# Real Options: Institutional Implications for Vertical Integration of Supply Chains in Competitive Environments 

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#### Abstract

Summary Participants along a production chain which exchange intermediate products on spot markets face price risks, such as the transmission of price fluctuations of the final product. In a real options environment this uncertainty may cause investment reluctance on the different steps of the production chain. This paper analyzes weather a stronger vertical integration along the production chain allows to reduce the investment reluctance. Therefore, an agent-based competitive model of the pork production chain is developed in which farmers use optimal investment strategies which are identified by genetic algorithms. Two production systems are compared: As example for a perfectly integrated system it is considered that every farmer can invest in closed systems in which piglets (the intermediate product) and finished hogs (the final product) are produced in equal amounts. In an alternative production system, farmers can either invest in farrowing (i.e. the production of piglets) or in hog finishing. The intermediate product, i.e. piglets, is traded on a spot market. Simulations show that the spot market solution and the closed system lead to the same production dynamics. The only precondition is that in the spot market system farrowers and hog finishers are aware of the investment strategies, respectively the production capacities of the market partners. This general finding is even independent of different depreciation rates on the production steps, though the price dynamics changes.


Keywords: real options, production chain, vertical integration, depreciation, agent-based models, genetic algorithms, stochastic simulation

## Real Options: Institutional Implications for Vertical Integration of Supply Chains in Competitive Environments

## 1. Introduction

According to the real options approach to investment (Henry, 1974, McDonald and Siegel, 1986, Pindyck, 1991) the Net Present Value (NPV) criterion in investment theory can be misleading under certain conditions. These conditions are: the returns of an investment are subject to an ongoing uncertainty, the investment is (at least partly) irreversible (i.e. the investment causes sunk costs), and the investor can defer the investment decision for some time. If all these conditions are fulfilled, even in case of risk neutrality, it is not necessarily optimal to invest if the expected present value of the future returns covers the investment outlays. Rather, one should assign a positive value to the preservation of the flexibility whether to invest or not; in other words, waiting for new information has a value.

Recently, several studies showed that the real options approach may also be relevant for investments in agricultural production such as for pork (Pietola and Wang 2000, Odening et al. 2003). Pork production requires irreversible investments in buildings, due to demand and supply fluctuations returns are uncertain, and usually investment decisions can be deferred. Based on price series for Finland, Pietola and Wang (2000) find arguments for significant investment reluctance for piglet production (farrowing) and for pork production (hog finishing). Since piglets are an intermediate product in the hog production chain and prices for piglets cause additional uncertainty, Pietola and Wang analyze the potential impact if farrowers and hog finishers would not trade piglets on spot markets but by contract production which defines piglet prices as a fixed multiple the actual pork price. Pietola and Wang find that contracting between farrowers and hog feeders would reduce the uncertainty and that investment reluctance could be reduced significantly. In addition, contracting creates welfare gains.

In this contribution we challenge this finding with regard to theoretical consistency. Instead of deriving investment strategies for each subsector individually on the basis of empirical price data, we explicitly model the subsectors and the spot market interaction. Because closed analytical solutions to determine optimal investment criteria only exist for rather simple situations, for example if the value of the project follows a geometric Brownian motion and the option never expires, Monte Carlo simulation is utilized. The main advantage of Monte Carlo simulation is its flexibility with respect to the stochastic process of the asset. However, instead of looking at the market at an aggregate level, we start with a bottom up approach by explicitly modeling the individuals (i.e. the farms) and their behavior. In a discrete-time model of market interaction, $N$ agents represent $N$ identical farms which compete in each subsector. Each of these farms can invest irreversibly into production assets (buildings) without knowing how the market environment will evolve in the future. Every farm invests according to its individual investment trigger which is derived by linking the agent-based model with a genetic algorithm (cf. Arifovic, 1994). Genetic algorithms (GA) can be understood as a certain form of computational intelligence which is based on a heuristic optimization technique
that is related to concepts of natural evolution, such as selection, crossover, and mutation. These mechanisms are repeatedly applied to a set (population) of solutions to the problem in order to find superior solutions. A fundamental advantage of using GA for complex optimization problems are the low prerequisites: Essentially, one just needs to specify the variables to be optimized, an environment that allows the evaluation of potential solutions, and the respective GA operators which breed the solutions.

