Habit Formation, the World Real Interest Rate, and the Present Value Model of the Current Account *

Takashi Kano†

International Department, Bank of Canada

and

Department of Economics, University of British Columbia

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†Contacting address: Special Studies Division, International Department, Bank of Canada, 234 Wellington, Ottawa, Ontario, CANADA, K1A 0G9. E-mail address: tkano@bank-banque-canada.ca

Abstract

A recent study reports that habit formation in consumption can improve the ability of the present value model of the current account (PVM) to forecast actual current account movements because habits make the current account more volatile. This paper, however, shows that the habit-forming PVM is observationally equivalent to the canonical PVM augmented with a transitory consumption shock: the sample test statistics of the habit-forming PVM are not informative to detect the role of habit formation in current account movements. This identification problem is resolved by conducting Monte Carlo experiments of two small open economy-real business cycle (SOE-RBC) models calibrated to Canadian postwar quarterly data: one with habit formation and the other with the stochastic world real interest rate. Results reveal that the SOE-RBC model with the stochastic world real interest rate dominates the SOE-RBC model with habit formation in explaining Canadian sample test statistics of the habit-forming PVM as well as the standard PVMs. This suggests that research on the current account should concentrate on the determinants of the world real interest rate rather than alternative specifications of utility.

Key Words: Current Account; Present Value Model; Habit Formation ; World Real Interest Rate; Small Open Model.

JEL Classification Number: F32, F41, E32.

Non-technical Summary

Habit formation in consumption is often employed to explain puzzles between macro models and aggregate data. One example is the present value model of the current account (PVM) that includes habit formation. A recent study by Gruber(2000) argues that habit formation improves the ability of the PVM to predict actual current account movements.

This paper shows that the habit-forming PVM is observationally equivalent to the canonical PVM augmented with a transitory consumption component that is serially correlated. This means that given the data for the present value test, any test statistic constructed from the former PVM takes the same value as that from the latter PVM. Hence, by looking at the sample test statistics, a researcher cannot identify whether or not habit formation plays an important role in actual current account movements.

To resolve this identification problem, this paper constructs two small open economy-real business cycle (SOE-RBC) models: one with habit formation and the other with a stochastic world real interest rate, as opposed to a constant world real interest rate, respectively. The two SOE-RBC models are calibrated to postwar Canadian quarterly data, and are used to generate artificial data to replicate the test statistics of the habit-forming PVM. The idea is: if the sample test statistics of the habit-forming PVM really reflect habit formation in consumption, the theoretical test statistics replicated by the SOE-RBC model with habit formation should be closer to the sample test statistics than those replicated by the SOE-RBC model with the stochastic world real interest rate.

Results from the Monte Carlo experiments reveal that to explain the sample test statistics of both the habit-forming and standard PVMs, the SOE-RBC model with the stochastic world real interest rate dominates the SOE-RBC model with habit formation; in other words, the former model does a better job of replicating the data. This suggests that research in this literature should concentrate on the determinants of the world real interest rate rather than on alternative specifications of utility.

Contents

1 Introduction

A small open economy model endowed with rational, forward-looking agents serves as a benchmark for studying current account dynamics in the recent literature. This model, as known as the intertemporal approach to the current account, stresses the consumption-smoothing behavior of economic agents in the determination of the current account in a small open economy[1]. When they expect changes in future income, forward-looking agents smooth their consumption by borrowing or lending in international financial markets and hence by generating current account movements. This role of consumption-smoothing behavior in current account determination is clearly expressed by the present value model (PVM) of the current account, which is a closed-form solution of the intertemporal approach. For example, the PVM predicts that the current account moves into deficit when a country's income is expected to decline temporarily, while no change in the current account occurs if the decline in income is expected to be permanent[2].

Many empirical studies including Sheffrin and Woo(1990), Otto(1992), Ghosh(1995) and Bergin and Sheffrin(2000), however, fail to find empirical support for the standard PVM of the current account in postwar data of the G-7 economies. The cross-equation restrictions the standard PVM imposes on the unrestricted vector autoregression (VAR) are statistically rejected for all of the G-7 economies except the U.S. Moreover, the forecasts of the standard PVM are too smooth to track actual current account movements. The empirical failures of the standard PVM have led some researchers to explore the role of consumption-tilting motives in current account movements: the current account might be adjusted to factors that deviate consumption away from the random-walk, permanent income level, for example, stochastic variations in the world real interest rate[3].

[1]Obstfeld and Rogoff(1995) provide a recent and most detailed survey of the intertemporal approach to the current account.

[2]A crucial prediction of the PVM is that only country-specific shocks matter for the current account of a small open economy. A global shock does not give a small open economy an opportunity to borrow or lend in international financial markets because all economies have identical preferences, technologies and endowments and hence react to a global shock symmetrically. All that occurs is that the world real interest rate adjusts to the global shock.

[3]For example, by using a structural VAR approach to identify global and country-specific shocks,

One way to introduce the consumption-tilting motive into the standard PVM is habit formation in consumption. Habit formation makes optimal consumption decisions depend not only on permanent income but also on past consumption. The household tends to maintain its past consumption level against unexpected shocks to permanent income; therefore, habit formation makes consumption smoother and more sluggish than in the basic permanent income hypothesis (PIH). The sluggishness of consumption in turn implies more volatile current account movements than the standard PVM predicts. Gruber(2000) uses habit formation in consumption to improve the ability of the PVM to track actual current account movements in the postwar quarterly data of the G-7 economies, of the Netherlands, and of Spain. He concludes that habit formation plays an important role in determining current account dynamics.

This paper shows that the habit-forming PVM is *observationally equivalent* to the canonical PVM augmented with a serially-correlated transitory consumption shock. In other words, given the information set studied by Gruber(2000), the two PVMs yield the same values of the sample test statistics. Because of this identification problem, Gruber's tests of the habit-forming PVM are not informative to detect the role of habit formation in current account movements.

In this paper, the source of the serially-correlated transitory consumption shock is specified with stochastic movements in the world real interest rate because of two reasons[4]. First, the stochastic real interest rate is a well-known way to introduce a consumption-tilting motive into the PVM of the current account as well as the permanent income hypothesis of consumption[5]. Expected future changes in the world real

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Kano(2003) shows that almost all of Canadian current account movements are dominated by country-specific shocks unrelated to variations in the smoothed, permanent income. This result empirically suggests the importance of consumption-tilting motives in Canadian current account movements.

[4]Another sources of the transitory consumption shocks are a transitory government expenditure shock affecting the utility function and the stochastic terms of trade.

[5]See Campbell and Mankiw(1989) for tests of the permanent income hypothesis (PIH), and Bergin and Sheffrin(2000) and Kano(2003) for tests of the current account PVM. In particular, Bergin and Sheffrin(2000) extend the standard PVM by introducing stochastic variations in the world real interest rates as well as real exchange rates, which yield a serially-correlated transitory consumption component

interest rate tilt the consumption path away from the random-walk, permanent income level and, as a result, introduce the consumption-tilting component into the PVM of the current account. Second, recent studies on small open economy-real business cycle (SOE-RBC) model, Blankenau, Kose and Yi(2001) and Nason and Rogers(2003), provide evidence that the world real interest rate shocks play a crucial role in explaining net trade balance/current account movements in a small open economy.

To solve the identification problem, this paper conducts Monte Carlo experiments based on a small open-real business cycle model (SOE-RBC) that incorporates with either habit formation or the stochastic world real interest rate. To this end, the SOE-RBC model of Nason and Rogers(2003) is extended by introducing habit formation. The extended model is then used to generate artificial data that yield theoretical distributions of "moments" to be explained in this paper.

As in a standard calibration exercise, moments of the artificial data generated by SOE-RBC models are compared with their sample counterparts. However, as examined by Nason and Rogers(2003), the "moments" this paper studies are not standard unconditional variances and covariances of the sample. Instead, they are the sample statistics conditional on the habit-forming and standard PVMs of the current account: the sample estimate of the habit-formation parameter, the cross-equation restrictions implied by the habit-forming and standard PVMs, and the current account forecasts of the habit-forming and standard PVMs.

It is worth noting that by construction, the theoretical distributions have the null hypothesis of the underlying SOE-RBC model as the data-generating process (DGP) of the moments. This paper generates the theoretical distributions under two different null hypotheses. First, setting the structural parameters of the SOE-RBC model to rule out stochastic variations of the world real interest rate derives the theoretical distributions under the null of the SOE-RBC model with habit formation. Second, setting the habit parameter equal to zero provides the theoretical distributions under the null of the SOE-

independent of permanent income. They observe that the extension improves the PVM prediction in Canada. Kano(2003) also shows the PVM of the current account in the presence of the stochastic world real interest rate using a different approach.

RBC model with the stochastic world real interest rate. The two different SOE-RBC models are evaluated from the viewpoint of classical statistics; that is to say, the sample statistics are used as critical values to derive empirical p-values. For example, if a sample statistic drops into the five percent tail of the theoretical distribution, the null is rejected at the five percent significance level.

The results from the Monte Carlo experiments support the SOE-RBC model with stochastic world real interest rates. Although the SOE-RBC model with habit formation can replicate a part of the empirical facts of the habit-forming PVM, the SOE-RBC model with the stochastic world real interest rate mimics all the relevant sample moments. The superiority of the SOE-RBC model with stochastic world real interest rates casts doubt on habit formation as the significant source of the consumption-tilting behavior needed to explain Canadian current account movements.

The structure of this paper is as follows. The next section introduces the habit PVM and discusses the observational equivalence problem. The sample moments conditional on the habit and standard PVMs are reported in section 3. Section 4 introduces the SOE-RBC models of this paper to mimic the sample moments. Section 5 reports the results of the Monte Carlo experiments. Concluding remarks are made in section 6.

2 The PVMs with Habit Formation and Transitory Consumption: Observational Equivalence

Gruber (2000) extends the standard PVM by introducing habit formation in consumption. Let C_t, B_t and NO_t denote consumption, international bond holding, and net output at period t, respectively. As in the standard literature, net output, which is defined as output minus domestic investment minus government expenditure, follows a nonstationary process having a country-specific, random-walk technology shock as the driving force[6]. The period utility function is specified as a quadratic form

$$u(C_{t+i} - h\bar{C}_{t+i-1}) = C_{t+i} - h\bar{C}_{t+i-1} - \frac{1}{2}(C_{t+i} - h\bar{C}_{t+i-1})^2, \quad 0 < h < 1$$

where h represents the habit parameter. $\bar{C}_t$ represents aggregate consumption unaffected by any representative household decision. This specification of habit formation is related to *external* habit formation or the *catching up with the Joneses*, as in Abel(1990) and Campbell and Cochrane(1999)[7]. Note that $\bar{C}_t = C_t$ in equilibrium.

