Habit Formation, Catching up with the Joneses, and Non-Scale Growth

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Abstract

Our objective is to provide some understanding on how alternative assumptions about preferences affect the process of economic growth. In this line, we solve a one-sector non-scale growth model under three alternative preference specification i) time separable, ii) catching up with the Joneses and iii) habit formation. Departing from the time separable specification leads to important differences in the dynamic structure, the adjustment path followed by key economic variables, the correlation patterns implied by the time series generated by the model, and the speed of convergence to the new steady state. In the catching up with the Joneses economy the differences arise from a consumption externality, while in the habit formation economy the difference arises from the fact that agents not only smooth consumption but also its rate of change.

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1. Introduction

The concepts of habit and status have long been acknowledged as being important characteristics of human behavior. The idea that the overall level of satisfaction derived from a given level of consumption depends, not only on the (current) consumption level itself, but also on how it compares to some benchmark level, is not new. Origins of this proposition can be traced as far back as Smith (1759), Veblen (1899), and Keynes (1936), although it was not until Duesenberry (1949), that an effort was made to provide these ideas with some micro theoretic foundations.[1]

Subsequent literature has identified two types of reference consumption levels that may characterize these "time non-separable" preference functions. The first is based on an *external* criterion, expressed in terms of the past consumption of some outside reference group, typically the average consumption of the overall economy. This is often referred to as "catching up with the Joneses" or "utility-interdependence" and the agent described as being "outward-looking". The second is an *internal* criterion based on the individual's own past consumption levels. It is often referred to as characterizing "habit formation," and the agent described as being "inward-looking".

A growing body of empirical evidence has confirmed the importance of non-separabilities and interdependence in preferences. Van de Stadt, Kapteyn, and van de Geer (1985) model both habit formation and utility interdependence. They estimate an indirect utility function using panel data for the Netherlands. Their results are compatible with the hypothesis that utility depends upon relative consumption, although they cannot exclude the possibility that utility reflects both relative and absolute consumption. Osborn (1988) introduces a consumption specification that allows for seasonal variations and habit persistence. Using UK seasonally unadjusted data she finds the habit persistent terms to be jointly significant.

More recently Fuhrer (2000) uses maximum likelihood to estimate an approximate linear consumption function derived from time non-separable preferences. He strongly rejects the hypothesis of time separable preferences. Employing a utility function that assigns relative weights to both current consumption and an internal benchmark, he finds 80% of the weight should be

[1] Duesenberry (1949) is well known for introducing the so-called "relative income hypothesis" for consumption.

attached to the latter.[2] In addition, Fuhrer and Klein (1998) present empirical evidence suggesting that habit formation is a relevant characterization of consumption behavior among the G-7 countries.

An extensive literature on asset-pricing anomalies, most notably the equity premium puzzle, lends further credence to the level of benchmark consumption being a significant determinant of consumption behavior. Habit-forming consumers dislike large and rapid cuts in consumption. As a result, the premium that they will require to hold risky assets that might force a rapid cut in consumption will be large relative to that implied by the time-separable utility model. This feature of time non-separable preferences is exploited by Abel (1990), Constantinides (1990), Ferson and Constantinides (1991), Gali (1994), and Campbell and Cochrane (1995) among others.

Despite this evidence supporting the relevance of benchmark consumption levels for current consumption decisions, relatively few attempts have been made to introduce time non-separable preferences into the growth literature, where the specification of preferences as time-separable functions remains standard. One notable early exception is Ryder and Heal (1973), who introduced habit formation into the basic neoclassical growth model. The focus of their paper is to study the role of habit formation in determining the generic nature of the transitional adjustment path, rather than in analyzing how habit formation influences the impact of structural changes on the evolution of the economy. More recently, this approach has been pursued by Carroll, Overland, and Weil (1997, 2000), Fisher and Hof (2000), Alonso-Carrera, Caballe, and Raurich (2001a,b), although under very rigid production conditions that characterize the simplest endogenous growth model.

However, time separable utility may yield misleading conclusions if in fact preferences are characterized by a high degree of complementarity between consumption at successive moments, as the empirical evidence suggests. Thus, given the acknowledged limitations of the endogenous growth model, it is important to analyze further the role of interdependent preferences under more general production conditions.[3] To do so is the objective of the present paper. Specifically, we

[2] At the same time, his evidence is inconclusive with respect to the weight assigned to past consumption levels in forming the benchmark level. Using discrete time, he cannot reject the proposition that it is completely determined by the previous period's consumption.

[3] These restrictions have drawn an important set of criticisms. Solow (1994) criticizes the constraints that this model imposes on the underlying technologies. Jones (1995) and Backus, et al. (1992) criticize some of the empirical implications, involving "scale effects" which are not supported by the data. These considerations led to the development of the "non-scale" growth model introduced by Jones (1995), although he used the term "semi-endogenous growth".

consider the implications of time non-separable preferences using the one-sector version of the non-scale growth model studied by Eicher and Turnovsky (1999a, 1999b, 2001), which permits a much more flexible production structure, particularly with respect to returns to scale.

Of the studies we have cited, our analysis is closest to Carroll et al. (1997). One of the objectives of their analysis was to compare the introduction of time non-separable preferences with traditional (time-separable) preferences, and to isolate the role of preferences, they intentionally restrict the production side to the simplest possible form. This approach indeed provides important insights into the role of time non-separable preferences. Most notably they show that whereas with conventional preferences the basic AK technology always places the economy on its balanced growth path, the introduction of time non-separable preferences introduces sluggishness into the system, so that the economy approaches its balanced growth equilibrium along a transitional path.

While this is an important contribution, at least in one case the implied transitional dynamics appears to be counter-factual. Specifically, Carroll et al. consider the consequences of a cut in the initial capital stock, and find that, whereas with traditional preferences this has no impact on the growth rate, with time non-separable preferences (both inward- and outward-looking) it involves an initial *reduction* in the growth rate. The growth rate then increases along the transition path and eventually returns to its pre-shock level. However, evidence (and intuition) would seem to suggest precisely the opposite, namely that following a disaster such as an earthquake that destroys capital, the growth rate generally *increases* in the very short run as investment is increased to restore the lost capital stock, with the growth rate eventually declining as the restoration approaches completion.

One of the findings of this paper is that introducing time non-separable preferences, in conjunction with the non-scale technology and the more flexible transitional dynamic adjustment paths it permits, can easily generate a short-run increase in the growth rate during the early stages of the transition following an initial loss in the capital stock, with the growth rate declining over time, thereby replicating this more plausible dynamic behavior. In effect, the transitional dynamics obtained by Carroll et al. are a manifestation of the constant productivity of capital imposed by the AK production technology. This substantially restricts the dynamic behavior of the system, leading to a monotonic adjustment process driven largely by preference parameters. Thus, one of the more general conclusions we draw is the potential importance of combining (i) the more general preferences with (ii) the more flexible production technology, in replicating observed behavior. But the fact that our equilibrium can generate more flexible dynamic paths comes at an inevitable price. This is because the more flexible paths reflect a higher order dynamic system, which is too intractable to be studied analytically, but instead must be analyzed using numerical simulations.

We employ the utility function introduced by Abel (1990) in the context of asset pricing and used by Carroll et al. (1997). Following these authors we shall consider both externally and internally generated consumption benchmarks. We shall compare their implications for the dynamic adjustment of the non-scale growth model, both to one another as well as to those of the conventional time separable specification of preferences. Departing from the basic growth model in these two dimensions -- production structure and preferences -- we find important differences in the equilibrium dynamics, the adjustment process of key economic variables, the correlation patterns implied by the model, and the speed of convergence to the new steady state.

There are several key results that we wish to stress at the outset. The first and most general finding is that the differences between assuming the conventional time separable utility function, on the one hand, and time non-separable preference functions, on the other, are substantial. By contrast, the difference between assuming that the reference consumption level is formed by looking outwards or inwards is relatively small, although it does depend upon the shock imposed upon the economy.

Second, in contrast to the AK model, introducing time non-separable utility may increase, rather than decrease, the speed of convergence. This depends upon how rapidly the reference stock adjusts relative to the intrinsic adjustment speed in the rest of the economy. Third, the introduction of consumption habits causes substantial intertemporal shifts in the time paths for consumption and savings following structural shocks to the economy. In the case of a productivity increase it leads to a smaller short-run increase in consumption and a larger increase in saving, which over time generates an eventual larger increase in consumption. The impact of habit is even more dramatic in the case where the shock takes the form of a destruction of capital. Fourth, the time path of welfare resulting from a structural change can be decomposed into the effect on the *absolute* consumption level, together with the effect on current consumption *relative* to the reference level. This can lead to substantially different welfare implications from those obtained for conventional preferences, depending upon how rapidly the reference stock is assumed to adjust. Consequently, the policy and welfare implications of structural changes, conducted under the conventional assumption of time separable preferences may turn out to be quite misleading if in fact preferences are time non-separable. Fifth, the initial stages of the dynamics are particularly sensitive to the speed of adjustment of the reference consumption level; they are less sensitive to the weight assigned to the reference consumption level in utility. Sixth, the presence of a reference consumption level can have a very different effect on the transitional dynamics in a non-scale model from its effect in the endogenous growth model. This depends upon how non-monotonic the transitional paths are in the former, which in part is sensitive to the adjustment speed of habits. Finally, time non-separable preferences provide interesting insight into the growth-saving relation. In contrast to the conventional model where saving is seen as the engine of growth, our model reverses this causal relation, suggesting that growth leads to saving. This behavior is consistent with empirical evidence summarized by Carroll et al. (2000).

The paper is organized as follows. Section 2 sets out the basic structure of the model, introducing our two versions of time non-separable preferences. Section 3 then characterizes the corresponding macroeconomic behavior of the economy. Section 4 conducts a numerical analysis, comparing the dynamic responses of the economy under the alternative specifications of preferences, while Section 5 carries out some sensitivity analysis. Section 6 compares the implications of the present non-scale model with those obtained under the more restrictive AK production structure. The conclusions are summarized in Section 7, while an appendix provides some technical details.

2. The Model

Consider an economy populated by N identical and infinitely lived households that grows at the exogenous rate $\dot{N}/N = n$. At any point in time, households derive utility from the comparison of their current consumption level relative to a reference consumption level. The individual household's objective is to maximize the intertemporal iso-elastic utility function:

$$\Omega \equiv \int_0^\infty \frac{1}{1-\varepsilon}\left[\frac{C_i}{H_i^\gamma}\right]^{1-\varepsilon} e^{-\beta t} dt = \int_0^\infty \frac{1}{1-\varepsilon}\left[C_i^{(1-\gamma)}\left(\frac{C_i}{H_i}\right)^\gamma\right]^{1-\varepsilon} e^{-\beta t} dt \tag{1}$$

where C_i and H_i are household i's current consumption and reference consumption levels, respectively[4]. Following Ryder and Heal (1973) we impose non-satiation in utility, restricting γ to lie in the range $0 \le \gamma < 1$.[5] As we can see from the second expression in (1), agents derive utility from a geometric weighted average of absolute and relative consumption, with γ assigning the weights. If $\gamma = 0$, (1) reduces to the conventional specification in which preferences are time separable and therefore only the absolute level of consumption matters. As $\gamma \to 1$, only relative consumption matters and the absolute level of consumption becomes irrelevant. In general, ε and γ interact to determine the (consumption) intertemporal elasticity of substitution (IES), having the property that it varies with the horizon considered. As the time horizon shrinks to zero, habits are predetermined and fixed, and therefore the IES converges to the expression for the conventional time separable case, $1/\varepsilon$. At the other extreme, as the time horizon increases to infinity, habits fully adjust to a change in consumption. Setting $H_i = C_i$ in (1) this implies a long-run intertemporal elasticity of substitution equal to $1/\left(\gamma + \varepsilon(1-\gamma)\right)$.[6] This contrasts with the conventional case, $\gamma = 0$, where the intertemporal elasticity of substitution, $1/\varepsilon$, remains constant even along the transitions. Thus we see that the long-run IES under time non-separable preferences exceeds the conventional IES if and only if $1/\varepsilon < 1$, as empirical evidence suggests.

Individual output is determined by the individual's private capital stock, K_i, the aggregate capital stock, $K = NK_i$, and the level of inelastically supplied labor, L_i. Assuming a Cobb-Douglas production function, individual output is determined according to,

$$Y_i = \alpha L_i^\sigma K_i^{(1-\sigma)} K^\eta \qquad 0 < \sigma < 1 \tag{2}$$

[4] Since the utility specification is not concave in both C_i and H_i, the question is raised whether or not the necessary conditions that we derive are in fact optimal in the habit formation case. This problem is characteristic of all the literature that employs the utility function in (1). Although we have not been able to provide a definitive proof that the first-order conditions are in fact an optimum, all of our extensive simulations suggest that this is in fact so. We thus conclude that if there are any superior paths to those we focus on, they would have very unusual parameter values and likely be of little economic interest.

[5] Non-satiation is guaranteed if an increase in a uniformly maintained consumption level increases utility, i.e. if $U_C(C_i, C_i) + U_H(C_i, C_i) > 0$.

[6] See also Carroll et al. (2000).