Two production systems are compared: As example for a perfectly integrated system, it is considered that every farmer can invest in closed systems in which piglets (the intermediate product) and finished hogs (the final product) are produced in equal amounts. In an alternative production system, farmers can either invest in farrowing (i.e. the production of piglets) or in hog finishing. The intermediate product, i.e. piglets, is traded on a spot market. Simulations show that the spot market solution and the closed system lead to the same production dynamics. The only precondition is that for the spot market system farrowers and hog finishers are aware of the investment strategies, respectively the production capacities of the market partners. This general finding is even independent of different depreciation rates on the production steps, though these modified assumptions change the price dynamics.

## 2. The Model

### 2.1. The investment problem

Consider a number of $N=50$ firms, each having repeatedly the opportunity to invest in identical assets or a fraction thereof, i.e. the assets are divisible. Initially, no firm is invested. The asset has a maximum size of 1 and can be used by firm $n$ to produce up to $x_{t, n} \leq 1$ units of output per production period. Size, investment outlay and production are proportional, i.e. there are no economies of scale. If a firm invests for the first time, its maximum initial investment outlay $M_{t, n}^{\max }$ is $I$. The investment outlay $M_{t, n}$ is considered to be totally sunk after the investment is carried out. For every future period, we consider a geometrical decay of the asset. The asset's productivity declines to $(1-\lambda)$ of the previous period's output, i.e. we consider a depreciation rate $\lambda$ such that $x_{t+\Delta t, n}=(1-\lambda) \cdot x_{t, n} .{ }^{1}$ However, in every period, each firm can invest or reinvest in order to increase production or to regain a production capacity of up to one unit of output. The outlay $M_{t, n}$ then has a maximum amount $M_{t, n}^{\max }$ depending on the missing production capacity, i.e.

$$
\begin{equation*}
M_{t, n}^{\max }=\left[1-(1-\lambda) \cdot x_{t, n}\right] \cdot I \tag{1}
\end{equation*}
$$

such that $x_{t+\Delta t, n}^{\max }=1$. Each firm's investment decisions aim to maximize the expected net present value of the future cash flows by choosing a specific investment trigger $P_{n}^{*}$, i.e. the goal of firm $n$ can be formulated as

[^0]\[

$$
\begin{equation*}
\max _{P_{n}^{*}}\left\{\hat{\Pi}_{n}\left(P_{n}^{*}\right)=E\left[\sum_{l=0}^{\infty}\left(x_{l \cdot \Delta t, n} \cdot P_{l \cdot \Delta t}-M_{l \cdot \Delta t, n}\left(x_{l \cdot \Delta t, n}, P_{n}^{*}, \nabla_{l \cdot \Delta t,-n}\right)\right) \cdot(1+r)^{-l \cdot \Delta t}\right]\right\} \tag{2}
\end{equation*}
$$

\]

with $P_{t}$ as the output price in period $t$ and $\nabla_{t,-n}$ denoting a certain market operator that captures demand developments which are assumed to be stochastic as well as to be dependent on the behavior of the other firms. ${ }^{2}$ Accordingly, we consider that the firms compete and interact on a market. To capture competition, the firms and their interaction are represented in an agent-based setting in which the firms are represented as agents that perceive their environment and respond to it individually and autonomously (e.g. Russel and Norvig 1995).

The environment of a firm $n$ can be considered as consisting of two parts. One is the behavior of the other firms. The other is the demand for outputs, which is modeled in terms of a demand function. The environment can be described as follows:

Total supply in period $t$ is

$$
\begin{equation*}
X_{t}^{S}=\sum_{n=1}^{N} x_{t, n} \tag{3}
\end{equation*}
$$

and demand is

$$
\begin{equation*}
X_{t}^{D}=\frac{\alpha_{t}}{P_{t}} \tag{4}
\end{equation*}
$$

For identity of demand and supply, we get

$$
\begin{equation*}
P_{t}=\frac{\alpha_{t}}{X_{t}^{D}}=\frac{\alpha_{t}}{X_{t}^{S}} \tag{5}
\end{equation*}
$$

Consider now that the demand parameter $\alpha_{t}$ follows geometric Brownian motion (GBM). Assuming discrete time this can be modeled as

$$
\begin{equation*}
\alpha_{t}=\alpha_{t-\Delta t} \cdot \exp \left[\left(\mu-\frac{\sigma^{2}}{2}\right) \cdot \Delta t+\sigma \cdot \varepsilon_{t} \cdot \sqrt{\Delta t}\right] \tag{6}
\end{equation*}
$$

with a volatility $\sigma$, a drift rate $\mu$, a standard normally distributed random number $\varepsilon_{t}$, and a time step length $\Delta t$.

