Monetary Fundamentals and Exchange Rate Dynamics Under Different Nominal Regimes*

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Abstract

We investigate the dynamic relationship between the US dollar exchange rate and its fundamentals across different exchange rate regimes using data going back to the late 1800s or early 1900s for six industrialized countries. For these countries there is evidence of a long-run relation between the nominal exchange rate and monetary fundamentals consistent with conventional theories of exchange rate determination. We employ a Markov-switching vector equilibrium correction model that allows for regime shifts in the entire set of parameters and the variance-covariance matrix. Our results suggest that the relative importance of exchange rates and fundamentals in restoring the long-run equilibrium level implied by the exchange rate-monetary fundamentals model varies significantly over time and is affected by the nominal exchange rate regime in operation.

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1 Introduction

A large literature has examined the relationship between the nominal exchange rate and the fundamentals suggested by conventional theories of exchange rate determination using data for the modern floating exchange rate period. In general, these studies have found little support for this relationship. Specifically, on the one hand, empirical work has often failed to find evidence of cointegration between the nominal exchange rate and monetary fundamentals - such as income differentials and money differentials (e.g. see Meese, 1986; Baillie and Selover, 1987; McNown and Wallace, 1989; Baillie and Pecchenino, 1991; Neely and Sarno, 2002). On the other hand, those studies that have found evidence of cointegration between the nominal exchange rate and monetary fundamentals provide little support for the theoretical restrictions predicted by the exchange rate-monetary fundamentals relationship (e.g. Cushman, 2000) or suggest that fundamentals are unable to predict more than a small amount of the variation in exchange rates (e.g. Mark, 1995; Kilian, 1999; Berkowitz and Giorgianni, 2001; Mark and Sul, 2001).

One reason for the inability to find evidence of a long-run relationship between the nominal exchange rate and monetary fundamentals is the low power of conventional statistical tests to reject a false null hypothesis of no cointegration with a sample span corresponding to the length of the recent float. Following the literature testing the validity of purchasing power parity (see Sarno and Taylor, 2002, and the references therein), two responses to the low power of these tests have been brought forth in the extant literature. First, some researchers have sought to increase test power by using panel cointegration tests applied jointly to a number of exchange rate series over the recent float. In some of these studies, the no-cointegration null hypothesis can be rejected for several countries, thereby providing support for the existence of a meaningful long-run relationship between exchange rates and monetary fundamentals (e.g. Groen, 2000; Mark and Sul, 2001).

Second, other researchers have sought to increase the power of their tests by increasing the length of the sample period under investigation. Rapach and Wohar (2002) take this alternative approach and apply a battery of unit root and cointegration tests to annual time series dating back to the late 19th century for 14 industrialized countries, reporting mixed results on the validity of the
exchange rate-monetary fundamentals relationship. Rapach and Wohar (2002) also investigate whether exchange rates or fundamentals adjust when there is a deviation away from long-run equilibrium. They find that departures from equilibrium may be restored via movements in the exchange rate or in fundamentals or both, suggesting that fundamentals may not be weakly exogenous with respect to the exchange rate. This evidence lends some support to the argument, put forth in a recent provocative paper by Engel and West (2002), that for countries and data where exchange rates and fundamentals appear to be linked by a long-run relationship, it may be the case that exchange rates help predict fundamentals, rather than the other way around.

In light of this evidence, there remain at least two important questions in this line of research. First, regardless of the relative success of some recent panel cointegration studies (e.g. Groen, 2000; Mark and Sul, 2001), it is controversial whether the exchange rate-monetary fundamentals relationship is validated by the data. Second, it is debatable whether, when the exchange rate-monetary fundamentals relationship is validated by the data, adjustment towards the long-run equilibrium level defined by this relationship is driven primarily by the exchange rate or by the fundamentals. The latter question is not only relevant empirically, given the lack of consensus in the literature on whether fundamentals and/or exchange rates are exogenous to each other, but also theoretically. Indeed, Engel and West (2002) show analytically that in a stylized rational expectations present value model, the exchange rate follows a near random walk if fundamentals are nonstationary and the discount factor is close to unity; under these conditions, therefore, the exchange rate is exogenous but an exchange rate-monetary fundamentals relationship may still exist where fundamentals bear the burden of adjustment towards long-run equilibrium.