The technology exhibits diminishing marginal product to each private factor and constant returns to scale in the two factors. But total returns to scale, $1+\eta$, are decreasing, constant, or increasing, according to whether the spillover from aggregate capital is negative, zero, or positive. Normalizing L_i to 1, and summing across households, yields the aggregate production function,

$$Y = \alpha N^{\sigma_N} K^{\sigma_K} \qquad \sigma_K \equiv 1-\sigma+\eta,\ \sigma_N = \sigma \tag{3}$$

We define a balanced growth path as being one along which all variables grow at a constant rate. With capital being accumulated from final output, the only balanced solution is one in which the capital-output ratio, K/Y, remains constant. This is consistent with the stylized empirical facts reported by Kaldor (1961) and Romer (1989). As we will show below, one of the stability conditions is $\sigma_K < 1$, a condition that we henceforth impose.[7]

Differentiating the aggregate production function (3), we obtain the equilibrium growth rates of capital and output, $\hat{K}^*$ and $\hat{Y}^*$,

$$\hat{Y}^* = \hat{K}^* = \frac{\sigma_N}{1-\sigma_K} n \equiv gn \tag{4}$$

yielding the standard result that because of the non-scale nature of our production technology, the equilibrium growth rate is completely determined by technological factors, together with the population growth rate, and is independent of any demand parameter.

Final output can be either consumed currently, or saved and transformed into additional capital to yield future consumption. Assuming that the existing capital stock depreciates at a rate, δ, agent i's capital stock evolves according to the accumulation relationship

$$\dot{K}_i = Y_i - C_i - (n+\delta) K_i \tag{5}$$

Finally we need to describe the evolution of the reference stock. The two time non-separable specifications differ only in how the reference stock is determined. In both cases the reference stock is an exponentially declining weighted average of past levels of consumption. In the outward-

[7] Turnovsky (2000) show in the basic one-sector non-scale growth model with conventional utility, that $\sigma_K < 1$, is necessary and sufficient for stability. It is seen from (3) that this imposes the restriction $\eta < \sigma$ on the externality.

looking case, households compare their current consumption with the past economy-wide average level. Since agents are atomistic, they ignore the effect of their individual consumption decisions on the evolution of the reference stock, taking it as exogenous. Therefore, for the outward-looking economy the current level of the reference consumption stock is determined by,

$$H_i(t) = \rho \int_{-\infty}^{t} e^{\rho(\tau - t)} \overline{C}(\tau)\, d\tau \qquad \rho > 0 \tag{6}$$

where $\overline{C} = \sum_{i=1}^{N} C_i / N$ denotes the economy-wide average consumption of agents.

Inward-looking agents compare their current consumption with an average of their own past consumption. When they decide over current consumption they fully internalize the effects of their decisions on the future evolution of their reference stock. Therefore in the inward-looking economy the current level of habit is determined by,

$$H_i(t) = \rho \int_{-\infty}^{t} e^{\rho(\tau - t)} C_i(\tau)\, d\tau \qquad \rho > 0 \tag{7}$$

The differences in behavior between the two economies arise from the fact that outward-looking agents ignore the externality that their own consumption decisions induce on other agents' utility, while the inward looking agents fully internalize this effect.

Differentiating (6) and (7) with respect to time implies the following rates of adjustment for the reference stock, for the outward-looking and the inward-looking case, respectively

$$\dot{H}_i = \rho\left(\overline{C} - H_i\right) \tag{8}$$

$$\dot{H}_i = \rho\left(C_i - H_i\right) \tag{9}$$

For both specifications the speed of adjustment, ρ, parameterizes the relative importance of recent consumption in determining the reference stock. For instance, if $\rho = 0.2$ (as in most of our simulations), the reference stock will adjust half way to a permanent change in C after three and a half years. Therefore, higher values of ρ lead to a higher influence of current consumption in the determination of the future reference stock, or alternatively to a lower level of persistence in habits.

3. Macroeconomic equilibrium under alternative preference specifications

We now proceed to derive the macroeconomic equilibria under alternative assumptions regarding the specification of preferences, beginning with the conventional case of time separable preferences to serve as a benchmark with which the more general representations can be compared.

3.1. Time separable preferences

The benchmark conventional preferences are obtained by setting $\gamma = 0$ in (1). In this case the representative agent is assumed to choose his consumption and rate of capital accumulation to maximize (1), so modified, subject to the production function, (2) and the accumulation equation, (5). This yields the conventional optimality conditions:[8]

$$U_{c_i} \equiv C_i^{-\varepsilon} = \lambda_i \tag{10a}$$

$$\alpha(1-\sigma)L_i^{\sigma_N} K_i^{\sigma_K - 1} - \delta - n = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{10b}$$

where λ_i denotes the agent's shadow value of capital, together with the transversality condition,

$$\lim_{t \to \infty} \lambda_i K_i e^{-\beta t} = 0 \tag{10c}$$

The interpretations of (10a) and (10b) are standard; (10a) equates the marginal utility of consumption to the shadow value of capital, while (10b) is the intertemporal allocation condition equating the marginal product of capital to the rate of return on consumption.

Taking the time derivative of (10a), combining with (10b), and aggregating across households, the optimal path for aggregate consumption is,

$$\hat{C} \equiv \frac{\dot{C}}{C} = \frac{1}{\varepsilon}\left[\alpha(1-\sigma)N^{\sigma_N} K^{\sigma_K - 1} - \delta - \beta - \left(1-\varepsilon\right)n\right] \tag{11}$$

Following our definition of the balanced growth path, it is convenient to write the system in terms of the following stationary variables $k \equiv K/N^g$, $c \equiv C/N^g$, which we characterize as being "scale-

[8] In performing the optimization, the individual agent takes the aggregate capital stock as given.

adjusted" per capita quantities, and which under constant returns to scale $(g=1)$ reduce to standard per capita quantities. Combining these definitions with (3), (5), and (11), the dynamic behavior of the economy can be described by the pair of differential equations in k and c

$$\dot{k} = \alpha k^{\sigma_K} - c - (gn + \delta)k \tag{12a}$$

$$\dot{c} = \frac{c}{\varepsilon}\left\{(1-\sigma)\alpha k^{\sigma_K - 1} - \delta - \beta - \left[(1-\varepsilon) + \varepsilon g\right]n\right\} \tag{12b}$$

Imposing the steady state condition, $\dot{c} = \dot{k} = 0$, we can solve (12) for the steady-state values of the scale-adjusted variables, k^* and c^*, as follows,

$$k^* = \left[\frac{\beta + \delta + [(1-\varepsilon) + \varepsilon g]n}{(1-\sigma)\alpha}\right]^{\frac{1}{\sigma_K - 1}} \tag{13a}$$

$$c^* = \alpha\left(k^*\right)^{\sigma_K} - (gn + \delta)k^* \tag{13b}$$

Turnovsky (2000) conducts a detailed analysis of the dynamic behavior of (12) in the neighborhood of (13), showing that it exhibits local saddle path-stable behavior if and only if $\sigma_K < 1$.

3.2. Outward-looking Consumption Benchmark

We now consider the representative agent who makes his consumption-investment choice to maximize (1) subject to (5), in the case that utility depends upon a benchmark consumption level ($\gamma > 0$). We assume initially that this is generated externally, in accordance with (6), so that in making his decisions, the individual ignores the impact of this external influence on his welfare.

The first order conditions for an optimum are

$$U_{c_i} \equiv \frac{C_i^{-\varepsilon}}{H_i^{\gamma(1-\varepsilon)}} = \lambda_i \tag{10a'}$$

$$\alpha(1-\sigma)L_i^{\sigma_N} K_i^{\sigma_K - 1} - \delta - n = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{10b}$$

$$\lim_{t\to\infty} \lambda_i K_i e^{-\beta t} = 0 \tag{10c}$$

The only modification is to (10a') which takes account of the fact that utility depends upon current consumption relative to the external benchmark.

Taking the time derivative of (10a'), combining with (10b) and (8) and imposing the equilibrium condition $C_i = \bar{C} = C/N$, the equilibrium path for aggregate consumption is

$$\hat{C} \equiv \frac{\dot{C}}{C} = \frac{1}{\varepsilon}\left[\alpha(1-\sigma)N^{\sigma_N}K^{\sigma_K -1} - \delta - \beta - (1-\varepsilon)n - \gamma(1-\varepsilon)\rho\left(\frac{C}{H}-1\right)\right] \qquad (14)$$

which relates the growth rate of consumption to the parameters of the model and the growth rate of the reference stock, which is exogenous from the consumer's point of view.

Analogous to the definitions of c,k given above, we define $h \equiv H/N^g$. This enables us to rewrite expression (3), (5), (8) and (14) in terms of the scale adjusted variables, k,c and h ,

$$\dot{k} = \alpha k^{\sigma_K} - c - (\delta + gn)k \qquad (12a)$$

$$\dot{c} = \frac{c}{\varepsilon}\left\{(1-\sigma)\alpha k^{\sigma_K -1} - \rho\gamma(1-\varepsilon)\frac{c}{h} - \left[\beta + \delta + (1-\varepsilon)n - \rho\gamma(1-\varepsilon)\right] - \varepsilon gn\right\} \qquad (12b')$$

$$\dot{h} = \rho(c-h) + (1-g)nh \qquad (12c)$$

Note that if either $\gamma = 0$, so that the reference stock is irrelevant to utility, or $\rho = 0$, so that the reference stock is fixed, (12a) and (12b') collapse to the system of equations that describes the dynamics under the conventional utility specification, as described in Section 3.1, with the dynamics of *h*, still determined by (12c), becoming irrelevant.

Imposing the steady state condition, $\dot{c} = \dot{k} = \dot{h} = 0$, we can solve for the steady state values of capital, consumption, and habit as follows,

$$k^* = \left[\frac{\beta + \delta - (\gamma - 1)(1-\varepsilon)n + \varepsilon gn + \gamma(1-\varepsilon)gn}{(1-\sigma)\alpha}\right]^{\frac{1}{\sigma_K - 1}} \qquad (13a')$$

$$c^* = \alpha\left(k^*\right)^{\sigma_K} - (gn + \delta)k^* \qquad (13b)$$

$$h^* = \frac{c^*}{\left(1 + (g-1)n/\rho\right)} \qquad (13c)$$

In the standard neoclassical case when the production function exhibits constant returns to scale, $g=1$ and (13c) implies $c^*=h^*$, so that the stationary consumption level coincides with the reference level. In that case (13a') and (13a) both reduce to the standard modified golden rule stock of capital, consistent with the early result of Ryder and Heal (1973). The equilibrium stock of capital will be independent of γ, the relative weight attributed to habit in utility, and coincides with the result of the conventional case. In the event that the production function exhibits increasing returns to scale, $g>1$, $c^*>h^*$ by an amount that is inversely related to ρ, the speed with which the reference stock adjusts to recent consumption experience. In that case the introduction of an externally formed benchmark reduces the equilibrium scale-adjusted per capita stock of capital if and only if the short-run intertemporal elasticity of substitution is less than unity $(1/\varepsilon<1)$.

Linearizing (12a), (12b') and (12c) around the steady state, the dynamics can be approximated by the third-order system:

$$\begin{pmatrix} \dot{k} \\ \dot{h} \\ \dot{c} \end{pmatrix} = \left(\begin{matrix} \alpha\sigma_K k^{\sigma_K-1}-gn-\delta & 0 & -1 \\ 0 & -\rho\dfrac{c}{h} & \rho \\ \dfrac{\alpha(1-\sigma)(\sigma_K-1)k^{\sigma_K-2}}{\varepsilon}c & \left(\dfrac{c}{h}\right)^2\left(\dfrac{\gamma(1-\varepsilon)\rho}{\varepsilon}\right) & -\rho\dfrac{c}{h}\left(\dfrac{\gamma(1-\varepsilon)}{\varepsilon}\right) \end{matrix} \right)_{\substack{c=c^*,k=k^*,\\ h=h^*}} \begin{pmatrix} k-k^* \\ h-h^* \\ c-c^* \end{pmatrix} \quad (15)$$

With k and h being sluggish variables while c is free to jump instantaneously, in order for this system to have a unique stable adjustment path (i.e. be saddle-path stable) we require that it has 2 negative (stable) and 1 positive (unstable) eigenvalues. It can be easily verified that the sign of the determinant of the matrix in (15) is the same as that of $(1-\sigma_K)$. Thus a necessary condition for stability is that $\sigma_K<1$. This, however, is consistent with there being either 2 negative and 1 positive root, or 3 positive roots. A sufficient condition to rule out the latter, and therefore to ensure a unique stable path, is that the trace of the matrix be negative and a plausible condition to ensure that this is so is that $c^*<(1-\sigma_K)\alpha\left(k^*\right)^{\sigma_K}$.[9] Indeed all of our simulations area characterized by saddle-point

[9] This condition is equivalent to requiring the equilibrium consumption-income ratio be less than $1-\sigma_K$

stability, which we feel confident, prevails over plausible parameter sets.[10]

3.3. Inward-looking Consumption Benchmark

We now turn to the second case where the agent's reference consumption level is determined by his own past consumption, in accordance with (7), so that the agent fully internalizes the impact of his current decisions on the future evolution of the reference stock. The agent's optimization therefore takes full account of (7), and we shall identify the second co-state variable, λ_2, associated with the internally generated reference stock. The optimality conditions are now modified to

$$\frac{C_i^{-\varepsilon}}{H_i^{\gamma(1-\varepsilon)}} + \rho\lambda_{2i} = \lambda_{1i} \tag{16a}$$

$$\alpha(1-\sigma)L_i^{\sigma_N} K_i^{\sigma_K - 1} - \delta - n = \beta - \frac{\dot{\lambda}_{1i}}{\lambda_{1i}} \tag{16b}$$

$$U_{H_i} \equiv -\gamma \frac{C_i^{(1-\varepsilon)}}{H_i^{\gamma(1-\varepsilon)+1}} = \lambda_{2i}\left(\beta + \rho\right) - \dot{\lambda}_{2i} \tag{16c}$$

together with the transversality conditions

$$\lim_{t\to\infty} e^{-\beta t}\lambda_{1i}K_i = \lim_{t\to\infty} e^{-\beta t}\lambda_{2i}H_i = 0 \tag{16d}$$

The key differences from the previous case are to (16a) and (16c). The former equates the utility of an additional unit of consumption adjusted by its impact on the future reference stock to the shadow value of capital, while the latter is an intertemporal allocation condition equating at the margin, the disutility of an additional unit of habit measured in terms of its shadow value with the cost of habit.