Firm $n$ invests in period $t$ if the expected price $\hat{P}_{t+\Delta t} \geq P_{n}^{*}$ with

$$
\begin{align*}
& \hat{P}_{t+\Delta t}=\frac{\alpha_{t+\Delta t}}{X_{t+\Delta t}} \text { and } \hat{X}_{t+\Delta t}=\sum_{n=1}^{N} x_{t+\Delta t, n} \text { with }  \tag{7}\\
& x_{t+\Delta t, n}=\left\{\begin{array}{cl}
1 & \text { if } n \text { invests } M_{t, n}^{\max } \\
(1-\lambda) \cdot x_{t, n}+\frac{M_{t, n}}{I} & \text { if } n \text { invests } 0<M_{t, n}<M_{t, n}^{\max } \\
(1-\lambda) \cdot x_{t, n} & \text { if } n \text { invests } M_{t, n}=0
\end{array}\right. \tag{8}
\end{align*}
$$

The questions now are: Which firms invest? And how much do they invest? Therefore, let us assume that firms with lower trigger prices $P_{n}^{*}$ have a stronger tendency to invest. Conse-

[^1]quently, all firms can be sorted according to their trigger prices, starting with the lowest investment trigger, i.e. $P_{n}^{*} \leq P_{n+1}^{*}$. The following propositions are straightforward:

Proposition 1: If firm $n$ does not invest in $t$ then firm $n+1$ will also not invest in $t$, i.e.

$$
M_{t, n}=0 \Rightarrow M_{t, n+1}=0
$$

Proposition 2: If firm $n$ does invest in $t$ then firm $n-1$ will invest $M_{t, n-1}^{\max }$ in $t$, i.e.

$$
M_{t, n}>0 \Rightarrow M_{t, n-1}=M_{t, n-1}^{\max } \Rightarrow x_{t+1, n-1}=1
$$

Proposition 3: In every period $t$, a marginal (or last) firm $n_{t}^{o}$ exists which invests $M_{t, n^{o}}$ such that the expected price for the next period is equal to the investment trigger of firm $n_{t}^{o}$, i.e. $P^{n^{\circ *}}=E\left(P_{t+\Delta t}\right)$ with $0<M_{t, n^{o}} \leq M_{t, n^{\circ}}^{\max }$ and $0 \leq n_{t}^{o} \leq N .^{3}$

The investment of firm $n_{t}^{o}$ can be computed according

$$
\begin{align*}
& P_{n^{\circ}}^{*}=\mathrm{E}\left(P_{t+\Delta t}\right)=\frac{\alpha_{t+\Delta t}=\alpha_{t} \cdot \exp (\mu \cdot \Delta t)}{x_{n^{\circ}, t+\Delta t}+\left(n^{\circ}-1\right)+(1-\lambda)^{\Delta t} \sum_{n=n^{\circ}+1}^{N} x_{t, n}}  \tag{9}\\
\Leftrightarrow \quad & x_{n^{\circ}, t+\Delta t}=\frac{\alpha_{t} \cdot \exp (\mu \cdot \Delta t)}{P_{n^{\circ}}^{*}}-\left(\left(n^{\circ}-1\right)+(1-\lambda) \sum_{n=n^{\circ}+1}^{N} x_{t, n}\right)  \tag{10}\\
\Leftrightarrow \quad & \frac{M_{t, n^{\circ}}}{I}=\left(\frac{\alpha_{t} \cdot \exp (\mu \cdot \Delta t)}{P_{n^{\circ}}^{*}}\right)-\left(n_{t}^{o}-1\right)-(1-\lambda) \cdot \Delta t \cdot \sum_{n=n_{t}^{o}}^{N} x_{t, n} \tag{11}
\end{align*}
$$

Now, $n_{t}^{o}$ can be identified by iteratively testing all firms for $P_{n_{t}^{o}}^{*} \leq \hat{P}_{t+\Delta t}^{n^{o}}$. The last firm with a positive investment is $n_{t}^{o}$.