The problem the representative household faces is to maximize its expected discounted lifetime utility

$$E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i} - h\bar{C}_{t+i-1})$$

subject to the budget constraint

$$B_{t+1} = (1+r)B_t + NO_t - C_t$$

where r is the world real interest rate assumed to be constant and equal to the subjective discount rate. In this case, the first-order necessary conditions together with the transversality condition yield an optimal consumption decision rule. Letting ϵ_t denote a disturbance orthogonal to information at period t-1 and adding ϵ_t to the optimal consumption decision rule provide

$$C_t = \left(\frac{h}{1+r}\right) C_{t-1} + \left(1 - \frac{h}{1+r}\right)\left(\frac{r}{1+r}\right)\left[(1+r)B_t + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t NO_{t+i}\right] + \epsilon_t \tag{1}$$

where the equilibrium condition $\bar{C}_t = C_t$ is imposed[8]. With habit formation, consumption is determined by a weighted average of permanent income and past consumption

[6]The basic SOE-RBC model, which is well-known as the intertemporal approach to the current account, is a single-shock model containing a country-specific, unit-root technology shock. See Obstfeld and Rogoff(1995), Glick and Rogoff(1995), and Nason and Rogers(2003). Under this assumption, the intertemporal approach has the standard PVM as a closed-form solution.

[7]If habits are *internal*, as in Constantinides(1990), they depend on the household's own consumption and the household takes habits into account when choosing the amount of consumption.

[8]Campbell(1987) argues that a transitory consumption error uncorrelated with lagged information improves the ability of the PIH to fit the U.S. data.

with the weight $h/(1+r)$. This fact makes adjustments of consumption to permanent income shocks more sluggish than in the standard PIH.

Substituting the resulting consumption equation into the current account identity $CA_t \equiv rB_t + NO_t - C_t$ produces the PVM with habit formation

$$CA_t = hCA_{t-1} + \left(\frac{h}{1+r}\right)\Delta NO_t - \left(1 - \frac{h}{1+r}\right)\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i} - \epsilon_t. \quad (2)$$

Notice that the current account depends on its own past value. This makes the process of the current account more persistent than in the standard PVMs of Sheffrin and Woo(1990) and Otto(1992). Furthermore, the current account becomes sensitive to the current change in net output: the current account depends on not only the expected present value of future declines of net output but the current change of net output as well. This makes the current account more volatile than in the standard PVM.

An important point is that the present value formula (2) is observationally equivalent to the PVM derived from a multiple-shock model. Let C_t^T denote arbitrary transitory consumption that follows an exogenous AR(1) process

$$C_t^T = \rho_c C_{t-1}^T + \omega_t \quad |\rho_c| < 1 \quad (3)$$

where C_t^T may be observable or may not, and ω_t is a white noise shock. Assume that consumption C_t is linearly decomposed into the transitory consumption C_t^T and permanent income C_t^P:[9]

$$C_t = C_t^T + C_t^P \quad (4)$$

where permanent income C_t^P is determined by the standard PIH formula

$$C_t^P = \left(\frac{r}{1+r}\right)\left[(1+r)B_t + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t NO_{t+i}\right]. \quad (5)$$

Appendix 1 shows that the non-habit-forming, multiple-shock model specified by eqs.(3), (4), and (5) has the following present value representation of the current account

$$CA_t = \rho_c CA_{t-1} + \left(\frac{\rho_c}{1+r}\right)\Delta NO_t - \left(1 - \frac{\rho_c}{1+r}\right)\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i} - v_t \quad (6)$$

[9]Because the underlying SOE-RBC model has the unique stochastic trend, i.e. the country-specific, permanent, technology shock, it is possible to decompose consumption into a random-walk component C_t^P and a transitory component C_t^T: see King, Plosser and Rebelo(1988).

where υ_t is a disturbance orthogonal to information at period t-1, which satisfies $E_{t-i}\upsilon_t = 0$ for $i \geq 1$.

Notice that the non-habit-forming PVM (6) is equivalent to the habit-forming PVM (2). Therefore, given the data of CA_t and ΔNO_t, any statistics based on eq.(2), for instance, an estimate of h, take the same values as those statistics from eq.(6). The habit-forming PVM is observationally equivalent to the non-habit PVM augmented with the AR(1) transitory consumption component. This implies that the statistics based on the habit-forming PVM (2) are not informative to identify whether or not habit formation helps to explain current account movements.

3 Sample Moments Conditional on the Habit-Forming and Standard PVMs

This section reports the sample moments conditional on the habit-forming and the standard PVMs. As mentioned in the introduction, this paper considers the sample test statistics of the two PVMs as the sample "moments" explained by SOE-RBC models. The next subsection discusses econometric issues related to estimation and test of the habit-forming PVM. The following subsection reports the sample moments.

3.1 Econometric Issues

Gruber(2000) exploits the generalized method of moments (GMM) procedure to estimate the habit parameter h in the habit-forming PVM (2). Define a variable $D_t \equiv CA_t - \Delta NO_t - (1+r)CA_{t-1}$ and rewrite the PVM (2) as

$$D_t = hD_{t-1} - \epsilon_t + (1+r)\epsilon_{t-1} + e_t \tag{7}$$

where ϵ_t, ϵ_{t-1} and e_t are disturbances orthogonal to the information set at period $t-2$, Ω_{t-2} [See Appendix 2 for the detailed derivation of eq.(7).]. Let W_{t-2} denote a $k \times 1$ vector that contains k different variables in Ω_{t-2}. Eq.(7) then implies unconditional moment conditions

$$EW_{t-2}(D_t - hD_{t-1}) = 0 \tag{8}$$

where E is the unconditional expectation operator. Eq.(8) makes it possible to estimate h by the GMM/two step-two stage least square (2SLS) procedure by West(1988). Let $\hat{h}_{2SLS}$ be the 2SLS estimate of h. When $k > 1$, $\hat{h}_{2SLS}$ is overidentified. The J-statistic of Hansen(1982) tests the orthogonality conditions (8). Given $k(> 1)$ instruments, the J-statistic is asymptotically distributed χ^2 with $k-1$ degrees of freedom.

This paper proposes a more efficient estimate of the habit parameter than the 2SLS estimate $\hat{h}_{2SLS}$. In addition to the unconditional moment conditions (8), other theoretical restrictions the habit-forming PVM imposes on a p-th order bivariate vector autoregressive (VAR) of CA_t and ΔNO_t are used to estimate the habit parameter. Recall that a VAR(p) process has a corresponding first-order representation with a companion matrix $\mathcal{A}$:

$$\mathcal{Y}_t = \mathcal{A}\mathcal{Y}_{t-1} + \mathcal{U}_t \tag{9}$$

where $\mathcal{U}_t$ is a $2p \times 1$, zero mean, homoskedastic, serially uncorrelated error vector such that $\mathcal{U}_t \equiv [u_t^{\Delta NO} \quad 0 \quad \cdots \quad 0 \quad u_t^{CA} \quad 0 \quad \cdots \quad 0]'$, and $\mathcal{Y}_t$ is a $2p \times 1$ vector constructed as

$$\mathcal{Y}_t \equiv [\Delta NO_t \quad \Delta NO_{t-1} \quad \cdots \quad \Delta NO_{t-p+1} \quad CA_t \quad CA_{t-1} \quad \cdots \quad CA_{t-p+1}]'.$$

By assumption of the VAR, $\mathcal{Y}_{t-1}$ is orthogonal to the VAR disturbances $U_t = [u_t^{\Delta NO}$ $u_t^{CA}]$. That is, the following unconditional moment conditions are satisfied:

$$E\mathcal{Y}_{t-1} \otimes U_t = 0 \tag{10}$$

where $\otimes$ is the operator of the Kronecker product.

Define a $1 \times 2p$ vector $\mathbf{e}_i$ that includes zeros except for the ith element equal to 1, i.e.

$$\mathbf{e}_i = [0 \cdots \underbrace{0}_{i-1st} \quad \underbrace{1}_{ith} \quad \underbrace{0}_{i+1st} \cdots 0].$$

The habit-forming PVM (2) then implies that under the null hypothesis, the following cross-equation restrictions should be the case:

$$\mathbf{e}_{p+1}\mathcal{A}\mathcal{Y}_t = \mathcal{K}^h \mathcal{A}\mathcal{Y}_t \tag{11}$$

where $\mathcal{K}^h$ is a $1 \times 2p$ vector such that

$$\mathcal{K}^h = h\mathbf{e}_{p+2} + \left(\frac{h}{1+r}\right)\mathbf{e}_1 - \left(1 - \frac{h}{1+r}\right)\left(\frac{1}{1+r}\right)\mathbf{e}_1\mathcal{A}\left[I - \left(\frac{1}{1+r}\right)\mathcal{A}\right]^{-1}.$$

Note that the cross-equation restriction (11) can be considered as an unconditional moment condition

$$E(\mathbf{e}_{p+1} - \mathcal{K}^h)\mathcal{A}\mathcal{Y}_t = 0. \tag{12}$$

Eq.(12) holds under the null hypothesis of the habit-forming PVM (2).

As a result, if the joint probability distribution of CA_t and ΔNO_t is specified by the unrestricted VAR (9), the habit-forming PVM (2) yields the unconditional moment conditions (10) and (12) in addition to (8)[10]. Construct a $(4p + k + 1) \times 1$ vector $g_t(\theta)$ such that

$$g_t(\theta) = \begin{bmatrix} W_{t-2}(D_t - hD_{t-1}) \\ \mathcal{Y}_{t-1} \otimes U_t \\ (\mathbf{e}_{p+1} - \mathcal{K}^h)\mathcal{A}\mathcal{Y}_t \end{bmatrix}$$

where θ is a vector constructed by stacking the habit parameter h and the elements of the companion matrix $\mathcal{A}$, i.e. $\theta \equiv [h \quad vec(\mathcal{A})']'$. The sample analogs of the theoretical moment conditions (8), (10), and (12) are given as

$$G(\theta) = T^{-1}\sum_{t=1}^{T} g_t(\theta) = 0$$

where T is the sample number. To obtain an efficient estimate of θ, this paper conducts the two-step GMM procedure of West(1988)[11]. Let $\hat{\theta}_{GMM}$ be the resulting two-step GMM estimate of θ with the asymptotic covariance matrix $\hat{V}_{\theta_{GMM}}$. In this case, the J-statistic $\mathcal{J}_T$ for the overidentifying restriction test, which satisfies

$$\mathcal{J}_T = TG(\hat{\theta}_{GMM})'M^*G(\hat{\theta}_{GMM})$$

under the optimal weighting matrix M^*, asymptotically follows the χ^2 distribution with degrees of freedom k.

[10]Gruber(2000) does not use the moment conditions (10) and (12) to estimate h. This fact makes Gruber's estimation and specification test based only on the overidentifying restrictions (8) inefficient since his procedure does not use all of information the model provides potentially.

[11]Appendix 3 reviews the two-step GMM estimation in detail.

