

# User Cost of Capital & Cost Function : Does the Margin of Freedom in the Modelling Yield Robust Results?

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## Abstract

In this paper we report results of a rather successful attempt to provide evidence on the substitution possibilities between capital, labor, and energy in French manufacturing industry. In order to assess the sensitivity of results to the choice of the functional form and to data construction procedures, alternative specifications of the cost function are estimated, based on three flexible functional forms (i) the Translog, (ii) the Generalized Leontief, and (iii) the Symmetric Generalized McFadden while two different series of the user cost of capital are constructed (i) the implicit *ex ante* price (which includes fiscality parameters), obtained by extending the theoretical model of Jorgenson and (ii) the *ex post* price computed as the residual rate of return to capital, when zero profits are imposed at full equilibrium position. We show that the choice of the regressor is even important in the context of the choice of the functional form. We also consider the validity of the *ex post* approximation of the user cost of capital, frequently used in studies of production and investment behaviour, and find that our data do not support it. In cases where we can precisely estimate an elasticity, our results seem robust to the choice of the functional form and the method of calculating the user cost of capital. This explains the fact that regressions with the *ex post* price of capital yield elasticities that can diverge from the true values of elasticities and discrepancies between the estimates suggests that care should be taken during the economic interpretation of results. The issues analysed in the present paper are therefore not merely of theoretical interest, but are also of practical concern.

*Keywords* : user cost of capital, flexible cost functions, technical change, variability of the regressors, precision of fits, substitution possibilities, capital-energy elasticity.

*JEL Classification* : C51, D24, E22, H32.

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## ANNEXE I

### THEORETICAL CONSISTENCY PROPERTIES OF THE COST FUNCTION

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**1. Domain.**  $C(p, y, t)$  is a non-negative real-valued function.

$$C(p, y, t) \geq 0, \quad p, y > 0 \quad \text{and} \quad C(p, 0, t) = 0$$

**1. Monotonicity.**  $C(p, y, t)$  is non-decreasing in  $p$  and  $y$ .

$$\frac{\partial C}{\partial p_i} \geq 0 \quad \text{or} \quad x_i \geq 0, \quad i \quad \text{and} \quad \frac{\partial C}{\partial y} \geq 0$$

**2. Homogeneity.**  $C(p, y, t)$  is linear homogeneous in  $p$ . Hence Euler Theorem yields

$$\sum_{i=1}^n p_i \frac{\partial C}{\partial p_i} = C \quad (2.a), \quad \sum_{i=1}^n p_i \frac{\partial^2 C}{\partial p_i \partial p_j} = 0, \quad j \quad (2.b)$$

$$\sum_{i=1}^n p_i \frac{\partial^2 C}{\partial p_i \partial y} = \frac{\partial C}{\partial y} \quad (2.c), \quad \sum_{i=1}^n p_i \frac{\partial^2 C}{\partial p_i \partial t} = \frac{\partial C}{\partial t} \quad (2.d)$$

**3. Symmetry.**  $C(p, y, t)$  is twice continuous differentiable, young theorem implies

$$\frac{\partial^2 C}{\partial p_i \partial p_j} = \frac{\partial^2 C}{\partial p_j \partial p_i} \quad \text{or} \quad \frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i}, \quad i, j$$

i.e. that the Hessian matrix must be symmetric.

**4. Concavity.**  $C(p, y, t)$  is concave in  $p$  if

$$\left[ \frac{\partial^2 C}{\partial p_i \partial p_j} \right]_{i,j} \text{ is a negative semidefinite matrix,}$$

i. e. that the Hessian matrix must be negative semidefinite.

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*Note :* The homogeneity restrictions 2.a, 2.b, 2.c are known us the adding-up condition, the Cournot aggregation conditions, and the Engel aggregation condition, respectively.