Equation (11) is an equilibrium condition: All firms which fully invest and hence produce at maximum capacity have trigger prices which are less or equal to the trigger price of firm $n^{o}+1$ which is also equal to the expected price for $t+\Delta t$. All firms which do not invest have trigger prices which are higher than or equal to the expected price for $t+\Delta t$.
For a given set of trigger prices $P^{*}$ and arbitrary initializations for $\alpha_{0}$, the expected profitability of each strategy

$$
\begin{equation*}
\hat{\Pi}_{n}\left(P_{n}^{*}\right)=E\left\{\sum_{l=0}^{\infty}\left(x_{l \cdot \Delta t, n} \cdot P_{l \cdot \Delta t}-M_{l \cdot \Delta t, n}\left(x_{l \cdot \Delta t, n}, P_{n}^{*}, \nabla_{l \cdot \Delta t,-n}\right)\right)(1+r)^{-l \cdot \Delta t}\right\} \tag{12}
\end{equation*}
$$

can be determined simultaneously by a sufficiently high number of repeated stochastic simulations of the market. For our analysis, we consider 5000 repetitions to be sufficient.
As presented to this point, the model resembles a farm's investment problem for a closed system of pork production in which the intermediate product piglets and the final product pork are produced in appropriate amounts, such that trade of the intermediate product is not necessary. The investment costs $I$ cover the costs for both production assets, i.e. $I={ }^{p i} I+{ }^{h o} I$. The

[^2]superscript on the left side self-explanatory marks the piglet producers and the hog finishers, respectively.

What are the consequences for a spot market relationship between hog finishers and piglet producers for their investment triggers? Naturally, in such a system the production capacity of the hog finishers corresponds to the demand parameter of the piglet producers:

$$
\begin{equation*}
{ }^{h o} X_{t}={ }^{p i} \alpha_{t} \tag{13}
\end{equation*}
$$

Considering iso-elastic demand with demand elasticity -1 , in the market equilibrium for the piglet producers is valid:

$$
\begin{equation*}
{ }^{p i} P_{t}=\frac{{ }^{p i} \alpha_{t}}{{ }^{p i} X_{t}}=\frac{{ }^{h o} X_{t}}{{ }^{p i} X_{t}} \tag{14}
\end{equation*}
$$

Piglet producer $n$ invests, if the expected price for piglets ${ }^{p i} P_{t+\Delta t}$ is larger or equal her trigger price ${ }^{p i} P^{*}$. Total production of piglets in period $t+\Delta t$ is:

$$
\begin{equation*}
{ }^{p i} x_{t+\Delta t}^{n^{o}}=\frac{\alpha_{t}}{\left({ }^{h o} P_{n t}^{*}+{ }^{p i} P_{n}^{*}\right) \cdot{ }^{2} P_{n}^{*}}-\sum_{p i}^{p i}{ }_{n=1}^{p i}{ }_{n_{t}^{o}-1}{ }^{i} x_{p i}^{M a x}-{ }^{p i} \lambda \sum_{p i} \sum_{n=n^{0}+1}^{p i} N{ }^{p i} x_{t}^{p i} n_{n} . \tag{15}
\end{equation*}
$$

Note, in contrast to the description above the net return for hog finishers ${ }^{h o} G_{t}$ must be adjusted by the piglet price (i.e. the variable costs of pork production). ${ }^{4}$ Additionally since finishers would not spend more money on buying piglets than the expected return for pork, the net return is zero in these cases, i.e.

$$
{ }^{h o} G_{t}=\left\{\begin{array}{ll}
0, & \text { if }{ }^{p i} P_{t} \leq{ }^{h o} \hat{P}_{t}  \tag{16}\\
\frac{{ }^{h o} \alpha_{t}}{{ }^{h o} X_{t}}{ }^{p i} P_{t}, \text { otherwise }
\end{array} .\right.
$$

For hog finishers is valid:

$$
\begin{equation*}
{ }^{h o} X_{t}=\frac{{ }^{h o} \alpha_{t}}{{ }^{p i} P_{t}+{ }^{h o} P_{t}} \tag{17}
\end{equation*}
$$

The remaining question is, how to determine appropriate sets of trigger prices ${ }^{h o} P_{n}^{*}$ and ${ }^{p i} P_{n}^{*}$ ? For this, the multi-firm market models are combined with a genetic algorithm (GA).

