Clues about how Institutions Optimize: Market and Product Differentiation*

Nicolás Garrido†
University of Alcalá
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Abstract

Frequently in the economic literature models have been cast has optimization problems where the method to solve it has been algorithms that have little or nothing to do with economics. In this paper, an institution is studied as an optimization algorithm where the rules of the institutions are the steps that the algorithm follows in order to find out a solution. Thus, as an example the problem of product differentiation is faced and a particular market institution is studied in this way. Conclusions about how effective is the market at finding out the optimum and how it distributes the surplus offered by a good are presented.

1 Introduction

Many problems in economic literature are cast as optimization ones over a particular cost function. Broadly speaking the method to solve the problem follows two common approaches; the analytical in which success depend on a carefully structured model based on understanding and creativity of the modeler and the brute-force processing in which iteratively an algorithm goes through the space of solution driven by the measure obtained from the cost function.

In any case the cost function represents one key aspect about the complexity of the problem[1]. Macready and Wolpert (1997) [3] proved that any optimization algorithm performs only as well as the knowledge concerning the cost function is incorporated into the algorithm. In other words, if there is no information about the cost function, then the expected performance of all algorithms on that function is the same.

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†Contact Information: nicogarrido@hotmail.com

[1] Along this note, the complexity of the problem is assumed to be described as a landscape (see [9]) which will be explained latter on.

The common strategy in economic literature to deal with this problem has been to construct models based on parsimonious cost functions (i.e. convexities, decreasing returns and linear approximation). The optimizer over these parsimonious spaces are agents, broadly speaking, economic institutions that use strongly mathematical rooted method of optimization to solve the problem.

Moreover in any case it is not possible to guarantee that an algorithm will obtain the appropriate optimal solution even with relative good knowledge about the configuration space[2]. Indeed, undecidability and computational intractability are frequent in the literature of computation theory (see [10]) and in most of the cases rooted in the halting problem.

This paper attempts to provide a different (or complementary) approach to the parsimonious modeling world and to the non always predictable success of optimization algorithms.

Is possible to think that the society is organized through institutions that produce solutions –or attempt to optimize the solution– to frequent social problems. Indeed, in a broad sense, along this paper it is assumed that a *landscape* is a good description of a the possible solutions to a specific social problem and an institution is an algorithm that attempt to find out the best solution over such a landscape which ex-ante is unknown for the members of the society.

Although the members of the society do not have full knowledge about the whole landscape, the interaction among the agents produces the emergence of institutions that help the society to be better off (i.e. obtain higher welfare). Particularly markets emerge as institutions which contribute –following the mathematics analogies– to optimize an implicit cost function related with resource production and allocation.

This vision, allows to propose a methodological approach in the sense of the idea exposed by Roth (2002)[6] where economists are called not only to analyze markets, but also to design them.

Dealing with the problem of product differentiation, in this paper it is assumed that there exist two landscapes describing both, the unitary cost of production for the firms and the willingness to pay for consumers from a potential set of goods that could be produced and commercialized in the economy composed by the firms and consumers. Assuming that these landscapes are static, the properties of allocation of an economic institution, particularly in this case a market, will be studied. The main concern will be to describe how this institution (i.e. algorithm) works and what type of solution is obtained as consequence of its application.

The market is considered here as an algorithm that collects –coordinate– the behavior of two rationally bounded type of agents (i.e. firms and consumers) and produces as result a specific good allocation. Thus, in this paper the behavior of the market institution as a mechanism to coordinate and optimize the production of the goods with the preferences of the consumers is explored.

How markets select goods or technologies has been studied for a long time in the literature. One of the first works related to the problem of how market

[2]The busy beaver problem in computability theory is an example of this. See [10]

system selects commodities that are produced and sold in a market has been described by Spence (1976a,b)[4] [5]. Thus, according to the behavior of the demand, markets will choose the high elasticity product even when the low elasticity product could produce higher social welfare and on the other hand in the case in which the demand curve is inelastic but the cost structure is different, the market will discriminate against high-fixed-cost/low-marginal-cost commodities.

Technology allocation has been developed with different approaches. Particularly, Arthur [1] combines the fact that the adoption of a specific technology induce a network of complement that produce increasing returns in the use of this technology, producing occasionally that the economy remain locked-in in an inefficient technology.

In all these cases some properties of static market are described, however in this paper different aspect are emphasized; the set of possible alternatives is represented by a landscape which does not have any parsimonious property; the agents do not have knowledge about the landscape and they learn about it through mechanisms specified in the institution; there exist coordination problems as consequence of competence among firms; there are problem of diffusion of information among the consumers itself and between consumers and firms and finally the method of research and development has to be explicitly specified.

Essentially the institution is the method of optimization and the challenge for the researcher –or institution designer– is to have an institutions that manage to obtain best solutions in any landscape.

In the next section the model is described. In section 3 the market algorithm is presented in detail and the analytical solution to such an algorithm is provided. In section 5 the results of simulations are shown and analyzed. Finally there is the conclusion.

2 Model

There is a set I of consumers and each of them buys a unit of the good produced by one of the firms active in the market. Any firm $f \in F$ produces a good from the set G of possible goods to be produced.

