# The Welfare Gains from Stabilization in a Stochastically Growing Economy with Idiosyncratic Shocks and Flexible Labor Supply*

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**Abstract**

Stochastic models with economy-wide shocks imply that the welfare costs of aggregate volatility are negligible. In reality idiosyncratic shocks are important, and empirical evidence suggests that their volatility is several times that of aggregate shocks. This paper introduces both types of shocks. We find that if in the process of eliminating aggregate risk, the policymaker can reduce idiosyncratic risk by a modest amount, in accordance with available empirical evidence, the welfare gains from aggregate stabilization can become significant. The introduction of idiosyncratic risk has important implications for asset pricing, and in particular may reduce the risk-free rate substantially, through the precautionary savings motive. Many of our results are sensitive both to the degree of risk aversion, and to the flexibility of labor supply. The paper highlights the tradeoffs involved in analyzing the effects of risk on growth and welfare, on the one hand, and on asset pricing, on the other, clarifying the need to examine these issues within a unified stochastic general equilibrium framework.

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## 1. Introduction

Attempts to assess the impact of risk on resource allocation and macroeconomic performance have generated anomalies and have been plagued by puzzles. Two such puzzles are intimately related. The first concerns the growth and welfare effects of output volatility, the so-called costs of business cycles. Assuming complete markets and the simplest exogenous growth model, Lucas (1987) obtains very small effects of economy-wide volatility on growth and welfare. He shows that the welfare cost of identically and independently distributed aggregate fluctuations is of the order of 0.1% of initial consumption. Using a stochastic endogenous growth model, but with a larger menu of assets available to diversify risks, Turnovsky (2000) reaches a similar conclusion. He finds that for an economy such as the United States, the growth effects of economy-wide volatility are negligible, and the associated welfare costs are of the order of 0.2% of the initial capital stock.

The second issue concerns the equity premium and risk-free return puzzles of Mehra and Prescott (1985) and Weil (1989) respectively. Mehra and Prescott (1985) show that a plausibly parameterized representative agent stochastic exchange economy predicts an equity premium of at most 0.35%, in sharp contrast to the historically observed premium of about 6% in U.S. data. Weil (1989) points out that this is because the risk-free return generated by such a model is far in excess of the 0.8% average secular risk-free rate suggested by the data.

The fundamental problem is that the aggregate risk in a developed economy such as the United States is far too small to generate plausible equilibrium responses in the representative agent model. Empirical studies by a variety of individuals suggest that the annual standard deviation of aggregate output fluctuations in OECD economies averages around 4%, though in the United States it is somewhat lower, being around 2.5%; see Danthine and Donaldson (1993), Gali (1994), Gavin and Hausmann (1995). Aggregate consumption volatility is even lower, being of the order of 1–2%. Since in the standard stochastic growth model, aggregate output risk influences growth as a variance, the contribution of aggregate risk is essentially negligible. By the same token, the risk premium is obtained by "pricing" risk at the coefficient of relative risk aversion, $R$. Again, given the small aggregate risk, a meaningful risk premium requires that the coefficient of relative risk aversion be unrealistically high. Accordingly, Obstfeld (1994) bases his analysis, on values of $R$ =18, while Kandel and Stambaugh (1991) have proposed values of $R$ as high as 30.

However, the assumption that all risk is economy-wide and can be diversified is restrictive. Atkeson and Phelan (1994) criticize the Lucas method of focusing on aggregate shocks, claiming instead that incomplete markets are a potential source of large growth effects of output volatility. Indeed, while aggregate risk in the US economy may be small, empirical evidence suggests that idiosyncratic risk has a standard deviation that is several times larger in magnitude; see Deaton and Paxson (1994), Pischke (1995), and Storesletten, Telmer and Yaron (1999, 2001). Anecdotally, the gyrations in the stock market certainly suggest that, while individuals trading among themselves may each experience large volatility in returns and therefore in income, since one person's gains is often another person's losses, at the aggregate level these will largely cancel out. Likewise, as individuals move in and out of employment, each agent experiences volatility in his own income, although in aggregate these shocks will also be largely offsetting. [1]

This paper develops a general equilibrium stochastic growth model of capital accumulation with both economy-wide and idiosyncratic shocks in the individual production process. The shocks are specified as Brownian motion processes, so that all shocks are permanent. We assume the absence of a risk-free asset, implying that individuals have to bear all risk inherent in their risky capital. We introduce elastically supplied labor in conjunction with an appropriate production technology, so that the equilibrium is one of endogenous stochastic balanced growth. The inclusion of labor income is an important feature of the model, since it has the desirable property of increasing the marginal propensity to consume out of wealth from around 0.06, in the absence of labor income, to more plausible values of over 0.2; see e.g. Carroll (2000), Carroll and Kimball (1996).[2] By endogenizing labor we can address another issue discussed in the literature, namely the effect of

[1] Relatively little effort has been devoted to trying to derive more plausible effects of risk on growth and aggregate welfare. Imrohorologlu (1989) calculates the costs of business cycles in a simple incomplete markets economy where each agent has a storage technology and finds the costs of aggregate fluctuations to be small, though larger than with complete markets. Recent exceptions include Clark, Leslie and Simmons (1994), and Barlevy (2000).

[2] This characteristic is shared by Turnovsky (2000) for the case of complete markets and no idiosyncratic shocks

labor flexibility on asset returns and its potential to stabilize consumption, enabling us to extend the work of Bodie et al (1992) to a general equilibrium framework.[3]

Our objective is to determine the welfare gains from stabilizing for aggregate shocks in the presence of idiosyncratic risk. Since we are concerned primarily with numerical magnitudes, we calibrate the model and thereby obtain a quantitative assessment of the effects of stabilizing for aggregate shocks on both a number of key economic variables, and especially the impact on economic welfare. The general conclusion is that idiosyncratic shocks in individual productivity under incomplete insurance may be an important factor in determining significant magnitudes of the growth and welfare effects of stabilizing for aggregate volatility**.**

A key aspect of our model is to allow for the potential dependence of individual risk upon aggregate market volatility. Intuitively it seems reasonable to argue that the risk specific to an individual is likely to vary with the aggregate risk in the economy. Indeed empirical evidence suggests that that a 1 percentage point increase in aggregate volatility is likely to be accompanied by a 1-2 percentage point increase in individual volatility. Atkeson and Phelan (1994) and Krusell and Smith (1999) calculate the welfare gains of eliminating aggregate volatility in the presence of incomplete markets and consumer heterogeneity respectively, but do not necessarily reduce individual volatility in the computation of these welfare effects. At the other extreme, Storesletten et al (2001) and Beaudry and Pages (2001) assume that individual volatility depends upon realizations of aggregate shocks, so that idiosyncratic risk is higher in recessions and lower in expansions. This dependence of idiosyncratic risk upon aggregate shocks implies that eliminating aggregate volatility leads to the simultaneous elimination of individual volatility, so that their measure of welfare gains reflects the complete elimination of both sources of risk. In this paper, we adopt a new intermediate position and calibrate our model based on empirical evidence on the relationship between individual and aggregate volatility.

[3] Bodie et al (1992) examine the introduction of labor supply flexibility in a partial equilibrium framework and find that it helps smooth consumption in the presence of risk. Basak (1999) examines a stochastic equilibrium related model with labor and human capital but does not consider endogenous leisure choice. Bianconi (2001) discusses the effects of labor/leisure choice and market completeness on asset prices in a static general equilibrium framework.

As expected, the model continues to yield the conclusion that the gains from stabilizing for economy-wide fluctuations alone are negligible, even for high degrees of risk aversion. However, for plausible sensitivity of idiosyncratic risk to aggregate volatility, much larger welfare gains from stabilizing for aggregate risk are obtained. Moreover, most of the gains come from the associated reduction in the idiosyncratic risk than in the elimination of the aggregate risk itself. For example the gains from reducing idiosyncratic risk by 0.025 from 0.15 to 0.125 are approximately 12 times those resulting from the comparable reduction of aggregate risk from 0.025 to zero. While the welfare gains are of course sensitive to the degree of risk aversion, our numerical analysis suggests that welfare gains of 2-4% are not implausible. These are obviously significant quantities and are consistent with the empirical estimates of the costs of recession with idiosyncratic risk obtained by Clark, Leslie and Simmons (1994). Labor flexibility is shown to decrease the costs of business cycles and thus reduce correspondingly the welfare gains from their stabilization. This is because it introduces an additional margin along which an individual can buffer productivity shocks so that as risk increases, additional labor supply (less leisure) can compensate for potential losses in income.

