1.

Table 12: **Dynamic Correlations between job destruction and employment**

$Corr(des_{t+k}, n_t)$	-3	-2	-1	0	1	2	3
U.S. data (DHS)	-0.65 (0.077)	-0.64 (0.077)	-0.47 (0.102)	-0.12 (0.075)	0.18 (0.098)	0.40 (0.103)	0.51 (0.098)
U.S. data (Ritter)	-0.54 (0.121)	-0.49 (0.085)	-0.36 (0.082)	-0.10 (0.114)	0.24 (0.094)	0.45 (0.082)	0.55 (0.087)
Benchmark model	-0.59	-0.80	-0.92	-0.75	-0.43	-0.18	0.01
Extended model	-0.56	-0.54	-0.43	-0.18	0.20	0.40	0.48

All series are logged and HP filtered. Standard errors presented in parenthesis are computed by the den Haan and Levin's [18] GMM-VARHAC procedure. DHS: job destruction series is constructed by Davis, Haltiwanger and Schuh [16] and corresponding employment series ( $n$ ) is obtained by  $LPM6/PO16$  where  $LPM6$  : employees in manufacturing sector,  $PO16$  : population. The data are taken from DRI-Webstract (former CITIBASE). Sample period is 1972Q2-1993Q4. Ritter: job destruction series and corresponding employment series ( $n$ ) are constructed by Ritter [38]. Sample period is 1972Q1-1993Q1.

Table 13: **Dynamic Correlations between job flows**

$Corr(cre_{t+k}, des_t)$	-3	-2	-1	0	1	2	3
U.S. data (DHS)	-0.38 (0.084)	-0.45 (0.092)	-0.55 (0.095)	-0.46 (0.140)	-0.11 (0.111)	0.19 (0.074)	0.32 (0.079)
U.S. data (Ritter)	-0.24 (0.123)	-0.32 (0.164)	-0.61 (0.099)	-0.83 (0.054)	-0.45 (0.095)	-0.19 (0.069)	-0.04 (0.063)
Benchmark model	-0.20	-0.13	-0.04	0.09	0.70	0.80	0.70
Extended model	-0.01	-0.13	-0.32	-0.58	-0.63	-0.30	-0.07

All series are logged and HP filtered. Standard errors presented in parenthesis are computed by the den Haan and Levin's [18] GMM-VARHAC procedure. DHS: job flows series are constructed by Davis, Haltiwanger and Schuh [16]. Sample period is 1972Q2-1993Q4. Ritter: job flow series are constructed by Ritter [38]. Sample period is 1972Q1-1993Q1.

Table 14: **Standard Deviations**

	U.S. data	Model	
		Benchmark	Extended
$\sigma_{cre}/\sigma_n$	4.40 (0.022)	5.77	2.84
$\sigma_{des}/\sigma_n$	6.63 (0.012)	5.22	4.57
$\sigma_v/\sigma_u$	1.26 (0.052)	0.79	0.71
$\sigma_n/\sigma_Y$	0.63 (0.002)	0.53	0.69
$\sigma_Y$	1.85 (0.081)	1.20	1.72

All series are logged and HP filtered. Standard errors presented in parenthesis are computed by the den Haan and Levin's [18] GMM-VARHAC procedure. First two rows: job creation and job destruction series are constructed by Davis, Haltiwanger and Schuh [16] and corresponding employment series is obtained by  $LPM6/PO16$  where  $LPM6$ : employees in manufacturing sector,  $PO16$ : population. The data are taken from DRI-Webstract (former CITIBASE). Sample period is 1972Q2-1993Q4. Remaining rows:  $u = LHUR \cdot LHPAR/10000$ ,  $v = LHELX \cdot LHUR \cdot LHPAR/10000$ ,  $Y = GDPQ/POP16$  where  $LHUR$ : unemployment rate,  $LHPAR$ : labor force participation rate,  $LHELX$ : help-wanted ads as percentage of unemployed,  $GDPQ$ : real GDP,  $POP16$ : population. The data are taken from DRI-Webstract. Sample period is 1972Q1-1993Q1.