The set of active goods presents in the market is a subset of goods $\Gamma \subset G$ that the firms in the set F are producing and selling. Note that the numerosity of Γ can be equal or lower from F because it is possible that two firms produce the same good.

Consumers have preferences over G. Indeed every consumer $i \in I$ has a willingness to pay over any good $g \in G$ and is represented by $v_i(.) : G \to \mathbb{R}$. It is assumed that g is an experimental good, i.e. a consumer does not know their willingness to pay until he does not experiment the good or until he does not receives information of the good from other consumer. Consumers other than experimenting the good to know $v_i(.)$, obtain information to know the value of $v_i(.)$ through processes of diffusion of information which will be specified below.

For the sake of simplicity it is assumed that consumers are homogeneous in their preferences, so from now on $v()$ will be used without the subscript.

There is imperfect information in the sense that neither consumers nor firms know all the possible $g \in G$ that could be produced in the market. Moreover $G \ggg F$, such that the number of potential goods in the economy is big compared to the number of firms present or supported by the market.

Note that the consumer buy just one unit of the good and it is assumed that they have the same income y such that $0 < p_f \leq y$ for $f \in F$ in order to assure that consumers can afford any good in the market.

Any good $g \in G$ is consumed after a period of time represented by the stochastic variable ρ drawn from a distribution Υ.In other words, consumers periodically consume the good and therefore they have to go shopping, and every time that the consumer goes shopping she buys one unit of good.

With the information that consumers manage to gather in any period, he will decides to buy from the firm f that maximize its surplus, $s = v(g_f) - p_f$, where g_f and p_f are the good produced and the price established by the firm

From the production side, any good $g \in G$ has an unitary cost of production given by the unitary cost function $c() : G \to \mathbb{R}$.

Firms know neither the unitary cost function for all the goods $g \in G$ nor $v()$, the consumer willingness to pay over the same set of goods.

Indeed firms will gather information about the cost function through two processes; the first is imitating, i.e. asking one of the active firms about the cost function for the specific good, and the other is through a research and development process which will be specified in detail below.

Once a firm makes the choice of producing a good g it can not change its production during its period of existence in the economy. What this assumption means is that just new entrant firms can make research of new goods in the space G.

Firms make their decision, using all the information that they manage to obtain, in order to maximize their profit π.

Even when detailed specifications about the mechanism through which new entrants discover goods will be specified, it is important to note that in the process of research there is path dependence over the set of active goods.

The path dependence in this case is present due to that when a firm make research and development it will take a subset of Γ from which the research will be done. The presence of path dependence implies that there exists the possibility that the market stop its research through the space G in some local optimum.

Thus, the firms have to make two different decisions; the first is in its start-up where they have to make the choice about the good that they will produce, and the second is the price at which they will sell it.

The difference between the willingness to pay and the unitary cost function for a given good $g \in G$, $s() = v() - c()$ is defined as the surplus that any good offer to the consumers.

The surplus describes the static landscape that firms have to face to find out what is the good that produce to them the maximum profit. In order to

obtain such a higher profit –which not necessarily mean the maximum value in the surplus landscape, firms have to face competence, imperfect information in $v()$, $c()$ and G, and path dependence in the research (search) process.

All these elements interact together in an specific market institutions and according to different market structures it is less or more likely that all the agents stay in local optimum or find out higher peaks.

Even when clues about how the institution allocate the good in G it is possible to be obtained, along this paper it will mainly be used algorithms to study the behavior of the it.

3 Markets Algorithms Description

Without market, every consumer would produces the good that he consumes. Thus, consumers would be distributed around the landscape of the surplus and some of them would have different surplus. On other hand there will not exist systematic research process that allows the society to go through the whole space of surplus in G; indeed the unique agents with incentives to make research would be the ones with negative surplus.

The presence of a social planner does not help to solve the problem mainly for two reasons; first the great size of G, and second the landscape complexity represented by $s()$ is unknown. In other words, the result that the social planner manages to obtain will be strongly related to the algorithm that he uses to find out the solution. Given the reasons exposed precedently and the No Free Lunch Theorem by Macready and Wolpert (1997) there is no guarantees about the quality of the solution.

Modern society are organized through market institutions to make exchange, research process, diffusion of information and coordination of activities. However the specific characteristic of each market institutions is strongly related to attributes of the good that is trading.

In order to understand how well is the performance of the market to allocate the maximum value in the surplus landscape it is necessary to describe in details how these activities are carried on. Once these activities are specified it is possible to study the whole process through simulated experiments.

The main activities that have to be specified to describe what from now on will be called the *market algorithm* are,

- Shopping process; in which the mechanism of diffusion of information among the consumers and how consumers make the decision about where to buy is specified,
- firms research and development; where the mechanism of research over the space of goods G for the set of firms is described. On other hand, it is defined the structure of neighborhood or accessibility on G,
- how firms establish their prices in order to obtain the maximum profit,

- specifics characteristics of the cost function of the good to be produced and finally
- entry/exit process of firms.

Note that in any case it is always assumed that all the agents active in the market have bounded rationality both; procedural and informational.