As Constantinides and Duffie (1996) argue, the introduction of permanent idiosyncratic shocks offers a promising approach to enriching the asset pricing implications of the representative agent model.[4] This paper also has interesting implications for asset pricing. Using basic asset market equilibrium relationships we derive the implicit return on the risk-free asset. As in previous models, aggregate risk has a negligible impact on asset pricing. By contrast, as in Saito (1998), the introduction of idiosyncratic risk reduces the risk-free rate substantially. This is because it has a significant impact on precautionary savings, putting downward pressure on the rates of return. With the productivity of capital, and thus its mean rate of return, determined by labor supply, which is largely insensitive to idiosyncratic production risk, the bulk of the adjustment is borne by a reduction in the risk-free rate. This in turn is reflected by a substantial increase in the implied risk premium. Indeed, we find that for a slightly higher, but plausible, degree of idiosyncratic risk, if the coefficient

[4] Campbell (1999) provides a related result in the context of idiosyncratic labor income pointing out that with plausible standard deviation of idiosyncratic labor income, risk aversion must be unrealistically large to give meaningful equity premiums. Saito (1998) provides the connection between the Constantinides and Duffie (1996) model and the simple Merton (1969) framework showing that the impact on risk premiums can be obtained without the complex pattern of time variation in the conditional variance of idiosyncratic shocks suggested by Constantinides and Duffie (1996).

of relative risk aversion is increased to 9, the premium on the return to capital increases to 6% and the risk-free rate declines to 0.8%, consistent with the empirical evidence. Moreover, this parameterization may be consistent with the empirical estimates obtained by Clark, Leslie, and Simmons of the welfare gains of stabilization of aggregate shocks, as long as their elimination is associated with only a modest reduction in idiosyncratic risk.

The key to these asset pricing implications, (besides that idiosyncratic shocks being permanent), is that the idiosyncratic risk is tied to capital which is nonmarketable, making the risk nondiversifiable. In this respect our implications for asset pricing with nondiversifiable risk are consistent with those obtained by previous authors, though using somewhat different frameworks. [5] We therefore do not mean to suggest that they provide a serious resolution to the equity premium puzzle, which relates to the returns on marketable securities. More importantly, because of the individual's ability to use his labor/leisure choice to buffer risk, we also show that labor supply flexibility works in the opposite direction to the idiosyncratic shocks, thus reducing the risk premium. But despite labor supply flexibility, nondiversifiable permanent shocks can still have substantive effects on the risk premium. Our analysis highlights the tradeoffs involved in analyzing the effects of risk on growth and welfare, on the one hand, and on asset pricing, on the other, thereby clarifying how the two issues are intimately related and emphasizing the need to examine them within a unified stochastic general equilibrium framework.

The paper is organized as follows. Section 2 presents the basic macroeconomic structure while Section 3 assess the relative importance of economy-wide and idiosyncratic risk on the key macroeconomic issues. Section 4 provides a brief discussion of the asset pricing implications, while Section 5 concludes. Details of the solutions and specific derivations are relegated to an Appendix.

[5] Heaton and Lucas (1992) and Lucas (1994) examine the effects of nondiversifiable risk in economies without production and find that individuals would self-insure when faced with idiosyncratic transitory risk. Our analysis considers permanent idiosyncratic shocks in a production economy. Saito (1998) examines the effect of idiosyncratic risk on the riskless return in a simple stochastic growth model and in this respect our approach represents an extension of his work.

## 2. Basic Macroeconomic Structure

We present the basic macroeconomic structure of our model. We start with the production side, and then consider the individual agent's optimal allocation problem and the resulting macroeconomic equilibrium.

### 2.1 Production

The economy is populated by a large number, $I$, of individuals indexed by $i$. Each individual $i$, is endowed with one unit of time that he allocates between leisure, $l$, and labor, $(1-l)$. There is only one good in this economy. A typical individual $i$, produces output, $dQ_i$, in firm $i$, in accordance with the stochastic Cobb-Douglas production function

$$dQ_i = A\big((1-l)B_iK\big)^b K_i^{1-b}(dt+dy+dz_i) \equiv Z_i(dt+dy+dz_i) \qquad 0<b<1 \qquad (1)$$

where $K_i$ is the individual instantaneous stock of capital, $(1-l)B_iK$ is the individual labor supply in efficiency units, and $K \equiv \sum_i K_i / I$ measures the average per individual economy-wide stock of capital. The parameter $b$ determines the magnitude of the labor share in total output and the extent of the external effect on production as in Arrow (1962) and Romer (1986), and more recently Corsetti (1997) and Turnovsky (2000). We show below that the adjusted efficiency term $B_i K$ yields a hybrid of exogenous stochastic labor-augmenting and externality spillover technologies.

The technology is subject to two types of stochastic shocks. First, $dy$ is a normally distributed temporally independent *economy-wide* total factor productivity shock common to all individuals and having mean zero and constant variance $s_y^2 dt$ over the instant $dt$. Second, the agent is subject to a normally distributed temporally independent, *individual-specific*, total factor productivity shock, $dz_i$, with mean zero and constant variance $s_z^2 dt$ over the instant $dt$, common to all agents. In order to focus on the diversification of the two sources of shocks, we assume that they are uncorrelated. We assume that agents are identical in all respects, except in the random drawing they receive of the idiosyncratic shock. Thus, since the labor supply decision is based on common information it is identical for all agents, and thus need not be indexed by the individual agent.

An important feature of the model is the assumption that, although the shocks themselves are uncorrelated, in general equilibrium the volatility of the idiosyncratic shocks (measured by its standard deviation say) is an increasing function of the volatility of the economy-wide disturbances. Intuitively, it is plausible to argue that economy-wide risks are likely to exacerbate the individual-specific risks. Indeed, we find compelling empirical evidence to support such a relationship.

The stochastic production function exhibits constant returns to scale in both the private decisions, the fraction of time devoted to work and the individual capital stock, as well as in the individual and aggregate capital stocks. The labor-augmenting technology comprises two multiplicative components, $B_i K$. $B_i$ parameterizes an internal effect generated by the accumulated effects of the idiosyncratic total factor productivity shocks and its variance. In the absence of such an internalized effect we will find that the aggregate production function will be incapable of generating equilibrium ongoing growth. However, we also find that a specific, but plausible specification of $B_i$ is able to restore ongoing growth at both the individual and aggregate levels.

Aggregating (1) over the $I$ individuals, the economy-wide (average) stochastic output is represented by

$$dQ \equiv \frac{\sum_i dQ_i}{I} = A((1-l)K)^b \frac{\sum_i B_i^b K_i^{1-b}}{I}(dt + dy + dz_i) \qquad (2)$$

In addition, for a large number of agents we assume that by the law of large numbers, $\sum_i dz_i / I \to 0$, as $I \to \infty$, i.e. the individual risk vanishes upon aggregation. Thus for sufficiently large $I$, (2) may be approximated by

$$dQ \equiv \frac{\sum_i dQ_i}{I} = A(1-l)^b K \frac{\sum_i B_i^b (K_i/K)^{1-b}}{I}(dt + dy) \qquad (2')$$

As we shall show below, the macroeconomic equilibrium is one in which the aggregate (average) capital and individual capital stocks grow in accordance with

$$\frac{dK}{K} = y\,dt + dw \qquad (3a)$$

$$\frac{dK_i}{K_i} = y_i dt + dw + dx_i \equiv y\, dt + dw + dx_i \qquad \text{(3b)}$$

respectively, where $y$ , $y_i$ are the mean economy-wide and individual growth rates, and $dw$, $dx_i$ are the mean economy-wide and individual shocks to the equilibrium growth rate. Thus, in equilibrium, all agents accumulate capital at the same average rate, though subject to idiosyncratic shocks that reflect the underlying shocks to productivity.