Figure 1: **Timing and event summary of the benchmark model**

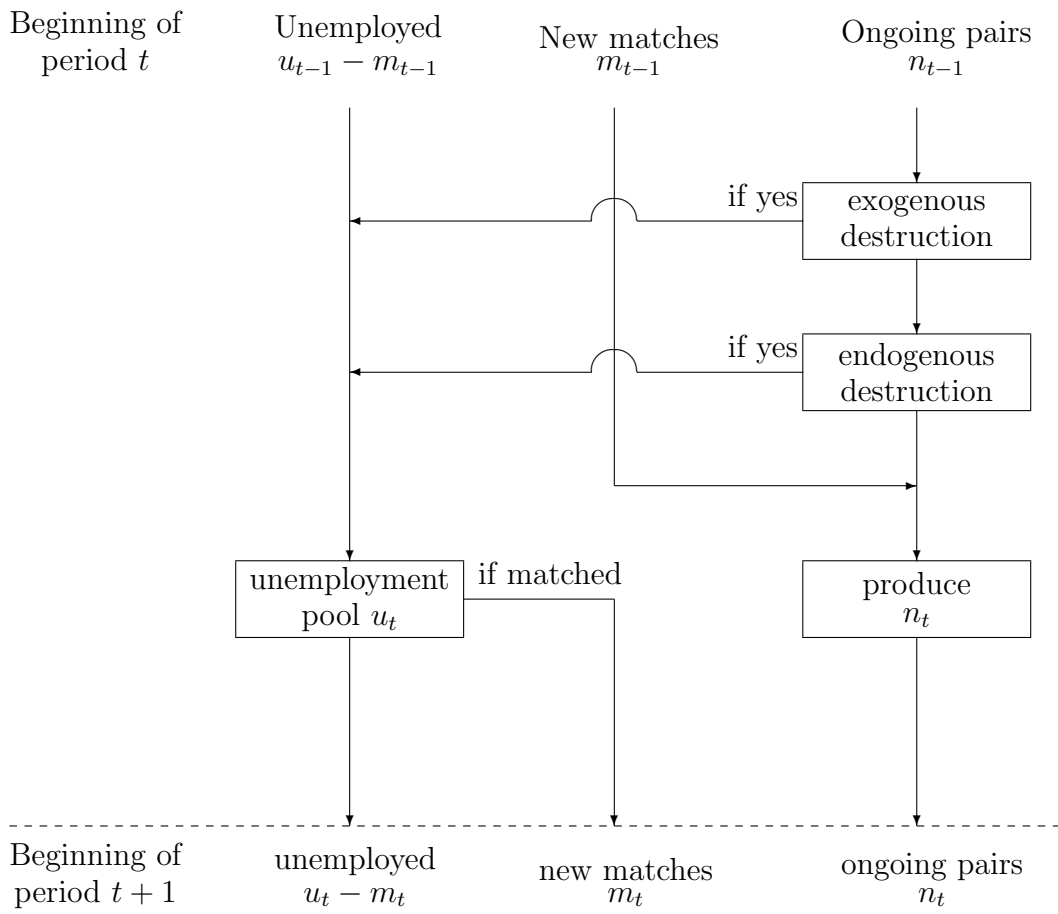




Figure 2: **Impulse responses of the benchmark model** (one standard deviation negative aggregate shock)