3.1 Consumers

3.1.1 Shopping

Every period a random proportion ρ of consumers consume the good that they bought before, so they go shopping to acquire one unit of the good produced by one of the active firms F.

Given the experimental characteristics of the good, the consumer does not know ex-ante what is his willingness to pay for a given g. Hence in order to obtain information about his willingness to pay three different methods are used;

Consumer Memory As the consumer has been consuming in precedent periods a specific good, he knows the willingness to pay of that good. However the consumers has just memory for the last period consumption such that consumers will not remember the willingness to pay of goods bought and consumed in period precedent to the last one.

Sharing Information Given the homogeneity in preferences among the consumers, it is possible to obtain information about the willingness to pay for a given g asking to other consumers.

Thus, as it is assumed that there exist a stochastic full connected network graph that link all the consumers, when the agent goes shopping he randomly asks to one of the other consumer in the network, information about the willingness to pay of the good that this neighbor is consuming[3].

Market Research When a new entrant arrives into the market, the firm makes a market research over a random sample of consumers. The consumers which participate in the process will obtain information about their willingness to pay for a good that the entrant will produce.

Given that consumer i is consuming one unit of the good produced by firm f, and that through any of the method mentioned previously he obtains information about the good produced by the firm h, he will switch from the brand f to the good produced by the firm h if,

$$v(g_f) - p_{f,t} < v(g_h) - p_{h,t} - \omega$$

[3] It is also possible to say that the consumer could obtain information asking to more than one consumer.

where $v(.)$ represents the willingness to pay that the consumer has over the goods $g_.$, $p_{.,t}$ is the price that firms f and h are applying during the period t and finally ω represent the switching cost that the consumer has to pay in order to start using a new good.

Switching cost introduces the idea that as consumers get used with a particular brand, in order to start consuming a new one, they will ask for some additional amount of surplus.

It could occur that the firm f in which the consumer i was buying before broke and it had to leave the market. In this case when the consumer i go shopping he selects the firm h unconditionally.

In other hand, it could also be the case that the firm h does not exist in the market any more, in such a case the consumer will stay with its old firm f given that it still exist.

Finally, consumer will not buy any good in two cases; if he faces negative surplus from both firms and in the case in which neither firm f nor h remain in the market.

3.2 Firms

3.2.1 Price

Firms set prices in order to obtain a targeted profit. This target is the animal spirits that the entrepreneur has when he opens the firm. The animal spirit for the entrepreneur of the firm i is simulated as a uniform random number a_i draw from the support $[0, A]$.

Thus, the price of the firm i, which is producing the good g is given by,

$$p_{i,t} = \frac{a_i}{q^e_{i,t}} + c(g)$$

where $c(g)$ represent the unitary cost of production associated with g and $q^e_{i,t}$ is the expected demand that the firm calculates for the period t.

Firms compute the expected demand using the following adaptive process,

$$q^e_t = q^e_{t-1} + \alpha(q_{t-1} - q^e_{t-1})$$

where α is the speed of adjustment to the difference between the expected and the actual demand in the last period.

Note that taking the animal spirits as a fixed cost, there exist increasing returns (i.e. average cost is fall as more products are sold) therefore the market favor the existence of natural monopolies.

Nevertheless, it is possible that a new entrant with a better product offers higher surplus for the consumers than the one offered by the monopolist, so eventually the new entrant could take over the market.

3.2.2 Exit

As long as a firm obtains profits higher than the animal spirit of its owner, the firm stay in the market. However, when the profits are lower than the animal spirits it is a signal that the firm is in troubles and eventually could leave the market.

Profits lower than the animal spirits can be obtained because there is a temporal change in the market or due to a mistake in the computation of the expected demand. Thus, the firm will not close as soon as he obtains profits lower than the animal spirits but if bad figures are repeated likely it will do it.

To simulate this behavior it is assumed that a firm i will leave the market with a given probability θ in case that the profit obtained is lower than the animal spirits,

$$\pi_t = (p_t - c(g))q_t - a < 0$$

where the subscript i is avoided.

3.2.3 Entry

Given the assumption that the number of firm in the market is constant, new entrance occurs just to replace the exit of other firm.

The entry process is key in the model, because given to the assumption that incumbents can not explore G, the new firm is the unique able to carry on research and development over the set of possible goods.

Three are the task that a new entrant develops,

1. it has to obtain its animal spirits from the support $(0, A]$
2. it decides whether to explore (i.e. do R&D) for a new good in G or to copy the good already produced by the firm with the higher share in the market[4] and. The research and development process determines how firms explore the space G in order to allocate the goods g.
3. its develops a market research such to make its good (or brand) known by the consumers and to calculate the initial expected demand.

Animal Spirits The entrepreneur obtains the animal spirit drawing a random number from the range $(0, A]$. Entrepreneurs which get into the market with high animal spirits have lower probabilities to survive in the long run.

[4] Note that this is equivalent to say that the best good in the market is the most consumed one.

Good to be produced In any period of time t the firms in the market can be distinguished in two groups; the proportion of firms ϕ_t that when they entered into the market made R&D and the proportion $(1 - \phi_t)$ that have imitated the best good present in the market in the entry period.