Taking the stochastic differential of $K_i/K$ and using (3a) and (3b) implies that agent $i$'s relative stock of capital evolves according to

$$\frac{d(K_i/K)}{K_i/K} = dx_i$$

Assuming that all agents begin with the same initial endowment of capital, $K_{i,0} = K_0$, the solution to this equation is (given the lognormality of the underlying shocks)

$$\frac{K_i(t)}{K(t)} = e^{-(1/2)s_x^2 t + x_i(t) - x_i(0)} \qquad \text{(4)}$$

so that the ratio of agent $i$'s stock of capital to the economy-wide average capital stock reflects the *accumulation* of the individual shocks to his stock of capital, as well as the volatility through time. Taking expected values of (4) yields

$$E\left(\frac{K_i(t)}{K(t)}\right) = 1$$

so that the expected value of agent $i$'s capital stock to the average capital stock equals unity.

Substituting (4) into the aggregation relationship $K \equiv \sum_i K_i / I$, we infer

$$\frac{1}{I}\sum_i e^{(x_i(t) - x_i(0))} = e^{(1/2)s_x^2 t} \qquad \text{(5)}$$

and multiplying the individual shocks $x_i(t) - x_i(0)$ by $(1-b)$ implies

$$\frac{1}{I}\sum_i e^{(1-b)(x_i(t) - x_i(0))} = e^{(1/2)(1-b)^2 s_x^2 t} \qquad \text{(5')}$$

Thus, using (4) we obtain

$$\frac{1}{I}\sum_i \left(K_i(t)/K(t)\right)^{(1-b)} = e^{-(1/2)(1-b)s_x^2 t}\frac{1}{I}\sum_i e^{(1-b)(x_i(t)-x_i(0))} = e^{-(1/2)(1-b)bs_x^2 t} \qquad (6)$$

Assume for the moment the absence of exogenous stochastic component, so that $B_i \equiv 1$ say. Then substituting (6) into (2') we see that the aggregate production function (2') becomes

$$dQ \equiv \frac{\sum_i dQ_i}{I} = Ae^{-(1/2)(1-b)bs_x^2 t}(1-l)^b K(dt+dy) \qquad (7)$$

That is, the aggregate production function is linear in the accumulating stock of capital, with productivity increasing with employment. However, the presence of idiosyncratic risk in the capital accumulation process causes the productivity of capital to decline with time, and thus precludes the existence of a stochastic balanced growth path.[6]

A stochastic balanced growth can be restored by introducing an exogenous stochastic labor-augmenting component that has the property

$$E\left(B_i^b\right) = e^{-(1/2)(1-b)bs_x^2 t} \qquad (8)$$

There are several ways this may be achieved, the most natural being to assume

$$B_i = e^{-(1/2)s_x^2 t + x_i(t) - x_i(0)} \qquad (9)$$

This specification asserts that, $B_i$, the stochastic labor-augmenting technological change impinging on individual's *i* technology comprises two components. First, the accumulation of past idiosyncratic exogenous total factor productivity shocks enhances the labor-augmenting technology and has a positive permanent impact on labor efficiency. On the other hand, the volatility associated with the idiosyncratic exogenous productivity shocks has an adverse impact on labor efficiency. Our production technology is a hybrid of the exogenous labor-augmenting technological change, $B_i$ and the externality from spillovers, *K*, familiar from Arrow (1962) and Romer (1986). Substituting (9) into (1) and using (4) we see that in equilibrium, individual output follows the process

[6] The reason for this is the fact that for Brownian motion processes variances are first order terms.

$$dQ_i = A(1-l)^b K_i\,(dt + dy + dz_i) \equiv Z_i(dt + dy + dz_i) \tag{10a}$$

while substituting (9) directly into (2') and evaluating, equilibrium aggregate output evolves according to

$$dQ = A(1-l)^b K(dt + dy) \equiv Z(dt + dy) \tag{10b}$$

Thus, the introduction of the stochastic labor-augmenting technological change with the externality from spillovers ensures that in equilibrium both individual and aggregate output are generated by stochastic "AK" technologies and therefore are consistent with an equilibrium stochastic balanced growth path. It is important to stress that both (10a) and (10b) are equilibrium relationships.

We assume that the wage rate, $r_{L_i}$, over the period $(t, t+dt)$ paid by producer *i* is determined at the start of the period and is set equal to the expected marginal physical product of labor in that firm over that period. The total return to labor, $dR_{L_i}$, over the period is thus specified nonstochastically by

$$dR_{L_i} = r_{L_i} dt = E\left(\frac{\P Z_i}{\P(1-l)}\right)dt = bA(1-l)^{b-1}K_i dt \tag{11a}$$

which is directly proportional to the capital stock employed by the firm. In equilibrium, firms having more capital and therefore more productive workers pay proportionately higher wages.

We assume that capital depreciates non-stochastically at the rate $d$ per unit of time. The rate of return to capital in firm *i* is thus determined residually, by

$$dR_{K_i} = \frac{dQ_i - dK_i dt - (1-l)dA_i}{K_i} \equiv r_K dt + du_{K_i} \tag{11b}$$

where

$$r_K \equiv (1-b)A(1-l)^b - d; \quad du_{K_i} \equiv \frac{Z_i}{K}(dy + dz_i) = A(1-l)^b\, dy$$

The average economy-wide wage rate and return to capital are thus respectively

$$dR_L = r_L dt = E\left(\frac{\P Z}{\P(1-l)}\right)dt = bA(1-l)^{b-1}Kdt \tag{11a'}$$

$$dR_K = \frac{dQ - dKdt - (1-l)dA}{K} \equiv r_K dt + du_K \qquad (11b')$$

where $r_K$ is defined above and

$$du_K \equiv \frac{Z}{K} dy = A(1-l)^b \, dy$$

Individual and aggregate returns to capital have the identical means, though the former is more volatile, since the idiosyncratic risk is eliminated in the aggregate. [7]

According to this specification, the wage rate is fixed over the period $(t, t+dt)$, with all short-run fluctuations in output being reflected in the stochastic return to capital. While this allocation of risk may seem extreme, it may be rationalized with the argument that wages are sluggish due to contractual arrangements. Furthermore, casual empirical evidence suggests that the returns to capital are more volatile than are wages.[8] Equations (11) imply further that the mean rate of return to capital is constant through time, while the average wage rate grows with the aggregate capital stock. The characteristics are consequences of the aggregate AK technology.

### 2.2 Individual Consumption, and Capital Accumulation

The individual agent is assumed to choose his consumption and rate of capital accumulation to maximize the expected value of the intertemporal constant elasticity utility function

$$E\int_0^\infty \frac{1}{g}\left(C_i l^q\right)^g e^{-rt} dt \qquad -\infty < g < 1, q > 0,, r > 0 \qquad (12a)$$

subject to the stochastic accumulation equation

$$dK_i = \left(r_K K_i + r_{L_i}(1-l) - C_i\right)dt + K_i du_{K_i} \qquad (12b)$$

[7] The source of idiosyncratic risk in our model is individual total factor productivity as opposed to labor income. This is a plausible and alternative source of variability given for example the possibility of household productive activities and individual-specific human capital accumulation not captured by the nonaccumulating factor.

[8] In the United States, for example, the relative volatility of real stock returns over the period 1855-1995 have been around 32% per annum, while the relative volatility of wages have been comparable to that of output, 2%.

where we assume that consumption and leisure are chosen at the nonstochastic rates $C_i dt$, $ldt$, respectively. It is important to note that the agent, being atomistic, treats his wage as evolving exogenously, although in equilibrium it is tied to his capital stock in accordance with (11a).