Internal preferences require the monitoring of two state variables. Letting $q_i \equiv \lambda_{2i}/\lambda_{1i}$ denote the relative price of habit to physical capital, after summing across households we can express the dynamics of the scale adjusted variables in terms of the fourth order system:

$$\dot{k} = \alpha k^{\sigma_K} - c - \left(gn + \delta\right)k \tag{17a}$$

[10] The stable roots may quite plausibly turn out to be complex, in which case the dynamics involves cyclical behavior.

$$\dot{c} = \frac{c}{\varepsilon}\left\{\frac{\alpha(1-\sigma)k^{\sigma_K - 1}}{1-\rho q} + \rho\gamma\varepsilon\frac{c}{h} + \gamma\rho\left(1-\varepsilon\right) + \frac{\rho q\left(\beta+\rho\right)-\beta-\delta-n}{1-\rho q} + \left(1-g\right)\varepsilon n\right\} \tag{17b}$$

$$\dot{h} = \rho\left(c-h\right) + \left(1-g\right)nh \tag{17c}$$

$$\dot{q} = q\left\{\alpha(1-\sigma)k^{\sigma_K - 1} + \gamma\frac{c}{h}\left(\frac{1}{q}-\rho\right) + \rho - \delta - n\right\} \tag{17d}$$

Imposing the stationary conditions, $\dot{c} = \dot{h} = \dot{q} = \dot{k} = 0$ we can determine the steady-state values of our scale-adjusted variables in the following recursive manner. First, (17c) yields the consumption-habit ratio, precisely as in the outward-looking case. Second, given c/h, (17d) determines the ratio of the shadow values in terms of capital. Third, substituting (17d) into (17b) yields a quadratic equation in scale-adjusted capital, one of the roots of which can be eliminated by imposing the transversality condition (16d).[11] Finally (17a) determines the steady-state level of normalized consumption.

$$k^* = \left[\frac{\beta+\delta-(\gamma-1)(1-\varepsilon)n + \varepsilon gn + \gamma\left(1-\varepsilon\right)gn}{(1-\sigma)\alpha}\right]^{\frac{1}{\sigma_K - 1}} \tag{18a}$$

$$c^* = \alpha\left(k^*\right)^{\sigma_K} - \left(gn+\delta\right)k^* \tag{18b}$$

$$h^* = \frac{c^*}{\left(1+\left(g-1\right)n/\rho\right)} \tag{18c}$$

$$q^* = \frac{\gamma\left(1+\left(g-1\right)n/\rho\right)}{\rho(\gamma-1)-\beta+n\varepsilon(1-\gamma)(1-g)} \tag{18d}$$

By comparing (18a) - (18c) with (13a'), (13b), and (13c) we can see that the steady-state values of capital, consumption, and habits are the same whether the reference consumption level is formed internally or externally. On the other hand, the adjustment process will differ between the two time non-separable specifications. Note from (18d) that $g \geq 1$, ensures that $q^* < 0$. Because in general an increase in the level of habits, given current consumption, is welfare-reducing, the shadow value of the reference stock is negative.

[11] Appendix A provides a detailed treatment of this issue.

The dynamics can be approximated by the fourth-order system presented below:

$$\begin{pmatrix}\dot{k}\\ \dot{h}\\ \dot{c}\\ \dot{q}\end{pmatrix}=\begin{bmatrix}\alpha\sigma_K k^{\sigma_K-1}-gn-\delta & 0 & -1 & 0\\ 0 & -\rho\left(\frac{c}{h}\right) & \rho & 0\\ \frac{\alpha(1-\sigma)(\sigma_K-1)k^{\sigma_K-2}}{1-q\rho}\left(\frac{c}{\varepsilon}\right) & -\gamma\rho\left(\frac{c}{h}\right)^2 & \gamma\rho\left(\frac{c}{h}\right) & \left(\frac{\rho c}{\varepsilon}\right)\left[\frac{\alpha(1-\sigma)k^{\sigma_K-1}+\rho-\delta-n}{(1-\rho q)^2}\right]\\ \alpha q(1-\sigma)(\sigma_K-1)k^{\sigma_K-2} & -\gamma\frac{c}{h^2}(1-\rho q) & \frac{\gamma}{h}(1-q\rho) & -\left(\frac{\gamma}{q}\right)\left(\frac{c}{h}\right)\end{bmatrix}_{\substack{k=k^*,h=h^*,\\ c=c^*,q=q^*}}\begin{bmatrix}k-k^*\\ h-h^*\\ c-c^*\\ q-q^*\end{bmatrix} \quad (19)$$

In this case with two "sluggish" variables (h,k) and two "jump" variables (c,q) we require two positive and two negative roots for a unique stable saddle-path solution. As in the previous case, it can be easily verified that the sign of the determinant of the matrix in (19) is the same as that of $(1-\sigma_K)$, so that $\sigma_K < 1$ is again a necessary condition for stability. To ensure that we do in fact have two positive roots requires extra conditions, which unfortunately turn out to be intractable.[12] Again, for plausible parameters we find that (19) exhibits saddlepoint behavior (possibly with complex roots), and we shall focus our attention on that case.

Since the three economies we have introduced differ only in terms of their demand characteristics, a closer analysis of the behavior of consumption will provide a better understanding of the differences in the adjustment processes that we will highlight in the next section. Even though consumption under conventional, outward-looking and inward-looking preferences is determined by the interaction of the systems, (12), (12a, 12b', 12c), and (17) respectively, for expositional purposes we are going to focus our attention on the dynamic equations for consumption. Assuming for convenience $n=\delta=0$ and letting $\hat{c}$ denote $\dot{C}/C$, (12b), (12b') and (17b) become[13]

$$\hat{c}=\frac{1}{\varepsilon}\left\{(1-\sigma)\alpha k^{\sigma_K-1}-\beta\right\} \quad (20a)$$

$$\hat{c}=\frac{1}{\varepsilon}\left\{(1-\sigma)\alpha k^{\sigma_K-1}-\rho\gamma(1-\varepsilon)\left(\frac{c}{h}-1\right)-\beta\right\} \quad (20b)$$

[12] Ryder and Heal (1973) discuss alternative generic dynamic paths in the case of the neoclassical production function.

[13] The derivation of (20c), follows an alternative solution method. In line with Carroll, Overland and Weil (1997), we eliminate the co-state variables through repeated differentiation of the first order conditions, reaching a second order differential equation in consumption. Under their assumptions about the production structure (20c) reduces to their expression.

$$\ddot{c} = \varepsilon\left(\frac{\dot{c}}{c}\right)^2 + \left(\frac{\dot{c}}{c}\right)\left[(2\beta+\rho-F_K) - 2\rho\gamma(1-\varepsilon) + \frac{F_{KK}\dot{k}}{\rho+F_K}\right] - \left(\frac{c}{h}\right)^2\left[\rho^2\gamma\left(\gamma(1-\varepsilon)+1\right)\right] \tag{20c}$$

$$+\left(\frac{\dot{c}}{c}\right)\left(\frac{c}{h}\right)\left[2\rho\gamma(1-\varepsilon)\right] + \left(\frac{c}{h}\right)\left(\frac{\rho\gamma}{\varepsilon}\right)\left[\rho\gamma(1-\varepsilon)(2\varepsilon-1)+\beta+\rho-\varepsilon\left(2\beta-F_K+\frac{F_{KK}\dot{k}}{\rho+F_K}\right)\right]$$

$$+\frac{1}{\varepsilon}\left[(\beta+\rho)\left(\beta-F_K+\frac{F_{KK}\dot{k}}{\rho+F_K}\right)\left(\rho\gamma(1-\varepsilon)\right)\right]\left(\rho\gamma(1-\varepsilon)\right)\left[\rho\left(\gamma(1-\varepsilon)+1\right)-\left(2\beta+2\rho-F_K+\frac{F_{KK}\dot{k}}{\rho+F_K}\right)\right]$$

In the conventional case, (20a), the rate of growth of consumption is determined by the interaction between the real interest rate and the rate of time preference, weighted by the intertemporal elasticity of substitution, a measure of the agent's willingness to shift consumption across periods. A high marginal product of capital will lead to a high level of consumption growth and therefore to a lower level of current consumption, and we call this the "rate of return effect".[14] With outward-looking agents, an additional variable interacts in the determination of the rate of growth of consumption, the reference stock. For empirically plausible values of ε, if consumption is below habit, consumption in the outward-looking economy will grow slower than in the conventional economy, and vice versa. This is what we call the "status effect" and it counteracts the "rate of return effect", constraining the deviations in consumption from its historical level.

As first observed by Carroll, Overland and Weil (1997), inward-looking agents are not only concerned with consumption smoothing, but they also smooth the rate of consumption growth and therefore the second derivative of consumption enters the dynamic equation. This is so because inward-looking agents acknowledge the fact that increases in consumption today will affect the future evolution of the reference stock, and therefore the utility derived in future periods.

4. Numerical analysis of some transitional paths.

To understand better the nature of the transitional dynamics we calibrate our models to reproduce some key features of actual economies. Table 1 summarizes the parameters upon which our simulations are based. Some of these are standard and non-controversial. In this regard,

[14] This effect is the combination of the "Solow effect", substitution effect and the human-wealth effect. As described by Mulligan and Sala-i-Martin (1993), the "Solow effect" implies that, given a constant saving rate, a low level of capital will lead to a high rate of growth simply because the average product of capital is high. The other two effects tend to increase current savings (decrease current consumption) increasing investment and growth.

$\sigma = 0.65$, implying a labor share of income of 65%, the rate of time preference $\beta = 0.04$, the instantaneous intertemporal elasticity of substitution, $1/\varepsilon = 0.4$, population growth rate $n = 0.015$, and depreciation rate, $\delta = 0.05$ are well documented, while being a non-scale model, the normalization $\alpha = 1$ is unimportant.

The critical parameters pertain to the relative importance of the reference stock, γ, the speed with which it is adjusted, ρ, and the production externality parameter, η. With respect to the two preference parameters we follow Carroll et. al (1997) and set $\gamma = 0.5, \rho = 0.2$ as benchmark values. However, information on these parameters is sparse, and we therefore conduct some sensitivity analysis based on other circumstantial evidence. Thus, based on the estimates provided by Fuhrer (2000), we also consider $\gamma = 0.8$. In addition, his results suggest a much faster speed of adjustment in the determination of the reference stock, although this estimate is obtained with a low degree of precision. A faster speed of convergence is also suggested by the application of this model to the equity premium puzzle literature, and in light of this we increase ρ to 0.8 and 2.[15]

Finally, for expositional purposes it is convenient to treat the benchmark technology as one of constant returns to scale ($\eta = 0$). This has the advantage that steady-state values are identical across specifications. But since one of the features of our production function is its flexibility with respect to returns to scale, we also consider the case of increasing returns $(\eta = 0.2)$ and decreasing returns $(\eta = -0.2)$, respectively. In this case all long-run responses, apart from the equilibrium growth rate, will vary across the preference structure. Nonetheless extensive sensitivity analysis found that our qualitative results are consistent with alternative structures of the returns to scale. Along the transition we express each variable as a ratio of its initial steady-state level to make the interpretation of the results easier.

Table 2 presents the base equilibrium for three economies, constant returns to scale, increasing and decreasing returns to scale for the three specifications of preferences. In the case of constant returns to scale, all three specifications yield common values for the output-capital ratio of 0.3, a savings rate of 21.6% (consumption-output ratio of 78.4%), and a growth rate of 1.5%. Under

[15]There is some difficulty in translating empirical estimates of these parameters, which are based on discrete-time models to our continuous-time formulation

increasing returns to scale the equilibrium output-capital ratios increase, while the savings rates decrease, the responses being greater where preferences are time separable. Under decreasing returns to scale, these responses are reversed.

4.1 Speed of convergence

A particularly interesting aspect of the results relates to the eigenvalues. These are crucial in determining the economy's speed of convergence, which in turn is important in determining the relative significance of the steady state versus the transitional dynamics, and has been the subject of both extensive empirical and theoretical analysis. The empirical evidence on convergence speeds is mixed. Early influential work by Barro and Sala-i-Martin (1992, 1995), Mankiw, Romer, and Weil (1992) yielded estimates of around 2-3% per annum, which became a benchmark estimate, although it conflicts with the predictions of the simplest neoclassical models, of around 10%. Subsequent work suggests that the rates of convergence are more variable, being sensitive to the time period and the set of countries, and a wider range of empirical estimates have since been obtained.[16]

The convergence of endogenous growth models with physical and human capital and one-sector neoclassical models, analyzed by Ortigueira and Santos (1997), of one-sector non scale models analyzed by Turnovsky (2000) and of one-sector AK models under time non-separable preference specifications analyzed by Carroll, Overland and Weil (1997) is determined by a one-dimensional stable manifold corresponding to the unique negative eigenvalue. That structure imposes the restriction that all the variables converge to their respective steady states at the same constant speed that equals the magnitude of the unique stable eigenvalue.