Functional Forms	Unit Cost Function	Input Demand System	
translog	$\log C^{TL}(p, t)^a \equiv \alpha_0 + \alpha_t t + \sum_{i=1}^n \alpha_i \log p_i + \sum_{i=1}^n \alpha_{it} t \log p_i$ $+ 0.5 \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} \log p_i \log p_j + 0.5 \alpha_{tt} t^2, \quad \alpha_{ij} = \alpha_{ji} \quad i, j$	$S_i^{TL}(p, t) = \alpha_i + \sum_{j=1}^n \alpha_{ij} \log p_j + \alpha_{it} t$ <p>were <math>\sum_{i=1}^n S_i^{TL}(p, t) = 1</math></p>	$\epsilon_i^T$ <p>où <math>\delta</math></p>
Generalized Leontief	$C^{GL}(p, t) \equiv \sum_{i=1}^n \sum_{j=1}^n \beta_{ij} \sqrt{p_i p_j} + \sum_{i=1}^n \beta_{it} p_i t, \quad \beta_{ij} = \beta_{ji} \quad i, j$	$a_i^{GL}(p, t) = \sum_{j=1}^n \beta_{ij} \sqrt{p_j / p_i} + \beta_{it} t$	$\epsilon_{ij}^G$
Symmetric Gneralized McFadden	$C^{MF}(p, t)^b \equiv \sum_{i=1}^n \gamma_i p_i + 0.5 \sum_{i=1}^n \sum_{j=1}^n \frac{\gamma_{ij} p_i p_j}{p} + \sum_{i=1}^n \gamma_{it} p_i t, \quad \gamma_{ij} = \gamma_{ji}$ <p>where <math>p^c = \sum_{i=1}^n \theta_i p_i, \quad 0 &lt; \theta_i &lt; 1 \quad i</math></p>	$a_i^{MF}(p, t) = \gamma_i - 0.5 \theta_i \sum_{k=1}^n \sum_{j=1}^n \frac{\gamma_{kj} p_k p_j}{p^2}$ $+ \sum_{j=1}^n \frac{\gamma_{ij} p_j}{p} + \gamma_{it} t$	$C_{ij}^{MF}(p, t)$

Notations :  $S_i(p, t) = p_i x_i / \sum_{i=1}^n p_i x_i = \partial \log C(p, t) / \partial \log p_i$ ,  $a_i = x_i / y = \partial C(p, t) / \partial p_i$ ,  $C_{ij}(p, t) = \partial^2 C(p, t) / \partial p_i \partial p_j$ ,  $\epsilon_{ij} = (\partial \log x_i \cdot \partial \log p_i) = (p_j C_{ij}(p, t)) / a_i(p, t) = (p_i p_j C_{ij}(p, t)) / (C(p, t) S_i(p, t))$ , where  $x_i$ , for  $i = 1, \dots, n$  are input demands,  $S_i$  are cost shares,  $a_i$  are input-output ratios and  $\epsilon_{ij}$  are price elasticities.

### ANNEXE III : TABLES AND FIGURES

TABLE I  
SAMPLE SUMMARY STATISTICS : FRENCH MANUFACTURING 1970 1989

Branch		$y$	$p_K^A$	$p_K^P$	$p_L$	$p_E$	$a_K^A$	$a_K^P$	$a_L$	$a_E$	$S_K^A$	$S_K^P$	$S_L^A$	$S_L^P$	$S_E^A$	$S_E^P$	$CU^A$	$CU^P$
Intermediate	Mean	229881	1.271	1.165	1.070	0.912	0.150	0.234	0.592	0.217	0.194	0.269	0.605	0.547	0.201	0.183	0.944	1.025
	Std.	17416	0.802	0.639	0.652	0.516	0.005	0.008	0.133	0.006	0.042	0.050	0.040	0.026	0.036	0.040	0.466	0.482
	Min.	189446	0.403	0.516	0.249	0.237	0.140	0.218	0.409	0.206	0.127	0.194	0.538	0.498	0.150	0.130	0.348	0.393
	Max.	257505	2.581	2.564	2.133	1.739	0.164	0.255	0.854	0.226	0.260	0.349	0.681	0.594	0.258	0.246	1.543	1.733
Equipement	Mean	205760	1.262	1.188	1.093	0.964	0.098	0.188	0.837	0.042	0.125	0.223	0.835	0.741	0.040	0.035	0.948	1.047
	Std.	29111	0.814	0.469	0.694	0.550	0.010	0.019	0.225	0.004	0.037	0.037	0.038	0.033	0.004	0.005	0.456	0.467
	Min.	141406	0.387	0.674	0.246	0.272	0.086	0.165	0.539	0.036	0.071	0.150	0.772	0.685	0.034	0.028	0.377	0.443
	Max.	244236	2.651	2.303	2.283	1.754	0.115	0.220	1.284	0.051	0.190	0.281	0.891	0.808	0.045	0.043	1.596	1.795
Consumer	Mean	147219	1.277	1.043	1.106	0.958	0.097	0.267	0.717	0.054	0.137	0.281	0.808	0.673	0.055	0.046	0.856	1.012
	Std.	12078	0.817	0.453	0.708	0.556	0.005	0.013	0.176	0.009	0.032	0.019	0.029	0.016	0.005	0.005	0.419	0.468
	Min.	115010	0.399	0.478	0.251	0.255	0.090	0.247	0.496	0.041	0.084	0.251	0.764	0.645	0.045	0.039	0.340	0.409
	Max.	161662	2.602	1.807	2.325	1.765	0.107	0.294	1.076	0.070	0.188	0.311	0.858	0.697	0.063	0.053	1.495	1.703