Our contribution in this paper does not relate to the question as to whether or not there is a link between exchange rates and fundamentals. We take as given the mixed evidence on the existence of this link as well as the evidence presented by Rapach and Wohar (2002) that for some countries there is strong evidence in favor a long-run relationship between exchange rates and fundamentals over a long span of data. The questions we address are instead related to the second controversial

1 Specifically, they find strong support for France, Italy, the Netherlands, and Spain; moderate support for Belgium, Finland and Portugal; weaker support for Switzerland; and no support for Australia, Canada, Denmark, Norway, Sweden, and the UK.
issue in this literature and, hence, we ask whether, for countries for which there appears to be robust evidence in favor of a long-run comovement of exchange rates and fundamentals, the exchange rates and/or the fundamentals drive the adjustment towards long-run equilibrium. In particular, we investigate the relative importance of exchange rates and fundamentals in restoring equilibrium across different exchange rate regimes, including the gold standard, the Bretton-Woods period, the Exchange Rate Mechanism, and the recent float.

One concern with much literature in this context, and especially with studies employing long spans of data, is a potential problem of structural instability due to the several different monetary regimes that characterize the last century or so\(^2\). In this paper we extend the long-span data used by Rapach and Wohar (2002) and apply a general modelling methodology in which regime changes in the data generating process are explicitly allowed for. With over a century of data and different exchange rate arrangements over our sample period, we tentatively hypothesize that during floating exchange rate periods, the nominal exchange rate may be relatively more important in restoring departures from long-run equilibrium, while monetary fundamentals should restore long-run equilibrium during fixed exchange rate periods. The time-invariant, linear framework generally adopted by the earlier literature does not allow for changes in the equilibrium correction coefficients over time and hence it is not possible to test such a hypothesis in that framework.

A related literature, which we take seriously into account in this paper, is the one that has provided mounting evidence that the conditional distribution of nominal exchange rate changes is well described by a mixture of normal distributions and that a Markov-switching model may be a good characterization of exchange rate behavior (e.g. see Engel and Hamilton, 1990; LeBaron, 1992; Engel, 1994; Engel and Hakkio, 1996; Engel and Kim, 1999; Clarida, Sarno, Taylor and Valente, 2001). We investigate whether allowing for regime-switching in the underlying data-generating process for the exchange rate-monetary fundamentals model is an adequate characterization that is capable of capturing the impact of the different monetary regimes of the last century on the dynamics of exchange rates and fundamentals\(^3\). This is done through estimating a Markov-switching

\(^{2}\)Structural instability has been recorded in the exchange rate-monetary fundamentals relation by a number of authors also when using data for the recent float alone (e.g. see Wolff, 1987, 1988; Schinasi and Swamy, 1989; Canova, 1993; Rossi, 2001).

\(^{3}\)Other researchers have uncovered evidence of nonlinearities of various kinds in the monetary model of exchange
vector equilibrium correction model (MS-VECM) for the nominal exchange rate and a set of monetary fundamentals. Given the evidence of significant regime-switching behavior in exchange rate movements, this seems a natural extension of the relevant literature, and an updated version of the data of Rapach and Wohar (2002) is the obvious data set to examine.

We employ annual data dating back to the late 1800s or early 1900s for Belgium, Finland, France, Italy, Portugal and Switzerland. These are countries for which evidence of a long-run relationship between exchange rates and monetary fundamentals is very robust (Rapach and Wohar, 2002). We select a specification of the MS-VECM to characterize the dynamic relationship between exchange rates and fundamentals which allows for regime shifts in the intercept, the variance-covariance matrix, and the entire set of parameters (autoregressive components and equilibrium correction terms). We show that the conventional linear VECM often used in this literature is rejected when tested against the alternative of an MS-VECM. The results are, in general, supportive of our conjecture that during fixed exchange rate regimes fundamentals adjust to restore deviations from long-run equilibrium, while exchange rates bear most of the burden of adjustment during flexible exchange rate regimes. Our transition probabilities are also consistent with the general result that the relative importance of exchange rates and fundamentals in restoring the long-run equilibrium level of the exchange rate varies over time and is affected by the nominal exchange rate arrangement in operation. We find these results to hold up in all of the countries we consider.4