By contrast, if the stable manifold is two-dimensional (as for either of the habit formation cases) the speed of convergence of any variable at any point of time is a weighted average of the two stable eigenvalues. Over time, the weight of the smaller (more negative) eigenvalue declines, so that the larger of the two stable (negative) eigenvalues describes the asymptotic speed of convergence.[17]

[16]For example, Islam (1995) estimates the rate of convergence to be 4.7% for nonoil countries and 9.7% for OECD countries. Caselli, Esquivel, and Lefort (1996), use a GMM estimator to correct for sources of inconsistency due to correlated country-specific effects and endogenous explanatory variables and obtain a convergence rate of around 10%. Evans (1997) using an alternative method to generate consistent estimates of convergence finds them to be around 6%.
[17] See Eicher and Turnovsky (1999) for more discussion of this point.

The flexibility provided by the additional eigenvalue allows the system to match some features of the data related with the timing of key variables and growth rates along the transitional path.

In the case of conventional time separable preferences the two eigenvalues are -0.0626 and -0.20. However, the dynamics of capital (and consumption), on the one hand, and the reference consumption level, on the other, decouple. As a result, capital and consumption converge at the constant rate of 6.26%, while the reference stock, which by assumption is irrelevant to the consumption-investment allocation, converges at 20%. However, when preferences depend upon benchmark consumption, the stable dynamics of the entire system becomes interdependent, and the convergence of capital and the reference consumption level occur jointly.[18] Moreover, in this case for both specifications of preferences, the two stable roots are complex, indicating that the stable adjustment path is one of cyclical behavior. However, because the imaginary component is small, the cycles occur only toward the end of the adjustment and are not apparent in the illustrations.

The interesting and perhaps counter-intuitive observation is that the introduction of a reference consumption stock, which one can view as a source of sluggishness, actually speeds up the dynamics. The speed of adjustment, -0.106, implicit in the real part of the two complex roots in the external case is essentially some kind of average of the two eigenvalues -0.0626 and -0.20 of the conventional system. Intuitively, the interaction of the capital dynamics with the more rapid dynamics of benchmark consumption in the indecomposable economy means that the convergence speed of the former is increased, while that of the latter slows down.

This can be seen by letting ρ vary monotonically. Taking the case $\rho = 0.02$, for conventional preferences, capital converges at 6.26%, while reference consumption now converges at only 2%. With external preferences, the eigenvalues are now both real (-0.0557, -0.022), so that asymptotically, the entire system -- capital, output, and benchmark consumption -- converge at 2.2%. The slow evolution of benchmark consumption slows everything down. At the other extreme, as $\rho \to \infty$, so that $h \to c$, the time non-separable utility model converges to the standard model, although with a higher IES. As a consequence, the convergence speed again converges to that of the standard model, although adjusted now for the higher IES.

[18] In early terminology of dynamic systems, the system would be said to be "indecomposable".

Table 2 also shows that external consumption benchmark lead to more rapid convergence than does the inward-looking economy. This is because from the perspective of an inward-looking agent, who takes account of the impact of his current consumption on the reference level, the utility function becomes more concave. This decreases the intertemporal elasticity of substitution, and slows down the rate of convergence. In the outward-looking economy, when the initial shock leaves the capital-habit ratio below its equilibrium level, the savings rate increases more than in the externality-free economy, and therefore the transition is characterized by under-consumption. On the other hand if the after-shock capital-habit ratio is above its equilibrium level, the transition of the outward-looking economy is characterized by over-consumption. As a result, over-consumption, when the stable growth path requires capital decumulation, and under-consumption, when the stable growth path is characterized by a higher level of capital, both accelerate the convergence process. Under increasing returns, speeds of convergence are reduced for all cases, while they are faster for decreasing returns to scale, consistent with Eicher and Turnovsky (1999b).

We now focus on the dynamic response to two shocks: (i) a 25% increase in the productivity parameter, (ii) a 10% destruction in capital. In both cases, by virtue of the non-scale characteristic of the model, the steady-state growth rate remains unchanged.

4.2 A 25% increase in the productivity parameter α.

We begin by considering the evolution of the three economies in response to a permanent 25% increase in the productivity parameter α from 1 to 1.25. Table 3 summarizes the short-run and long-run effects of the change on key economic variables for the three specifications of preferences. While we treat constant returns to scale as the benchmark case, Panels B and C also report the cases of increasing and decreasing returns to scale.

Table 3 also summarizes the effects of the shocks on two measures of welfare. The long-run (intertemporal) level of welfare, reported in the final column, measures the representative agent's optimized utility function Ω [given in (1)], when C_i, H_i are evaluated along the equilibrium path. The welfare gains reported are equivalent variation measures, calculated as the percentage change in the permanent flow of consumption necessary to equate the initial levels of welfare to what they would be following the structural changes. Details of this calculation are provided in the Appendix.

In addition, we report the short-run (instantaneous) welfare effect. In the Appendix we show that converting the utility measures into equivalent permanent changes in consumption flows, we can express the change in instantaneous utility from the base pre-shock level in terms of

$$\frac{dZ_a}{Z_b} = \frac{dC}{C_b} + \frac{\gamma}{1-\gamma}\frac{d\left(C/H\right)}{C/H} \tag{21}$$

In the case of conventional preferences, $\gamma = 0$, the time path for instantaneous utility simply mirrors that of instantaneous consumption. However, when utility depends in part upon benchmark consumption, the percentage change in instantaneous utility consists of two components; the percentage change in absolute consumption, plus the percentage change in consumption relative to its benchmark level. In the case we consider $\gamma = 0.5$, in which case (21) is just the sum of these two components. Decomposing the change in instantaneous utility in this way enhances our understanding of the comparative welfare effects of the technology shock for the three preferences.

Figure 1 illustrates the resulting transitions for the alternative preference specifications. We focus on four aspects: the evolution of key quantity variables, growth rates, consumption-savings behavior, and welfare. We proceed as follows. We first describe the features shared by our three specifications and then we highlight their differences. Since the qualitative responses are not too sensitive to returns to scale, we illustrate only the constant returns to scale case, when the steady-state equilibria are unchanged across the specifications of preferences.

From Panel A in Table 3, we see that a permanent increase in productivity of 25% raises the steady-state levels of capital, output, and consumption by 41%. The fact that these long-run changes are independent of the utility specification, is consistent with the formal steady-state equilibria summarized in Section 3. With the capital stock being fixed instantaneously, the immediate effect of the 25% increase in α is to raise short-run output by 25%, this being so for all three utility specifications. At the same time, the higher productivity of capital encourages the accumulation of more capital so that the additional output leads to immediate increases in both savings and consumption.[19] The higher level of saving results in an increase in the rate of capital accumulation that leads to a progressive increase in output, capital, and consumption. This accumulation process continues until the new steady-state levels of capital, output, and consumption are achieved.

From Figure 1, we see that the economy evolves along qualitatively similar paths in response to this productivity shock, for all three forms of utility specification, this being characterized by steadily increasing capital, output, consumption, and welfare, together with initial increases in the savings and growth rates, which thereafter decline steadily over time. But despite these qualitative similarities, there are striking differences in the magnitudes between the two specifications of time non-separable preferences, on the one hand, and the conventional time separable preferences, on the other. This can be seen clearly from the various panels of Figure 1 where the time paths for both inward-looking and outward-looking economies track each other closely, but are quite distinct from those of the conventional case.

In the standard case of time separable utility, where only the "rate of return effect" is in operation, the higher marginal product of capital leads to an immediate 25% increase in output, relative to its initial steady-state level, leading to a 23% increase in the consumption level and a corresponding 32% increase in the level of savings. This translates to an initial increase in the savings rate, $s(t)/y(t)$, from 21.6% to around 23%.

When preferences are conditioned by the presence of a reference consumption level, the initial response is a relatively larger increase in savings and a relatively smaller increase in consumption. The initial increase in consumption is limited by the "status effect", meaning that the utility associated with any short-run increase in consumption relative to the reference stock is dampened, thereby reducing the incentive to consume. In the case of inward-looking agents, the initial increase in the consumption level is reduced to 17%, thus allowing a 52% increase in savings, relative to its initial level, and an increase in the savings rate from 21.6% to 26.7%. This leads to a substantially faster growth of capital, as seen in Figs 1b and 1d, relative to the conventional case. After around 10 years the capital stock in the inward-looking economy will exceed that of the

[19]Since we are dealing with a closed economy without a government sector savings and investment coincide and we use the terms savings and investment interchangeably.

conventional economy by a sufficient amount so that its consumption level will begin to overtake it as well, as seen in Fig. 1c.

The case of an externally generated reference consumption level operates in much the same way, though with one difference. When the reference stock is generated externally, the agent ignores the fact that current consumption also contributes positively to the evolution of his reference consumption stock reducing his future satisfaction. As a result, the transition in this case is characterized by initial over-consumption relative to the inward-looking economy, followed by subsequent under-consumption, during later phases of the transition (see Fig. 1c). However, these differences are very small, the initial increase in consumption being 17.4% rather than 17.0%.

The time paths for instantaneous welfare corresponding to the alternative specifications for preferences are illustrated in Fig. 1f. We immediately see from (21) that the initial 23% increase in consumption under conventional preferences leads to a corresponding initial 23% increase in welfare. Both forms of time non-separable preferences lead to a smaller increase in initial consumption of only around 17%, which raises utility by the same percentage amount. But in addition, with habits being sluggish (and fixed instantaneously) this raises short-run relative consumption by around 17.5%, so that overall, welfare increases by around 37% in the short run.[20] Over time, as habits begin to catch up to current consumption, the second term in (21) declines to zero and the steady-state welfare changes all converge to the long-run change in the consumption level, 41%. The different intertemporal measures of welfare thus reflect the differences along the transitional time paths and it is interesting to observe that with rapidly increasing consumption and more slowly changing relative consumption with internal habits, means that instantaneous welfare actually overshoots its long-run level during much of the transition. Indeed, both internal and external preferences imply welfare gains of around 40%, very close to the 41% that would be attained if the adjustment to the new steady-state occurred instantaneously, and significantly greater than 33.7% implied by conventional preferences.

These results have two interesting implications. First, despite the fact that agents having time

[20] With external habits leading to slightly more consumption in the short run, they are therefore associated with slightly higher short-run welfare.

non-separable preferences enjoy smaller short-run absolute consumption gains than those having conventional preferences, in response to the productivity shock, they nevertheless enjoy larger short-run utility gains.[21] This is because they also derive benefits from the relative change involved. Second, if preferences are in fact time non-separable as much recent empirical evidence suggests, the welfare conclusions obtained under the assumption of time separability could be highly misleading.

4.3 Destruction of Capital of 10%

Table 4 summarizes the effects of a temporary 10% destruction of capital, brought about by a natural disaster such as an earthquake. Figure 2 illustrates the dynamics in the benchmark case of constant returns to scale. Focusing on this case, we see that for all three specifications of preferences this leads to an initial reduction in output of 3.6%. In the case of conventional preferences, we see that this causes an initial reduction in consumption of around 4.3% and savings of around 1%, leading to an initial *increase* in the savings rate by around 3% to around 22.3%, and to a gradual restoration of the capital stock at its original level. The initial decline in consumption leads to an equivalent initial reduction in welfare of around 4.3%. However, the monotonic increase in consumption back to its pre-shock level reduces the present value of the overall welfare loss throughout the transition to approximately 1.7%.

The introduction of an internally generated reference stock leads to substantial differences in the adjustment paths following a temporary destruction of capital. The initial reduction in output is again around 3.6%. This time, the existence of the reference stock inhibits the decline in initial consumption, which now falls by only 3.3%, leading to an immediate *decrease* in the savings rate of close to 1%, reducing it to 21.4%. In the short run, following the initial destruction, the "status effect" limits the capacity of consumption to adjust. Therefore, savings increases faster than does output so that the savings rate begins to rise. After about 12 years, the growth rate of savings catches up to that of output and the savings rate peaks at around 22.1%. Thereafter, as the effects of the past increases in consumption are incorporated into the reference stock, current consumption

[21] This comparison needs to be interpreted with care. We do not mean to compare the welfare of agents having time-separable utility functions with those of agents having time-dependent utility functions. Instead, we are suggesting that an analysis based on time-separable utility would understate the short-run benefits derived by an agent having time-dependent utility

increases and the savings rate declines, doing so monotonically until the new equilibrium is reached.

In the presence of the preference externality, agents ignore the fact that a reduction in current consumption lowers the future reference stock, this leads to a transition characterized by under-consumption. The initially larger reduction in consumption allows for an immediate increase in the saving rate of 1%, thereafter the adjustment path is qualitatively similar to the inward-looking case.

The time paths for instantaneous welfare corresponding to the alternative specifications for preferences are illustrated in Fig. 2f. Using (21), we see that for conventional preferences, $\gamma = 0$, the time path for instantaneous utility simply mirrors that of instantaneous consumption, declining by around 4% initially. For the assumed value of $\gamma = 0.5$, the initial decline in welfare for both internal and external shocks reflects the decline in absolute and relative consumption. Thus welfare in the two cases immediately declines by around 6.5% and 7.5% respectively. But with capital adjusting more rapidly with internal habits and most rapidly with external habits the initial decline in welfare is eliminated more rapidly as we move from the conventional, to the internal, to the external habits cases. After about 15 years, the initial welfare ranking will be reversed. Despite the rather different time profile of welfare costs, these are more or less offsetting, so that the intertemporal welfare losses of the destruction of capital with either inward-looking or outward-looking agents is around 2%, slightly higher than the 1.7% for the conventional utility function.