Note : All input prices are normalized to unity for 1980. All input and output quantities are expressed in 1980 French francs.  $y$  is the output quantity index,  $p_i$  is the price index of input  $i$  ( $i = K, L, E$  which stands for capital, labour and energy respectively),  $a_i$  is the  $i$ th input-output ratios,  $S_i$  is the  $i$ th input cost share.

$CU^A$  means the unit cost of production evaluated with the ex ante price of capital ( $p_K^A$ ). The same criterion is applied for other entries.

$CU^P$  means the unit cost of production evaluated with the ex post price of capital ( $p_K^P$ ). The same criterion is applied for the other entries.

TABLE II  
ANNUAL PERCENTAGE CHANGES IN INPUT PRICES AND IN THE UNIT COST OF  
PRODUCTION, FRENCH MANUFACTURING, 1970-1989

Branch		$\Delta p_K^A$	$\Delta p_K^P$	$\Delta p_L$	$\Delta p_E$	$\Delta CU^A$	$\Delta CU^P$
Intermediate	Mean	0.093	0.095	0.120	0.104	0.084	0.083
	Std.	0.166	0.160	0.053	0.184	0.089	0.070
	Min.	-0.253	-0.345	0.036	-0.234	-0.046	0.007
	Max.	0.540	0.354	0.213	0.651	0.274	0.320
Equipement	Mean	0.093	0.073	0.125	0.102	0.079	0.077
	Std.	0.169	0.139	0.048	0.114	0.051	0.033
	Min.	-0.261	-0.200	0.041	-0.099	-0.014	0.024
	Max.	0.539	0.365	0.213	0.366	0.154	0.155
Consumer	Mean	0.093	0.069	0.125	0.107	0.082	0.079
	Std.	0.161	0.073	0.051	0.139	0.046	0.039
	Min.	-0.238	-0.083	0.051	-0.114	0.010	-0.004
	Max.	0.516	0.201	0.256	0.508	0.159	0.158

TABLE III  
CONCAVITY VIOLATIONS

Branch	ex ante price of capital ( $p_k^A$ )			ex post price of capital ( $p_k^P$ )		
	Generalized		Generalized	Generalized		Generalized
	Translog	Leontief		McFadden	Translog	
Intermediate Goods	16	20	20	20	5	0
Equipment Goods	4	0	0	6	0	0
Consumer Goods	5	0	0	0	0	0

*Note :* Concavity violations are sample points at which the estimated cost function is not concave. The sample size is 20.

TABLE IV  
PRICE ELASTICITIES AND ALLEN PARTIAL ELASTICITIES OF  
SUBSTITUTION FOR INTERMEDIATE GOODS,  
FRENCH MANUFACTURING, 1970-1989