The remainder of the paper is as follows. Section 2 provides a brief outline of the theoretical background underlying the link between the nominal exchange rate and monetary fundamentals. In Section 3 we set out the econometrics of Markov-switching multivariate models as applied to nonstationary processes and cointegrated systems. In Section 4 we describe our data set, while in

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4 Other research somewhat related to the present study is the research focusing on the behavior of the real exchange rate over long spans of data (e.g. see Grilli and Kaminsky, 1991; Lothian and Taylor, 1996). A separate strand of research examines the time series properties of exchange rates and fundamentals across different exchange rate regimes. Notably, Flood and Rose (1995) find that fixed exchange rates are less volatile than floating rates, but the volatility of macroeconomic variables such as money and output does not change very much across exchange rate regimes. See also the work on structural instability of interest rate parity relationships by Granger and Siklos (1999) and the work on the relationship between prices and financial stability by Bordo, Dueker and Wheelock (2002).
the following section we report and discuss our empirical results. A final section concludes.

2 Exchange Rates and Monetary Fundamentals: A Brief Overview

A large literature has investigated the relationship between the nominal exchange rate and monetary fundamentals. This research focuses on the deviation, say $u$, of the nominal exchange rate from its fundamental value:

$$u_t = s_t - f_t,$$

where $s$ denotes the log-level of the nominal bilateral exchange rate (the domestic price of the foreign currency); $f$ is the long-run equilibrium of the nominal exchange rate determined by the monetary fundamentals; and $t$ is a time subscript.

The fundamentals term is given by:

$$f = (m_t - m^*_t) - \phi(q_t - q^*_t),$$

where $m$ and $q$ denote the log-levels of money supply and income respectively; $\phi$ is a constant; and asterisks denote foreign variables. Here $f$ may be thought of ‘as a generic representation of the long-run equilibrium exchange rate implied by modern theories of exchange rate determination’ (Mark and Sul, 2001, p. 32). For example, equation (2) is implied by the monetary approach to exchange rate determination (Frenkel, 1976; Mussa, 1976, 1979; Frenkel and Johnson, 1978) as well as by Lucas’ (1982) equilibrium model and by several ‘new open economy macroeconomic’ models (Obstfeld and Rogoff, 1995, 2001; Lane, 2001). Hence, the link between monetary fundamentals and the nominal exchange rate is consistent with both traditional models of exchange rate determination based on aggregate functions as well as with more recent microfounded open economy models.

While it has been difficult to establish the empirical significance of the link between monetary fundamentals and the exchange rate due to a number of cumbersome econometric problems (Mark, 1995; Kilian, 1999; Berkowitz and Giorgianni, 2001), some recent research suggests that the fundamentals described by equation (2) comove in the long run with the nominal exchange rate and
therefore determine its equilibrium level (Groen, 2000; Mark and Sul, 2001; Rapach and Wohar, 2002).

Equation (1) says that, if the departure from the exchange rate-monetary fundamentals relationship \(u_t\) is stationary, given \(s_t, f_t \sim I(1)\), the nominal exchange rate and the fundamentals exhibit a common stochastic trend and are cointegrated with cointegrating vector \([1, -1]\). Then, by the Granger Representation Theorem (Engle and Granger, 1987), the nominal exchange rate and the fundamentals must possess a VECM representation in which \(u_t\) plays the part of the equilibrium error. We investigate this framework and use exactly a linear VECM representation to shed light on the relative importance of the nominal exchange rate and the fundamentals in restoring equilibrium in the long-run relationship linking exchange rates and fundamentals across different exchange rate regimes since the 19th century. Indeed, we use a generalization of a standard linear VECM which is capable of allowing all of the VECM parameters to change over time and to identify the various regimes that characterize the long sample periods examined in this study.

3 Markov-switching equilibrium correction

In this section we outline the econometric procedure employed in order to model regime shifts in the dynamic relationship between the nominal exchange rate and monetary fundamentals. The procedure essentially extends Hamilton’s (1988, 1989) Markov-switching regime framework to non-stationary systems, allowing us to apply it to cointegrated vector autoregressive (VAR) and VECM systems (see Krolzig, 1997, 1999).