In the Appendix we show that the solution to this problem implies the following equilibrium for the individual agent:

$$\frac{dK_i}{K_i} = \frac{1}{1-g}\left(r_K - r + \frac{1}{2}g(g-1)\left(s_w^2 + s_x^2\right)\right)dt + dw + dx_i \equiv y_i dt + dw + dx_i \tag{13a}$$

$$\frac{C_i}{K_i} = \frac{1}{1-g}\left(r - gr_K + (1-g)(1-l)\frac{r_{L_i}}{K_i} - \frac{1}{2}g(g-1)\left(s_w^2 + s_x^2\right)\right) \tag{13b}$$

$$\frac{C_i}{K_i} = \frac{l}{q}\frac{r_{L_i}}{K_i} \tag{13c}$$

$$r_K = (1-b)A(1-l)^b - d \tag{13d}$$

$$\frac{r_{L_i}}{K_i} = bA(1-l)^{b-1} \tag{13e}$$

$$dw = A(1-l)^b\, dy; \quad s_w^2 = A^2(1-l)^{2b} s_y^2 \tag{13f}$$

$$dx_i = A(1-l)^b\, dz_i; \quad s_x^2 = A^2(1-l)^{2b} s_z^2 \tag{13h}$$

In addition the equilibrium must satisfy the transversality condition, which for the constant elasticity utility function is given by

$$\lim_{t\to\infty} E\left[K_i^g e^{-rt}\right] = 0 \tag{13j}$$

and in the Appendix we show that this condition reduces to $C_i/K_i > A(1-l)^b b$.[9]

Equations (13a) and (13b) describe the individual's mean growth and consumption-capital ratio, while (13c) is the marginal rate of substitution between consumption and leisure. Substituting for (13d) – (13h) equations (13a) – (13c) jointly determine $y_i, C_i/K_i$, and $l$, in terms of parameters

[9]This condition asserts that consumption/capital ratio exceeds labor income, a condition that is met in all of our simulations. It reduces to the original condition $C/K > 0$, due to Merton (1969), in the absence of labor income (as assumed by Merton).

that are assumed to be identical for all agents, thus validating our assumption that each agent's labor supply is identical. The key point to observe about these equations is that the agent's equilibrium depends upon the overall volatility of wealth, as measured by the sum of the economy-wide and the individual-specific variances. It is only with respect to specific realizations of the idiosyncratic shocks that individual agents may differ. In particular, the consumption/capital ratio in (13b,c) is identical for all *i* so that perfect aggregation across individuals is feasible. The term involving the overall volatility, $s_w^2 + s_x^2$ in the consumption/capital ratio in (13b) represents the precautionary saving component of the marginal propensity to consume.

## 2.3 Macroeconomic Equilibrium

Averaging (13a) and (13b) over the *I* individuals in the economy and substituting for the equilibrium returns to capital and labor we see that the key equilibrium quantities, namely the equilibrium economy-wide growth rate, the consumption-capital ratio, fraction of leisure time, and aggregate and individual volatilities are given by

$$\frac{dK}{K} = \frac{1}{1-g}\left(A(1-l)^b(1-b) - d - r + \frac{1}{2}g(g-1)A^2(1-l)^{2b}\left(s_y^2 + s_z^2\right)\right)dt + dw \equiv y\,dt + dw \qquad (14a)$$

$$\frac{C}{K} = \frac{1}{1-g}\left(r - g\left(A(1-l)^b(1-b) - d\right) - \frac{1}{2}g(g-1)A^2(1-l)^{2b}\left(s_y^2 + s_z^2\right)\right) + bA(1-l)^b \qquad (14b)$$

$$\frac{C}{K} = \frac{l}{q}bA(1-l)^{b-1} \qquad (14c)$$

$$s_w^2 = A^2(1-l)^{2b}s_y^2; \quad s_w^2 + s_x^2 = A^2(1-l)^{2b}(s_y^2 + s_z^2) \qquad (14d)$$

Comparing (14a) with (13a) we see that the economy-wide mean growth rate is identical to the individual's rate of capital accumulation; both depend upon the economy-wide and the individual-specific risk. However, because the individual-specific shocks average out in the aggregate, the volatility of the aggregate growth rate is reduced to $s_w^2$, in contrast to $s_w^2 + s_x^2$, for the individual's rate of capital accumulation. Written in the form (14b), we see that effect of the net return to capital on consumption depends upon $-g$ , reflecting the fact that it has both a positive income effect and a negative substitution effect. In contrast, labor income, $bA(1-l)^b$ , is fully reflected in consumption. Thus (14b) is a generalization of the conventional expression for the consumption-capital ratio, to which it reduces in the absence of labor income.

Of particular significance is the welfare of the representative agent, as the economy evolves along its stochastic equilibrium growth path. In the Appendix we show that

$$\Omega \equiv \int_0^\infty \frac{1}{g} C_i^g e^{-rt} dt = \frac{K_0^g \left( (C_i/K_i) l^q \right)^g}{g \left[ C_i/K_i - bA(1-l)^b \right]} \qquad (15)$$

Given the transversality condition (15) implies $\Omega g > 0$ . Equation (15) forms the basis for analyzing the impacts of changes in volatility on economic welfare. We do so by converting the changes implied by (11b) into certainty equivalent measures of initial capital stock.

## 3. Economy-Wide and Individual Risk and Economic Performance

We turn now to the main issue, namely the impact of economy-wide and idiosyncratic risks on the growth rate and economic welfare.

### 3.1 Qualitative Effects

The qualitative effects of an increase in either economy-wide risk, $s_y^2$, or idiosyncratic risk, $s_z^2$, on the key equilibrium quantities can be immediately determined from equations (14) and (15). Since both sources of risk affect $y, C/K, l,$ and $\Omega$ additively, they have the same qualitative impact. Thus, in the more plausible case where $g < 0$, an increase in either source of risk will reduce the consumption to capital ratio, increase the time devoted to labor (reduce $l$) and thus raise the productivity of capital, its growth rate, and volatility. This is because with sufficiently risk averse agents, higher capital variability requires a higher rate of return on investment which is associated with higher growth rates. The reduction in the consumption-capital ratio is also a reflection of the positive precautionary savings effect, a further manifestation of the higher volatility on growth. This relationship, generally typical of linear stochastic growth models of this type, runs counter to some recent empirical evidence suggesting that volatility and growth are negatively related.[10] Irrespective of the impact on consumption and growth, higher volatility has an adverse effect on welfare. From (11b), together with the transversality condition, and letting $c \equiv C/K$, we see that

$$\frac{\partial \Omega}{\partial s_j^2} = \frac{\left[(c - bA(1-l)^b)g - c\right]c^{g-1}}{(c - bA(1-l)^b)^2} < 0 \quad for\ g < 1 \qquad j = y,z$$

### 3.2 Calibration

The qualitative effects just noted are straightforward. It is obvious that the idiosyncratic shocks, by influencing the equilibrium additively with the economy-wide shocks, provide a reinforcing effect. The interesting issue is one of the magnitudes and to this we now turn.

We calibrate the model using the following parameters characteristic of the US economy.

| | |
|---|---|
| Production parameters | $b = 0.6,\ A = 0.65,\ d = 0.04$ |
| Preference parameters | $r = 0.04;\ g = -1.5, -4, -8$<br>$q = 1,\ 1.75,\ 2.5$ |
| Stochastic shocks | $s_y = 0,\ 0.025,\ 0.04;$<br>$s_z = 0,\ 0.15,\ 0.20,\ 0.26$ |

The production parameters are standard. The choice of $b$ implies the elasticity of labor in production is 0.6, while the choice of $A$ and $q$ implies a *K/Y* ratio in the range of 3.2. Setting the rate of depreciation at 4% implies the (mean) net return to capital in the economy is 8.6%. The rate of time preference of 4% is also standard. Empirical evidence on the coefficient of relative risk aversion, $R \equiv 1 - g$, is far-ranging. Epstein and Zin (1991) obtain values of $R$ clustering around unity, consistent with a logarithmic utility function, while at the other extreme, issues pertaining to the "equity premium puzzle" induce authors to take $R$ as high as 18 (Obstfeld, 1994) or even 30 (Kandel and Stambaugh 1991). However, Constantinides, Donaldson, and Mehra, (1998) present alternative empirical evidence to suggest that $R$ lies most plausibly in the range 2–5, a range that

[10] This same result is obtained by Obstfeld (1994) and Grinols and Turnovsky (1998). It runs counter to recent empirical evidence by Ramey and Ramey (1995) that finds growth and volatility to be negatively correlated. However, the empirical evidence is not unambiguous and early studies by Kormendi and Meguire (1985) obtain a positive relationship, more supportive of the qualitative implications of this model.