One interesting contrast from the productivity shock is that there is a greater divergence between the two specifications of time non-separable preferences, particularly during the early phases of the adjustments. This is most clearly evident in the behavior of the savings rate, $s(t)/y(t)$, which begin to converge only after around 20 years.

Finally, the analysis of this shock provides some interesting insight into the relationship between growth and savings. Empirical evidence summarized by Carroll et al. (2000) suggests that growth leads to savings. But conventional growth models have the property that growth and savings are contemporaneously related and cannot capture adequately this type of Granger-causal relationship. By contrast, comparing Figs. 2b and 2e in the two cases with benchmark consumption, we see that the increase in the growth rate from 1.5% to around 1.7% *precedes* the increase in the savings rate by several periods; i.e. growth leads savings, consistent with the empirical evidence.

Intuitively, an economy having a low ratio of capital to benchmark consumption reduces its consumption level only gradually as the benchmark is allowed to adjust. This process of belt-tightening combined with the higher marginal product of capital leads to high growth rates. Since agents by now are "habituated" to low levels of consumption, a large share of the additional output is saved, leading to a progressive increase in the savings rate. As capital recovers, the growth rate of output decreases and now the faster rate of growth of consumption leads to a progressive reduction of the saving rate that now converges monotonically to its equilibrium value. The "status effect" is behind this behavior, initially preventing the decrease in consumption, and then allowing for slow consumption growth. The time-varying growth rate of output does the rest.

5. Some Sensitivity Analysis

Our analysis has introduced three critical parameters, upon which we shall focus: (i) the speed of adjustment of the reference stock, ρ, (ii) the weight of habit in preferences, γ, and (iii) returns to scale, determined by the production externality, η. Of these, our results are most sensitive to ρ, so that (i) and shall be illustrated in more detail.

5.1 Speed of Adjustment of Reference Stock

Figures 3 and 4 illustrate the transitional dynamics of key variables in response to the two shocks as we increase the speed of adjustment of the reference stock, ρ, from the base value 0.2, through 0.8 and 2. We focus on consumption, savings, and welfare, since these are the variables most sensitive to this change in the preference structure, and we restrict ourselves to the initial stages of the transition, since this is the phase during which most of the changes occur. In all graphs the paths for time separable preferences remain unchanged. The issue is therefore, how the two types of habit formation change relative to this. We may note that as $\rho \to 0$ or $\rho \to \infty$ the paths of the time non-separable economies converge to the conventional case. In general, the relation between ρ and the speed of convergence of the real variables is non-monotonic. The asymptotic speed of convergence of the overall system increases with ρ for low values of ρ, and decreases with ρ for high values of ρ.

Since both inward-looking and outward-looking economies behave similarly, we shall focus on the former. Increasing ρ from 0.2 to 0.8 reduces the initial increase in consumption, raising the initial savings rate (following the shock) from 26.7% to over 28.5%. Lower short-run consumption reduces welfare in the short run, as greater resources are devoted to capital accumulation, and indeed, on impact, welfare increases by less than 30%, rather than 36.9%. In the short term this increases the growth rate of output and the savings rate falls. At the same time, the initial more rapid rate of adjustment of habits reduces the consumption-habits ratio, more than offsetting the positive effects of more consumption, so that after the initial increase, welfare falls during the first 2 periods. However, over time, the increase in absolute consumption more than offsets the decline in relative consumption and welfare begins to rise. As ρ increases further, the decline in welfare during the early phase becomes more pronounced.

Figure 4 conducts a similar sensitivity analysis in the case of a 10% destruction of capital. The most striking feature is that increasing the speed of adjustment of the reference stock causes consumption to continue to decline following the initial shock, in the case of time non-separable preferences, doing so for about 2 periods. This has two effects on the "hump" in the savings ratio; it both accentuates it and pushes it forward in time, so that if $\rho = 2$ it peaks after about two periods, rather than in about 10-12, as in the benchmark case.

5.2 Weight of Habits in Preferences

The changes in response to changes in γ are less pronounced and we simply report the main characteristics. As γ declines toward zero, the contribution of relative consumption declines and the time non-separable specification converges to the conventional time separable case. As γ increases, the time non-separable economies respond to an increase in productivity with smaller initial increases in consumption. Intuitively, as the weight of relative consumption increases, smaller increases in the level of consumption are enough to achieve larger increases in instantaneous welfare. The possibility of the time non-separable economies to substitute relative consumption for absolute consumption allows them to achieve larger increases in savings, capital accumulation and growth. If $\gamma = 0.8$, the weight assigned to relative consumption is so large that welfare initially increases by around 90% for both specifications, and then declines, along with relative consumption during the transition.

In the case of the destruction of capital, an increase in the weight of relative consumption reduces the initial decline in consumption following the shock, in the case of time non-separable preferences. At the same time, consumption continues to declines for several periods thereafter, particularly for the inward-looking economy. The combination of these two effects tends to postpone but accentuate the "hump" in the savings ratio, that this shock generates.

5.3 Returns to Scale

Panels B and C in Tables 2-4 summarize the quantitative effects of the two shocks under increasing returns and decreasing returns, respectively. In the case $\eta = 0.2$, the equilibrium savings ratio and under time non-separable preferences exceeds that under conventional preferences causing the long-run output-capital ratio to be smaller. The opposite applies for $\eta = -0.2$. The long-run output-capital and savings ratios are more sensitive to returns to scale under conventional preferences than they are under either form of time non-separable preferences. This is because the sensitivity of these two quantities to returns to scale is inversely related to the IES and for $1/\varepsilon < 1$, time non-separable preferences are equivalent to a larger IES.

With the capital stock fixed instantaneously, a 25% increase in productivity raises short-run output by 25% in all cases. With conventional preferences the initial increase in output is mainly absorbed by a large increase in consumption, leading to an immediate decrease in the savings rate that monotonically converges to its long-run equilibrium from below. For time non-separable preferences, the initial response of consumption is tied to the predetermined benchmark level of consumption and therefore the increase in output leads to an immediate increase in the savings rate, which thereafter converges monotonically to its long-run equilibrium from above.

The time paths for the key variables generally reflect the patterns illustrated in Figure 1 and are therefore not shown. But there are some differences worth noting for time non-separable preferences. First, for increasing returns to scale, instantaneous welfare increases monotonically, while for decreasing returns the overshooting illustrated in Fig. 1f is accentuated. Finally, for decreasing returns, both consumption and capital overshoot their long-run equilibria during the transition.

Under increasing returns to scale the reductions in short-run output due to the destruction of capital are exacerbated, declining uniformly by 5.6% in all three cases. Under conventional preferences consumption declines by 5.7%, while with time non-separable preferences, the declines are moderated due to the benchmark consumption level. The qualitative patterns of adjustment are generally similar to those illustrated in Fig. 2. The main difference is in the time paths for the savings ratio. The time paths in the case of time non-separable preferences shift down relative to that for conventional preferences in the case of decreasing returns to scale, and shift up relatively in the opposite case.

6. Time non-separable preferences: AK vs. non-scale technology.

As we noted at the outset, Carroll et al. (1997) examine the dynamics of the basic Rebelo (1991) endogenous growth model under time non-separable preferences. The introduction of a second state variable, benchmark consumption, introduces transitional dynamics, so that in contrast to the conventional AK model, the economy is no longer always on its balanced growth path, but now follows a transitional path. Nonetheless, the strong knife-edge conditions required to generate ongoing growth severely restrict the equilibrium dynamic behavior, essentially restricting it to monotonic adjustments exclusively driven by preference parameters.

In this section we briefly compare the results of that model, with its assumed constancy of the return on capital, with those of the present model, obtained under a more flexible production specification. To preserve comparability, we increase the population growth rate to $n = 2\%$, so as to generate an equilibrium growth rate of 2% as they do. We also follow them setting $\varepsilon = 2$. All other parameters remain unchanged. Since the behavior of each production technology -- AK and non-scale -- is qualitatively similar across preference specifications we restrict our comparison to the "catching up with the Joneses" case.

The key factor determining the similarity or divergence in the behavior of the two models in response to a shock is the extent to which the dynamics in the present, more flexible, model is non-monotonic. We conduct the comparison for both types of shocks. For the productivity shock, and slowly adjusting habits, the transitional adjustment path in the non-scale model is monotonic, and the behavior of the two economies are qualitatively similar during the transition. The only difference is that in the AK model, this shock leads to a higher permanent growth rate, whereas in the non-scale model the increase in growth rate is purely transitory.[22]

Differences are much more pronounced for the destruction of capital, even for slowly adjusting habits, as is strikingly evident from Figure 7. In the AK case, an economy with the initial capital-habit ratio below its steady-state level will have an initial low level of consumption. The saving, capital accumulation, output and consumption growth rates will initially be low, recovering monotonically their steady state levels after three decades. In contrast, the model with the more flexible production technology predicts a qualitatively different behavior. The initial destruction of capital raises its marginal product leading to an immediate decrease in the level of consumption. The higher level of savings leads to an increase in the rate of capital accumulation. This higher level of investment results in a progressive recovery of output, capital and consumption. With a constant rate of return to capital, saving and growth remain below their steady state levels along the transition, while if the rate of return to capital is endogenously determined, saving and growth approach their steady state from above.

The intuition behind these contradicting results rests on the different assumptions about the behavior of the marginal product of capital implied by each production technology. If the aggregate technology exhibits constant returns to capital, then its marginal product is independent of the level of capital, and therefore the saving-consumption decision is dominated by the predetermined reference stock. After a destruction of physical capital, an agent with a high reference stock will try to prevent consumption from falling relative to habit, "the status effect", consuming at an unsustainably high level while he allows the reference stock to adjust. This higher consumption-output ratio lowers the rates of saving, capital accumulation, and growth along the transition. On the other hand, if the aggregate technology exhibits diminishing returns to capital two counteracting

[22]We do not illustrate this case. We have also allowed for more rapidly adjusting habits (i.e. increasing ρ). This tends to introduce non-montonicity in the response to the productivity shock for internal habits in the present model, leading to greater divergence between its dynamics from that of the AK model.

effects drive the adjustment process. As under AK technology, the “status effect” is present, preventing a plunge in the level of consumption. As in the conventional model described in section 3.1, the “rate of return effect” stimulates saving and capital accumulation. In the non-scale framework the “rate of return effect” dominates leading to a transition characterized by above-equilibrium levels of growth and saving.

7. Conclusions

Recent empirical evidence has supported the importance of time non-separable preferences as an alternative to the conventional time separable utility function. Given the convincing nature of this evidence it is important that its consequences for the dynamics and growth of the macro economy be well understood. Previous research has focused almost entirely on the simplest AK growth model and this paper has examined the effects of introducing time non-separable preferences in the more flexible non-scale growth framework.

This analysis has been carried out with a twofold objective in mind. First, we have compared the adjustment process of the key variables in our model, under three alternative preference specifications: (i) conventional time separable preferences, (ii) outward-looking preferences reflecting attitudes of “catching up with the Joneses”, and (iii) inward-looking preferences that reflect habit formation. Because of the complexity of the more general specifications of preferences, most of our work has proceeded numerically, by calibrating a plausible macroeconomic model.

The analysis finds important differences in the behavior of consumption and saving under the two specifications of time non-separable preferences, (ii) and (iii), relative to the conventional specification, (i). These differences arise from the fact that the introduction of the reference stock ties the behavior of these variables to the past, thus limiting their ability to respond to a shock. How much (ii) and (iii) deviate from one another during a transition depends on the shock. Inward-looking and outward-looking preferences track each other remarkably closely in response to an exogenous increase in productivity, with both deviating substantially from the time path generated by conventional preferences. However, for the other shock we consider, a destruction in the initial capital stock, (ii) and (iii) are less closely tied during the transition.

The second aspect we consider is to contrast the effects of time non-separable preferences under alternative production structures. We do this by comparing our results, obtained under a more general production structure, and the results obtained under the restrictive AK framework. The main conclusion of this comparison is the surprisingly different response of both models to a 10% destruction of capital. In the AK structure consumption, saving and the growth rate approach the new steady state from below. Conversely, our model predicts an approach to the steady state from above for saving and the growth rate, while at the same time allows an immediate decrease in the consumption-habit ratio and a subsequent overshooting of its steady state level. This overshooting is made possible by the increased dimensionality of the dynamic system that allows for the possibility of non-monotonic adjustment processes. The intuition behind these results lies in understanding the interacting forces at work. The dynamics under time separable preferences are driven by what we have called the "rate of return effect". On the other hand, what we have called the "status effect" is the engine behind the adjustment process in the time non-separable AK model. In our specification both effects play an important role along the adjustment process, leading to a flexible framework able to account for a very rich dynamic behavior.

Another interesting consequence of the more flexible framework we have adopted is the fact that our time non-separable specifications provide interesting insight in the growth-saving relation. In contrast to conventional models where saving is seen as the engine of growth, our model inverts this causal relation. Preliminary analysis of the time series characteristic of our simulated data suggests Granger causality from growth to saving. This implication is consistent with empirical evidence summarized by Carroll et al. (2000) and is something that we intend to study in more detail in subsequent work.

Finally, this paper has abstracted from policy. But it is clear that the sharply contrasting macroeconomic behavior generated by time non-separable versus time separable preferences has potentially important implications for policy makers. If in fact the economy is better described by time non-separable preferences, then macroeconomic policies based on the assumption of conventional preferences are likely to be grossly inappropriate. Clearly the design of optimal fiscal policies under time non-separable preferences is an important issue that also merits investigation.