Consider the following \(M\)-regime \(p\)-th order Markov-switching vector autoregression (MS(\(M\))-VAR(\(p\))) which allows for regime shifts in the intercept term:

\[
y_t = \nu(z_t) + \sum_{i=1}^{p} \Pi_i y_{t-i} + \varepsilon_t, \tag{3}
\]

where \(y_t\) is a \(K\)-dimensional observed time series vector, \(y_t = [y_{1t}, y_{2t}, \ldots, y_{Kt}]'\); \(\nu(z_t)\) is a \(K\)-dimensional column vector of regime-dependent intercept terms, \(\nu(z_t) = [\nu_1(z_t), \nu_2(z_t), \ldots, \nu_K(z_t)]'\); the \(\Pi_i\)’s are \(K \times K\) matrices of parameters; \(\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Kt}]'\) is a \(K\)-dimensional vector
of Gaussian white noise processes with covariance matrix $\Sigma$, $\varepsilon_t \sim \text{NID}(0, \Sigma)$. The regime-generating process is assumed to be an ergodic Markov chain with a finite number of states $z_t \in \{1, \ldots, M\}$ governed by the transition probabilities $p_{ij} = \Pr(z_{t+1} = j \mid z_t = i)$, and $\sum_{j=1}^{M} p_{ij} = 1 \forall i, j \in \{1, \ldots, M\}$.

A standard case in economics and finance is that $y_t$ is nonstationary but first-difference stationary, i.e. $y_t \sim I(1)$. Then, given $y_t \sim I(1)$, there may be up to $K - 1$ linearly independent cointegrating relationships, which represent the long-run equilibrium of the system, and the equilibrium error (the deviation from the long-run equilibrium) is measured by the stationary stochastic process $u_t = \alpha' y_t - \beta$ (Granger, 1986; Engle and Granger, 1987). If indeed there is cointegration, the cointegrated MS-VAR (3) implies a Markov-switching vector equilibrium correction model or MS-VECM of the form:

$$\Delta y_t = \nu(z_t) + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \Pi y_{t-1} + \varepsilon_t,$$

where $\Gamma_i = -\sum_{j=i+1}^{p} \Pi_j$ are matrices of parameters, and $\Pi = \sum_{i=1}^{p} \Pi_i - I$ is the long-run impact matrix whose rank $r$ determines the number of cointegrating vectors (e.g. Johansen, 1995; Krolzig, 1999).

Although, for expositional purposes, we have outlined the MS-VECM framework for the case of regime shifts in the intercept alone, shifts may be allowed for elsewhere. The present application focuses on a multivariate model comprising, for each of the countries analyzed, the nominal exchange rate and the monetary fundamentals (hence $y_t = [s_t, f_t]'$, where $f_t = [(m_t - m^*_t) - (q_t - q^*_t)]$), for which, following the reasoning of Section 2, a unique cointegrating relationship, represented by $(s_t - f_t)$, should exist. As discussed in Section 5 below, in our empirical work, after considerable experimentation, we selected a specification of the MS-VECM which allows for regime shifts in the intercept, the variance-covariance matrix and the whole set of parameters (autoregressive component, $\Gamma_i$ and the speed of adjustment terms in the cointegration matrix, $\Pi$). This model, the Markov-Switching-Intercept-Autoregressive-Heteroskedastic-VECM or MSIAH-VECM, may be written as follows:

$$\Delta y_t = \nu(z_t) + \sum_{i=1}^{p-1} \Gamma_i (z_t) \Delta y_{t-i} + \Pi (z_t) y_{t-1} + u_t,$$
where $\Pi(z_t) = \alpha(z_t) \beta', u_t \sim NIID(0, \Sigma(z_t))$ and $z_t \in \{1, \ldots, M\}$.