	ex ante price of capital ( $p_k^A$ )			ex post price of capital ( $p_k^P$ )		
Elasticity	Translog	Generalized Leontief	Generalized McFadden	Translog	Generalized Leontief	Generalized McFadden
$\epsilon_{KK}$	-0.074 <sup>a</sup> (0.008)	-0.107 <sup>a</sup> (0.014)	-0.106 <sup>a</sup> (0.012)	-0.038 <sup>a</sup> (0.012)	-0.105 <sup>a</sup> (0.015)	-0.100 <sup>a</sup> (0.013)
$\epsilon_{KL}$	0.116 <sup>a</sup> (0.012)	0.102 <sup>a</sup> (0.019)	0.103 <sup>a</sup> (0.019)	0.044 <sup>a</sup> (0.017)	0.136 <sup>a</sup> (0.027)	0.130 <sup>a</sup> (0.024)
$\epsilon_{KE}$	-0.042 <sup>a</sup> (0.011)	0.005 (0.013)	0.003 (0.013)	-0.006 (0.011)	-0.031 <sup>a</sup> (0.015)	-0.030 <sup>a</sup> (0.014)
$\epsilon_{LK}$	0.037 <sup>a</sup> (0.004)	0.032 <sup>a</sup> (0.006)	0.031 <sup>a</sup> (0.006)	0.022 <sup>a</sup> (0.009)	0.064 <sup>a</sup> (0.013)	0.057 <sup>a</sup> (0.011)
$\epsilon_{LL}$	-0.055 <sup>a</sup> (0.008)	-0.020 (0.012)	-0.020 (0.014)	-0.021 (0.018)	-0.084 <sup>a</sup> (0.026)	-0.077 <sup>a</sup> (0.024)
$\epsilon_{LE}$	0.018 <sup>a</sup> (0.007)	-0.012 (0.008)	-0.011 (0.009)	0.001 (0.011)	0.020 (0.015)	0.020 (0.014)
$\epsilon_{EK}$	-0.040 <sup>a</sup> (0.010)	0.004 (0.012)	0.003 (0.013)	-0.009 (0.016)	-0.042 <sup>a</sup> (0.020)	-0.042 <sup>a</sup> (0.020)
$\epsilon_{EL}$	0.054 <sup>a</sup> (0.020)	-0.036 (0.027)	-0.037 (0.027)	-0.002 (0.032)	0.063 (0.047)	0.064 (0.045)
$\epsilon_{EE}$	-0.014 (0.022)	0.031 (0.024)	0.034 (0.025)	0.011 (0.022)	-0.021 (0.031)	-0.022 (0.030)
$\sigma_{KL}$	0.191 <sup>a</sup> (0.020)	0.165 <sup>a</sup> (0.032)	0.166 <sup>a</sup> (0.030)	0.080 <sup>a</sup> (0.032)	0.236 <sup>a</sup> (0.047)	0.227 <sup>a</sup> (0.043)
$\sigma_{KE}$	-0.206 <sup>a</sup> (0.053)	0.026 (0.070)	0.015 (0.066)	-0.033 (0.060)	-0.174 <sup>a</sup> (0.083)	-0.166 <sup>a</sup> (0.078)
$\sigma_{LE}$	0.090 <sup>a</sup> (0.032)	-0.059 (0.044)	-0.059 (0.044)	-0.004 (0.058)	0.112 (0.083)	0.111 (0.079)

Note : Elasticities are evaluated at means of exogeneous variables. Standard errors are in parentheses.  $\epsilon_{ii}$  ( $i = K, L, E$ ) means the own price elasticity of demand for input  $i$ ,  $\epsilon_{ij}$  ( $i, j = K, L, E$ ) means the elasticity of demand for input  $i$  with respect to price of input  $j$ ,  $\sigma_{ij}$  means the Allen elasticity of substitution between inputs  $i$  and  $j$ .

<sup>a</sup> Significant at the 95 % level.

TABLE V  
PRICE ELASTICITIES AND ALLEN PARTIAL ELASTICITIES OF  
SUBSTITUTION FOR EQUIPMENT GOODS,  
FRENCH MANUFACTURING, 1970-1989