Notice that the J-statistic jointly tests the overidentifying restrictions implied by the unconditional moment conditions (8), (10), and (12), but does not test the exact cross equation restrictions (11). To do so, define a $1 \times 2p$ vector $\mathcal{F}(\theta)$ as $\mathcal{F}(\theta) \equiv (\mathcal{K}^h - \mathbf{e}_{p+1})\mathcal{A} + \mathbf{e}_{p+1}$. Let θ_0 denote the true parameter vector under the null of the habit-forming PVM. Eq.(11) implies that $\mathcal{F}(\theta_0) = \mathbf{e}_{p+1}$ under the true parameter vector θ_0, i.e. the $p+1$st element of the vector $\mathcal{F}(\theta_0)$ should be one, while the others should be zero. The GMM estimate of the vector $\mathcal{F}(\theta)$, $\mathcal{F}(\hat{\theta}_{GMM})$, makes possible piecewise tests of the $2p$ cross-equation restrictions by the standard t-statistics, as well as joint test of those restrictions by the Wald statistic. The asymptotic standard error of the estimate $\mathcal{F}(\hat{\theta}_{GMM})$ is calculated from its covariance matrix numerically derived by the Delta method

$$\frac{\partial \mathcal{F}(\hat{\theta}_{GMM})}{\partial \theta'} \hat{V}_{\theta_{GMM}} \frac{\partial \mathcal{F}(\hat{\theta}_{GMM})}{\partial \theta'}'.$$

Let $\mathbf{k}(\theta) \equiv \mathbf{e}_{p+1} - \mathcal{F}(\theta)$. Then the estimates $\hat{\theta}_{GMM}$ and $\hat{V}_{\theta_{GMM}}$ yield the Wald statistic $\mathcal{W}_T$ satisfying

$$\mathcal{W}_T = \mathbf{k}(\hat{\theta}_{GMM}) \left[\frac{\partial \mathbf{k}(\hat{\theta}_{GMM})}{\partial \theta'} \hat{V}_{\theta_{GMM}} \frac{\partial \mathbf{k}(\hat{\theta}_{GMM})}{\partial \theta'}' \right]^{-1} \mathbf{k}(\hat{\theta}_{GMM})'.$$

Under the null hypothesis of $\mathbf{k}(\theta_0) = 0$, the Wald statistic $\mathcal{W}_T$ asymptotically follows the χ^2 with degrees of freedom $2p$.

Finally, the predictions of the habit-forming PVM on actual current account movements, denoted by $\mathcal{CA}_t^f$, are constructed as $\mathcal{CA}_t^f \equiv \mathcal{F}(\hat{\theta}_{GMM})\mathcal{Y}_t$. Under the null, it is the case that $\mathcal{CA}_t^f = CA_t$. Therefore, comparing the predictions with actual current account series provides another information to test the null hypothesis of the habit-forming PVM (2).

3.2 Empirical Results

This paper studies the quarterly, real, seasonally-adjusted Canadian data that spans the sample periods Q1:1963 and Q4:1997. The data construction follows Otto(1992) and Nason and Rogers(2003)[12]. The current account series and the first difference series of net

[12] All the data are distributed by Statistics Canada.

output are demeaned to construct the sample vector $\mathcal{Y}_t$. The fourth lag $p = 4$ is chosen as the optimal lag by the general-to-specific likelihood ratio (LR) tests. To construct the series D_t, this paper uses the calibrated value of the constant world real interest rate $r = 0.0091$ [or equivalently 3.70 percent point on an annual basis: $r = (1.037)^{0.25} - 1$].

A crucial point for conducting the GMM/2SLS estimation is how to choose the instrument variables W_{t-2}. Theoretically, any variables in the information set Ω_{t-2} can be included in W_{t-2}. This paper lags the instruments more than one period and includes in W_{t-2} the fourth and fifth lagged values of CA_t and ΔNO_t to avoid potential correlation between $D_t - hD_{t-1}$ and any variable at period $t-2$ or $t-3$. In this case, W_{t-2} is a 4×1 vector satisfying

$$W_{t-2} = [\Delta NO_{t-4} \quad \Delta NO_{t-5} \quad CA_{t-4} \quad CA_{t-5}]'.$$

Therefore, $p = k = 4$ are chosen in the following analysis.

Table 1(a) summarizes the empirical results. First, the two estimates of the habit parameter, $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$, are reported in the first two columns. The 2SLS estimator based only on the unconditional moment conditions (8) yields $\hat{h}_{2SLS} = 0.931$ with the asymptotic standard error 0.192. This number is close to the estimate Gruber(2000) obtains ($\hat{h}_{2SLS} = 0.902$ and $s.e. = 0.257$, respectively). On the other hand, the GMM estimator based on the full moment conditions (8), (10), and (12) provides $\hat{h}_{GMM} = 1.002$ with the asymptotic standard error 0.152. Therefore, the GMM estimate based on the full moment conditions draws an inference of a larger habit parameter than the 2SLS estimate[13]. Although it is safe to claim that h is non-zero, either $\hat{h}_{2SLS}$ or $\hat{h}_{GMM}$ has a 95 % confidence interval including $h = 1$ [14]. This inference violates the constraint $h < 1$.

The statistic $\mathcal{J}_T$ is 0.455 with a p-value of 0.978, which means that the overidentifying restrictions out of the unconditional moment conditions (8), (10), and (12) cannot be jointly rejected even at 97.8 % significance level. However, the Wald statistic $\mathcal{W}_T$ for the

[13] It is worth while mentioning that the standard error of the GMM estimate is smaller than that of the 2SLS. This means that the sampling uncertainty of the GMM estimate is smaller that that of the 2SLS estimate.

[14] If $h = 1$, the utility function implies that the household wants to smooth change in consumption, rather than level of consumption, across periods.

cross-equation restrictions is 37.128 with a small p-value. This means that the cross-equation restrictions $\mathbf{k}(\theta_0) = 0$ are jointly rejected at any standard significance levels. Furthermore, the piecewise tests of the eight elements in the vector $\mathcal{F}(\theta)$ reflect this joint rejection of the cross equation restrictions. Recall that under the null, the fifth element $\mathcal{F}_5$ should be one, while all the other elements should be zero. The table reports that the GMM estimate $\hat{\mathcal{F}}_5$ is 1.276 with the asymptotic standard error 0.226. Hence, the estimate is not significantly different from one. The observation that two estimates $\hat{\mathcal{F}}_1 = -0.302$ and $\hat{\mathcal{F}}_6 = -0.400$ are statistically significant, however, violates the respective single null hypotheses. All the other estimates $\hat{\mathcal{F}}_i$ for $i \neq 1, 5, 6$ are statistically insignificant based on the two standard error rule.

Figure 1(a) plots the actual current account series, the predictions of the habit-forming PVM $\mathcal{CA}_t^f$, and the corresponding asymptotic two standard error band. Observe that the predictions of the habit-forming PVM track the actual current account fairly closely. The narrow standard error band reflects small sampling uncertainty attached to the predictions. The standard error band includes the actual current account in all the sample periods. These observations support the inference that the habit-forming PVM explains actual movements of the Canadian current account fairly well, as Gruber(2000) reports.

Comparing the empirical results of the habit-forming PVM (2) with those of the standard PVM demonstrates how introducing habit formation improves the ability of the PVM to track actual current account movements. Setting $h = 0$ and $\epsilon_t = 0$ in the habit-forming PVM (2) provides the following cross-equation restrictions imposed on the unrestricted VAR (9) under the null of the standard PVM

$$\mathbf{k}^*(\theta_0) \equiv \mathbf{e}_{p+1} - \mathcal{F}^*(\theta_0) = 0$$

where

$$\mathcal{F}^*(\theta) = -\mathbf{e}_1(1+r)^{-1}\mathcal{A}[I_8 - (1+r)^{-1}\mathcal{A}]^{-1}.$$

Note that θ includes only the VAR parameters. Hence, the unbiased estimate of θ is obtained by OLS. Let $\hat{\theta}_{OLS}$ denote the OLS estimate.

Table 1(b) reports the Wald statistic $\mathcal{W}_T^*$ to test the cross-equation restrictions

$\mathbf{k}^*(\theta_0) = 0$ jointly, and the estimates of the eight elements of the vector $\mathcal{F}^*(\hat{\theta}_{OLS})$ to test the cross-equation restrictions piecewisely. First, the Wald statistic $\mathcal{W}_T^*$ is 20.589 with the asymptotic p-value 0.009. Therefore, the cross-equation restrictions are jointly rejected at any standard significance levels. The failure of the standard PVM is clearer in the piecewise tests of the null hypotheses. If the standard PVM holds, the fifth element of the vector $\mathcal{F}^*(\hat{\theta}_{OLS})$ should be one, while the other elements be zero. The estimate of the fifth element $\hat{\mathcal{F}}_5^*$ is -0.115 with the asymptotic standard error 0.408. Hence, the single null $\mathcal{F}_5^* = 1$ is strictly rejected by the standard t-statistic. All of the other estimates are statistically insignificant.

Figure 1(b) plots the actual Canadian current account series, the predictions of the standard PVM $\mathcal{CA}_t^{*f} = \mathcal{F}(\hat{\theta}_{OLS})\mathcal{Y}_t$, and the asymptotic two standard error band. The predictions are too smooth to track the actual series. The standard error band excludes the actual series at almost all periods. Hence, the standard PVM cannot predict the position of the Canadian current account. These observations clearly reveal the superiority of the habit-forming PVM to the standard PVM at least in the predicting ability.

The empirical results of this paper track those of Sheffrin and Woo(1990), Otto(1992), and Gruber(2000). Tables 2(a) and (b) summarize the empirical facts - the sample moments - of both the habit-forming and standard PVMs. In particular, this paper shares with Gruber(2000) the observation that taking habit formation into account greatly improves the PVM's prediction on the Canadian current account. The empirical results of both Gruber and this paper appear to support the claim that habit formation helps to explain Canadian current account movements.

However, the observational equivalence between the PVMs with habit formation and serially-correlated transitory consumption makes a researcher unable to identify whether the successful aspects of the habit-forming PVM are actually attributed to habit formation or other factor that generate consumption-tilting motives. A leading example for a small open economy is the stochastic world real interest rate. The next section discusses this paper's strategy to solve the identification problem.

4 Monte Carlo Investigation

Facing the identification problem, this paper conducts calibration-Monte Carlo exercises based on the SOE-RBC models with habit formation and the stochastic world real interest rate. The first task is to extend the SOE-RBC model of Nason and Rogers(2003) by introducing habit formation in consumption, as discussed in the next subsection.

4.1 The Small Open Economy Real Business Cycle Model

The lifetime utility function of the representative household is

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i u(C^*_{t+i}, L_{t+i}) \tag{13}$$

where $C^*_t = C_t - h\bar{C}_{t-1}$ and L_t is leisure at period t. Eq.(13) implies that the lifetime utility is non-separable not only across periods but also between consumption and leisure in each period. In particular, the period utility function $u(C^*, L)$ is parameterized as a constant relative risk aversion type

$$u(C^*, L) = \frac{(C^{*\phi} L^{1-\phi})^{1-\gamma} - 1}{1-\gamma}$$

for $\gamma \neq 1$. For $\gamma = 1$,

$$u(C^*, L) = \phi \ln C^* + (1-\phi) \ln L$$

and in either case $0 < \phi < 1$. Therefore, in the case of $\gamma = 1$ the preferences are separable between consumption and leisure.