An MS-VECM can be estimated using a two-stage maximum likelihood procedure. The first stage of this procedure essentially consists of the implementation of the Johansen (1988, 1991) maximum likelihood cointegration procedure in order to test for the number of cointegrating relationships in the system and to estimate the cointegration matrix. In fact, in the first stage use of the conventional Johansen procedure is legitimate without modelling the Markovian regime shifts explicitly (see Saikkonen, 1992; Saikkonen and Luukkonen, 1997). The second stage then consists of the implementation of an expectation-maximization (EM) algorithm for maximum likelihood estimation which yields estimates of the remaining parameters of the model (Dempster, Laird and Rubin, 1977; Hamilton, 1993; Kim and Nelson, 1999; Krolzig, 1999).\footnote{In the present application, however, the first stage is not carried out since we impose the theoretical restriction that $\phi = 1$ in equation (2), which has been shown to be valid on the data set employed here (Rapach and Wohar, 2002). This is why in equation (5) we define $\Pi(z_t) = \alpha(z_t) \beta'$, with $\beta'$ restricted consistent with the unity restrictions and regime independent.}

We now turn to a brief discussion of our data set.

4 Data

The data set used in this study comprises annual observations for the nominal exchange rate (domestic price of foreign currency), the money supply relative to the US, and real gross domestic product (GDP) relative to the US for six countries: Belgium, Finland, France, Italy, Portugal and Switzerland. The time series used are obtained from updating the data set used by Rapach and Wohar (2002) using data taken from the International Financial Statistics of the International Monetary Fund.\footnote{In turn, the nominal exchange rate series are from Taylor (2002), and the money supply and real GDP series are from Bordo and Jonung (1998) and Bordo, Bergman, and Jonung (1998).} The countries considered in this paper are the ones for which Rapach and Wohar find evidence supporting the exchange rate-monetary fundamentals relationship and for which the $[1, -1]$ restrictions implied by the theoretical framework discussed in Section 2 are not rejected.\footnote{Although for Spain the $[1, -1]$ restriction could not be rejected, we do not report results from Spain below since we experienced problems in achieving convergence when estimating a Markov-switching model.} From these data we could construct both $s_t$ and $f_t$, as defined in equation (2), and hence the equilibrium error $u_t$, as defined in equation (1).
The data run from the late 19th or early 20th century to the late 20th century and thus cover a variety of international monetary arrangements, including the classical gold standard, the Bretton Woods era, the modern float and, for some countries, the Exchange Rate Mechanism of the European Monetary System. The start date of the sample, dictated by data availability is, 1880 for Belgium, France, Italy and Switzerland, 1911 for Finland, and 1890 for Portugal. Except for Switzerland, whose sample period ends in 2000, the end date of the sample is 1998 for all countries since these countries joined the European Monetary Union on 1 January 1999, when the euro replaced their national currencies.

We now turn to our empirical analysis.

5 Empirical results

In this section we report and discuss our empirical results. We begin by reporting the results from a battery of tests designed to test a linear VECM for the exchange rate and the fundamentals against a Markov-switching model and to identify the appropriate number of regimes to be allowed for. We then proceed to estimating an MS-VECM for each exchange rate considered. Finally, we discuss our empirical results in light of the historical chronology of exchange rate regimes of the last century, providing evidence on the relative importance of exchange rates and fundamentals in restoring the long-run equilibrium implied by the exchange rate-monetary fundamentals relationship across different regimes.

5.1 Regime identification

We begin by estimating a standard linear VECM using full-information maximum likelihood (FIML) methods:

\[ \Delta y_t = \nu + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \Pi y_{t-1} + u_t, \]  

where \( y_t = [s_t, f_t]' \), assuming a maximum lag length \( p = 5 \), as suggested by both the Akaike Information Criterion (AIC) and the Schwartz Information Criterion (SIC). Employing the conventional general-to-specific procedure, we obtained fairly parsimonious models for each exchange
rate, and found no evidence of significant residual serial correlation\(^8\). We then proceed to investigating the presence of nonlinearities through the estimation of a fairly general Markov-switching model of the form:

\[
\Delta y_t - \delta (z_t) = \alpha (z_t) [\beta' y_{t-1} - \mu (z_t)] + \sum_{i=1}^{p-1} \Gamma_i (z_t) [\Delta y_{t-i} - \delta (z_t)] + \omega_t, \tag{7}
\]

where \(y_t = [s_t, f_t]'\), \(\delta (z_t)\) is the regime-dependent vector of means of the short-run dynamics, \(\mu (z_t)\) is the regime-dependent mean of the long-run equilibrium relationships, \(\omega_t \sim N(0, \Sigma(z_t))\), and \(z_t = 1, 2, 3\).