	ex ante price of capital ( $p_k^A$ )			ex post price of capital ( $p_k^P$ )		
Elasticity	Translog	Generalized Leontief	Generalized McFadden	Translog	Generalized Leontief	Generalized McFadden
$\epsilon_{KK}$	-0.187 <sup>a</sup> (0.018)	-0.054 <sup>a</sup> (0.010)	-0.052 <sup>a</sup> (0.009)	-0.065 <sup>a</sup> (0.015)	-0.069 <sup>a</sup> (0.009)	-0.050 <sup>a</sup> (0.008)
$\epsilon_{KL}$	0.177 <sup>a</sup> (0.015)	0.032 <sup>a</sup> (0.013)	0.039 <sup>a</sup> (0.010)	0.050 <sup>a</sup> (0.015)	0.052 <sup>a</sup> (0.012)	0.035 <sup>a</sup> (0.012)
$\epsilon_{KE}$	0.010 (0.007)	0.022 <sup>a</sup> (0.007)	0.013 <sup>a</sup> (0.006)	0.015 <sup>a</sup> (0.005)	0.017 <sup>a</sup> (0.005)	0.015 <sup>a</sup> (0.005)
$\epsilon_{LK}$	0.026 <sup>a</sup> (0.002)	0.005 <sup>a</sup> (0.002)	0.005 <sup>a</sup> (0.002)	0.015 <sup>a</sup> (0.005)	0.015 <sup>a</sup> (0.003)	0.009 <sup>a</sup> (0.003)
$\epsilon_{LL}$	-0.043 <sup>a</sup> (0.003)	-0.016 <sup>a</sup> (0.004)	-0.017 <sup>a</sup> (0.003)	-0.020 <sup>a</sup> (0.007)	-0.014 <sup>a</sup> (0.006)	-0.012 <sup>a</sup> (0.006)
$\epsilon_{LE}$	0.017 <sup>a</sup> (0.002)	0.011 <sup>a</sup> (0.002)	0.012 <sup>a</sup> (0.002)	0.005 (0.005)	-0.001 (0.004)	0.003 (0.004)
$\epsilon_{EK}$	0.032 (0.022)	0.061 <sup>a</sup> (0.021)	0.038 <sup>a</sup> (0.019)	0.098 <sup>a</sup> (0.032)	0.093 <sup>a</sup> (0.030)	0.084 <sup>a</sup> (0.031)
$\epsilon_{EL}$	0.360 <sup>a</sup> (0.042)	0.225 <sup>a</sup> (0.057)	0.247 <sup>a</sup> (0.052)	0.100 (0.095)	-0.023 (0.092)	0.064 (0.094)
$\epsilon_{EE}$	-0.392 <sup>a</sup> (0.038)	-0.286 <sup>a</sup> (0.052)	-0.285 <sup>a</sup> (0.050)	-0.198 <sup>a</sup> (0.068)	-0.070 (0.067)	-0.148 <sup>a</sup> (0.070)
$\sigma_{KL}$	0.212 <sup>a</sup> (0.018)	0.038 <sup>a</sup> (0.016)	0.046 <sup>a</sup> (0.012)	0.067 <sup>a</sup> (0.020)	0.067 <sup>a</sup> (0.015)	0.045 <sup>a</sup> (0.015)
$\sigma_{KE}$	0.257 (0.172)	0.580 <sup>a</sup> (0.196)	0.324 <sup>a</sup> (0.160)	0.440 <sup>a</sup> (0.143)	0.498 <sup>a</sup> (0.158)	0.432 <sup>a</sup> (0.157)
$\sigma_{LE}$	0.430 <sup>a</sup> (0.051)	0.276 <sup>a</sup> (0.070)	0.292 <sup>a</sup> (0.062)	0.134 (0.128)	-0.030 (0.123)	0.083 (0.124)

Note : Elasticities are evaluated at means of exogenous variables. Standard errors are in parentheses.  $\epsilon_{ii}$  ( $i = K, L, E$ ) means the own price elasticity of demand for input  $i$ ,  $\epsilon_{ij}$  ( $i, j = K, L, E$ ) means the elasticity of demand for input  $i$  with respect to price of input  $j$ ,  $\sigma_{ij}$  means the Allen elasticity of substitution between inputs  $i$  and  $j$ .

<sup>a</sup> Significant at the 95 % level.

TABLE VI  
PRICE ELASTICITIES AND ALLEN PARTIAL ELASTICITIES OF  
SUBSTITUTION FOR CONSUMER GOODS,  
FRENCH MANUFACTURING, 1970-1989