Define Y_t, I_t, G_t and r_t to be output, investment, government consumption expenditure, and the real interest rate the representative household faces at period t. The household's budget constraint is

$$B_{t+1} = (1 + r_t)B_t + Y_t - I_t - G_t - C_t. \tag{14}$$

Output Y_t is produced by a Cobb-Douglas production function

$$Y_t = K_t^{\psi} [A_t N_t]^{1-\psi} \quad 0 < \psi < 1 \tag{15}$$

where K_t, A_t and N_t are capital stock, county-specific, labor-augmenting technology, and labor input at period t. Since the household is endowed with a unit hour to allocate between labour and leisure, the restriction $L_t + N_t = 1$ must be satisfied. The law of motion for capital is represented as

$$K_{t+1} = (1-\delta)K_t + \left(\frac{K_t}{I_t}\right)^{\varphi} I_t \quad 0 < \varphi < 1 \tag{16}$$

where $0 < \delta < 1$ is the depreciation rate. Eq.(16) includes adjustment costs of investment with the parameter φ. This specification of the adjustment costs follows Baxter and Crucini(1993).

As studied by Nason and Rogers(2003) and Schmitt-Grohé and Uribe(2003), the real interest rate r_t is decomposed into two components. The first component q_t is the exogenous and stochastic return that is common across the world. In this paper, q_t follows a covariance stationary process. The other component is the risk premium specific to this small open economy. The risk premium is given as a linear function of the economy's bond-output ratio. Following Nason and Rogers(2003), this paper specifies the stochastic real interest rate r_t to be

$$r_t = q_t - \eta \frac{B_t}{Y_t}, \quad 0 < \eta. \tag{17}$$

Eq.(17) implies that if the small open economy is a debtor (i.e. $B_t < 0$), the economy must pay a premium above q_t.[15]

The processes of the three exogenous variables G_t, A_t and q_t are specified as follows. Government consumption expenditure G_t is proportional to output Y_t with a constant ratio g:[16]

$$G_t = gY_t. \tag{18}$$

The country-specific, labor-augmenting technology A_t is a random walk with drift

$$A_t = A_{t-1}\exp(\alpha + \epsilon_t^a), \quad \alpha > 0, \quad \epsilon_t^a \sim i.i.d.\mathbf{N}(0, \sigma_a^2). \tag{19}$$

[15] The endogenous risk premium in eq.(17) excludes an explosive/unit root path of international bonds in the linearized solution of the equilibrium. Moreover it solves the famous problem in the SOE-RBC model that the deterministic steady state depends on the initial condition.

[16] For example, consider the government budget that G_t is financed by lump-sum tax T_t satisfying $T_t = gY_t$. This assumption means that G_t and Y_t share not only a common trend but also a common cycle. Although this restriction is strict, it is reasonable for the Monte Carlo exercise in this paper because any shock to G_t can be considered as a shock to induce the consumption-smoothing motive, rather than the consumption-tilting motive.

Finally, the world real interest rate q_t follows an AR(1) process

$$1 + q_t = (1 + q^*)^{(1-\rho_q)}(1 + q_{t-1})^{\rho_q} \exp(\epsilon_t^q), \quad |\rho_q| < 1, \quad \epsilon_t^q \sim i.i.d.\mathbf{N}(0, \sigma_q^2) \tag{20}$$

where q^* is the deterministic steady state value of q_t. In the following analysis, ϵ_t^a and ϵ_t^q are assumed to be uncorrelated at all leads and lags.

4.2 The Optimality Conditions and Interpretations

The problem of the representative household is to maximize eq.(13) subject to eqs.(14)-(17), given the processes of the exogenous variables, eqs.(18)-(20), and the initial conditions $\bar{C}_{t-1} > 0, K_t > 0$, and $B_t \gtreqqless 0$. The optimality conditions are

$$\Gamma_{t+1} = \beta \left(\frac{C_{t+1} - h\bar{C}_t}{C_t - h\bar{C}_{t-1}} \right)^{\phi(1-\gamma)-1} \left(\frac{1 - N_{t+1}}{1 - N_t} \right)^{(1-\gamma)(1-\phi)}, \tag{21}$$

$$1 = E_t \Gamma_{t+1} \left[1 + r_{t+1} - \eta \left(\frac{B_{t+1}}{Y_{t+1}} \right) \right], \tag{22}$$

$$\left(\frac{1-\phi}{\phi} \right) \left(\frac{C_t - h\bar{C}_{t-1}}{1 - N_t} \right) = (1 - \psi) \frac{Y_t}{N_t} \left[1 + \eta \left(\frac{B_t}{Y_t} \right)^2 \right], \tag{23}$$

and

$$\frac{1}{1-\varphi} \left(\frac{I_t}{K_t} \right)^{\varphi} =$$
$$E_t \Gamma_{t+1} \left\{ \psi \frac{Y_{t+1}}{K_{t+1}} \left[1 + \eta \left(\frac{B_{t+1}}{Y_{t+1}} \right)^2 \right] + \left[\frac{1-\delta}{1-\varphi} + \frac{\varphi}{1-\varphi} \left(\frac{I_{t+1}}{K_{t+1}} \right)^{1-\varphi} \right] \left(\frac{I_{t+1}}{K_{t+1}} \right)^{\varphi} \right\}. \tag{24}$$

Recall that in equilibrium, the level of aggregate consumption must equal that of the representative household's consumption: $\bar{C}_t = C_t$. Any equilibrium path must satisfy the optimality conditions (21)-(24), the constraints (14)-(17), and the exogenous processes (18)-(20) with the transversality conditions

$$\lim_{i\to\infty} \beta^i E_t \lambda_{B,t+i} B_{t+i+1} = 0 \quad \text{and} \quad \lim_{i\to\infty} \beta^i E_t \lambda_{K,t+i} K_{t+i+1} = 0$$

where $\lambda_{B,t}$ and $\lambda_{K,t}$ are the shadow prices for the constraints (14) and (16), respectively.

Eq.(21) shows the stochastic discount factor, which turns out to be a familiar form $\beta(C_{t+1}/C_t)^{-1}$ when $h = 0$ and $\gamma = 1$. When $h \neq 0$ and $\gamma \neq 1$, the stochastic discount factor depends further on past consumption C_{t-1} and leisure at periods t and $t+1$, L_t and L_{t+1}. The higher C_{t-1} is, the lower Γ_{t+1} is because the marginal utility of consumption at period t rises due to habit formation and the marginal rate of the intertemporal substitution falls[17]. Similarly, the higher L_t is, the lower Γ_{t+1} is because the marginal utility of consumption at period t positively depends on leisure.

Eq.(22) is the optimality condition for holding the international bonds, i.e. the Euler equation. Notice that if $\eta = 0$, $h = 0$, $\gamma = 1$, and the world real interest is constant, under the assumption of $\beta(1 + r) = 1$, the Euler equation requires perfect smoothness of consumption across periods. Habit formation $h > 0$, the non-separable period utility over consumption and leisure $\gamma \neq 1$, and stochastic variations in the world real interest rate tilt consumption from the perfectly smoothed level through their effects on the stochastic discount factor[18]. The optimal consumption deviates away from the perfect smoothed level, i.e. permanent income. Hence, the deviation can be considered as the consumption-tilting motive or the transitory consumption component.

Eq.(23) is the optimality condition for the intratemporal substitution between consumption expenditure and leisure. It implies that the marginal rate of substitution between C_t and L_t should be equal to the marginal product of labour gross of the response of the endogenous risk premium to a change in labour. The Euler equation for capital, (24), has the interpretation that the expected loss of holding one more capital (represented by the LHS) should be equal to the expected benefit of the additional capital (represented by the RHS). The benefit consists of increased production gross of the risk premium, depreciation and smaller future adjustment costs of investment. On the other hand, the household needs to pay the cost that consists of the current utility loss due to investment in capital.

[17]A rise in C_t increases the stochastic discount factor Γ_{t+1} as in the standard case.

[18]Habit formation makes the household want to smooth not only consumption level but also consumption growth. The non-separable utility over consumption and leisure makes the household desire to smooth not only consumption but also leisure. Finally, if the real interest rate is expected to rise the future, the household wants to tilt consumption toward the future by lending out in international capital markets.

4.3 The Numerical Solution and Calibration

To derive the numerical solution of the equilibrium path, this paper takes linear approximation of the equilibrium conditions. First, all of the endogenous variables except for N_t and Γ_t are stochastically detrended by dividing them by the random walk technology shock A. Define the stochastically detrended variables $c_t \equiv C_t/A_t$, $i_t \equiv I_t/A_t$, $y_t \equiv Y_t/A_t$, $\varpi_t \equiv C_{t-1}/A_{t-1}$, $k_t \equiv K_t/A_{t-1}$ and $b_t \equiv B_t/A_{t-1}$. Next, a first-order Taylor expansion of each of the equilibrium conditions (14)-(17) and (21)-(24) is taken around the deterministic steady state. Let $\tilde{x}_t \equiv x_t - x$ and $\hat{x}_t \equiv x_t/x - 1$ for any variable x_t with the steady state x. Define vectors $\mathcal{P}_t$ and $\mathcal{S}_t$ by

$$\mathcal{P}_t = [\hat{c}_t \quad \hat{i}_t \quad \hat{y}_t \quad \hat{N}_t]' \quad \text{and} \quad \mathcal{S}_t = [\hat{\varpi}_t \quad \hat{k}_t \quad \hat{b}_t \quad \widetilde{\Delta \ln A_t} \quad \ln\widetilde{(1+q_t)}]'.$$

Then the solution method of Sims(2000) shows that there exists the unique equilibrium path and the vectors $\mathcal{P}_t$ and $\mathcal{S}_t$ follow the processes

$$\mathcal{P}_t = \mathcal{H}_1 \mathcal{S}_t \quad \text{and} \quad \mathcal{S}_t = \mathcal{H}_2 \mathcal{S}_{t-1} + \mathcal{H}_3 \epsilon_t \tag{25}$$

where $\epsilon_t = [\epsilon_t^a \quad \epsilon_t^q]$. Eq.(25) is the state space representation of the SOE-RBC model of this paper(see Appendix 4 in detail).

Recall that there are fourteen structural parameters in the model. Table 3 gives the calibrated values of the structural parameters used in Monte Carlo experiments. This paper conducts two types of Monte Carlo experiments as discussed below. The baseline parameters β, γ, ϕ, ψ, φ, δ, η, g, α, σ_a and q^* are fixed across the experiments and set as the mean values of the prior distributions of Nason and Rogers(2003). In particular, across the experiments, the risk premium parameter η is chosen to be a very small number 0.000071 in order to cut the effect of the endogenous risk premium on the consumption-tilting motive/the transitory consumption component. In this case, the real interest rate r_t is almost equivalent to the world common real interest rate q_t[19].

The first Monte Carlo experiment is related to the SOE-RBC model with habit formation. This case sets the habit parameter depending on the estimated value. Although

[19] As Nason and Rogers(2003) study, the specific number 0.000071 implies that the risk premium in Canada is one basis point at an annual rate at the steady state.

there are two candidates from two different estimations, the GMM estimate from the full moment conditions, $\hat{h}_{GMM}$, is suitable because it is more efficient than $\hat{h}_{2SLS}$. The problem is that $\hat{h}_{GMM}$ is greater than one, under which there exists no steady state in the SOE-RBC model. Therefore, in this experiment, the habit parameter is chosen to be 0.990, which is close to the estimate and included in the corresponding 90 % confidence interval. This experiment does not allow the world real interest rate to vary stochastically in order to maintain the assumptions of the habit-forming PVM: there is only a country-specific, unit-root technology shock. To this end, the persistence of the world real interest rate, ρ_q, and its standard deviation σ_q are set to be negligible: $\rho_q = \sigma_q = 1.00 \times 10^{-7}$. Therefore, the resulting theoretical distributions of the text statistics of the PVMs have the SOE-RBC model with habit formation as the null hypothesis.