appears to be gaining increasing acceptance. Since one of the key issues concerns the role of the coefficient of risk aversion, we allow $g$ to lie in the range –1.5 to –8, with the corresponding values of *R* being between 2.5 and 9.[11]

The parameter $q$ describes the degree of substitution between leisure and consumption in utility. The value $q = 1.75$ corresponds to the value chosen in the business cycle literature and implies equilibrium fractions of time devoted to leisure of around 0.7, consistent with the empirical evidence. In effect, $q$ may be related to measures of the elasticity of labor supply with respect to the real wage. Thus, $q = 1,\ q = 2.5$ correspond to low substitution for leisure (or low elasticity of labor supply) and high substitution for leisure (or high elasticity of labor supply), respectively. [12]

The critical parameters of our numerical simulations are the relative volatility of average per-capita income and individual idiosyncratic shocks. Gali (1994) provides estimates of $s_y$ for OECD countries, measured as percentage variations of GDP about trend output. The mean figure he obtains using this measure is around 6%; the figure for the US being 3.6%. Other authors, using different measures obtain somewhat smaller estimates, around 2.5% being typical for the US; see Gavin and Hausmann (1995), Ramey and Ramey (1995), Danthine and Donaldson (1993). Estimates obtained for $s_z$ are much larger. Pischke (1995) finds the standard deviation of idiosyncratic shocks to be around 6.5 times as large as the standard deviation of average per capita income. Deaton and Paxson (1994) find a similar pattern using the volatility of consumption data. Storesletten et. al. (1999) use the same methodology of Deaton and Paxson (1994) to provide direct GMM-based estimates of the standard deviation of idiosyncratic shocks that are much larger than the previous estimates.

A key factor that we wish to consider concerns the potential co-reduction of the individual volatility with the aggregate volatility. The seminal work by Lucas (1987) focused entirely on the welfare gains of eliminating aggregate volatility, with no consideration of individual volatility.

[11] Note that by using the constant elasticity utility function, $g$ is related to both the coefficient of relative risk aversion *R* and the elasticity of intertemporal substitution $e$ by $R = 1 - g = 1/e$. Thus pegging $g$ at –1.5, -4, -8, is also equivalent to assuming $e = 0.4,\ 0.2,\ 0.11$, respectively, which is also consistent with the empirical evidence. Introducing a recursive preference function enables us to disentangle the two parameters $e, R$.

[12] See e.g. Hansen and Wright (1992) and Cooley (1994) for business cycle models that include endogenous labor/leisure choice. The elasticity of labor supply that we refer to is evaluated in general equilibrium across balanced growth paths and, due to the nonseparability of labor and leisure in the utility function, it is a function of the endogenous choice of leisure, $l/(1-l)$ as well as $q$. However, nonseparability in utility has the desirable property of guaranteeing a balanced growth path with constant leisure, i.e. income and substitution effects cancel out, see e.g Caballe and Santos (1993).

Subsequent authors such as Atkeson and Phelan (1994) and Krusell and Smith (1999), who do introduce idiosyncratic risk, nevertheless also analyze the welfare gains of stabilizing for aggregate shocks, holding idiosyncratic risk unchanged.

On the other hand, it is plausible to argue that idiosyncratic risk is in part a function of aggregate risk, a relationship that can be formulated in different ways. Two recent papers by Storesletten et. al, (2001) and Beaudry and Pages (2001) argue that the idiosyncratic risk varies with the realization of the aggregate shocks, being higher when the economy suffers an adverse aggregate shock. In this case, stabilizing for aggregate shocks implies that all volatility -- aggregate and idiosyncratic -- is eliminated.

We adopt an intermediate position and argue that idiosyncratic risk is likely to be a positive function of aggregate volatility. To test for this dependence we have run regressions of the cross sectional standard deviation of individual earnings from PSID data on the standard deviation of aggregate GDP growth for the U.S. economy.[13] Specifically, we ran a regression similar to Gourinchas (2000), controlling for aggregate unemployment and the level of GDP growth, but adding the standard deviation of the GDP growth rate to the right-hand-side as a measure of aggregate volatility:

$$\underset{}{s_z} = \underset{(0.013)}{0.447} + \underset{(0.199)}{4.040}\, u + \underset{(0.112)}{1.256}\, gdp + \underset{(0.193)}{0.906}\, s_y \; ; \qquad R^2 = 0.54 \tag{16a}$$

where numbers in parentheses are standard errors of the estimates and $s_z$ is the standard deviation of the cross sectional individual earnings, $u$ is unemployment, $gdp$ is GDP growth and $s_y$ is the standard deviation of GDP growth where data is from 1979 to 1992, see Gourinchas (2000) for further details of the data. We also ran a regression without controlling for aggregate unemployment and the level of GDP growth:[14]

[13] The data used was kindly provided by Pierre-Olivier Gourinchas from his paper, Gourinchas (2000).

[14] The regressions in Gourinchas (2000, p20) use the variance of the cross sectional individual earnings rather than its standard deviation as the dependent variable, regressing it on *u* and *gdp*. We have also run regressions adding the variance of the growth of GDP to the right-hand-side of his equation, as well as an equation analogous to (16b) in terms of the variances, that does not control for aggregate unemployment and the level of GDP growth. In both cases, adding the variance of GDP is statistically significant and shows a positive dependence between individual and aggregate volatility as measured by variances. However, for our purposes we find the relationship in terms of standard deviations to be more convenient.

$$s_z = 0.725 + 2.253\ s_y; \qquad R^2 = 0.15 \qquad (16b)$$
$$(0.005)\ \ (0.229)$$

All regression coefficients are statistically significant, implying in particular that aggregate volatility does indeed exert a significant impact on individual volalility. These two regressions suggest that it is quite plausible to associate a 1 percentage point reduction in aggregate risk, $s_y$, with a 1–2 percentage point reduction in idiosyncratic risk, $s_z$.

With these data on volatility in mind, we begin by considering a benchmark economy in which there is no production risk. We then introduce an economy-wide production risk of 2.5%, consistent with the Ramey-Ramey and other aggregate studies. We next add idiosyncratic risk of 6 times the size of the aggregate risk, consistent with the Pischke (1995) evidence, setting $s_z = 0.15$, and in order to obtain some idea of the sensitivity to $s_z$, we increase it to 0.20. Finally, we consider a slightly riskier economy in which $s_y = 0.04, s_z = 0.26$, the relative magnitudes of the two types of risk again being consistent with the empirical evidence.

Equilibrium values for key quantities are reported in Table 1. In each case we compute the welfare gains from eliminating all the aggregate risk under varying assumptions regarding the extent to which the reduction in aggregate risk is accompanied by a reduction in idiosyncratic risk. These results are reported in Table 2. The striking conclusion of these results is that the reduction of aggregate risk need be accompanied by only a modest elimination of idiosyncratic risk -- certainly well within the degree suggested by the empirical evidence -- in order for aggregate stabilization to yield significant welfare improvement.

### 3.3 Numerical Results

Table 1 presents equilibrium values pertaining to some of the main production variables. In all cases we focus on the labor flexibility parameter $q = 1.75$ as representing the most plausible case, and subsequently consider the impact of variations in this parameter. Panel A reports the benchmark case of zero risk. For a coefficient of risk aversion $R = 2.5$, we obtain an equilibrium growth rate of 1.81%, with 0.704 of the agent's time being allocated to leisure, implying an output-capital ratio of around 0.31, and a consumption to capital ratio of over 25%. The ratio $C/K$ of around 0.25 is reasonably close to the empirical evidence suggested by Carroll (2000), this being due to the inclusion of labor income, in the absence of which $C/K$ would otherwise drop to around 0.07. In a riskless economy, an increase in $g$ represents a decrease in the intertemporal elasticity of substitution. Thus we see that $g = -4, g = -8$ are associated with increases in consumption and reductions in the growth rate.

Panel B introduces aggregate risk of 2.5%. The main point to observe is that aggregate risk of this magnitude has a negligible impact on the equilibrium, even for values of the coefficient of risk aversion as high as 9. This finding is consistent with Lucas (1987) and Turnovsky (2000).