Table 1.
Benchmark parameters

Production parameters	$\alpha = 1, \sigma = 0.65, \eta = -0.2, 0, 0.2, \delta = 0.05$
Preference parameters	$\beta = 0.04, \varepsilon = 2.5,$ $\rho = 0.2, 0.8, 2; \gamma = 0.5, 0.8$
Population growth	$n = 0.015$

Table 2
Base Equilibria

Constant Returns to Scale, $\eta = 0$								
Conventional				External and Internal Habits				
Y/K	I/Y %	$\hat{Y}$ %	stable eigenvalues	Y/K	I/Y %	$\hat{Y}$ %	stable eigenvalues external	stable eigenvalues internal
0.30	21.7	1.50	-0.0626, -0.20	0.30	21.7	1.50	-0.1057 -0.1057	-0.0932 -0.0932
Increasing Returns to Scale, $\eta = 0.2$								
Conventional				External and Internal Habits				
Y/K	I/Y %	$\hat{Y}$ %	stable eigenvalues	Y/K	I/Y %	$\hat{Y}$ %	stable eigenvalues external	stable eigenvalues internal
0.35	20.6	2.16	-0.0383 -0.20	0.33	21.5	2.16	-0.0557 -0.1319	-0.0567 -0.1085
Decreasing Returns to Scale, $\eta = -0.2$								
Conventional				External and Internal Habits				
Y/K	I/Y %	$\hat{Y}$ %	stable eigenvalues	Y/K	I/Y %	$\hat{Y}$ %	stable eigenvalues external	stable eigenvalues internal
0.27	22.37	1.14	-0.0942 -0.20	0.28	21.8	1.14	-0.1222 -0.1222	-0.1093 -0.1093

Table 3
25% Increase in α

Constant Returns to Scale $\eta = 0$										
	Short-run					**Long-run**				
	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %
Conventional	0	23.1	25.0	0.229	23.04	41	41	41	0.217	33.71
External habits	0	17.4	25.0	0.264	37.82	41	41	41	0.217	40.19
Internal habits	0	17.0	25.0	0.267	36.91	41	41	41	0.217	40.18
Increasing Returns to Scale $\eta = 0.2$										
	Short-run					**Long-run**				
	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %
Conventional	0	27.5	25.0	0.190	27.48	64	64	64	0.206	42.54
External habits	0	21.8	25.0	0.235	48.26	64	64	64	0.215	55.09
Internal habits	0	21.3	25.0	0.239	47.05	64	64	64	0.215	55.26
Decreasing Returns to Scale $\eta = -0.2$										
	Short-run					**Long-run**				
	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %
Conventional	0	19.6	25.0	0.257	19.61	30	30	30	0.224	27.12
External habits	0	14.3	25.0	0.284	30.75	30	30	30	0.218	31.07
Internal habits	0	14.0	25.0	0.286	30.01	30	30	30	0.218	30.95

Table 4
10% Destruction in *K*

Constant Returns to Scale $\eta = 0$										
	Short-run					**Long-run**				
	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %
Conventional	-10	-4.3	-3.6	0.223	-4.36	0	0	0	0.217	-1.72
External habits	-10	-3.8	-3.6	0.218	-7.50	0	0	0	0.217	-2.02
Internal habits	-10	-3.3	-3.6	0.214	-6.52	0	0	0	0.217	-2.01
Increasing Returns to Scale $\eta = 0.2$										
	Short-run					**Long-run**				
	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %
Conventional	-10	-5.7	-5.6	0.207	-5.72	0	0	0	0.206	-3.27
External habits	-10	-4.9	-5.6	0.209	-9.55	0	0	0	0.215	-3.71
Internal habits	-10	-4.5	-5.6	0.205	-8.70	0	0	0	0.215	-3.84
Decreasing Returns to Scale $\eta = -0.2$										
	Short-run					**Long-run**				
	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %	Cap. %	Cons. %	Output %	Sav. Ratio	Welf. %
Conventional	-10	-3.5	-1.6	0.239	-3.47	0	0	0	0.224	-0.90
External habits	-10	-3.1	-1.6	0.230	-6.19	0	0	0	0.218	-1.19
Internal habits	-10	-2.6	-1.6	0.226	-5.09	0	0	0	0.218	-1.12