The second experiment is related to the SOE-RBC model with the stochastic world real interest rate. In this case, the world real interest rate is allowed to vary stochastically. Nason and Rogers(2003) also estimate the persistent parameter ρ_q and the standard deviation σ_q of the common component of the world real interest rate[20]. They give 0.903 and 0.004 as the means of the prior distributions of ρ_q and σ_q, respectively. This paper uses these values, and also set the habit parameter to zero to rule out the effect of the habit formation. The resulting theoretical distributions of the statistics of the PVMs have the null hypothesis of the multi-shock SOE-RBC model - the SOE-RBC model with the stochastic world real interest rate.

Each of the experiments generates 1000 sets of artificial data by which the theoretical distributions of the test statistics, $\hat{h}_{2SLS}$, $\hat{h}_{GMM}$, $\mathcal{W}_T$, $\mathcal{F}(\hat{\theta}_{GMM})$, $\mathcal{W}_T^*$, and $\mathcal{F}^*(\hat{\theta}_{OLS})$, are constructed. The GMM procedure is repeatedly applied to the sets of the artificial data, and the resulting 1000 replications of $\hat{\theta}_{GMM}$ are used to construct the theoretical distributions of the statistics. The matching of the theoretical moments with the sample moments is evaluated as in Christiano(1989) and Gregory and Smith(1991). That is, taking the sample statistics as critical values, this paper counts the proportion of times that the simulated number exceeds the corresponding sample point estimate. This pro-

[20]They calculate the world real interest rate by using Fisher's equation, the three-month Euro-dollar deposit rate, the Canadian dollar-U.S.dollar exchange rate, and the implicit GDP deflator of Canada.

portion is considered as the empirical p-value of the corresponding sample point estimate under the null hypothesis that the data generating process - the underlying SOE-RBC model - is true. Extreme values below 5 % or above 95 %imply a poor fit in the dimension examined.

5 Results

This section reports the results of the Monte Carlo experiments. The first experiment is related to the SOE-RBC model with habit formation. Three successful aspects of the habit-forming SOE-RBC model should be mentioned. The third column of Table 4 summarizes the empirical p-values of the sample estimates. First, observe that the p-values of $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$ are 0.7245 and 0.3824, respectively. Figures 2(a) and (b) show the nonparametrically smoothed theoretical distributions of $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$[21]. Notice that the modes of the theoretical distributions are close to the sample estimates, especially in $\hat{h}_{GMM}$. Second, Table 4 reveals that there are no elements of the vector $\mathcal{F}(\hat{\theta}_{GMM})$ that take extreme p-values above 0.95 or below 0.05. The third successful aspect is observed in the predictions of the habit-forming PVM, $\mathcal{CA}_t^f$. Figure 4(a) plots the estimated predictions of the habit-forming PVM and the 90 % theoretical confidence band. Note that all the point estimates fall inside the confidence band. The probability that the sample predictions are inside the band through the whole periods is actually equal to 1. Hence at least from these observations, it is hard to reject an inference that the true distributions of $\hat{h}_{2SLS}$, $\hat{h}_{GMM}$, $\mathcal{F}(\hat{\theta}_{GMM})$ and $\mathcal{CA}_t^f$ are the theoretical distributions under the null of the SOE-RBC model with habit formation.

The habit-forming SOE-RBC model, however, fails to replicate the sample estimates $\mathcal{W}_T$, $\mathcal{W}_T^*$, $\mathcal{F}^*(\hat{\theta}_{OLS})$ and $\mathcal{CA}_t^{*f}$. The third column of Table 4 reports that the empirical p-values of the Wald statistics for both the habit-forming and standard PVMs, $\mathcal{W}_T$ and $\mathcal{W}_T^*$, are 0.0696 and 0.0141, respectively. The p-value of $\mathcal{W}_T^*$ implies that at the significance level of 5 %, the sample estimate rejects the habit-forming SOE-RBC model as the

[21]The smoothed distribution is obtained by the nonparametric kernel density estimation with the normal kernel.

underlying DGP, while the p-value of $\mathcal{W}_T$ means rejection of the habit-forming SOE-RBC model on boundary and at least at 10 % significance level. The nonparametrically smoothed theoretical distributions of $\mathcal{W}_T$ and $\mathcal{W}_T^*$ in Figures 2(c) and (d) visually show the failure of the habit-forming SOE-RBC model to replicate the test statistics of the habit-forming and standard PVMs, $\mathcal{W}_T$ and $\mathcal{W}_T^*$: the sample estimates are at the far right tails of the theoretical distributions. Moreover, all the p-values of the elements of the vector $\mathcal{F}^*(\hat{\theta}_{OLS})$ take extreme values above 0.95 or below 0.05, except for $\hat{\mathcal{F}}_7^*$ equal to 0.0605. Finally, Figure 4(b) plots the sample predictions of the standard PVM and the corresponding 90 % theoretical confidence band. Observe how frequently the sample predictions fall outside the confidence band. The probability that the sample predictions are inside the confidence band through the whole period equals to 0.3972.

The next Monte Carlo experiment is based on the SOE-RBC model with the stochastic world real interest rate. The surprising result of this experiment is that there is no clear evidence to reject the null hypothesis that the true DGP is the SOE-RBC model with the stochastic world real interest rate. The fourth column of Table 4 reports the empirical p-values of the sample estimates in this experiment. First, note that the empirical p-values of $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$ are 0.115 and 0.1070, which in turn imply that the underlying SOE-RBC model cannot be rejected even at 10 % significance level. Figures 3(a) and (b) draw the smoothed theoretical distributions of $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$. Although the dispersion of the theoretical distribution of $\hat{h}_{2SLS}$ is large, and the theoretical distribution of $\hat{h}_{GMM}$ is heavily skewed toward the left, their modal values are close to the sample estimates. Regarding the vector $\mathcal{F}(\hat{\theta}_{GMM})$, the empirical p-values of all the elements except for the first one support the SOE-RBC model with the stochastic world real interest rate as the true DGP. As shown in Figure 5 (a), even with a couple of exceptions, almost all of the sample predictions on the current account, $\mathcal{CA}_t^f$, fall inside the theoretical 90 % confidence band. The probability that the sample predictions are inside the band is equal to 0.9858.

The result of the Wald statistic $\mathcal{W}_T$ is the first clear difference between the two Monte Carlo experiments. In the SOE-RBC model with the stochastic world real interest rate, the empirical p-value of the Wald statistic $\mathcal{W}_T$ is 0.5499. This implies that the sample estimate is fairly close to the median of the theoretical distribution, and the underlying null cannot be rejected at any standard significance levels. Its smoothed theoretical distribution in Figure 3(c) visually repeats this inference. Furthermore, striking differences are observed regarding the sample statistics related to the standard PVM. The empirical p-value of the Wald statistics for the standard PVM, $\mathcal{W}_T^*$, is 0.3259, which in turn implies together with the smoothed theoretical distribution in Figure 3(d) that the null of the SOE-RBC model with the stochastic world real interest rate cannot be rejected in this dimension. Except for $\hat{\mathcal{F}}_4$, all the estimates of the elements of the vector $\mathcal{F}^*(\hat{\theta}_{OLS})$ have the p-values between 0.05 and 0.95. Moreover, Figure 5(b) shows that the sample predictions are inside the 90 % theoretical confidence band in greater number of periods than in the case of the habit-forming SOE-RBC model. Indeed, the probability that the sample predictions are inside the band through the whole periods is 0.8156. This observation echoes the main finding of Nason and Rogers(2003): stochastic variations in the world real interest rate can explain the rejections of the standard PVM observed in the literature.

The results of the two Monte Carlo experiments are summarized in Table 5. This paper therefore reveals the superiority of the SOE-RBC model with the stochastic world real interest rate to the habit-foming SOE-RBC model to explain the broad empirical facts of the habit-forming and standard PVMs. Better than habit formation in consumption, stochastic variations in the world real interest rate explain the transitory consumption component/the consumption-tilting behavior, which is a crucial factor of the DGP of the Canadian current account.

6 Conclusion

This paper issues a caution about interpreting the empirical results from the habit-forming PVM as evidence that habit formation in consumption plays a significant role in explaining current account movements. One reason is that the habit-forming PVM is observationally equivalent to the non-habit PVM associated with serially correlated transitory consumption. This makes identification of the habit-forming PVM of the current account problematic.

Monte Carlo simulations based on SOE-RBC models are one to avoid this identification problem. The simulation exercises study the ability of different SOE-RBC models to mimic the sample moments or the empirical facts conditional on the habit-forming and standard PVMs. Two SOE-RBC models are hypothesized as the true DGPs of the sample moments: the one with with habit formation and the other with the stochastic world real interest rate. The Monte Carlo simulations make it possible to construct the theoretical distributions of the sample moments from the two hypothesized DGPs.

The results of the matching exercise based on the post-war Canadian data support the SOE-RBC model with the stochastic world real interest rate. The model matches all the key sample moments of the habit-forming and standard PVMs. The SOE-RBC model with habit formation mimics only a part of the empirical facts of the habit-forming PVM. This model fails to mimic the cross-equation restrictions predicted by the habit-forming PVM and all the empirical facts related to the standard PVM. Thus, the SOE-RBC model with a world real interest rate shock dominates the habit forming SOE-RBC model. Recent studies of Lettau and Uhlig(2000) and Otrok, Ravikumar and Whiteman(2002) claim counterfactual predictions of habit formation on several aspects of macroeconomics, e.g. consumption volatility and the equity premium puzzle. This paper also casts doubts on habit formation as an important source for the Canadian current account movements.

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Appendices

Appendix 1: Derivation of Eq.(6)

Let CA_t^P denote the standard PVM under $h = 0$ and $C_t^T = 0$:

$$CA_t^P \equiv rB_t + NO_t - C_t^P = -\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t \Delta NO_{t+i}. \tag{A.1.1}$$

Substituting the decomposition $C_t = C_t^P + C_t^T$ into the current account identity and using eq.(A.1.1) yield

$$\begin{aligned} CA_t &\equiv rB_t + NO_t - C_t \\ &= rB_t + NO_t - C_t^P - C_t^T \\ &= CA_t^P - C_t^T. \end{aligned} \tag{A.1.2}$$

Applying the AR(1) process of C_t^T to eq.(A.1.2) gives

$$\begin{aligned} CA_t &= CA_t^P - C_t^T \\ &= CA_t^P - \rho_c C_{t-1}^T - \omega_t \\ &= \rho_c CA_{t-1} + CA_t^P - \rho_c CA_{t-1}^P - \omega_t \end{aligned} \tag{A.1.3}$$

Several calculations easily show that the term $CA_t^P - \rho_c CA_{t-1}^P$ has the following representation

$$\begin{aligned} CA_t^P - \rho_c CA_{t-1}^P = \left(\frac{\rho_c}{1+r}\right)\Delta NO_t - \left(1 - \frac{\rho_c}{1+r}\right)\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t \Delta NO_{t+i} \\ - \left(\frac{\rho_c}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i (E_t - E_{t-1})\Delta NO_{t+i}. \end{aligned}$$

Note that the last term of the RHS represents revision of expectation for future changes in net output between periods t and $t-1$. Let this term be, say, ξ_t, and notice that expectation of ξ_{t+s} conditional on the information set at period t is zero for any $s \geq 1$ by law of the iterated expectation. Substituting back the term $CA_t - \rho_c CA_{t-1}$ into eq.(A.1.3) and setting $v_t = \xi_t + \omega_t$ provide eq.(6).