Certain aspects of the equilibrium change dramatically with the introduction of idiosyncratic risk, $s_z = 0.15$ in Panel C. In Part (i), for a moderate degree of risk aversion $R = 2.5$, the growth rate jumps to just 2%, and while the aggregate volatility remains low at 0.8%, the individual volatility increases dramatically to 4.8%. On the other hand, labor supply, the capital-output ratio, and the consumption-capital ratio are all relatively insensitive to the degree of risk in the economy -- either aggregate or idiosyncratic – even for a relatively high degree of risk aversion.

In Part (ii) of Panel C, the idiosyncratic risk is increased from 0.15 to 0.20, e.g. Storesletten, et al (1999). This change has a substantial impact on the equilibrium growth rate and its volatility. The same pattern continues further in Part (iii) of Panel C which considers a slightly riskier economy in which $s_y = 0.04, s_z = 0.25$.

One important feature common to the three cases in Panel C is the non-monotonicity of the mean growth rate with respect to *R*. This is in contrast to the monotonic decrease in (mean) growth with *R* in Panels A and B. The reason for this is that, for the constant elasticity utility function, an increase in (the magnitude of) $g$ reflects both a decrease in the intertemporal elasticity of substitution and an increase in the degree of risk aversion, i.e. both effects are entangled. The effect of the lower intertemporal substitution is to increase consumption and to reduce the growth rate, while higher risk aversion has the exact opposite effect because there is only risky capital available for investment (incomplete insurance). Thus, for the cases of no risk or low levels of risk in Panels A and B, the former intertemporal substitution effect dominates and we get the observed monotonic decline in (mean) growth. But, for the case of higher levels of risk (including idiosyncratic risk) in

Panel C, as R increases, the two effects trade-off and we obtain the observed non-monotonic behavior of the mean growth rate.

In all cases the equilibrium is highly sensitive to the labor elasticity parameter $q$. Panels A-C illustrate the effects of an increase in the substitution between leisure and consumption (or increases in the elasticity of labor supply) measured by alternative values of $q$. As $q$ increases say from 1 to 2.5, *c/k* and $y/k$ decline as *l* increases. This is because increased substitution between leisure and consumption allows an agent to substitute toward more leisure and less consumption along the indifference curve across balanced growth paths. This reduces the productivity of capital, reducing the output capital ratio and the consumption to capital ratio. The reduction in work reduces both the aggregate and idiosyncratic risk and the corresponding mean growth rate. Most notably, more flexibility in labor supply is associated with reduced volatility at both the individual and aggregate levels. This is because adding the labor/leisure margin allows an individual to use labor supply flexibility to buffer the uninsurable risk.

### 3.4 Gains from Aggregate Stabilization

We now turn to the key results, presented in Table 2. This table reports the effects of eliminating the aggregate volatility, accompanied by varying degrees of reductions in idiosyncratic risk. Again, we shall focus our remarks on the benchmark case $q = 1.75$. Suppose that the economy is in an initial equilibrium with aggregate risk $s_y = 0.025$ and idiosyncratic risk $s_z = 0.15$, [Panel C(i) of Table 1]. Suppose also that the stabilization authority decides to eliminate all the aggregate risk. In this case, if doing so has no effect on idiosyncratic risk, then if the coefficient of risk aversion $R = 2.5$, the welfare gain so obtained is only 0.06% of the initial aggregate capital stock. Moreover this increases to only 0.14% if the coefficient of risk aversion reaches $R = 9$, an upper bound on its plausibility values. These essentially negligible welfare effects confirm the familiar results of Lucas (1987).

Now suppose that the reduction in aggregate risk of 0.025 is accompanied by a corresponding reduction in idiosyncratic risk by 0.025 from 0.15 to 0.125, as suggested by the regression equation (16a). In this case we find that the welfare gains from stabilization increase dramatically. If $R = 2.5$ they increase approximately 12-fold to 0.73%, and to over 1% for higher, but plausible degrees of risk aversion. Moreover, the gains are due overwhelmingly to the reduction in the idiosyncratic risk rather than to the elimination of the aggregate risk. In other words a reduction in idiosyncratic risk from 0.15 to 0.125 is far more beneficial than an equivalent reduction in aggregate risk from 0.025 to 0.

If the reduction in aggregate risk is accompanied by an approximate doubling in the reduction of idiosyncratic risk from 0.15 to 0.10, as suggested by (16b), then the gains from the stabilization of the aggregate risk increase to between 1.3% to 2.8%, depending upon the degree of risk aversion. If further, in the process of eliminating the aggregate risk, the idiosyncratic risk is halved to 0.075, the gains increase further to between 1.7% and 3.7%. Finally, if in the process of stabilizing the aggregate risk, the idiosyncratic risk is also eliminated entirely as in Storesletten et. al, (2001) and Beaudry and Pages (2001), the welfare gains increase to between 2.3% and 4.9%. In these extreme cases, the welfare gains from eliminating moderate aggregate risk are unquestionably substantial. But it is interesting to note that *over half* of the maximum potential welfare gains can be achieved by eliminating just *one third* of the idiosyncratic risk. This is an obvious consequence of the fact that the risk appears as a variance in the equilibrium.

Looking across Panel 2 (i) we see that since low values of $q$ are associated with increased risk at both the aggregate and individual levels, the gains from stabilization are correspondingly increased, and similarly reduced as $q$ increases. We have already noted that leisure increases with $q$. But the positive welfare effect of additional leisure is dominated by the negative effects on growth and consumption, reducing both overall welfare and the gains from stabilization as well.[15]

Panels (ii) and (iii) conduct a similar exercise for the higher degrees of risk and yield the same pattern of benefits. In all cases, we find that the welfare gains from eliminating aggregate risk with no reduction in idiosyncratic risk are extremely low. If one looks at this table overall, it seems reasonable to suggest that welfare gains from aggregate stabilization of the order of 2-3%, consistent with the empirical evidence by Clark, Leslie, and Symons (1994) are not implausible, even with

[15] In computing the certainty equivalent measures of changes in the initial capital stock underlying this welfare comparison, we also need to take account of the change in $q$ involved.

relatively modest accompanying reductions in idiosyncratic risk. Certainly it would seem highly likely that the welfare gains are at least 1%, and this certainly cannot dismissed as being negligible.

## 4. Some Implications for Asset Pricing

We now briefly examine the implications of the model for equilibrium asset pricing. To do this we consider the implicit pricing of a risk-free asset. Suppose such an asset (a bond) pays a return $r$. Following Saito (1998) and others, with all agents being identical the only equilibrium is the no trade equilibrium, the implication of which is that the equilibrium risk-free rate of return is

$$r = r_K - (1-g)(s_y^2 + s_z^2) \equiv (1-b)A(1-l)^b - d - (1-g)A^2(1-l)^{2b}(s_y^2 + s_z^2) \qquad (17a)$$

Thus, the risk premium on (untraded) capital is

$$r - r_K = (1-g)A^2(1-l)^{2b}(s_y^2 + s_z^2) \qquad (17b)$$

Table 3 reports the rates of return, the premium on the return to capital, and the savings rate, $s/y$, for the degrees of aggregate and idiosyncratic risk considered previously. The following patterns can be identified.

(i) The mean return on capital is relatively insensitive to both the degrees of risk aversion and to the degrees of aggregate and idiosyncratic volatility. This is because it is determined by the labor supply, which is insensitive to these parameters.

(ii) In the presence of idiosyncratic risk, the mean return on capital and the savings rate are both nonmonotonically related to the degree of risk aversion. This reflects the corresponding nonmonotonicity in the growth rate, noted earlier.

(iii) Aggregate volatility has a negligible effect on the riskless rate. In contrast, idiosyncratic risk has a substantial negative impact on the riskless rate, particularly when the degree of risk aversion is large. This is because high savings associated with the precautionary motive drives the rates of return down, and with the mean rate of return on capital insensitive to idiosyncratic shocks, the adjustment is borne by the riskless rate.

(iv) In the case of relatively high volatility in Panel (iii) for $R = 9$ and $q = 1.75$ the premium to capital reaches 6%, although the riskless rate is still around 2.4%. Moreover, if for this last case one increases the share of labor in production to $b = 0.65$, then the riskless rate drops to 0.8%, with the risk premium increasing to 6.1%, consistent with the empirical evidence. The implied welfare gains from eliminating the aggregate risk remain around 3% as long as they are associated with a modest reduction in the idiosyncratic risk from 0.26 to 0.24.