### 2.2. The Genetic Algorithm and its implementation ${ }^{5}$

GA are a heuristic optimization technique which has been developed in analogy to the concepts of natural evolution and the terminology used reflects this. Even though there is no "standard GA" but many variations of GA, there are some basic elements which are common to all GA (cf. Holland, 1975, Goldberg, 1989, Forrest, 1993, Mitchell, 1996). ${ }^{6}$ The first task

[^3]of an application of GA is to specify a way of representing each possible solution or strategy as a string of genes which is located on one ore more chromosomes. Usually this is achieved by representing solutions (e.g. strategies, numbers, etc.) as binary bits, i.e. zeroes or ones, which form the genes. Since our problem is relatively simple, i.e. we just search for a single value (i.e. every strategy just consists of a certain trigger price), we take the investment trigger as a real value and apply the GA operators to the trigger price itself. The second task is to define a population of $N$ genomes to which the genetic operators, i.e. selection, crossover and mutation, can be applied. The population size here is 50 genomes. This allows us to directly map the set of genomes to the firms' strategies, i.e. every firm's trigger price in our model is represented by one genome of the genome population. Vice versa every genome can be understood as the strategy of a certain firm.

Each application of the genetic operators to the population of genomes creates a new, modified generation of genomes. The number of generations depends on the problem to be solved. It can range from some 50 to a couple of thousand. In most GA applications the first generation of genomes is initialized by random values or it is set arbitrarily. During the following generations, the genome population passes through the following steps:

## a) Fitness Evaluation

Each time before the GA operators b) to d) are applied, the goodness of every genome is evaluated by applying a fitness function. This function assigns a score to each genome in the current population according to the capability of the genome strategy to solve the problem at hand. The better the strategy performs, the higher its fitness value. For our applications, the fitness value is directly derived from the strategy's average profitability $\Pi_{n}\left(P_{n}^{*}\right)$ or payoff in 5000 stochastic simulations of the market model.

## b) Selection and Replication

Selection determines the genetic material to be reproduced in the next generation. The fitter the genome (i.e. the better adapted it is to the problem) the more likely it is to be selected for reproduction. Selection can be implemented in many different ways. In this model the 20 most successful genomes always survive. The next 15 genomes are replaced with certain likelihood by the 15 most successful genomes of the last simulation series. The next 10 genomes are replaced by the 10 fittest genomes with a higher likelihood. And the least 5 successful genomes are always replaced by the 5 most successful genomes. Summarizing, the 5 most successful genomes can quadruplicate, the next 5 can triplicate, and the next 5 most successful strategies can double.

## c) Crossover

Figure 2 shows the simplest case of a 1-point-crossover, where the coded strings of two parent genomes are split at a randomly chosen locus and the sub-strings before and after the locus are exchanged between the two parent genomes resulting in two offspring. This technique is also used for our GA implementation. With a certain likelihood, for every genome $a$ a partner $b$ is randomly chosen from the selected genomes. The values are cut at a randomly chosen digit. If e.g., the numbers are cut after the third digit, offspring $a^{\prime}$ gets the first three digits of
parent $a$ and all further digits of parent $b$ and vice versa. Thus the triggers $a=1.2345678$ and $b=1.1111111$ become $a^{\prime}=1.2311111$ and $b^{\prime}=1.1145678$.

Figure 1: Example of a 1-point-crossover after the $3^{\text {rd }}$ digit.

## d) Mutation

Mutation also brings new genetic varieties into the population of genomes. Furthermore, mutation serves as a reminder or insurance operator because it is able to recover genetic material into the population which was lost in previous generations (Mitchell 1996). This insures the population against an early and permanent fixation on an inferior genotype. Mutation is implemented here by multiplying every solution with certain, but small likelihood with a random number between 0.95 and 1.05. The mutation likelihood as well as the range of the random number may be chosen according to experience as well as according to the already obtained results. A flow diagram can be found in the appendix.