Appendix 2: Derivation of Eq.(7)

Substituting the PVM (2) into the definition of D_t yields

$$
\begin{aligned}
D_t &\equiv CA_t - \Delta NO_t - (1+r)CA_{t-1} \\
&= -(1+r-h)CA_{t-1} - \left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i} - \epsilon_t \\
&= hD_{t-1} - hD_{t-1} - (1+r-h)CA_{t-1} - \left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i} - \epsilon_t.
\end{aligned} \tag{A.2.1}
$$

Substituting the definition $D_{t-1} \equiv CA_{t-1} - \Delta NO_{t-1} - (1+r)CA_{t-2}$ into the second term in the RHS of eq.(A.2.1) and using the PVM (2) to eliminate the resulting term CA_{t-1} further rewrite eq.(A.2.1) as

$$
\begin{aligned}
D_t &= hD_{t-1} - \epsilon_t + (1+r)\epsilon_{t-1} \\
&+ (1+r-h)\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_{t-1}\Delta NO_{t+i-1} - \left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i}.
\end{aligned} \tag{A.2.2}
$$

Note that the fourth term in the RHS of eq.(A.2.2) equals

$$
\left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_{t-1}\Delta NO_{t+i}.
$$

Therefore eq.(7) is the case:

$$
\begin{aligned}
D_t &= hD_{t-1} - \epsilon_t + (1+r)\epsilon_{t-1} - \left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i (E_t - E_{t-1})\Delta NO_{t+i} \\
&= hD_{t-1} - \epsilon_t + (1+r)\epsilon_{t-1} + e_t.
\end{aligned}
$$

Note that expectation of e_{t+s} conditional on the information set at period t is zero for any $s \geq 1$ because

$$
\begin{aligned}
E_t e_{t+s} &= -\left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t(E_{t+s} - E_{t+s-1})\Delta NO_{t+i+s} \\
&= -\left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i (E_t - E_t)\Delta NO_{t+i+s} \\
&= 0
\end{aligned}
$$

by the law of the iterated expectation.

Appendix 3: The Two-Step GMM Estimation

In the first step, the criterion function $J(\theta) = G(\theta)'MG(\theta)$ is minimized with respect to θ under the restriction that the weighting matrix M is the identity matrix I. The resulting estimate of θ, say θ^*, is used to construct the optimal weighting matrix M^* such that

$$M^* = \left[T^{-1}\sum_{t-1}^{T} g_t(\theta^*)g_t(\theta^*)'\right]^{-1}$$

when $g_t(\theta^*)$ follows an i.i.d. process. Because there is a possibility of serial correlation of $g_t(\theta^*)$ in the first step, this essay exploits the heteroskedasticity-autocorrelation consistent estimator of Newey and West(1987) to calculate the optimal weighting matrix M^*. In the second step, minimizing the criterion function $J(\theta)$ under the optimal weighting matrix M^* yields the second step estimate $\hat{\theta}_{GMM}$ with the asymptotic variance-covariance matrix

$$\hat{V}_{\theta_{GMM}} = T^{-1}\left[\frac{\partial G(\hat{\theta}_{GMM})}{\partial \theta'} M^* \frac{\partial G(\hat{\theta}_{GMM})}{\partial \theta'}'\right]^{-1}.$$

Appendix 4: The State Space Representation of the Equilibrium Path

The purpose of this appendix is to explain in detail the derivation of the state space representation from the system of stochastic difference equations, which contains eqs.(14)-(24). The first step is to convert the system to the stationary one. To do that, it is convenient to introduce a new variable ϖ_t satisfying

$$\varpi_t = C_{t-1}/A_{t-1}. \tag{A.4.1}$$

That is, ϖ_t is stochastically detrended consumption at period $t-1$.

A.4.1: Deriving the Stationary System

Using the stochastically detrended variables and eqs.(17), (18) and (A.4.1) rewrites the system of equations (14)-(16) and (21)-(24) as the following stationary system:

The Stationary System

$$b_{t+1} = \left[1 + q_t - \eta\left(\frac{b_t}{y_t}\right)\exp(-\Delta \ln A_t)\right]\exp(-\Delta \ln A_t)b_t + (1-g)y_t - i_t - c_t \tag{14'}$$

$$y_t = k_t^{\psi} N_t^{1-\psi} \exp(-\psi \Delta \ln A_t) \tag{15'}$$

$$k_{t+1} = (1-\delta)\exp(-\Delta \ln A_t) k_t + \left(\frac{k_t}{i_t}\right)^{\varphi} i_t \exp(-\varphi \Delta \ln A_t) \tag{16'}$$

$$\beta \exp\left\{[\phi(1-\gamma)-1]\Delta \ln A_{t+1}\right\} \left[\frac{c_{t+1} - h\exp(-\Delta \ln A_{t+1})\varpi_{t+1}}{c_t - h\exp(-\Delta \ln A_t)\varpi_t}\right]^{\phi(1-\gamma)-1} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\phi)(1-\gamma)}$$
$$= \Gamma_{t+1} \tag{21'}$$

$$1 = E_t \Gamma_{t+1} \left[1 + q_{t+1} - 2\eta \exp(-\Delta \ln A_{t+1}) \left(\frac{b_{t+1}}{y_{t+1}}\right)\right] \tag{22'}$$

$$\frac{1-\phi}{\phi}\left[\frac{c_t - h\exp(-\Delta \ln A_t)\varpi_t}{1-N_t}\right] = (1-\psi)\frac{y_t}{N_t}\left[1 + \eta \exp(-2\Delta \ln A_t)\left(\frac{b_t}{y_t}\right)^2\right] \tag{23'}$$

$$\begin{aligned}
&\frac{1}{1-\varphi}\left(\frac{i_t}{k_t}\right)^{\varphi} \exp(\varphi \Delta \ln A_t) = \\
&E_t \Gamma_{t+1} \left\{\frac{1-\delta}{1-\varphi} + \frac{\varphi}{1-\varphi}\exp[(1-\varphi)\Delta \ln A_{t+1}]\left(\frac{i_{t+1}}{k_{t+1}}\right)^{1-\varphi}\right\} \exp(\varphi \Delta \ln A_{t+1})\left(\frac{i_{t+1}}{k_{t+1}}\right)^{\varphi} \\
&\qquad + E_t \Gamma_{t+1} \psi \exp(\Delta \ln A_{t+1}) \frac{y_{t+1}}{k_{t+1}}\left[1 + \eta \exp(-2\Delta \ln A_{t+1})\left(\frac{b_{t+1}}{y_{t+1}}\right)^2\right]
\end{aligned} \tag{24'}$$

and eq.(A.4.1). The stationary system contains the eight equations, the eight endogenous variables and the two exogenous variables following the processes (19) and (20).

A.4.2: The Deterministic Steady State

Let c, y, i, N, k, b, Γ and ϖ denote the deterministic steady state values of the corresponding variables. From the stationary system, the deterministic steady state is characterized as follows. First, from eq.(21'), the steady state value of the stochastic discount factor, Γ, is given as

$$\Gamma = \beta \exp\{[\phi(\gamma-1)-1]\alpha\}$$

where α is the unconditional mean of $\Delta \ln A_t$. Eq.(A.4.1) shows that the steady state value ϖ is equal to c

$$\varpi = c.$$

From eqs.(16') and (22'), the steady state ratios i/k and b/y are determined by

$$\frac{i}{k} = [1-(1-\delta)\exp(-\alpha)]^{\frac{1}{1-\varphi}} \exp(\varphi\alpha)^{\frac{1}{1-\varphi}}$$

and

$$\frac{b}{y} = \left[\frac{1 + q^* - \frac{1}{\Gamma}}{2\eta \exp(-\alpha)} \right].$$

Given i/k and b/y, the steady state ratio y/k is determined as a solution of the equation

$$\frac{1}{1-\varphi} \left(\frac{i}{k} \right)^{\varphi} \exp(\varphi\alpha) = \Gamma \left\{ \frac{1-\delta}{1-\varphi} + \frac{\varphi}{1-\varphi} \exp[(1-\varphi)\alpha] \left(\frac{i}{k} \right)^{1-\varphi} \right\} \exp(\varphi\alpha) \left(\frac{i}{k} \right)^{\varphi} + \Gamma\psi \exp(\alpha) \frac{y}{k} \left[1 + \eta \exp(-2\alpha) \left(\frac{b}{y} \right)^2 \right].$$

Because i/k and y/k have been already derived, the steady state ratio i/y can be constructed by dividing i/k by y/k. Eqs.(15') and (23') then yield the steady state ratios k/N and c/y as

$$\frac{k}{N} = \left[\frac{y}{k} \exp(\psi\alpha) \right]^{\frac{1}{\psi-1}}$$

and

$$\frac{c}{y} = \left[1 + q^* - \eta \left(\frac{b}{y} \right) \exp(-\alpha) \right] \exp(-\alpha) \left(\frac{b}{y} \right) + (1-g) - \left(\frac{i}{y} \right) - \left(\frac{b}{y} \right).$$

Finally, eq.(23') determines the steady state level of N as a solution of the equation

$$\frac{1-\phi}{\phi} [1 - h \exp(-\alpha)] \frac{c}{y} = (1-\psi) \frac{1-N}{N} \left[1 + \eta \exp(-2\alpha) \left(\frac{b}{y} \right)^2 \right].$$

Given N, the steady state level k is obtained by multiplying the ratio k/N by N. The steady state level y is obtained by multiplying y/k by k. Similarly, the other steady state levels c and i are constructed by multiplying c/y and i/y by y, respectively.

A.4.3: Derivation of the State Space Representation

The next step is to take a first-order Taylor expansion of the system (A.4.1), (14')-(16') and (21')-(24') around the deterministic steady state. Applying Sim's (2000) method to the linearized rational expectation model shows that there exists the unique equilibrium path, and the vectors $\mathcal{P}_t$ and $\mathcal{S}_t$ follow the state space representation eq.(25).