As it is assumed that in the decision of R&D there exist herding, (i.e. more innovators imply more innovations) the decision of an entry firm about whether to make R&D is assumed to be taken with probability equal to the proportion ϕ_t present in the current period.

In order to avoid that the system stay locked in any extreme, the value considered is $\tilde{\rho}_t = \min(\varphi, \rho_t)$, where $1 - \varphi > 0$ is the probability of observing a mutation when either all the active firms are researcher or imitator.

Firm Research and Development Firms know neither the whole space of potential goods G nor the landscape $c()$, so it have to be specified what is the method used by firms to explore G.

Goods are represented as a string of binary digits $\{0, 1\}$, for instance, if the number of firms is 3 and the goods in G are described by 5 binary digits[5], in given period of time the set of active goods could be $\Gamma = \{\{00100\}, \{10001\}, \{11111\}\}$[6].

In the R&D process there exist path dependence because firms in any case use as starting point to its research process one of the goods active in the market. In fact the set of active goods in an economy establish what are the possible products to be produced in the very short run.

Thus when firms are engaged in a R&D process initially it starts with the product γ with higher market success from the set Γ. The success is given for the product more consumed in the market.

Once the firm obtain the binary representation of γ, the research process is represented as a flip in one of the digits of the string of bits chosen randomly. When the new firm is produced, the firm is able to calculate the unitary cost $c()$ of producing it[7].

For the sake of compactness in what follows it is described the sequence of steps the entry & exit process means, starting from the point in which a firm leave the market and is replaced by a new one,

1. All the consumers that were buying from the old firm, are set free,
2. a new entrant selects a random animal spirit a from $(0, A]$,
3. the proportion $\tilde{\phi}$ of innovators in the market is computed,
4. the firm with the biggest market share is selected in order to have its good γ as the prototype to be followed.
5. With probability $\tilde{\phi}$ the entrant will become a researcher and in that case it will do the following task,

[5] This means that the maximun number of goods in G is $2^5 = 32$.

[6] The decimal representation of this good is $\Gamma = \{4, 17, 31\}$.

[7] It is possible to incorporate cost of doing research and development, however in this first version of the work it will not take in account.

(a) Randomly the firm changes one of the bits in the binary description of γ

(b) The unitary cost using the new good γ' is computed from the landscape $c()$.

6. With probability $(1 - \tilde{\phi})$ the firm do the following

   (a) Firm copies the good γ.

7. The entrant makes a market research doing the following task,

   (a) It selects a random sample of customers

   (b) It asks to the customers using the price of the best firm or a price who offer zero surplus, according to whether the surplus using the best firm price is positive or not, if the customers want to change the firm[8].

   It is assumed that the consumers participating in a market research have high willingness to change the firm, such that they will rather prefer the entrant in the case in which the surplus of the good of its old firms and the one from the entrant are the same. Moreover it is also considered that the switching cost is not present during the market research process. The assumption is that entrants are more aggressive during its entrance process, through promotions or special discounts that they offer to the customers.

   In case that the consumer which is in the pool does not have a firm because his old firm close, this consumer will adopt the entrant unconditionally.

   (c) The number of customer from the sample of customers that accept to change become the q_i^e and the q_i of the new entrant firm.

   (d) The entrant set the price for its first period according to $p_t = \frac{a_i}{q_t^e} + c_g$

4 Analytical clues

The model could be studied as a two stage game where in the first stage firms select a good from the set of available goods G and in the second stage firms set the price at which the product will be sold in the market.

The total number of consumers is C and each of them buy just one unit of the good from one of the firms present in the market.

It is assumed perfect and complete information; consumers know what is the set Γ of active goods in the market, the willingness to pay $v(.)$ for any good in

[8] Note that here it is assumed that the entrant manage to konw the willingness to pay $v()$ of the sample of consumers that participates in the market research.

such a set, and the prices that firms are currently setting. Given that consumers do not have income constraints, they buy from the firm that gives the maximum positive surplus, $v(g) - p$.

On other hand, firms know $v(.)$ (i.e. the preference of the consumers over G) and also they know what is the unitary cost of production of any $g \in G$ and the complete space G.

In any case firms attempt to maximize their profit,

$$\begin{aligned} \pi &= qp - c(g) \\ sa.c(g) &\leq v(g) \end{aligned}$$

where the constrain specifies that the cost of production has to be at least lower than the willingness to pay of the consumer for the good g.

Monopoly The monopolist face a market size limited by the number C of homogenous customers, so given the constant return assumed in the production function of the model, in the first stage the firm will select the good that has the higher surplus –offer the greater unitary profit.

In the second period the firm will set the price that gives to it all the surplus of the consumer. In this case the firm act as a price discriminator.

Monopoly Solution	
Good	Price
$g^* = \arg\max_g v(g) - c(g)$	$p^* = v(g^*)$

Is possible the existence of multiple solutions to this problem. Indeed, according to the landscapes $v(.)$ and $c(.)$ more than one good could produces the same result in the first stage, therefore in the second stage too.