In one particular point our GA application deviates from conventional applications. Here, the GA is not just used to solve a more or less complex optimization problem in which the goodness of the solution and the problem at hand are directly related. In our case, the goodness of a solution rather depends on the alternative solutions generated by the GA. In other words: in conventional GA applications the fitness of a genome can be obtained directly from a comparison of payoffs of the different solutions because the payoffs are independent of the competing solutions. Here, a solution's payoff depends on the other solutions. Thus, we are applying the GA to a game theoretic setting and we are not searching for an optimal solution, but for an equilibrium solution, i.e. the Nash-equilibrium strategy. ${ }^{7}$

### 2.3. The scenarios

The model as it is presented above can be used for many different scenarios. One motivation is to validate our approach for the standard case of a one step production system, i.e. the closed farrowing-finishing system, by showing that it leads to the same conclusions as analytical approaches. The calculations are based on an interest rate of $r=6 \%$. The drift rate $\mu$ is assumed to be zero and the volatility $\sigma$ is assumed to be $20 \%$. Depreciation rate $\lambda$ equals $5 \%$. Thus, investment costs $I_{\lambda=5 \%}=8.36364$ imply total production costs of 1 per unit of output. The total time span $T$ simulated in every stochastic simulation is determined as 100 years. For later periods the expected returns are set equal to the returns in year 100. The possible error can be assumed to be negligible since later returns are discounted by more than $99.7 \%$.

In order to validate the agent-based model of multiple competing farms, it will now be shown that the agent-based approach leads for the closed system to the same dynamics like a direct

[^4]simulation of the price dynamics that would have to be expected. For these reference experiments, it is assumed that output prices directly follow GBM for competitive markets. This idea is based on the seminal finding of Leahy (1993) showing that the market impacts of e.g. depreciations and competition can be ignored in the way that myopic behavior leads to adequate decisions if volatilities and the drift rate of the price process are estimated properly. ${ }^{8}$

## 3. Results

### 3.1. Validation

Consider the existence of an equilibrium investment trigger $P^{*}$ at which all firms invest and assume that in period $t-\Delta t$ firms have invested according to $\hat{P}_{t}=P^{*}$. From equations (5) and (6) we know that after the investment decisions are made, $P_{t}$ purely depends on the relation of $\alpha_{t}$ and $\alpha_{t-\Delta t}$. Hence, the price in $t$ will be

$$
\begin{equation*}
P_{t}=P^{*} \cdot \exp \left[\left(\mu-\frac{\sigma^{2}}{2}\right) \cdot \Delta t+\sigma \cdot \varepsilon_{t} \cdot \sqrt{\Delta t}\right] \tag{18}
\end{equation*}
$$

Consider now that the actual price in period $t$ is $P_{t} \geq P^{*}$. Then the firms will respond and invest such that $\hat{P}_{t+\Delta t}=P^{*}$. Now consider $P^{*} \geq P_{t}$. Then, two cases have to be differentiated. If $P^{*} \geq P_{t}>(1-\lambda)^{\Delta t} \cdot P^{*}$ then some firms will reinvest, such that $\hat{P}_{t+\Delta t}=P^{*}$. Otherwise, if $P_{t} \leq(1-\lambda)^{\Delta t} \cdot P^{*}$ no firm will reinvest and $\hat{P}_{t+\Delta t}=P_{t} /(1-\lambda)^{\Delta t}$. With this knowledge and in accordance with equations (1) to (12) the price dynamics can be described as

$$
P_{t}=\left\{\begin{array}{l}
P^{*} \cdot \exp \left[\left(\mu-\frac{\sigma^{2}}{2}\right) \cdot \Delta t+\sigma \cdot \varepsilon_{t} \cdot \sqrt{\Delta t}\right] \quad \text { if } P_{t-\Delta t} \geq(1-\lambda)^{\Delta t} \cdot P^{*}  \tag{19}\\
\left\{\begin{array}{c}
\frac{P_{t-\Delta t}}{(1-\lambda)^{\Delta t}} \cdot \exp \left[\left(\mu-\frac{\sigma^{2}}{2}\right) \cdot \Delta t+\sigma \cdot \varepsilon_{t} \cdot \sqrt{\Delta t}\right]= \\
P_{t-\Delta t} \cdot \exp \left[\left(\mu-\log (1-\lambda)-\frac{\sigma^{2}}{2}\right) \cdot \Delta t+\sigma \cdot \varepsilon_{t} \cdot \sqrt{\Delta t}\right]
\end{array}\right\} \quad \text { otherwise }
\end{array}\right.
$$

With equation (19) price dynamics can be simulated directly, i.e. without the explicit representation of firms. Moreover, (19) can be used to determine the equilibrium investment trigger $P^{*}$. Repeated stochastic simulations of equation (19) for various values of $P^{*}$ should reveal that the zero-profit condition will only be fulfilled if $P^{*}$ is equal to the equilibrium investment trigger. If $P^{*}$ is higher, the dynamics should allow for profits. If $P^{*}$ is smaller, this should imply losses. Accordingly, the equilibrium trigger price $P^{*}$ can be determined by minimizing the square of the expected profits, i.e.