Figures and Tables

Table 1: The Sample Statistics of the PVMs

(a) **The Habit-Forming PVM**

$\hat{h}_{2SLS}$	$\hat{h}_{GMM}$	$\mathcal{J}_T$	$\mathcal{W}_T$
0.931	1.002	0.455	37.128
(0.192)	(0.152)	[0.978]	[0.000]

$\hat{\mathcal{F}}_1$	$\hat{\mathcal{F}}_2$	$\hat{\mathcal{F}}_3$	$\hat{\mathcal{F}}_4$	$\hat{\mathcal{F}}_5$	$\hat{\mathcal{F}}_6$	$\hat{\mathcal{F}}_7$	$\tilde{\mathcal{F}}_8$
-0.302	-0.068	0.017	0.006	1.276	-0.400	0.062	0.138
(0.130)	(0.059)	(0.073)	(0.049)	(0.226)	(0.157)	(0.073)	(0.079)

(b)**The Standard PVM**

$\mathcal{W}_T^*$	$\hat{\mathcal{F}}_1^*$	$\hat{\mathcal{F}}_2^*$	$\hat{\mathcal{F}}_3^*$	$\hat{\mathcal{F}}_4^*$	$\hat{\mathcal{F}}_5^*$	$\hat{\mathcal{F}}_6^*$	$\hat{\mathcal{F}}_7^*$	$\hat{\mathcal{F}}_8^*$
20.589	0.229	0.066	0.010	0.106	-0.115	0.046	-0.019	-0.095
[0.009]	(0.171)	(0.179)	(0.126)	(0.088)	(0.408)	(0.106)	(0.113)	(0.106)

Note: Table 1(a) reports the sample statistics of the PVM with habits. $\hat{h}_{2SLS}$ is the 2SLS estimate of the habit parameter based on the single unconditional moment conditions (8) while $\hat{h}_{GMM}$ is the GMM estimate of the habit parameter based on the full unconditional moment conditions (8), (10) and (12). $\mathcal{J}_T$ is the χ^2 statistic with the fourth degree of freedom for the overidentifying restriction test. $\mathcal{W}_T$ is the χ^2 statistic with the eighth degree of freedom for the cross-equation restrictions (13). The brackets below $\mathcal{J}_T$ and $\mathcal{W}_T$ show the corresponding asymptotic p-values. $\hat{\mathcal{F}}_i$ represents the estimate of the ith element in the vector $\mathcal{F}(\hat{\theta}_{GMM})$. The numbers in parentheses give the asymptotic standard errors for the corresponding estimates.

On the other hand, Table 1(b) shows the sample statistics for the standard PVM. $\mathcal{W}_T^*$ is the χ^2 statistic with the eighth degree of freedom for the cross-equation restrictions of the standard PVM. $\hat{\mathcal{F}}_i^*$ represents the estimate of the ith element in the cross-equation restrictions of the standard PVM .

Table 2: Empirical Facts of the Present Value Models

(a) **The Habit-Forming PVM**

1. The Habit Parameter is Close to One.
2. The Cross-Equation Restrictions are Jointly Rejected.
3. The Fifth Element of $\mathcal{F}(\hat{\theta}_{GMM})$ is Close to One.
4. The Predictions Track the Actual Series Closely.

(b) **The Standard PVM**

1. The Cross-Equation Restrictions are Jointly Rejected.
2. The Fifth Element of $\mathcal{F}^*(\hat{\theta}_{OLS})$ is Close to Zero.
3. The Predictions are Too Smooth.

Table 3: Calibrated Parameters of the SOE-RBC Models

Baseline Parameters

β	ϕ	γ	ψ	φ	δ
0.994	0.371	2.000	0.350	0.050	0.020

η	g	α	σ_a
0.071×10^{-4}	0.230	0.0024	0.012

Monte Carlo Experiments with Habit Formation

h	ρ_q	σ_q
0.990	1.000×10^{-7}	1.000×10^{-7}

Monte Carlo Experiments with the World Real Interest Rate

h	ρ_q	σ_q
0.000	0.903	0.004

Table 4: Sample Estimates and Empirical P-values under the Nulls of SOE-RBC Models

		Empirical P-values:	
	SampleEstimates	Habit Formation	World Real Interest Rates
$\hat{h}_{2SLS}$	0.931	0.7245	0.1150
$\hat{h}_{GMM}$	1.002	0.3824	0.1070
$\mathcal{W}_T$	37.128	0.0696	0.5499
$\hat{\mathcal{F}}_1$	-0.302	0.7326	0.9536
$\hat{\mathcal{F}}_2$	-0.068	0.6297	0.5217
$\hat{\mathcal{F}}_3$	0.017	0.4733	0.4571
$\hat{\mathcal{F}}_4$	0.006	0.4904	0.6670
$\hat{\mathcal{F}}_5$	1.276	0.2593	0.1493
$\hat{\mathcal{F}}_6$	-0.400	0.7841	0.7215
$\hat{\mathcal{F}}_7$	0.062	0.4198	0.3885
$\hat{\mathcal{F}}_8$	0.138	0.3481	0.1766
$\mathcal{W}_T^*$	20.589	0.0141	0.3259
$\hat{\mathcal{F}}_1^*$	0.229	0.0000	0.7164
$\hat{\mathcal{F}}_2^*$	0.066	0.0000	0.8073
$\hat{\mathcal{F}}_3^*$	0.010	0.0071	0.7952
$\hat{\mathcal{F}}_4^*$	0.106	0.0131	0.0363
$\hat{\mathcal{F}}_5^*$	-0.115	0.9980	0.4773
$\hat{\mathcal{F}}_6^*$	0.046	0.0151	0.7548
$\hat{\mathcal{F}}_7^*$	-0.019	0.0605	0.8295
$\hat{\mathcal{F}}_8^*$	-0.095	0.0111	0.9072

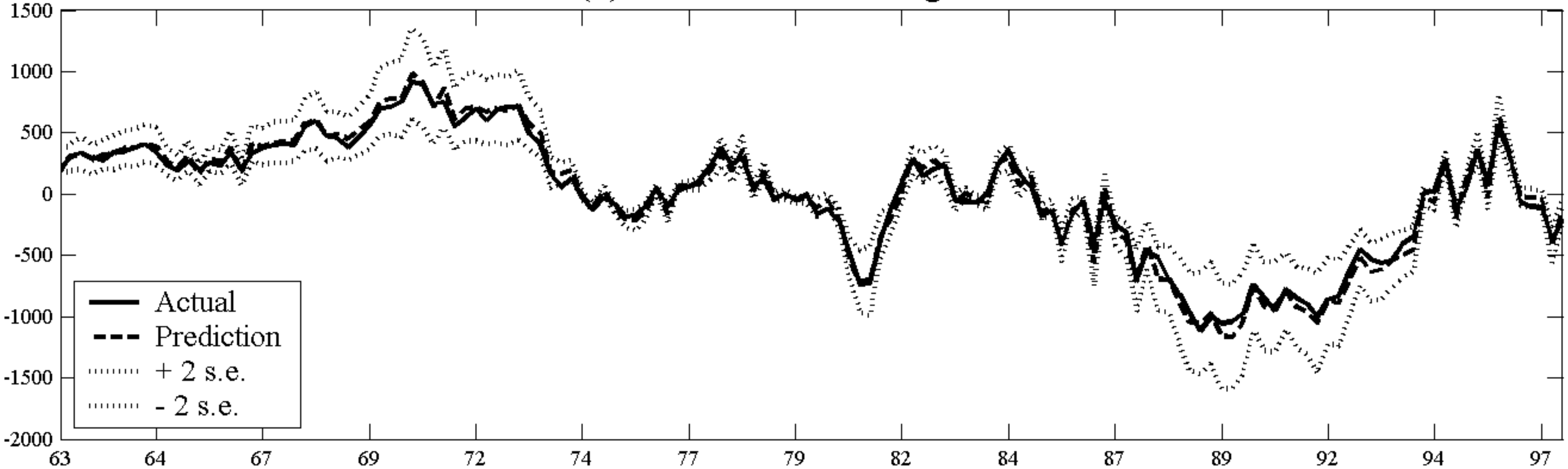
Note: Empirical p-values are constructed as the frequency that the simulated number exceeds the corresponding sample point estimate.

Table 5: The Monte Carlo Experiments: Which SOE-RBC Model Mimics the Empirical Facts in Table 2?

1. The SOE-RBC model with habit formation mimics the first, third and fourth facts of the habit-forming PVM.
2. The SOE-RBC model with habit formation fails to mimic the second fact of the habit-forming PVM: the Wald statistic for the cross-equation restrictions.
3. The SOE-RBC model with habit formation fails to mimic all the facts of the standard PVM.
4. The SOE-RBC model with the stochastic world real interest rate mimics all the facts of the habit-forming PVM.
5. The SOE-RBC model with the stochastic world real interest rate mimics all the facts of the standard PVM. In particular, the model does a better job to mimic the third fact of the standard PVM than the habit SOE-RBC model does.

Figure 1 The PVM Predictions on the Canadian Current Account

(a) The Habit-Forming PVM



(b) The Standard PVM

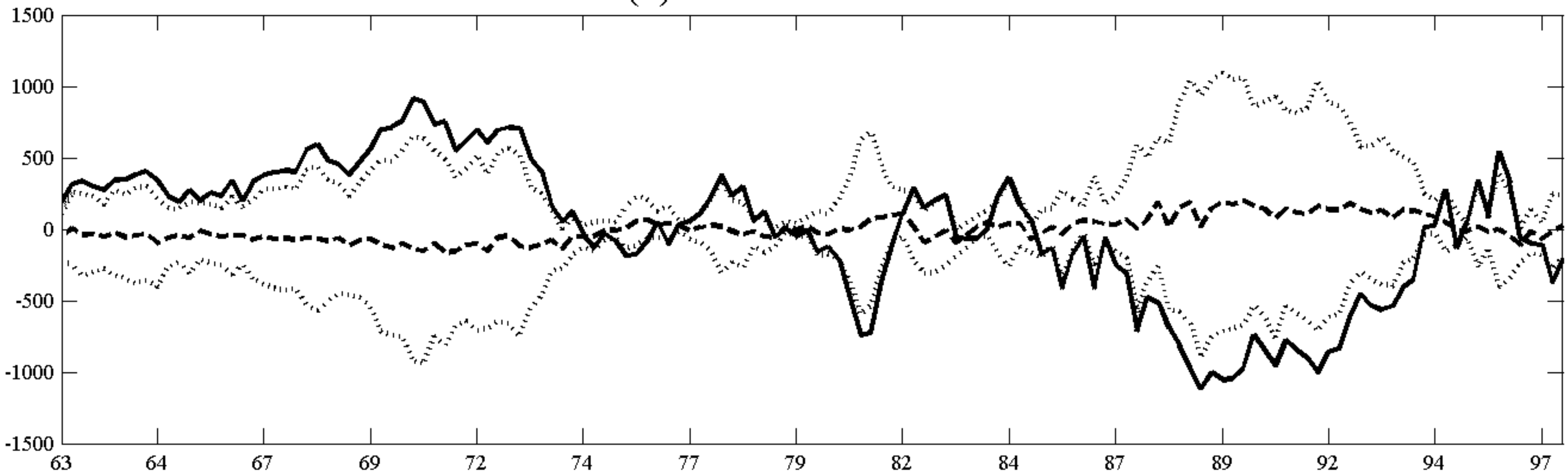


Figure 2 Theoretical Distributions of Test Statistics
The SOERBC with Habit Formation