However, qualitatively the result for the consumers is indifferent because in all the cases as the firm act as price discriminator it will take over all the consumer surplus.

Oligopoly[9] In this case, firms in the first period select the good to sell in the market and it is in the second stage of the game, they are involved in a Bertrand competence. Thus this time, when the firms set a price, it has to take in account also the prices that other firms are offering.

In the first stage it is selected the good that offer, as in the monopoly case, the maximum surplus to the consumer in order to be competitive in the second stage.

In the second stage firms compete a la Bertrand, setting their prices equal to the marginal cost, which is given by the cost of producing one additional unit $c()$.

[9] Introducing the animal spirit in this analysis would produce that even when many firms are active in the market, the one with the lower animal spirit will become a monopolist in the market.

Oligopoly Solution

Good	Price
$g^* = \arg\max_g v(g) - c(g)$	$p^* = c(g)$

Note that the difference between the monopoly and the oligopoly lies in who appropriate the surplus offer by the good g^*. In the monopoly the firm transforms the surplus in profit whereas in the oligopoly the surplus remain for the consumer.

As in the monopolistic case, according to the value of $v(.)$ and $c(.)$ it is possible the existence of multiple g^* which satisfy the oligopoly solution. Under the setting of this paper there is no market selection for any of the goods that satisfy the condition.

In the framework of this paper, and as it turns to be one of the most common critique to the economic mainstream, the main concern it has to be with *argmax*. Basically *argmax* is assumed to be an optimization algorithm which gives the solution to the problem without any economic meaning in the steps that its made.

The idea of this paper is to set an optimization procedure that with economic meaning in any step, shows how to problem is solved.

Thus, it is possible to study the *market algorithm* and to modify their properties in order to produce good solutions independently of the landscapes $v(.)$ and $c(.)$.

5 Simulations

Up to now the values of the willingness to pay and the unitary cost have been described as landscapes over the set G.

As the knowledge that we have in reality of any of these landscapes is poor and in most of the cases it is not possible to describe their properties, it is desirable to study the behavior of the *market algorithm* under landscapes with different level of complexities.

Kauffman's NK-Model (see [12]) are useful to this purpose because allows to describe landscapes that have different level of correlation within their highest values.

5.1 Landscape Description

According to Reidys and Stadler (2002) (see [9]) the landscape is defined as a triple (G, χ, f) consisting of:

1. a set G of configurations or the set of all the possible goods to be produced in the economy,

2. a notion χ of neighborhood, nearness, distance or accessibility on G, defined in the case of this paper by the method used by the firms to make

research and development (i.e. how to go from one existent good to a new one),

3. a fitness function $f : G \to \mathbb{R}$. In the model there are two fitness function; one represents the willingness to pay and the other the unitary cost of the goods.

It is possible to consider a good as composed for a set n of characteristics where costs and preferences are function of them. A change in one of these characteristics could produces minor variation in costs and preferences. These fitness landscapes are *correlated*, nearby points tend to have similar heights. Such a landscape allows to a simple optimization algorithm find out optimum values.

On other hand it is also possible that the single change in one of the characteristics produce abrupt changes in the value of cost and preferences over the goods G. Such a *random* landscape does not present any correlation and there are poor information available to be used by an optimization algorithm.

The NK-Model allows using one parameter to specify the level of correlation of the landscape. Thus given the value of N –the number of characteristics of the good– K determines the level of correlation of the landscape. A value of $K = 0$ means that the landscape is highly correlated whereas $K = N - 1$ means that landscape does not have correlation, in other words it is a random landscape.

A single good $g \in G$ is described by a set of n binary digits, and the notion of nearness, distance or adjacency of a such a good g is given by a single flip of one bit from its representation. Note that the concept of nearness is associated to how the research process is carry on by the firms.

Finally the fitness function describing the willingness to pay $v()$ and the unitary cost function $c()$ are computed in independent way[10] according to the specification of the NK-Model.

5.2 Single Experiment

In order to show one example of the results produced by the market algorithm a single experiment is run with the following parameters; the number of characteristics of the good is $n = 30$ (approx. 10^8 possible goods), in both landscapes the value of $K = 4$.The values obtained from the landscape to the willingness to pay are in the range $(0, 4]$, whereas the unitary cost goes from $(0, 3]$. There are $C = 10000$ consumers, the constant number of firms in the market is $F = 10$ and the maxima animal spirit is $A = 100$. There is neither switching nor research cost, the exit probability of a firm is given by $\theta = 1\%$, in every period the proportion of consumer $\rho = 5\%$ goes shopping, and every time that a new entrant arrives into the market, it makes the market research over the 5% of the population of consumers. Finally firms adapts their expectations about the

[10] Indeed in order to obtain these values two different contribution table are used. To see more details about the NK-model see [12] pag 169.

market demand with $\alpha = 0.25$ of the difference between the actual and the new information[11].

In Figure 1 the results of 2000 periods of interaction between consumers and firms is shown.