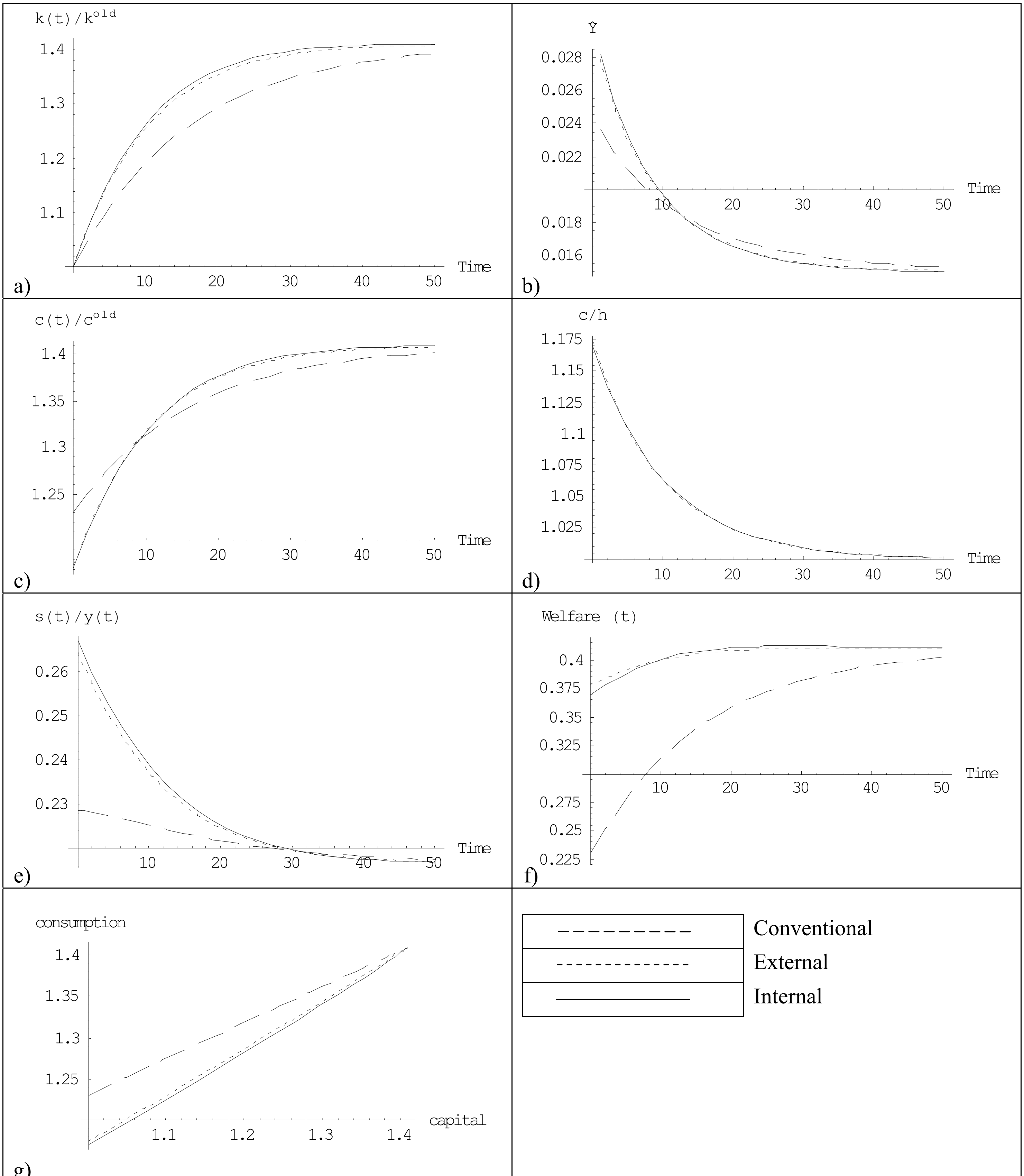

Figure 1. Transitional dynamics after a 25% increase in α , $\eta = 0$

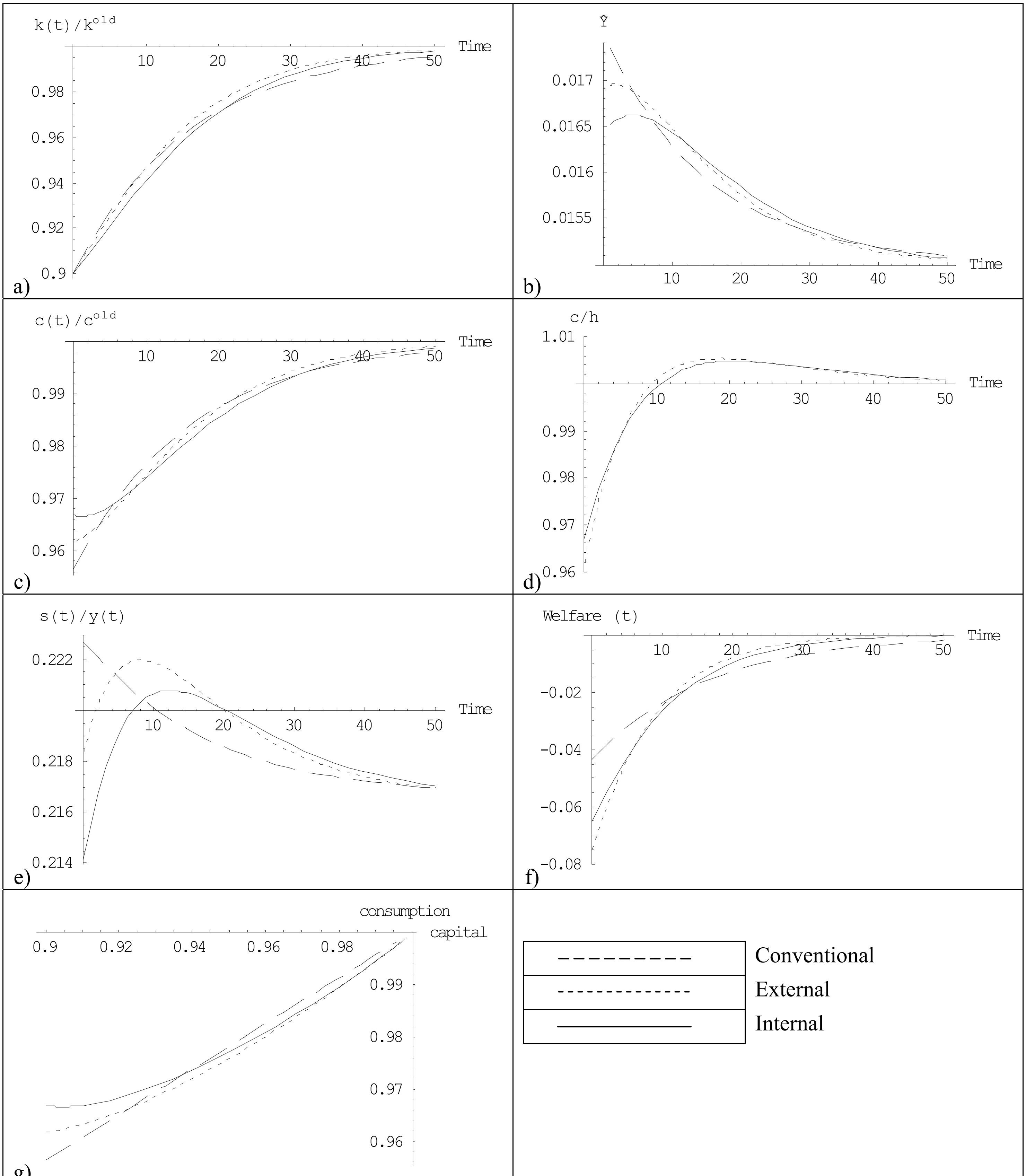

Figure 2. Transitional dynamics after a 10% destruction of capital, $\eta = 0$

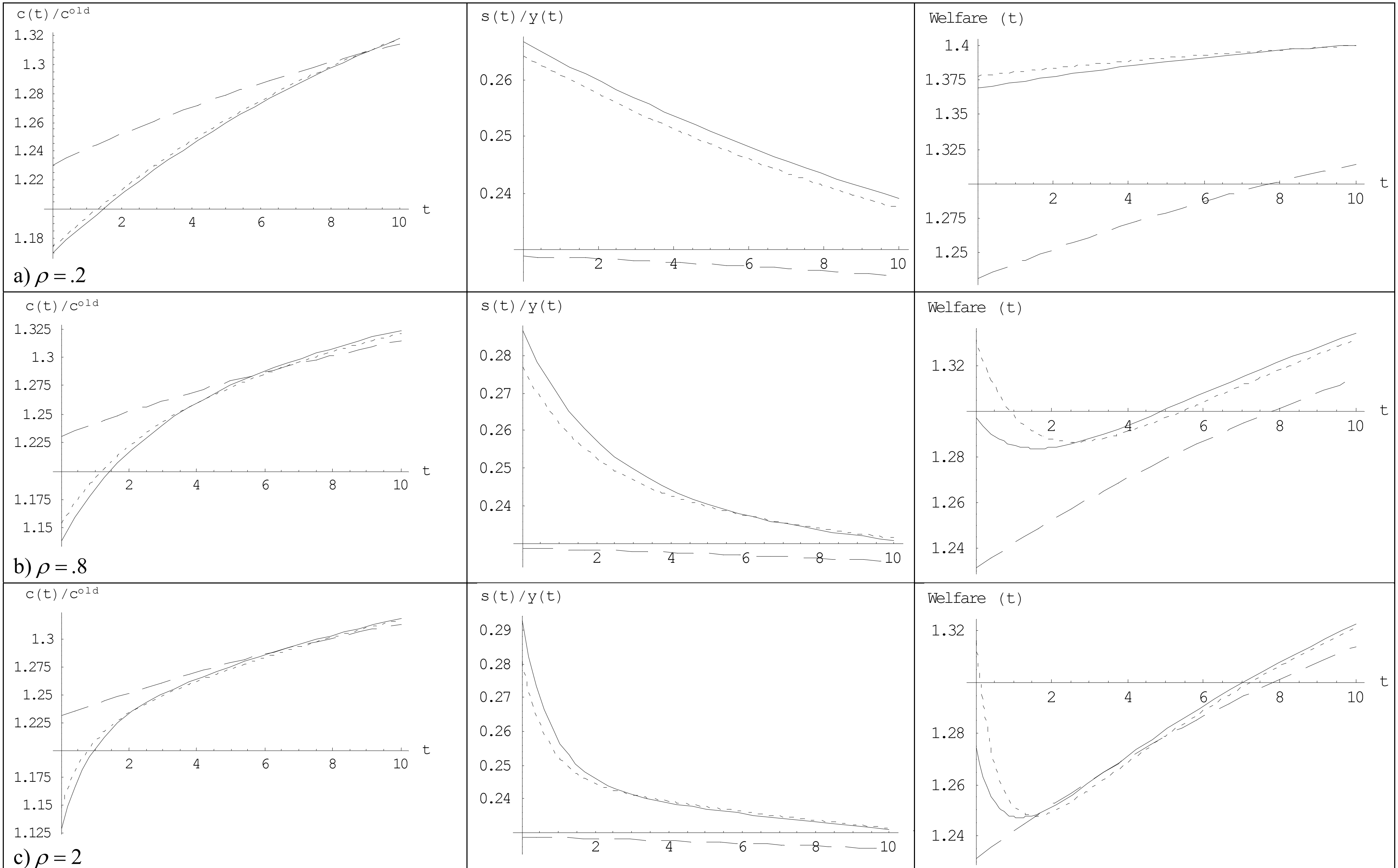

Figure 3. Transitional dynamic after an increase α , $\eta = 0$ for alternative values of ρ

– – – – – – –	Conventional	- - - - - - - - - - - -	External	————	Internal

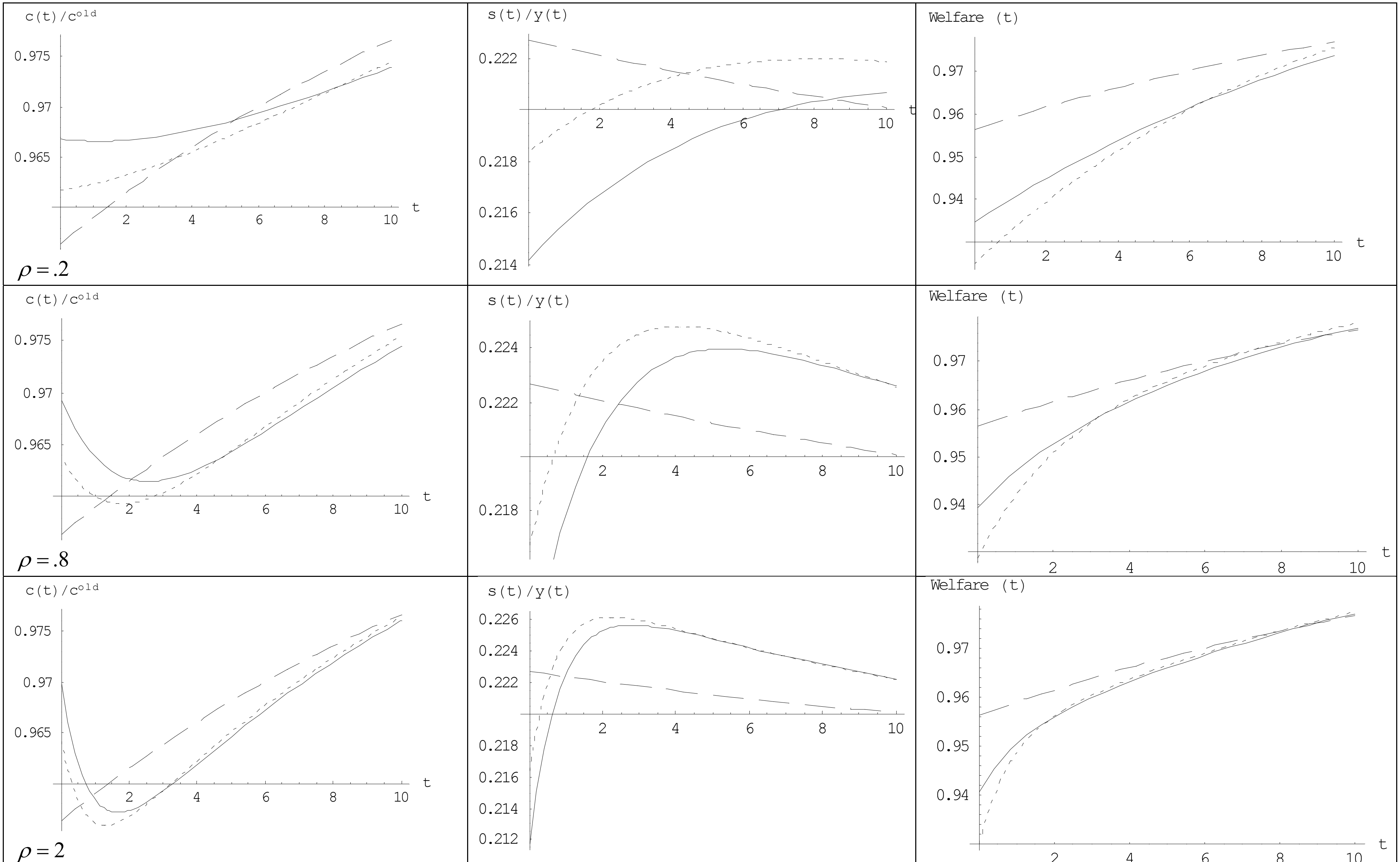

Figure 4. Transitional dynamic after a 10% destruction of capital, $\eta = 0$ for alternative values of ρ

– – – – – – –	Conventional	- - - - - - - - - - - -	External	————————	Internal

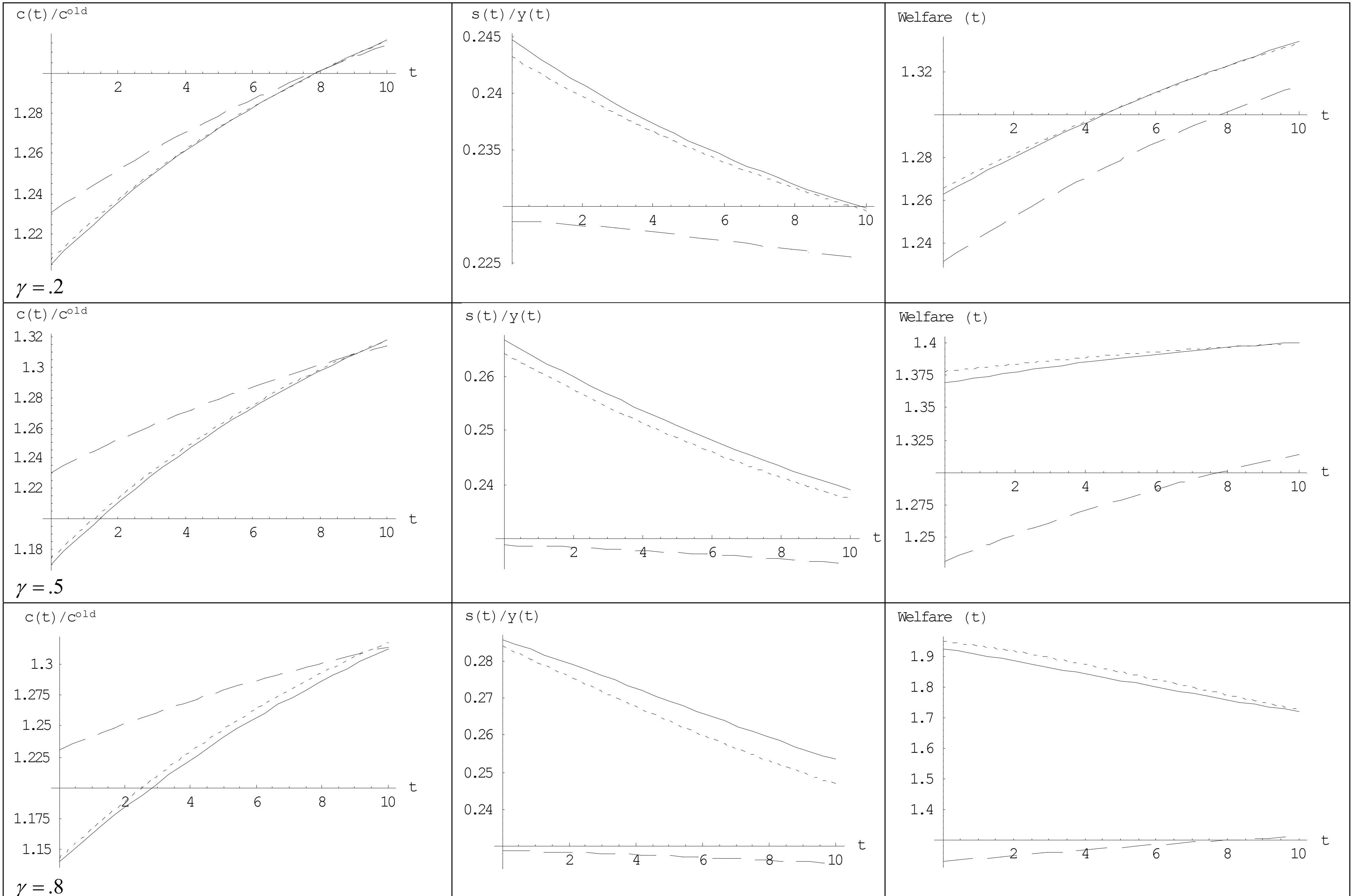

Figure 5 . Transitional dynamics after a 25% increase in α , $\eta = 0$. Alternative values for γ

– – – – – – –	Conventional	- - - - - - - - - - - -	External	________	Internal

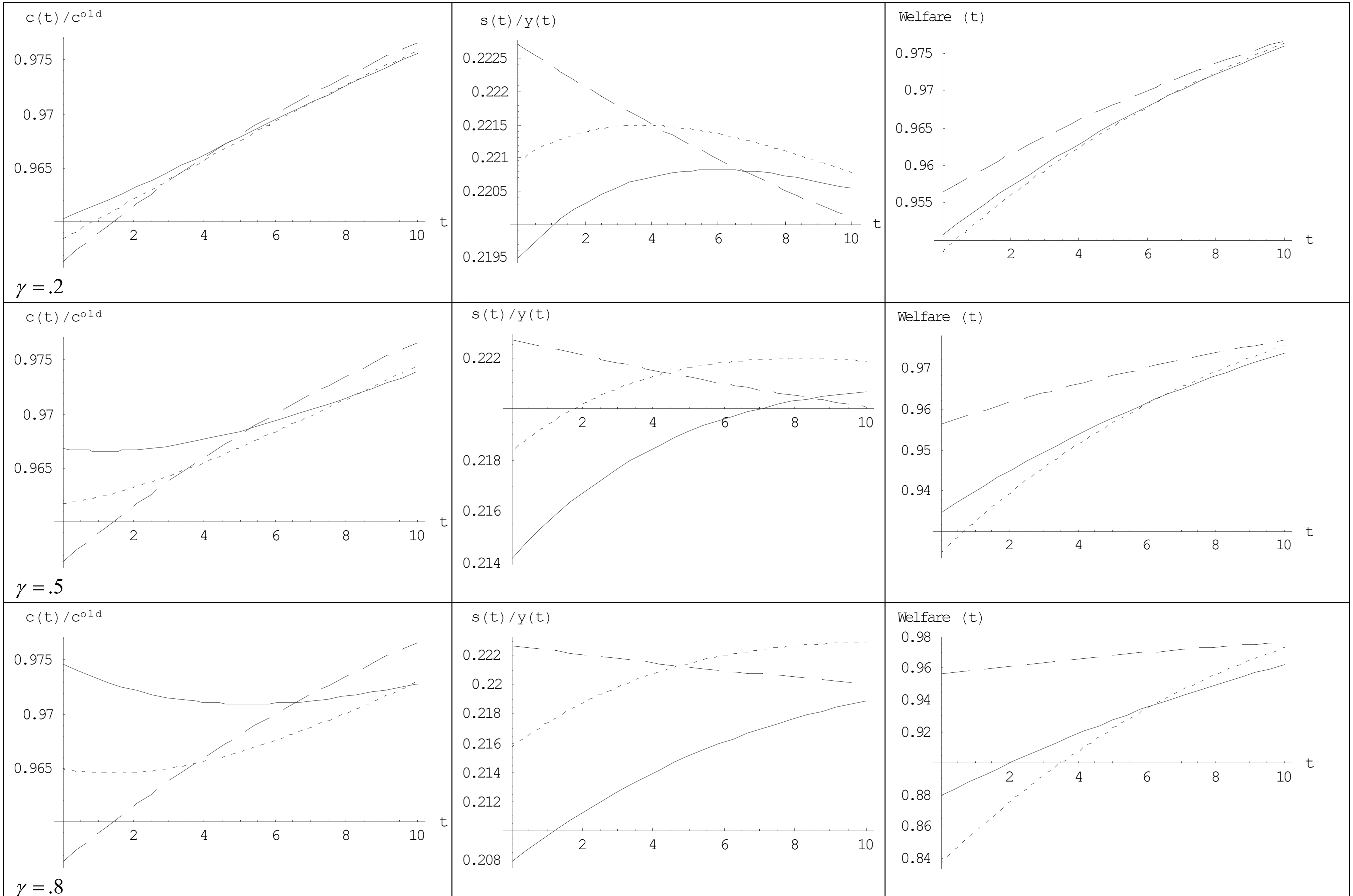

Figure 6. Transitional dynamics after a 10% destruction of capital, $\eta = 0$. Alternative values for γ