We applied the conventional ‘bottom-up’ procedure designed to detect Markovian shifts in order to select the most adequate characterization of an \(M\)-regime \(p\)-th order MS-VECM for \(\Delta y_t\).\(^9\) The VARMA representations of the time series (Poskitt and Chung, 1996) suggested in each case that the number of regimes was in the range between two and three.

However, for each MS-VECM estimated, the implicit assumption that the regime shifts affect only one of the intercept term, the variance-covariance matrix and the autoregressive component of the VECM was found to be inappropriate. In fact, we tested formally the significance of all these components by using likelihood ratio (LR) tests of the type suggested by Krolzig (1997, p. 135-6). The results, reported in the first three columns of Table 1 (\(LR_1\), \(LR_2\) and \(LR_3\)), indicate strong rejections of the null of no regime dependence, clearly suggesting that an MS-VECM that allows for shifts in the intercept, the variance-covariance matrix and the autoregressive component, namely an MSIAH-VECM, is the most appropriate model within its class in the present application.

Next, in the same spirit of the tests previously executed, we carried out another LR test in order to select the most parsimonious VECM characterizing the dynamic relationship between nominal exchange rates and monetary fundamentals. In particular, we tested the null of MSIAH-VECM(1) against the alternative of MSIAH-VECM\((p)\) and, as shown by the results in the fourth column

\(^8\)Full details on these estimation results are available from the authors upon request.
\(^9\)Essentially, the bottom-up procedure consists of starting with a simple but statistically reliable Markov-switching model by restricting the effects of regime shifts on a limited number of parameters and testing the model against alternatives. In such a procedure, most of the structure contained in the data is not attributed to regime shifts, but explained by observable variables, consistent with the general-to-specific approach to econometric modelling. For a detailed discussion of the bottom-up procedure, see Krolzig (1997).
of Table 1 (LR4), for all countries examined, we were not able to reject this null hypothesis at standard significance levels.

In order to discriminate between models allowing for two regimes against models governed by three regimes we also executed a further LR test. It is well known that the presence of nuisance parameters creates special problems in formally identifying the number of regimes as the scores associated with the parameters of interest under the alternative hypothesis may be zero under the null. In order to avoid this problem several testing procedures have been proposed\(^\text{10}\). The results reported in the fifth column of Table 1 (LR5) show large test statistics and the corresponding \(p\)-values, calculated as in Ang and Bekaert (1998), suggest that three regimes may be appropriate in all cases, with the exception of Belgium and Switzerland where two regimes are sufficient to describe the dynamics of the nominal exchange rate and monetary fundamentals. Finally, the linearity tests, reported in the last column of Table 1 (LR6), indicate in each case the rejection of the linear VECM in favor of a nonlinear, Markov-switching alternative model.

5.2 MS-VECM estimation results

The regime identification procedure carried out in the previous subsection suggests, for all countries examined, an MS-VECM governed by either two or three regimes. This model may be written as follows:

\[
\Delta y_t = \nu(z_t) + \Pi(z_t) y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i(z_t) \Delta y_{t-i} + \omega_t, \tag{8}
\]

where \(\Pi(z_t) = \alpha(z_t) \beta', \omega_t \sim NIID(0, \Sigma(z_t))\) and \(z_t = 1, 2, 3\). We estimated the MSIAH-VECM in equation (8) using an EM algorithm for maximum likelihood (Dempster, Laird and Rubin, 1977), for each of the countries under investigation. The estimation yields fairly plausible estimates of

\[^{10}\text{In particular, the regularity conditions under which the Davies (1977, 1987) test is valid are violated, since the Markov model has both a problem of nuisance parameters and a problem of ‘zero score’ under the null hypothesis. Moreover, even if the Davies bound is appropriate, it is possible that it will only be valid if the null model is a linear model with iid errors; in the present case, it is difficult to believe that this condition is met since exchange rate innovations are not homoskedastic, which would induce some distortion. Therefore, the distribution of the LR test is likely to differ from the adjusted \(\chi^2\) distribution proposed by Davies (1977, 1987). For extensive discussions of the problems related to LR testing in this context, see Hansen (1992, 1996) and García (1998). We are thankful to Bruce Hansen for clarifying several econometric issues related to LR testing in the present context.}\]
the coefficients for the VECMs estimated, including the regime-dependent adjustment coefficients in $\alpha(z_t)$.\textsuperscript{11} \textsuperscript{12}