$$
\begin{equation*}
\min _{P^{*}}\left\{E^{2}\left[\Pi\left(P^{*}\right)\right]=E^{2}\left[\sum_{l=0}^{\infty}\left(x_{l \cdot \Delta t, n} \cdot P_{l \cdot \Delta t}-M_{l \cdot \Delta t, n}\left(x_{l \cdot \Delta t, n}, P^{*}\right)\right)(1+r)^{-l \cdot \Delta t}\right]\right\} \tag{20}
\end{equation*}
$$

[^5]with $P_{0}=P^{*}$ and $P_{t}$ follows equation (15). ${ }^{9}$
Figure 2 shows that for identical trigger prices and identical $\alpha_{t}$, the agent-based model and the direct price simulation lead to an identical price path. Moreover, the direct price simulations lead to practically identical trigger prices. Hence direct price simulation allows to validate the results of the agent-based approach. Unfortunately, this approach is not as generally as the agent-based approach and cannot be applied directly to production chains in which farms interact on spot markets.

Figure 2: Price dynamics in the agent-based model and in the direct price simulation (identical trigger prices for all genomes)


### 3.2. Closed systems versus spot market interaction

Table 1 presents the trigger prices for investments under alternative assumptions. As a result the trigger prices of the closed systems correspond to the sum of the trigger prices of the spot market solution. Accordingly, one can conclude that stronger vertical integration does not increase investments and welfare. Moreover, vertical integration does not influence the production volume - even if farmers are risk avers. This is shown by Figure 3. For a given dynamics of demand for pork, the scenarios lead to identical price paths for pork.

Table 1: Trigger prices dependent on vertical integration and depreciation

|  | closed system | spot market |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  | piglet producer | pork producer | sum |
| risk neutrality | 2.362 | 1.018 | 1.345 | 2.363 |
| risik aversion ${ }^{1)}$ | 2.375 | 1.017 | 1.354 | 2.371 |

Considering the utility function $U=(a+X)^{1 / 2}$, with $a=2$ for piglet producers and $a=10$ for pork producers with $X$ as the present value of all cash flow streams achieved with the respective strategy in a simulation.

[^6]Figure 3: Price paths as results from alternative scenarios


This result contradicts the empirically based results of Pietola und Wang (2000). How to dissolve this contradiction? On the one hand the simulation experiments are based on several specific assumptions, e.g. identical useful lifetimes of barns for piglets and hogs by using a fixed depreciation rate of $5 \%$, rational expectations about the behavior of the market partners as well as on the assumed piglet market which is based on a price elasticity of -1 . On the other hand, the results are surprising. While real piglet prices show significant fluctuations, the piglet prices as represented in figure 3 are constant over wide phases. This can be explained by several assumptions of the model: the implicit synchronicity of the useful lifetime of the barns, the fixed depreciation rate, and the rational expectations hypothesis. These assumptions enable that the capacities of piglet production can be optimally adjusted to the hog finishing capacities.

A variation of the useful lifetime of the farrowing barns changes the price dynamics for piglets. Nevertheless, this has no significant effect on total pork production. According to table 2, variations of the depreciation rate for farrowing barns do not affect the sum of trigger prices for piglets and for pork. Higher depreciation rates for farrowing barns lower their trigger price, while equilibrium gross margins for finishing barns increase in the same amount. This is a consequence of the higher flexibility of the piglet production. Vice versa lower depreciation rates for farrowing barns leads to a higher volatility of the piglet prices and therefore to higher trigger prices. Simultaneous equilibrium gross margins of finishers can be reduced because finishers benefit from the farrowers' inflexibility. For high depreciation rates of farrowing barns the trigger prices are even smaller than 1 . The reason is that in the short run the piglet producers benefit from the small flexibility of hog finishers in the case of moderate demand declines for pork. In this case the trigger price forms a kind of lower reflecting barrier. Figure 4 and Figure 5 illustrate this effect by showing exemplary dynamics of prices for hogs and piglet for different depreciation rates for farrowing barns (i.e. ${ }^{p i} \lambda=10 \%$ and ${ }^{p i} \lambda=2.5 \%$ for ${ }^{h o} \lambda=5 \%$ ).