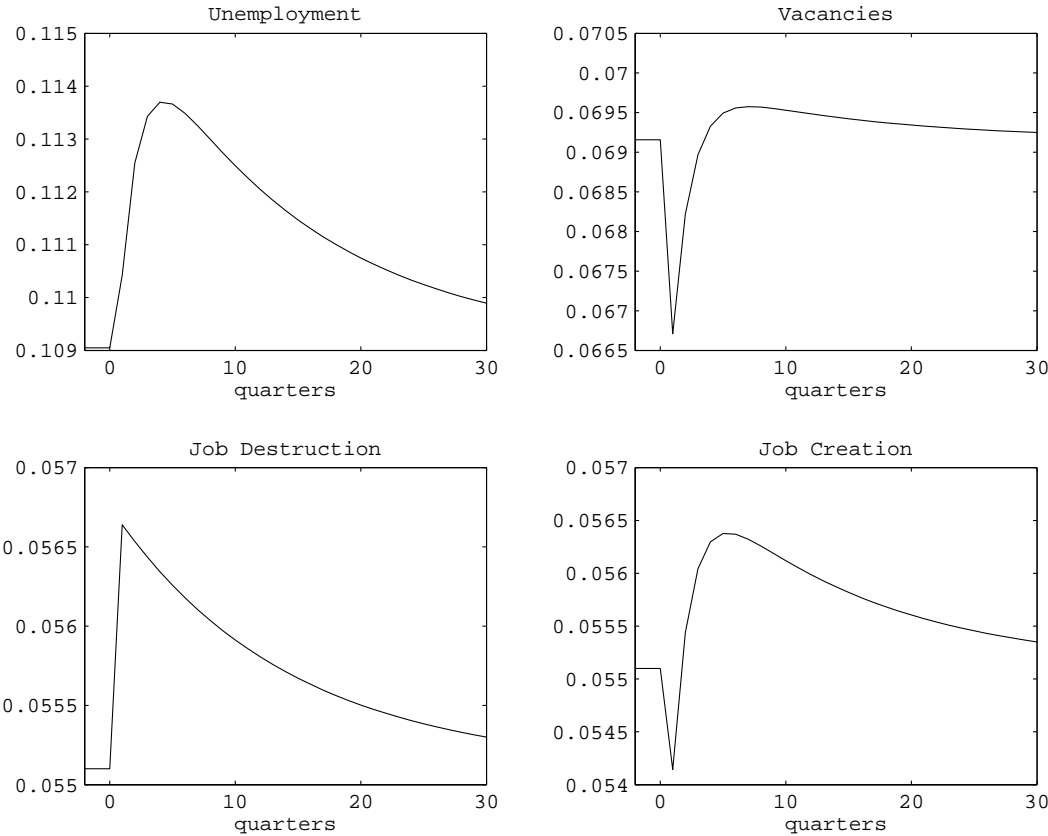




Figure 4: **Impulse responses of the extended model** (one standard deviation negative aggregate shock)