	ex ante price of capital ( $p_k^A$ )			ex post price of capital ( $p_k^P$ )		
Elasticity	Translog	Generalized Leontief	Generalized McFadden	Translog	Generalized Leontief	Generalized McFadden
$\epsilon_{KK}$	-0.109 <sup>a</sup> (0.023)	-0.072 <sup>a</sup> (0.013)	-0.069 <sup>a</sup> (0.010)	-0.095 <sup>a</sup> (0.021)	-0.101 <sup>a</sup> (0.033)	-0.125 <sup>a</sup> (0.025)
$\epsilon_{KL}$	0.139 <sup>a</sup> (0.024)	0.084 <sup>a</sup> (0.019)	0.085 <sup>a</sup> (0.015)	0.103 <sup>a</sup> (0.029)	0.080 <sup>a</sup> (0.041)	0.119 <sup>a</sup> (0.035)
$\epsilon_{KE}$	-0.030 <sup>a</sup> (0.014)	-0.012 (0.012)	-0.016 (0.011)	-0.008 (0.013)	0.021 (0.013)	0.007 (0.013)
$\epsilon_{LK}$	0.024 <sup>a</sup> (0.004)	0.014 <sup>a</sup> (0.003)	0.013 <sup>a</sup> (0.002)	0.043 <sup>a</sup> (0.012)	0.032 <sup>a</sup> (0.016)	0.043 <sup>a</sup> (0.012)
$\epsilon_{LL}$	-0.052 <sup>a</sup> (0.007)	-0.036 <sup>a</sup> (0.006)	-0.034 <sup>a</sup> (0.006)	-0.075 <sup>a</sup> (0.019)	-0.038 (0.023)	-0.057 (0.020)
$\epsilon_{LE}$	0.028 <sup>a</sup> (0.005)	0.022 <sup>a</sup> (0.005)	0.021 <sup>a</sup> (0.005)	0.032 <sup>a</sup> (0.009)	0.006 (0.009)	0.014 (0.009)
$\epsilon_{EK}$	-0.074 <sup>a</sup> (0.035)	-0.027 (0.027)	-0.040 (0.027)	-0.050 (0.079)	0.125 (0.078)	0.036 (0.080)
$\epsilon_{EL}$	0.416 <sup>a</sup> (0.076)	0.330 <sup>a</sup> (0.072)	0.322 <sup>a</sup> (0.067)	0.471 <sup>a</sup> (0.134)	0.095 (0.134)	0.216 (0.140)
$\epsilon_{EE}$	-0.342 <sup>a</sup> (0.064)	-0.303 <sup>a</sup> (0.064)	-0.282 <sup>a</sup> (0.062)	-0.422 <sup>a</sup> (0.081)	-0.220 <sup>a</sup> (0.074)	-0.253 <sup>a</sup> (0.079)
$\sigma_{KL}$	0.171 <sup>a</sup> (0.030)	0.102 <sup>a</sup> (0.023)	0.104 <sup>a</sup> (0.018)	0.154 <sup>a</sup> (0.043)	0.116 <sup>a</sup> (0.058)	0.169 <sup>a</sup> (0.050)
$\sigma_{KE}$	-0.541 <sup>a</sup> (0.257)	-0.220 (0.231)	-0.310 (0.208)	-0.177 (0.282)	0.473 (0.296)	0.144 (0.315)
$\sigma_{LE}$	0.516 <sup>a</sup> (0.094)	0.414 <sup>a</sup> (0.091)	0.394 <sup>a</sup> (0.084)	0.700 <sup>a</sup> (0.200)	0.138 (0.193)	0.308 (0.206)

Note : Elasticities are evaluated at means of exogeneous variables. Standard errors are in parentheses.  $\epsilon_{ii}$  ( $i = K, L, E$ ) means the own price elasticity of demand for input  $i$ ,  $\epsilon_{ij}$  ( $i, j = K, L, E$ ) means the elasticity of demand for input  $i$  with respect to price of input  $j$ ,  $\sigma_{ij}$  means the Allen elasticity of substitution between inputs  $i$  and  $j$ .

<sup>a</sup> Significant at the 95 % level.

TABLE VII  
DERIVATIVES OF PRICE ELASTICITIES FOR CONSUMER GOODS,  
FRENCH MANUFACTURING, 1980

Elasticity	ex ante price of capital ( $p_k^A$ )			ex post price of capital ( $p_k^P$ )		
	Translog	Generalized Leontief	Generalized McFadden	Translog	Generalized Leontief	Generalized McFadden
$\epsilon_{KE}$	-0.036 (0.016)	-0.013 (0.014)	-0.017 (0.013)	-0.007 (0.014)	0.023 (0.014)	0.025 (0.014)
$\partial\epsilon_{KE}/\partial p_K$	0.071 <sup>a</sup> (0.014)	0.006 (0.006)	0.005 <sup>a</sup> (0.001)	0.022 <sup>a</sup> (0.005)	-0.009 (0.006)	0.003 (0.004)
$\partial\epsilon_{KE}/\partial p_L$	-0.096 <sup>a</sup> (0.011)	0.001 (0.001)	-0.026 <sup>a</sup> (0.005)	-0.043 <sup>a</sup> (0.004)	-0.002 <sup>a</sup> (0.001)	-0.009 (0.008)
$\partial\epsilon_{KE}/\partial p_E$	0.024 <sup>a</sup> (0.005)	-0.008 (0.007)	0.004 (0.014)	0.021 <sup>a</sup> (0.005)	-0.011 (0.006)	0.031 <sup>a</sup> (0.011)

Note : Standard errors are reported in parenthesis.  $\partial\epsilon_{KE}/\partial p_K$  means the partial derivative of  $\epsilon_{KE}$  with respect to the price of capital ( $p_K$ ). The same criterion is applied for other entries.