The *Proportion of Innovators* plot represent how much research and development is carry on in the market in a given period of time. As it is possible to observe, there is a cycle between firms copying and firms doing research. After observing many experiments with different parameters, it was observed that the process has cycles until a firm manage to get some optimum. After this, as the unique firms which have chances of stay longer in the market are copiers the process tend to stay with higher proportions of copiers, while the monopolist does not make mistakes.

The concentration observed in the market is represented by the *Concentration* plot through the Herfindahl index. The plot represents the trend of this market to produces natural monopolies. However, the plot shows that periodically firms which have a monopoly in the market, make mistakes and they have to exit.

This dynamic can be explained as an aggregate effect, which is produced in the following sequence

1. Assume initially that there is a monopoly and almost all the new entrants attempt to innovate. However as the monopolist is in some local optimum (also it could be some global optimum), the market stay with the same monopolist for a while.

2. Randomly occurs that firms start imitating the monopolistic. These firms will take consumers away from the monopolist, because entrant are initially more aggressive than the incumbent firm. Given the natural monopoly structure the monopolist in the long run tend to keep the market. However, there is an adjustment time where the incumbent looses and recover an small proportion of customers.

3. As copiers increase, there is also an increase in the probability that a new firm become a copier and the adjustment time become even longer.

[11] The market is explored through different experiments and in all the cases the initial conditions are specified by the following characteristics,

- Consumers are distributed among all the firms uniformly.
- Initially the expected demand q_i^e, of the firms is set equal to C/F.
- Initially which firms is an imitator or a researcher is assigned randomly with a uniform distribution.
- Initially the good that the firm will sell in the market is assigned randomly from the space G.
- The animal spirit initially is randomly assigned as a number in the range $(0, A]$

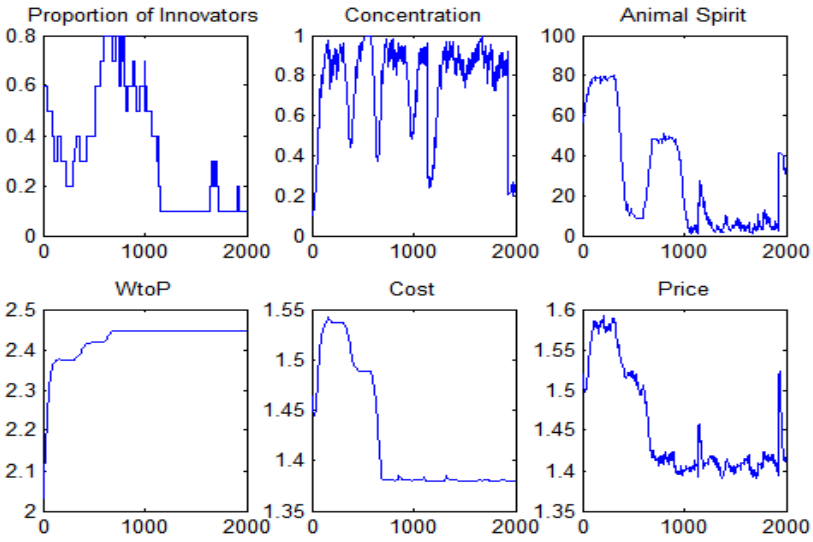


Figure 1: Single Experiments of the market.

4. Great number of copiers makes the monopolistic to make mistakes in their predictions so it will have profits below its animal spirit, and eventually it could occur that it has to leave the market!

Summing up, the idea is that new small entrants induce the monopolist to make mistake, and because of that it could occur that the monopolist eventually have to leave the market.

The *Animal Spirits*[12], give an idea about how hard is the competence among the firms in the market. If the competence is intensive the animal spirits in the market has to be low, otherwise firms could make great profits. As the plot shows, when a monopoly has low animal spirit it has higher chances of staying longer in the market.

The *WtoP* (Willingness to Pay) and the *Cost*[13] plots show how in the beginning the optimum for the consumer is allocated, next the firms optimize their production. As consequence of observing many experiments with different parameters it is not possible to sustain that it is a pattern of behavior, instead seems to be that it is consequence of the randomness in the landscape.

The efficiency of the process of allocation is not evaluated comparing against

[12] The line in the plot is the animal spirit of each firm weighted by the market share of the firm. $A^* = \sum_{i=1}^{F} s_i a_i$

[13] As in the case of the Animal Spirits, the index ploted for willingness to pay and cost are weight demand share values. $I^* = \sum_{j=1}^{F} s_j i_j$, where i is the cost and wtop index.

some global optimum[14]. However from the *WtoP* and *Cost* plot it is possible to claim that the algorithm manage to get some optimum value.

The market share weigthed *Price* keeps fluctuating above the value of 1.4 which is the unitary cost of producing a unit of the good. This result allows to advance the idea that even when the market structure induces monopolies, the price is equivalent to the one derived from the oligopoly analysis.

5.3 Monte Carlo Analysis

Three properties of the market algorithm are the focus of these simulations; search efficiency, surplus division and level of animal spirits that the market allows to survive.

Search efficiency focus on how good is the market institution at allocating the good which potentially provides the higher surplus.