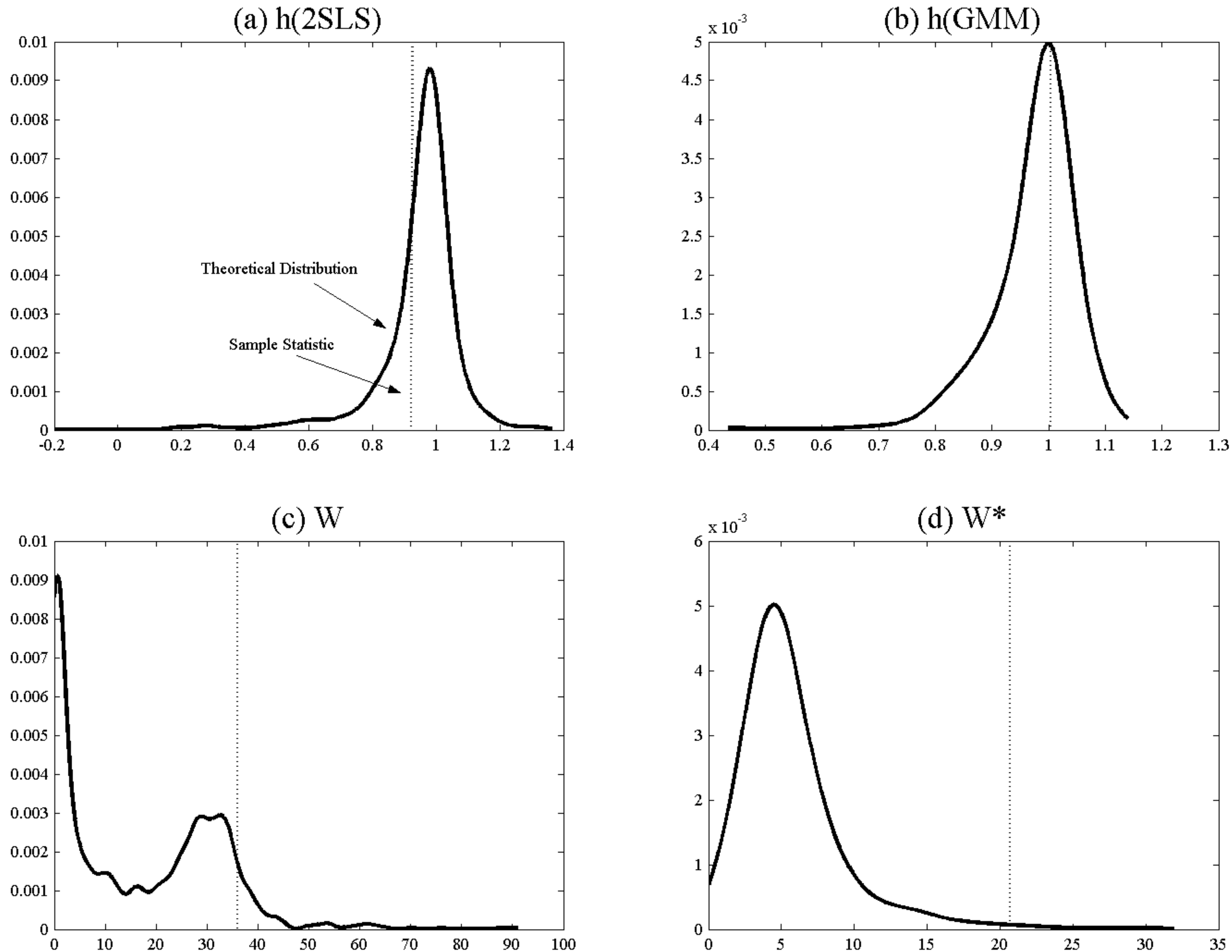


Figure 3 Theoretical Distributions of Test Statistics
The SOERBC with Stochastic World Real Interest Rates

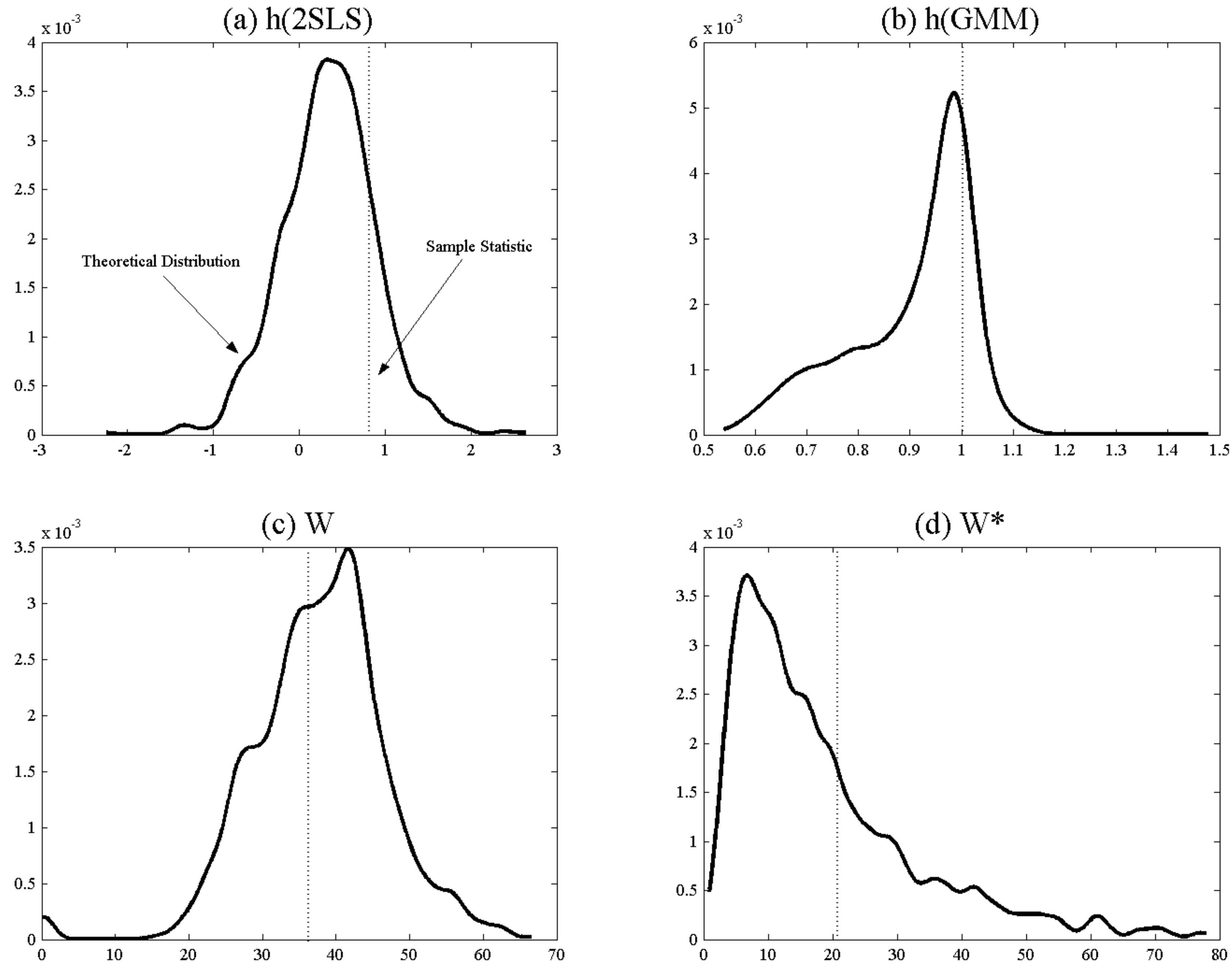


Figure 4 Sample Predictions and Theoretical Distributions
The SOERBC with Habit Formation

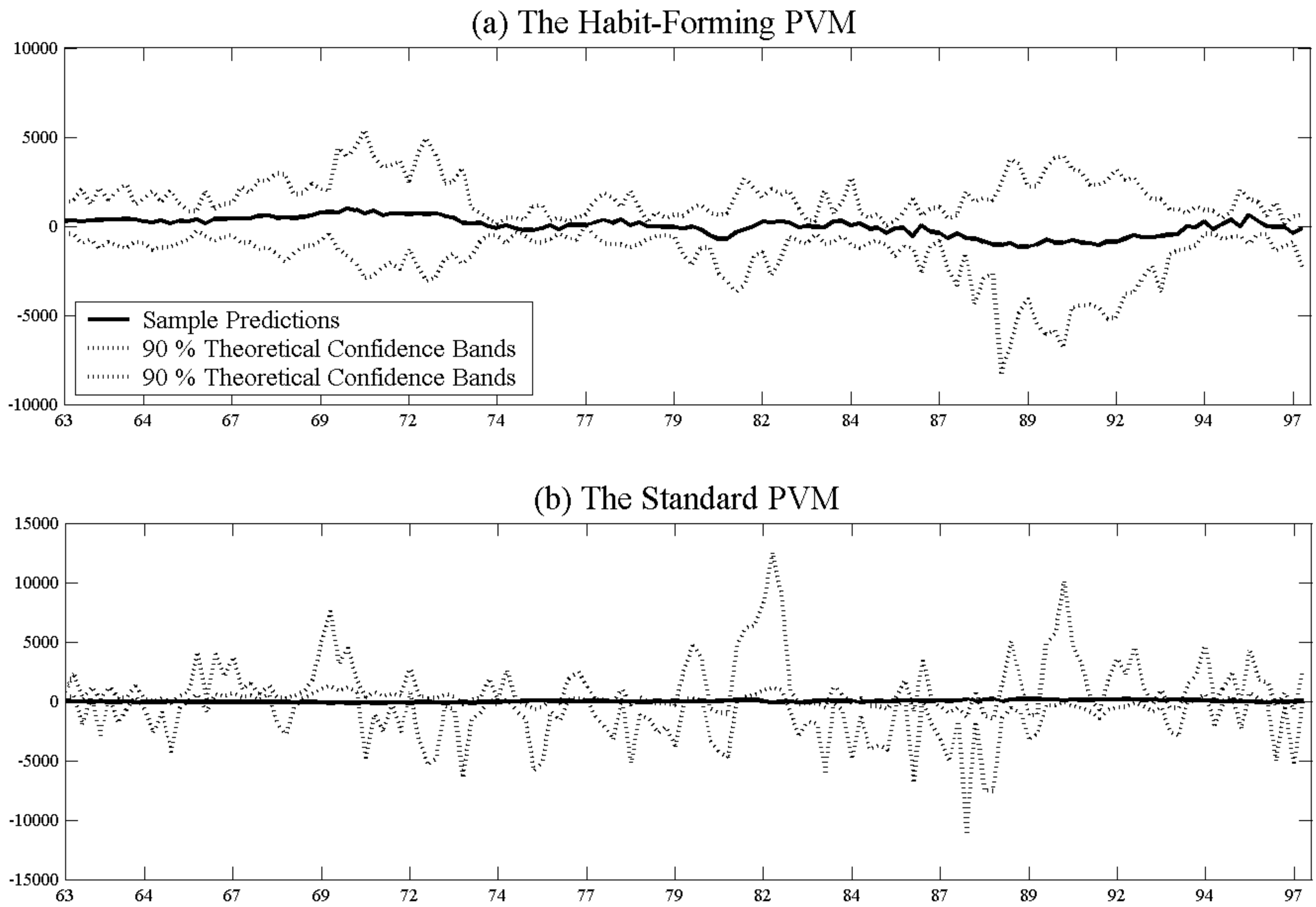


Figure 5 Sample Predictions and Theoretical Distributions
The SOERBC with the Stochastic World Real Interest Rate