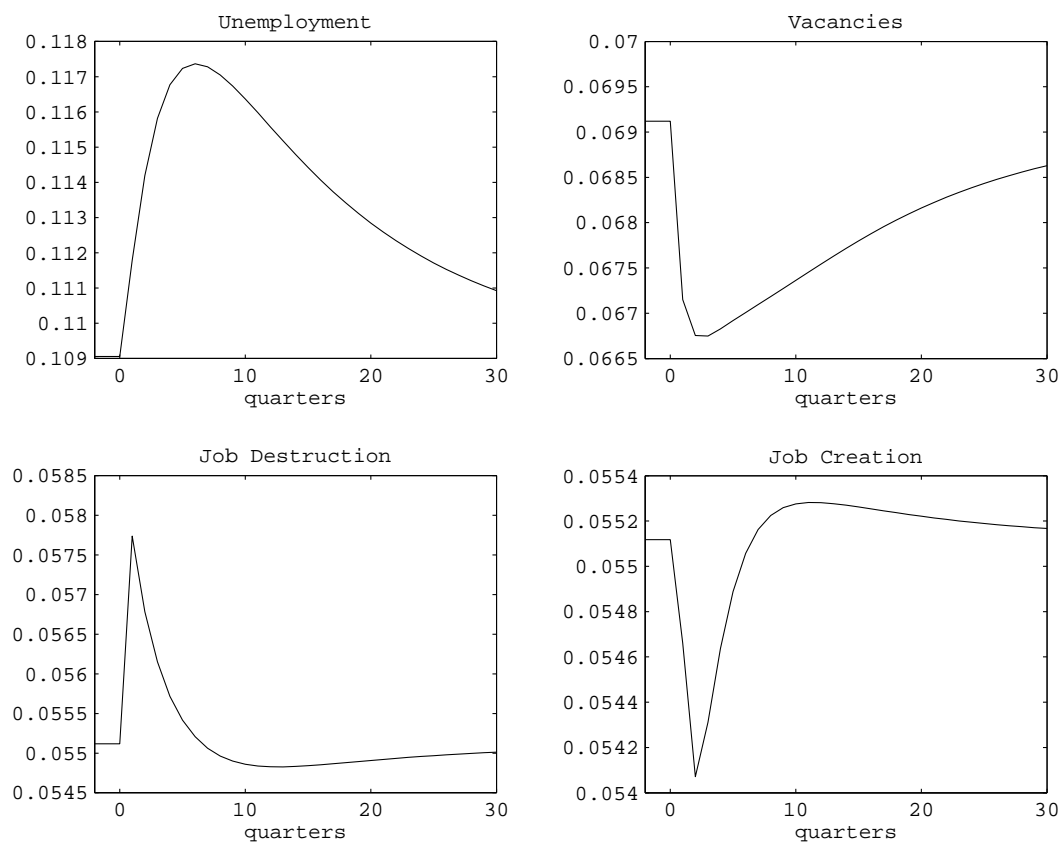


Figure 5: Flows into vacancy pool

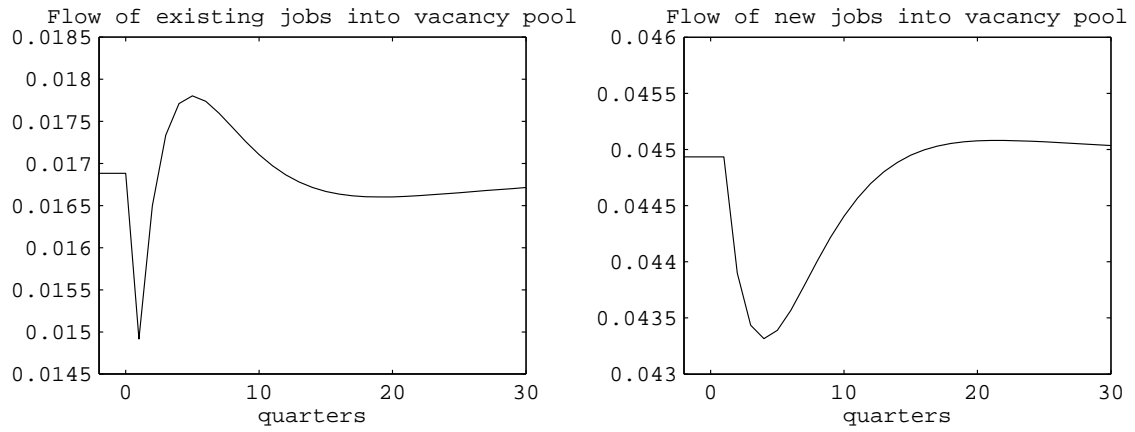


Figure 6: Impulse responses of aggregate output

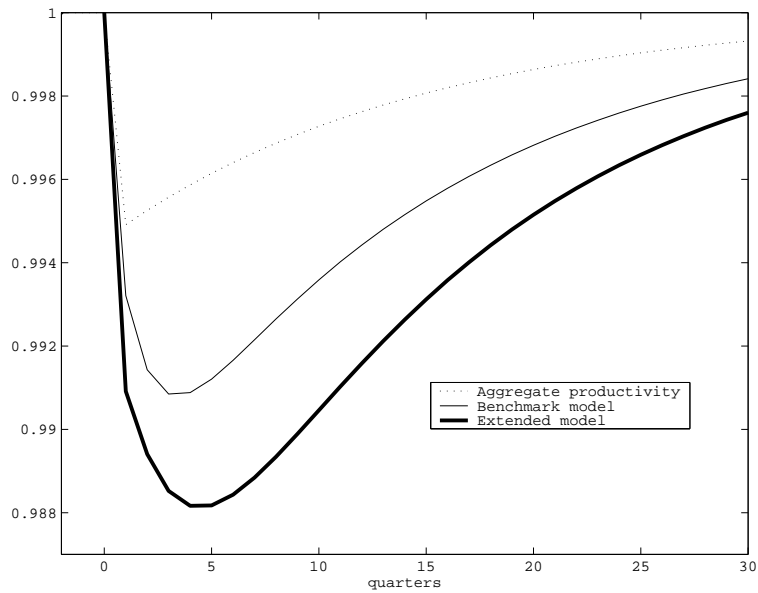


Figure 7: Impulse responses of  $\lambda_t^w$