Once the good with higher surplus has been allocated, the surplus division describes how this surplus is divided between consumers and firms. Firms use price to obtain part of the surplus that a good offer as profit, if price is equal to the willingness to pay of the consumers this means that firms transforms the whole surplus in profit. On other hand if price is equal to unitary cost it means that the surplus is enjoyed for the consumers.

The level of animal spirits in the market is a signal of how much profit the firms manage to obtain from the consumers. Indeed given the price policy of the model the animal spirits is complementary to the surplus division to understand how the surplus is allocated.

These three properties are studied for changes in the number of firms and the level of correlation of the landscapes. The number of firms represents the number of searchers actives competing in the market, and the roughness of the landscape represents how sensitive is the good to small changes in it characteristics.

For each combination of parameters, 100 experiments are run an the results are used to compute indexes to understand the properties of the process.

Every experiment is observed during 1200 periods; as it is assumed that a period of simulation is equivalent to a month, the market is observed during 100 years. The number of firms used in every experiments are $F = \{10, 50, 100, 150, 100\}$, the number of binary characteristics of the good is $n = 30$ and the correlation in the landscapes are $K = \{1, 10, 20, 29\}$.

The rest of the parameters are; the proportion of consumer buying in every period $\rho = 5\%$, the switching cost $s = 0$, the speed of learning of the firms $\alpha = 0.25$, the probability that a firm when make negative profits leave the market is $\theta = 1\%$, when a new entrant arrives into the market it will drive a market research over 5% of the population and the maximum animal spirit is $A = 100$.

[14] It was not possible to use and optimization algorithm to allocate the optimun surplus in order to compare the results.

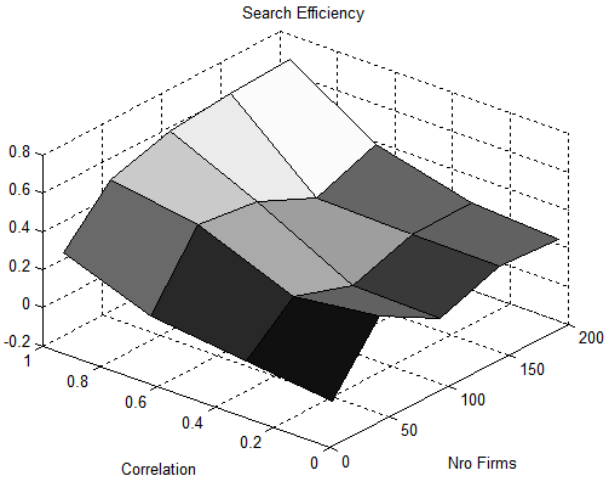


Figure 2: Comparation of market against random walk

Search Efficiency Thus in order to measure the efficiency of the market algorithm, instead of finding out the optimum value in both landscapes $c()$ and $v()$, and comparing such a value with the one obtained using the market algorithm it is used a benchmark process and the results obtained by the simulated market are compared against the one obtained for the benchmark process.

As the benchmark process is used a random walk represented by a firm which start randomly with a good and randomly flip one of the characteristics and accept the change just in case that the surplus obtained with the new good is higher than the previous good. This benchmark process is tested during the same period of time than the market and at the end the value of the surplus $s(b^r)$ is computed.

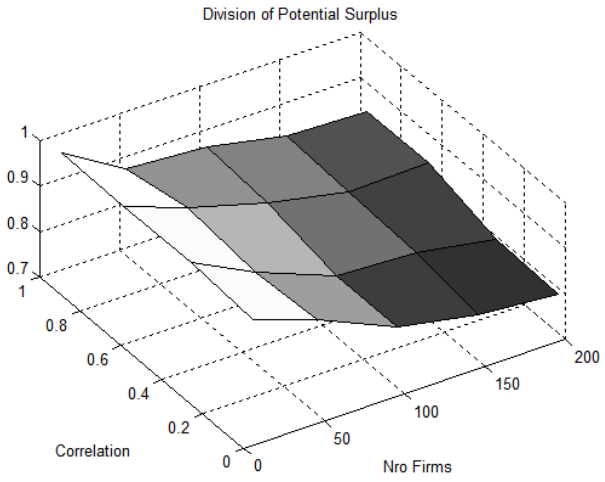
On other hand, the demanded share weighted surplus $s(b^m)$ at the end of the market experiment is computed.

In figure 2 the plot of the qualitative index $I_1 = s(b^m) - s(b^r)$ is represented[15]. The main properties of the market is that when higher is the correlation of the landscapes or higher the number of firms it will outperform clearly the random walk. However fewer the number of firms or less the level of correlation of the landscapes the market will behave almost as a random walk.

Note that the market in all the case will work remarkably better when the landscape is simple (i.e. high correlation).

[15] The point plotted in the graph –and in the following graphs– is the mean obtained throught all the experiments runned with one parameter.

Division of Surplus In section 4 is described that in the case of monopoly all the surplus is taken by the firms whereas in the oligopoly the surplus is obtained by the consumers.