(v) Both the rate of return on capital and the risk-free rate are highly sensitive to the flexibility of labor supply, $q$. As $q$ increases, the impact falls relatively more on the rate of return on capital causing the equity premium to be less sensitive to the degree of idiosyncratic risk. Again, more labor supply flexibility helps individuals buffer the uninsurable risk thus reducing the price of the riskless asset and increasing its rate of return. However, we note that even with labor supply flexibility, modest decreases in idiosyncratic risk can generate large gains from stabilization and plausible risk premium.

In effect, our results confirm the results of Costantinides and Duffie (1996) and Saito (1998) that idiosyncratic permanent nondiversifiable shocks increase the risk premium. Our contribution is to show that even with endogenous labor/leisure choice functioning as a buffer to this kind of risk, the effect of idiosyncratic shocks on asset pricing is quantitatively significant.

## 5. Conclusions

A plausible dynamic stochastic general equilibrium macroeconomic structure should be able to explain some anomalies observed in the real world. In this regard, stochastic models in which all shocks are economy-wide imply that the welfare costs of observed aggregate volatility are negligible. But in reality idiosyncratic shocks exist and are important, and indeed empirical evidence suggests that the volatility of such shocks is several times that of aggregate shocks. Thus in this paper we have derived an equilibrium growth path in which agents are subject to both types of shocks.

Although the existence of idiosyncratic shocks has little impact on the welfare gains obtained from eliminating aggregate shocks alone, again it seems plausible and is supported empirically that the magnitude of idiosyncratic risk is positively related to the degree of aggregate risk. Thus we find that if in the process of eliminating the aggregate risk, the policymaker can reduce (but not eliminate completely) the typical agent's idiosyncratic risk by an amount suggested by the empirical evidence, the welfare gains from aggregate stabilization are no longer insignificant. On the contrary they may plausibly be of the order 2-3%, consistent with the empirical evidence of Clark, Leslie, and Simmons (1994). Moreover, the bulk of the gains are obtained from the reduction of the idiosyncratic risk, by even modest amounts, rather than from the macro stabilization. This finding carries with it the policy implication that a large payoff to aggregate stabilization policy is to try and stabilize the environment in which individuals operate.

The introduction of idiosyncratic risk has important implications for asset pricing, and in particular may reduce the risk-free rate substantially. By impinging significantly on precautionary savings, it puts downward pressure on the rates of return. However, with the mean productivity of capital and its mean rate of return determined by labor supply, which is insensitive to idiosyncratic shocks, the adjustment is reflected primarily in the risk-free rate.

Many of our results are sensitive both to the degree of risk aversion, which we have restricted to lie within an empirically plausible range, and also to the flexibility of labor supply. As the flexibility of labor supply increases, volatility is reduced and the benefits from aggregate stabilization decline as well. We provide a general equilibrium version of the result of Bodie et al (1992) that labor supply flexibility helps smooth consumption by buffering risk.

In general, our model highlights the tradeoffs involved in analyzing the effects of risk on growth and welfare, on the one hand, and on asset pricing, on the other. These tradeoffs involve many dimensions, including the flexibility of the labor supply, thus emphasizing the need to examine these issues within a stochastic general equilibrium framework.

There are several fruitful avenues for future research. The framework presented here is general enough to analyze issues of government spending and taxes. An important avenue is in the direction of endogenizing the extent of the nondiversification of risk with the introduction of agency relationships in the mean-variance framework presented here.

## Appendix

### A.1 Derivation of Optimality Conditions

The representative agent's stochastic optimization problem is to choose consumption and the rate of capital accumulation to maximize:

$$E_0\int_0^{\infty}\frac{1}{g}\left(C_i(t)l(t)^q\right)^g e^{-rt}dt \tag{A.1a}$$

subject to the stochastic capital (wealth) accumulation equation:

$$dK_i = (r_K K_i(t) + r_{L_i}(t)(1-l) - C_i)dt + K_i du_{K_i} \tag{A.1b}$$

where the agent takes $r_K, r_{L_i}$ as given, and $du_{K_i} \equiv A(1-l)^b(dy + dz_i)$. These are functions of aggregate (average) labor supply and are also taken as given by the individual agent Dividing (A.1b) by $K_i$, yields

$$\frac{dK_i}{K_i} = \left(r_K + \frac{r_{L_i}(t)(1-l)}{K_i} - \frac{C_i}{K_i}\right)dt + du_{K_i} \equiv y_i dt + du_{K_i} \tag{A.1b'}$$

where $y_i$ denotes the agent's mean rate of capital accumulation.

Equation (11a') of the text specifies that the *equilibrium* wage rate is tied to the individual's capital, $K_i$ However, this is only the case in equilibrium and the individual in making his decisions, does not perceive this. Instead, he perceives his wage rate as growing exogenously with time, independently of his own capital, $K_i(t)$, and hence we write $r_L(t)$.

Since the individual perceives the state variable, $K_i$, and since time appears both additively (through $r_{L_i}(t)$) and through the exponential time discounting, we propose a value function of the time-separable form[16]

$$V(K_i, t) = e^{-rt}\left[X(K_i) + H(t)\right] \tag{A.2}$$

[16] It is also possible to solve the stochastic optimization problem by postulating a value function of the form $V(K_i, K, t) \equiv e^{-rt}X(K_i, K)$. This formulation involves two state variables and is equivalent to, but more cumbersome than, the approach adopted.

We define the differential generator of the value function $V(K_i, t)$ to be

$$\Psi[V(K_i,t)] \equiv \frac{\P V}{\P t} + \left(r_K K_i + r_L(t)(1-l) - C_i\right)\frac{\P V}{\P K_i} + \frac{1}{2} s_u^2 K_i^2 \frac{\P^2 V}{\P K_i^2} \qquad \text{(A.3)}$$

where for convenience we let $s_u^2 \equiv s_w^2 + s_x^2 = A^2(1-l)^{2b}(s_y^2 + s_z^2)$ denote the sum of the variances of the economy-wide and idiosyncratic shock.

The individual's formal optimization problem is to choose $C_i$ to maximize:

$$e^{-rt}\frac{1}{g}\left(C_i l\right)^g + \Psi\left[e^{-rt}[X(K_i) + H(t)]\right] \qquad \text{(A.4)}$$

Taking the partial derivative of (A.4) with respect to $C_i$ and $l$ and canceling $e^{-rt}$, yields

$$C_i^{g-1}(1-l)^{qg} = X_K(K_i) \qquad \text{(A.5a)}$$

$$qC_i^g(1-l)^{qg-1} = r_{L_i}(t)X_K(K_i) \qquad \text{(A.5b)}$$

where $X_K(K_i)$ is the marginal value of an extra unit of capital. Dividing (A.5b) by (A.5a) leads to equation (13c). In principle, we may solve equations (A.5a) and (A.5b) to obtain the following expressions for the individual’s consumption and labor supply

$$C_i \equiv C(K_i, r_{L_i}(t)) \qquad \text{(A.6a)}$$

$$l \equiv l(K_i, r_{L_i}(t)) \qquad \text{(A.6b)}$$

In addition, the value function must satisfy the Bellman equation

$$\max_{C_i}\left\{e^{-rt}\frac{1}{g}\left(C_i l^q\right)^g + \Psi\left[e^{-rt}[X(K_i) + H(t)]\right]\right\} = 0 \qquad \text{(A.7)}$$

which may be expressed as (where dot denotes time derivative):

$$\frac{1}{g}\left[C(K_i, r_L(t))l(K_i, r_L(t))^q\right]^g - r\left[X(K_i) + H(t)\right] + \dot{H}(t) + \left(r_K K_i + r_{L_i}(t)(1-l) - C_i(K_i)\right)X_{K_i}(K_i)$$

$$+\frac{1}{2}K_i^2 X_{K_i K_i}(K_i)s_u^2 = 0 \qquad \text{(A.8)}$$

This Bellman equation holds for all values of $K_i$, at all points of time *t*. Thus we can take the partial derivative of this equation with respect to $K_i$. In so doing, we note that $H(t)$ is independent of (the agent's) $K_i$, while (A.6) implies that $C_i$ (and potentially *l)* is a function of $K_i$. Performing this calculation yields:

$$C_i^{g-1}(1-l)^{qg}\,\partial C_i/\partial K_i - qC_i^{g}(1-l)^{qg-1}\,\partial l/\partial K_i - rX_{K_i} + (r_K - \partial C_i/\partial K_i + r_{L_i}\,\partial l/\partial K_i)X_{K_i}$$

$$+s_u^2 K_i X_{K_iK_i} + \frac{1}{2}s_u^2 K_i^2 X_{K_iK_iK_i} = 0$$

and using (A.6a, A6b) this reduces to

$$(r_K - r)X_K + \left[r_K K_i + r_{L_i}(t)(1-l) - C_i\right]X_{K_iK_i} + s_u^2 K_i X_{K_iK_i} + \frac{1}{2}s_u^2 K_i^2 X_{K_iK_iK_i} = 0 \qquad \text{(A.9)}$$