Table 2: Trigger prices dependent on different depreciation rates for farrowing barns ( ${ }^{p i} \lambda=5 \%$ )

|  | ${ }^{p i} \lambda=2.5 \%$ | ${ }^{p i} \lambda=5 \%$ | ${ }^{p i} \lambda=7.5 \%$ | ${ }^{p i} \lambda=10 \%$ |
| :--- | :---: | :---: | :---: | :---: |
| ${ }^{h o} P^{*}$ | 1.2555 | 1.3450 | 1.4013 | 1.4238 |
| ${ }^{p i} P^{*}$ | 1.1184 | 1.0180 | 0.9601 | 0.9393 |
| ${ }^{h o} P^{*}+{ }^{p i} P^{*}$ | 2.3739 | 2.3630 | 2.3614 | 2.3631 |

Figure 4: Price dynamics for ${ }^{h o} \lambda=5 \%$ and $^{p i} \lambda=10 \%$


Figure 5: Price dynamics for ${ }^{h o} \lambda=5 \%$ and $^{p i} \lambda=2.5 \%$


The considerations above show that certain assumptions have a strong effect on specific results, such as the investment triggers on the different production steps. However, the fundamental result that closed systems are not superior compared to market solutions is not affected, even if the assumptions are changed. Probably, the results would alter if one would assume some kind of bounded rationality, such as that farmers cannot observe the production
capacities of competitors and market partners in real time but with a certain time lag. However, this assumption would also affect stronger vertically integrated systems because one would need a kind of sectoral planning agency.

## 4. Summary and conclusions

Participants along a production chain which exchange intermediate products on spot markets face price risks, such as the transmission of price fluctuations of the final product. In a real options environment this uncertainty may cause investment reluctance on the different steps of the production chain. This paper analyzes whether a stronger vertical integration along the production chain allows to reduce the investment reluctance. Therefore, an agent-based competitive model of the pork production chain has been developed in which farmers use optimal investment strategies which are identified by genetic algorithms. Two production systems are compared: As example for a perfectly integrated system it is considered that every farmer can invest in closed systems in which piglets (the intermediate product) and finished hogs (the final product) are produced in equal amounts. In an alternative production system, farmers can either invest in farrowing (i.e. the production of piglets) or in hog finishing. The intermediate product, i.e. piglets, is traded on a spot market. Simulations show that the spot market solution and the closed system lead to the same production dynamics. The only precondition is that in the spot market system farrowers and hog finishers are aware of the investment strategies and the production capacities of the market partners. This general finding is even independent of different depreciation rates on the production steps, though the price dynamics for the intermediate product changes. Though this result is intuitively surprising, it is in accordance with several other insights of the real options theory such as that myopic investors which ignore impacts of competition behave efficient (Leahy 1993) and that real options theory does not justify political interventions such as price stabilization (Dixit and Pindyck 1993). However, as already mentioned, our findings are based on certain restrictive assumptions. Accordingly, the next steps of research will be to relax certain assumptions, such as the demand elasticity (respectively price flexibility) of -1 .

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## Appendix: Flow diagram of the agent-based simulation approach.




[^0]:    1 The use of the decay parameter $\lambda$ is analogous to the probabilistic approach presented in Dixit and Pindyck (1994, pp 200). To understand this, simply consider that any firm $n$ actually consists of an infinite number of identical infinitely small firms.

[^1]:    2 Note, that equation (2) implicitly assumes risk neutrality.

[^2]:    ${ }^{3}$ Notice, $n_{t}^{o}$ is zero if there is no investor in period $t$.

[^3]:    ${ }^{4}$ For simplification and without loss of generality, we abstract from additionally variable cost, i.e. costs for feed etc.
    5 The following representation of GA draws on Balmann and Happe (2001).
    ${ }^{6}$ For other GA-applications to real options cf. Balmann, Mußhoff and Odening (2001) and Diaz (2000).

[^4]:    ${ }^{7}$ A number of publications during the past 10 years show that agent-based GA approaches function quite well. Examples and discussions are given for instance in Arifovic (1994, 1996), Axelrod (1997), Balmann and Happe (2001), Dawid (1996), Dawid and Kopel (1998) and Chattoe (1998).

[^5]:    ${ }^{8}$ For a detailed analysis with particular regards to depreciation and demand elasticities cf. Odening et al. (2002).

[^6]:    9 This optimization problem can be solved by combining the required stochastic simulations with a GA.