— — — — — — —	Conventional	- - - - - - - - - - - -	External	————————	Internal

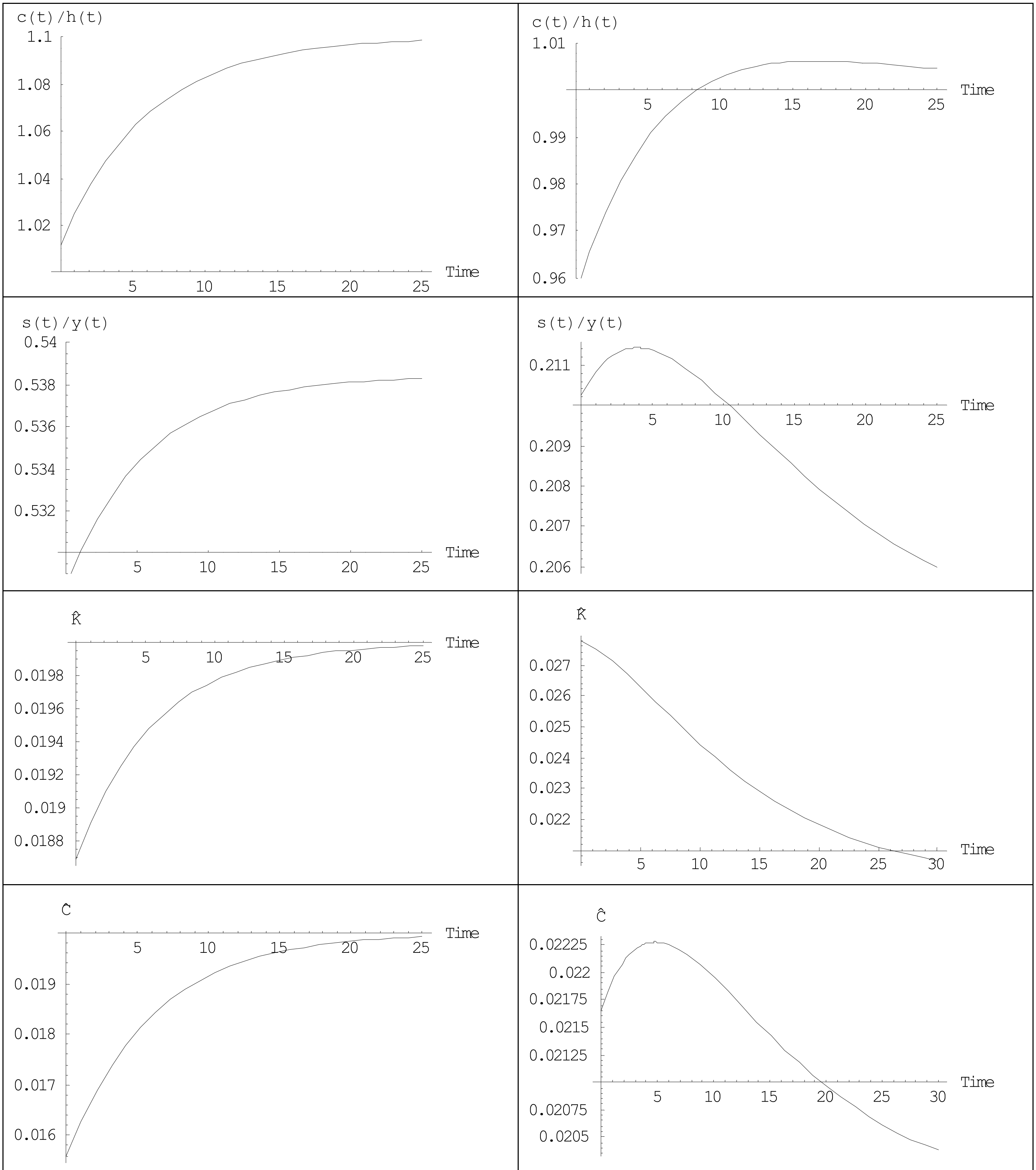

Figure 7. Transitional dynamics after a 10% destruction of capital, $\eta = 0$, under external habits. AK model in the left column, non-scale model in the right column

Appendix

A.1 Elimination of Non-Optimal Equilibrium

We show how one of the solutions for capital, in the internal habit formation case violates a transversality condition and therefore can be eliminated. We first reproduce (18d), rewritten as

$$q^* = \frac{\gamma\left(1+(g-1)n/\rho\right)}{\rho\gamma\left(1+(g-1)n/\rho\right)+n+\delta-\rho-\alpha(1-\sigma)\left(k^*\right)^{\sigma_K-1}} \tag{A.1}$$

Setting $\dot{c}=0$ in (17b) and multiplying the resulting equation by $(1-\rho q^*)$ we get

$$\alpha(1-\sigma)k^{*\sigma_K-1}+\rho q^*\left(\beta+\rho\right)-(\beta+n+\delta)+\left(\rho\gamma\varepsilon\left(\frac{c}{h}\right)^*+\gamma\rho\left(1-\varepsilon\right)+(1-g)\varepsilon n\right)\left(1-\rho q^*\right)=0 \tag{A.2}$$

Substituting (A.1) and (17c) into (A.2) leads to a quadratic expression in $(k^*)^{\sigma_K-1}$

$$\alpha(1-\sigma)\left(k^*\right)^{\sigma_K-1}+\rho\left(\frac{\gamma\left(1+(g-1)n/\rho\right)}{\rho\gamma\left(1+(g-1)n/\rho\right)+n+\delta-\rho-\alpha(1-\sigma)\left(k^*\right)^{\sigma_K-1}}\right)\left(\beta+\rho\right)-(\beta+n+\delta)+$$
$$\left(\rho\gamma\sigma\left(1+\frac{(g-1)}{\rho}n\right)+\gamma\rho\left(1-\eta\right)+\left(1-g\right)\varepsilon n-\varepsilon gn\right)\left(1-\rho\left(\frac{\gamma\left(1+(g-1)n/\rho\right)}{\rho\gamma\left(1+(g-1)n/\rho\right)-\rho+n+\delta-\alpha(1-\sigma)\left(k^*\right)^{\sigma_K-1}}\right)\right)=0$$

Rearranging terms, we can factorize this equation as follows

$$\left(\delta+n-\rho-\alpha(1-\sigma)\left(k^*\right)^{\sigma_K-1}\right)\left(\alpha(1-\sigma)\left(k^*\right)^{\sigma_K-1}-\rho\gamma\left(\frac{c}{h}\right)^*-(\beta+n+\delta)+\left(\rho\gamma\varepsilon\left(\frac{c}{h}\right)^*+\gamma\rho\left(1-\varepsilon\right)+\left(1-g\right)\varepsilon n\right)\right)=0 \tag{A.3}$$

The two solutions to which are:

$$k_1^* = \left(\frac{-\rho+n+\delta}{\alpha(1-\sigma)}\right)^{\frac{1}{\sigma_K-1}} \tag{A.4a}$$

$$k_2^* = \left(\frac{\beta+\delta-(\gamma-1)(1-\varepsilon)n+\left[\varepsilon+\gamma\left(1-\varepsilon\right)\right]gn}{(1-\sigma)\alpha}\right)^{\frac{1}{\sigma_K-1}} \tag{A.4b}$$

We now consider the transversality condition (15d), and note that it is equivalent to

$$\hat{\lambda}_1 + \hat{K} - \beta < 0 \tag{A.5}$$

where $\hat{\lambda}_1, \hat{K}$ are steady-state growth rates, and are given by:

$$\left(\frac{\dot{\lambda}_1}{\lambda_1}\right) \equiv \hat{\lambda}_1 = \beta + n + \delta - (1-\sigma)\alpha(k^*)^{\sigma_K - 1} \tag{A.6a}$$

$$\hat{K} = gn \tag{A.6b}$$

Substituting (A.6a) we find $\hat{\lambda}_1 + \hat{K} - \beta = \rho + gn > 0$. This violates the transversality condition, and therefore we can eliminate solution (A.6a). The second root, (A.6b), satisfies both transversality conditions provided the dynamic efficiency condition, $(1-\sigma)\alpha(k^*)^{\sigma_K - 1} > \delta + gn$, is satisfied. In light of this, the optimal solution for capital in the presence of habit formation is given by (16a) and is identical to the solution for the catching up with the Joneses case given by (13a').

A.2 Welfare Changes as Measured by Equivalent Variations in Income Flows

We assume that the economy is initially on a balanced growth path, (indexed by *b)* which is growing at the equilibrium growth rate, gn, and with the corresponding level of base welfare

$$\frac{1}{1-\varepsilon}\int_0^\infty \left(C_{i,b}H_{i,b}^{-\gamma}\right)^{1-\varepsilon} e^{-\beta t}dt = \frac{c_b^{1-\varepsilon}h_b^{-\gamma(1-\varepsilon)}\left(C_0 H_0^{-\gamma}\right)^{(1-\varepsilon)}}{1-\varepsilon}\int_0^\infty e^{[(1-\varepsilon)(g-1)(1-\gamma)n-\beta]t}dt \tag{A.7}$$

where c_b, h_b are the constant ratios along the initial balanced growth path and for simplicity we set $N_0 = 1$. Assuming further that we begin from an initial steady state,

$$H_0 = \frac{\rho}{(g-1)n+\rho}C_0 \equiv \phi C_0$$

where C_0 is the level of consumption at time 0, (A.8) can be evaluated

$$\frac{c_b^{1-\varepsilon}h_b^{-\gamma(1-\varepsilon)}\phi^{-\gamma(1-\varepsilon)}C_0^{(1-\varepsilon)(1-\gamma)}}{(1-\varepsilon)\left[\beta-(1-\varepsilon)(g-1)(1-\gamma)n\right]} \equiv W(c_b, h_b; C_0) \equiv W_b \tag{A.8}$$

Intertemporal welfare along an equilibrium path is given by

$$\frac{1}{1-\varepsilon}\int_0^\infty \left(C_i H_i^{-\gamma}\right)^{1-\varepsilon} e^{-\beta t} dt = \frac{\phi^{-\gamma(1-\varepsilon)} C_0^{(1-\varepsilon)(1-\gamma)}}{1-\varepsilon}\int_0^\infty c(t)h(t)^{-\gamma} e^{[(1-\varepsilon)(g-1)(1-\gamma)n-\beta]t} dt \equiv W(c_a, h_a; C_0) \equiv W_a \quad (A.9)$$

where c_a, h_a denote the time-varying trajectories along the resulting transitional path.

As a means of comparing these two levels of utility, we determine the percentage change in the initial consumption level, C_0, and therefore in the consumption flow over the entire base path, such that the agent is indifferent between c_b, h_b and c_a, h_a. That is, we seek to find ζ such that

$$W(c_b, h_b; \zeta C_0) = W(c_a, h_a; C_0) = W_a \tag{A.10}$$

Performing this calculation yields

$$\frac{c_b^{1-\varepsilon} h_b^{-\gamma(1-\varepsilon)} \phi^{-\gamma(1-\varepsilon)} \left(\zeta C_0\right)^{(1-\varepsilon)(1-\gamma)}}{(1-\varepsilon)\left[\beta-(1-\varepsilon)(g-1)(1-\gamma)n\right]} \equiv \zeta^{(1-\varepsilon)(1-\gamma)} W_b = W_a$$

and hence

$$\zeta - 1 = \left(W_a / W_b\right)^{1/(1-\varepsilon)(1-\gamma)} - 1 \tag{A.11}$$

(A.11) determines the change in the base consumption level, and thus in the consumption level at all points of time that will enable the agent's base level of intertemporal welfare to equal that following some structural change.

The short-run welfare gain (over the base level) is calculated analogously, by

$$\xi - 1 = \left(Z_a / Z_b\right)^{1/(1-\varepsilon)(1-\gamma)} - 1 \tag{A.12}$$

where $Z_b(t) \equiv \left(c_b h_b^{-\gamma} C_0^{1-\gamma}\right)^{1-\varepsilon}$, $Z_a(t) \equiv \left(c(t)h(t)^{-\gamma} C_0^{1-\gamma}\right)^{1-\varepsilon}$, so that

$$\zeta - 1 = \left(\frac{c(t)h(t)^{-\gamma}}{c_b(t)h_b(t)^{-\gamma}}\right)^{1/(1-\gamma)} - 1 \tag{A.13}$$

Taking differentials, we see that the change in instantaneous utility relative to the base is

$$\frac{dZ}{Z_b} = \frac{1}{1-\gamma}\left(\frac{dc}{c_b} - \gamma \frac{dh}{h_b}\right)$$

which we can write as

$$\frac{dZ}{Z_b} = \frac{dC}{C_b} + \frac{\gamma}{1-\gamma} \frac{d\left(C/H\right)}{\left(C/H\right)_b}$$

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