Consider now $X_{K_i} = X_{K_i}(K_i)$, the stochastic differential of which is:

$$dX_{K_i} = X_{K_iK_i}dK_i + \frac{1}{2}X_{K_iK_iK_i}(dK_i)^2 \qquad \text{(A.10)}$$

Taking expected values of (A.10), and dividing by *dt*, implies

$$\frac{E(dX_{K_i})}{dt} = \left[r_K K_i + r_{L_i}(t)(1-l) - C_i\right]X_{K_iK_i} + \frac{1}{2}s_u^2 K_i^2 X_{K_iK_iK_i} \qquad \text{(A.11)}$$

and substituting (A.11) into (A.9) leads to the relationship:

$$\frac{E(dX_{K_i})}{X_{K_i}dt} = (r - r_K) - s_u^2\frac{K_i X_{K_iK_i}}{X_{K_i}} \qquad \text{(A.12)}$$

The solution to this equation is by trial and error. Given the objective function (A.1a), we propose:

$$X(K_i) = eK_i^g \qquad \text{(A.13)}$$

where the parameter $e$ is to be determined. Evaluating the partial derivatives $X_{K_i}(K_i)$, $X_{K_iK_i}(K_i)$ and substituting into (A.12), the expected marginal utility evolves in accordance with:

$$\frac{E(dX_{K_i})}{X_{K_i}dt} = (r - r_K) + s_u^2(1-g) \qquad \text{(A.14)}$$

Combining with (A.10), the actual marginal utility follows the stochastic process

$$\frac{dX_{K_i}}{X_{K_i}} = \left[(r - r_K) + s_u^2(1-g)\right]dt - (1-g)du_i \qquad \text{(A.15)}$$

To determine the equilibrium growth path, we recall the optimality condition (A.6a). Taking the stochastic differential of this equation, with l being constant, implies

$$\frac{dC_i}{C_i} = \frac{1}{(g-1)}\frac{dX_{K_i}}{X_{K_i}} + \frac{1}{2}\frac{(2-g)}{(g-1)^2}\left(\frac{dX_{K_i}}{X_{K_i}}\right)^2$$

Using (A.15) to evaluate this expression leads to

$$\frac{dC_i}{C_i} = \frac{1}{1-g}\left(r_K - r + \frac{1}{2}g(g-1)s_u^2\right)dt + du_i \qquad \text{(A.16)}$$

Focusing on a stochastic balanced growth path along which $E(dC_i/C_i) = E(dK_i/K_i)$ and recalling the definition of $du_i = dw + dx_i$ leads to (13a) of the text:

$$\frac{dK_i}{K_i} = \frac{1}{1-g}\left(r_K - r + \frac{1}{2}g(g-1)\left(s_w^2 + s_x^2\right)\right)dt + dw + dx_i \qquad \text{(A.17a)}$$

Substituting this into (A.1b’) and focusing on the deterministic component leads to (13b) of the text:

$$\frac{C_i}{K_i} = \frac{1}{1-g}\left(r - gr_K + (1-g)(1-l)\frac{r_{L_i}}{K_i} - \frac{1}{2}g(g-1)\left(s_w^2 + s_x^2\right)\right) \qquad \text{(A.17b)}$$

## A.2 Solution for the Value function

Although the above solution has been obtained without completely solving the Bellman equation, we must nevertheless ensure that it is met. First, recall the optimality condition (A.5). Evaluating this for the value function (A.13) implies, so that

$$C_i = (eg)^{1/(g-1)}K_i \qquad \text{(A.18)}$$

Combining (A.18) with (A.17b) we obtain

$$\frac{C_i}{K_i} = (eg)^{1/(g-1)} = \frac{r - gr_K + (1-g)\frac{r_{L_i}(1-l)}{K_i} - \frac{1}{2}g(g-1)s_u^2}{1-g} \qquad \text{(A.19)}$$

Note that in equilibrium, when $r_{L_i} / K_i = bA(1-l)^{b-1}$ and therefore constant, the equilbrium

$$\frac{C_i}{K_i} = (eg)^{1/(g-1)} = \frac{r - gr_K + (1-g)bA(1-l)^b - \frac{1}{2}g(g-1)s_u^2}{1-g} \qquad \text{(A.19')}$$

and is constant, implying that $e$ is constant, consistent with the conjectured solution (A.13).

Now return to the Bellman equation, (A.8), written as

$$\frac{1}{g}C_i^g - r\left[X(K_i) + H(t)\right] + \dot{H}(t) + \left(r_K K_i + r_{L_i}(1-l) - C_i\right)X_{K_i}(K_i) + \frac{1}{2}K_i^2 X_{K_iK_i}(K_i)s_u^2 = 0$$

Substituting for (A.19) and recalling the assumed form of (A.13), we can write this in the form

$$K_i^g e\left[(1-g)(eg)^{1/(g-1)} - r + gr_K + g\frac{r_{L_i}(1-l)}{K_i} + \frac{1}{2}g(g-1)s_u^2\right] + \dot{H}(t) - rH(t) = 0 \qquad \text{(A.20)}$$

and substituting (A.19) into (A.20) the latter reduces to the following differential equation in $H(t)$,

$$\dot{H}(t) - rH(t) = -eK_i^{g-1}r_{L_i}(1-l) \qquad \text{(A.21)}$$

Since future values of $K_i$ are not yet known, the bounded solution to this equation is

$$H(t) = E_t\int_t^\infty eK_i(s)^{g-1}r_{L_i}(1-l)e^{-r(s-t)}ds \qquad \text{(A.22)}$$

Intuitively, (A.22) asserts that the (utility) value associated with the labor income stream, that the agent takes as exogenously given, equals the discounted expected stream of future wage income evaluated at the marginal utility of income, $eK_i^{g-1}$. The solution for the value function is thus :

$$V(K_i,t) = e^{-rt}\frac{1}{g}\left(\frac{C_i}{K_i}\right)^{g-1}\left\{K_i^g + E_t\int_t^\infty K_i(s)^{g-1}r_{L_i}(1-l)e^{-r(s-t)}ds\right\} \qquad \text{(A.23)}$$

## A.3 Evaluation of Welfare along the Equilibrium Path

The transversality condition is given by

$$\lim_{s\to\infty}E_0\left\{X_{K_i}K_i e^{-rs}\right\} = \lim_{s\to\infty}E_0\left\{eK_i(s)^g e^{-rs}\right\} = 0 \qquad \text{(A.24)}$$

Solving (A.17a), substituting into (A.18) and evaluating, this reduces to the condition

$$\frac{C_i}{K_i} > bA(1-l)^b \qquad \text{(A.25)}$$

Likewise, solving (A.16), substituting into (A.1a) and evaluating leads to

$$\Omega \equiv \int_0^{\infty} \frac{1}{g}\left(C_i l^q\right)^g e^{-rt} dt = \frac{K_0^g \left(C_i / K_i\right)^g l^{qg}}{g\left[C_i / K_i - bA(1-l)^b\right]} \qquad \text{(A.26)}$$

which is well-defined as long as the transversality condition is met.

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