Division of surplus between firms and consumers

In figure about the Division of Surplus between firms and consumers the index $I_2 = \frac{v^m(b) - p^m(b)}{s^m(b)}$, is plotted. It represents the proportion of surplus obtained by the consumer where $p^m(b)$, $v^m(b)$ and $s^m(b)$ are the demanded share weighted price, willingness to pay and surplus at the end of the experiments. Thus, close the value of the index to 1 means that the surplus is obtained for the consumer, (i.e. oligopoly results), close the value to 0 the surplus is transformed in profit for the firm (i.e. monopolistic result)[16].

As it is possible to observe, fewer firms independently of the correlation in the landscape produces that the consumers obtain almost all the surplus whereas when the number of firms increase, they manage to get an small proportion of the surplus for them. The presence of few firm make more hard the competence among the firms due to the more easily diffusion of information among the consumers and this enhance in the competence produces two effect; firms set prices closer to unitary cost and on other hand the animal spirits of the entrepreneurs is minimized.

Animal Spirits Selection The animal spirits has two different interpretation; as the profit the firms are able to get in the market, or as the level of fix cost that the market is able to sustain.

[16] Note that given the price policy of the model, if there is a monopoly and the number of consumers is $C = 10000$, in order to observe the value of the index $I_2 = 0$ the animal spirits of the monopolist had to be $a = s(g) * C$, where $s(g)$ is the surplus of the allocated good g.

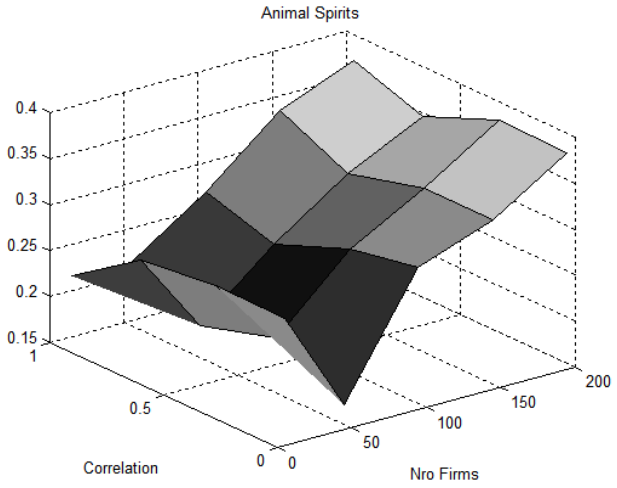


Figure 3: Animal Spirit

Using the index $I_3 = \frac{A-a^*}{A}$ where A is the maximum animal spirit available, and a^* is the demanded share weighted animal spirit of the firms in figure 3 is measure how this value change according to the number of firms and level of correlation. Close I_3 to one (zero) means that the market does not allows the animal spirits to be high (low).

The level of animal spirits is quite independent of the correlation of the landscapes. However higher the number of firms, the market offer more profit; indeed as the number of firms is greater, the flow of information among the consumers is lower and this produces that firms with higher animal spirits stay longer in the market.

6 Conclusion

The model of this paper is constructed under a set of key assumptions; there is no parsimonious representation of the reality; the quality of the solution to the problems that society faces can be represented as landscapes where higher peaks mean better solutions; society establishes institutions that rule the behavior of its components in order to find out good –or best– solutions; the members of the society do not have full knowledge about the landscapes, they are bounded rational agents and the mechanisms of landscape exploration are determined and conditioned within the institution.

Over this artificial representation of reality, the task that the researcher face is to design an institution, which could be seen as an optimization algorithm, where the procedures –or algorithm instructions– are explicitly defined as activities that the agents within the institutions have to do. The institution has

to be robust enough such to assure that it will manage to get good solutions independently of the reality (i.e. landscape) that it has to deal with.

In order to see these ideas working together an example concerning to the problem of product differentiation was addressed. Specifically a market where consumers interact with firms in order to find out a good to satisfy the society was considered.

The behavior of the market when the level of correlation in the landscape and the number of firms change was studied according to three properties; efficiency at searching over the space of goods (landscape), division of surplus and entrepreneur aggressiveness.

When higher is the number of firms and the level of correlation among the goods, the market studied produces higher efficiency, measure as the surplus allocated to the members of the society.

In other hand fewer the number of firms in the market, produces that the surplus allocated will be enjoyed by the consumers, whereas greater number of firms allows them to convert a portion of the surplus in profit, independently of the correlation among the goods.

Finally when the number of firms increases the market accepts entrepreneurs with higher animal spirits.

Further works has mainly two lines to developed, the first is related to the method and the second concern the specific problem studied in this paper.

Methodologically more problems using the same concepts have to be addressed in order to produces a crystallization of the method pointing out the advantages and disadvantages, dynamic landscape could give even better representation of the reality to express coevolution of agents, institutions and reality, finally the theory of optimization algorithm could be used as starting point to provide analytical solutions.

Concerning the product differentiation problem; the market model studied produces natural monopolies, so it reduces the level of competence among the firms; introducing heterogeneity will contribute to observe more properties of the allocation process; assuming different method of research and development could change the efficiency of the market and finally different specification of the mechanisms of information diffusion among the agents could also contribute to better understanding of the market algorithm.

6.1 Files Allocation

(Matlab) Models/Tech
(Document) Notes/MarkAlgo
(Fortran) Models/Tech/fort ->

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