<sup>a</sup> Significant at the 95 % level.

TABLE VIII  
SENSITVITY OF THE RESULTS TO THE CHOICE OF THE FUNCTIONAL FORM  
AND TO THE DATA CONSTRUCTION PROCEDURE

Branche	User cost of capital	Translog (TL)	Gnralized Leontief (GL)	Symmetric Generalized McFadden (MF)
Intermediate Goods	ex ante price ( $p_K^A$ )	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} \leq 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a < 0 \quad \sigma_{LE}^a > 0$ concavity violated in 80% of cases	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} \geq 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} \geq 0 \quad \sigma_{LE} \leq 0$ concavity violated in 100% of cases Separability between $E$ and $(K, L)$	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} \geq 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} \geq 0 \quad \sigma_{LE} \geq 0$ concavity globally violated Separability between $E$ and $(K, L)$
		$\epsilon_{KK}^a < 0 \quad \epsilon_{LL} \leq 0 \quad \epsilon_{EE} \geq 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} \leq 0 \quad \sigma_{LE} \leq 0$ concavity violated in 100% of cases Separability between $E$ and $(K, L)$	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} \leq 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a < 0 \quad \sigma_{LE} > 0$ concavity violated in 25% of cases $K$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} \leq 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a < 0 \quad \sigma_{LE} > 0$ concavity globally respected $L$ and $E$ are independents
	ex post price ( $p_K^P$ )	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} > 0 \quad \sigma_{LE}^a > 0$ concavity violated in 20% of cases $K$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a > 0 \quad \sigma_{LE}^a > 0$ concavity respected at each sample point	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} > 0 \quad \sigma_{LE}^a > 0$ concavity globally respected
		$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a > 0 \quad \sigma_{LE} > 0$ concavity violated in 33% of cases $L$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a > 0 \quad \sigma_{LE} < 0$ concavity respected at each sample point $L$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a > 0 \quad \sigma_{LE} > 0$ concavity globally respected $L$ and $E$ are independents
Equipement Goods	ex ante price ( $p_K^A$ )	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} > 0 \quad \sigma_{LE}^a > 0$ concavity violated in 20% of cases $K$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a > 0 \quad \sigma_{LE}^a > 0$ concavity respected at each sample point	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} > 0 \quad \sigma_{LE}^a > 0$ concavity globally respected
		$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a > 0 \quad \sigma_{LE} > 0$ concavity violated in 33% of cases $L$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a > 0 \quad \sigma_{LE} < 0$ concavity respected at each sample point $L$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE} < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a > 0 \quad \sigma_{LE} > 0$ concavity globally respected $L$ and $E$ are independents
Consumer Goods	ex ante price ( $p_K^A$ )	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE}^a < 0 \quad \sigma_{LE}^a > 0$ concavity violated in 25% of cases	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} < 0 \quad \sigma_{LE}^a > 0$ concavity respected at each sample point $K$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} < 0 \quad \sigma_{LE}^a > 0$ concavity globally respected and $E$ are independents
		$\epsilon_{KK}^a < 0 \quad \epsilon_{LL}^a < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} < 0 \quad \sigma_{LE}^a > 0$ concavity respected at each sample point $K$ and $E$ are independents	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL} < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} > 0 \quad \sigma_{LE}^a > 0$ concavity respected at each sample point Separability between $K$ and $(L, E)$	$\epsilon_{KK}^a < 0 \quad \epsilon_{LL} < 0 \quad \epsilon_{EE}^a < 0$ $\sigma_{KL}^a > 0 \quad \sigma_{KE} > 0 \quad \sigma_{LE} > 0$ concavity globally respected Separability between $E$ and $(K, L)$

Notes : Inputs  $i$  and  $j$  ( $i, j = K, L, E$ ) are Allen substitutes (complements) if  $\epsilon_{ij} > 0$  ( $\epsilon_{ij} < 0$ ). There are independents if  $\epsilon_{ij} = 0$ .

<sup>a</sup> Significant at the 95% level.