In order to assess the goodness of fit of the MSIAH-VECM, we calculated the ratio of the $R^2$, the residual variance (RV), the AIC and the SIC from each estimated MSIAH-VECM to the corresponding measure for its best linear VECM counterpart. The results in Table 2 show that, for each country examined, the estimated MSIAH-VECM outperforms the best alternative linear model, leading to a substantial reduction of the residual variance - up to about 42% for Italy and Portugal - and a marked improvement of the $\overline{R^2}$ - for all countries higher than 13%. This performance is not due to an increased number of parameters since both the AIC and SIC ratios suggest a better fit for the estimated MSIAH-VECM compared to the linear VECM.

5.3 An historical interpretation of the regimes

Table 3 reports the regime-dependent equilibrium correction coefficients. For Belgium, in Regime 1, the equilibrium correction coefficient is significant in the exchange rate equation but not in the fundamentals equation. This implies that, in Regime 1, the exchange rate adjusts to restore deviations from long-run equilibrium. This result is consistent with Figure 1, which plots the smoothed transition probabilities from estimating the MSIAH-VECM for Belgium: we see that up until about the beginning of the interwar period, the probability of being in Regime 2 is near or equal to unity. It is not surprising to find a large number of switches in transition probabilities during the interwar period as exchange rate arrangements were not stable during this period.

For Regime 2, we find that both equilibrium correction coefficients are significant, indicating that

\textsuperscript{11}We do not report all of our MS-VECM estimation results to conserve space. We do report, however, the estimated equilibrium correction coefficients for each equation of the MS-VECMs in Table 3, as discussed below. We also report in the appendix the full MS-VECM estimation output for one representative country.

\textsuperscript{12}We also looked at graphs of the standardized residuals, the smoothed residuals and the one-step prediction errors from each estimated MSIAH-VECM. The difference is concerned with the weighting of the residuals. Loosely speaking, the smoothed residuals are the closest to the sample residuals from a linear regression model; however, they overestimate the explanatory power of the Markov-switching model due to the use of the full-sample information covered in the smoothed regime vector. The standardized residuals are conditional residuals. The one-step prediction errors are based on the predicted regime probabilities. Unfortunately, many conventional diagnostic tests, such as standard residual serial correlation tests, may not have their conventional asymptotic distribution when the residuals come from Markov-switching models and are therefore not reported here. However, the graphs of standardized residuals, the smoothed residuals and the one-step prediction errors provided no visual evidence of residual serial correlation in any of the residuals series plotted.
both the exchange rate and monetary fundamentals adjust to restore long-run equilibrium. The transition probabilities indicate that during the fixed exchange rate period of Bretton Woods, the probability of being in Regime 2 is near or equal to unity. The probability of being in Regime 1 during the post-Bretton Woods floating rate period is close to unity for the post-1979 period.

For Finland we find that neither equilibrium correction coefficient is significant in Regime 1, implying that exchange rates and monetary fundamentals do not comove in this regime. However, in Regime 2 we find that it is the monetary fundamentals that adjust (as only the equilibrium correction coefficient in the fundamentals equation is significant). In contrast, we find that in Regime 3, only the exchange rate adjusts to restore deviations from long-run equilibrium (as the equilibrium correction coefficient in the exchange rate equation is significant while the equilibrium correction coefficient in the fundamentals equation is insignificant). This is consistent with the transition probabilities for Finland shown in Figure 2. The probability of being in Regime 2 is almost always unity during the Bretton Woods fixed exchange rate period, while the probability of being in Regime 3 is virtually unity during the post-Bretton Woods era of floating exchange rates.

For France we find that both the exchange rate and monetary fundamentals adjust to deviations from long-run equilibrium in each of Regimes 1 and 2. However, in Regime 3, only monetary fundamentals adjust to restore deviations from long-run equilibrium. Figure 3 plots the corresponding transition probabilities. We find that up until the interwar period the probability of being in Regime 3 is near or equal to unity. As would be expected, the transition probabilities switch often during the interwar period, while the probability of being in Regime 3 during the Bretton Woods fixed exchange rate period is near or equal to unity for most of the period. The probability of being in Regime 2 for the post-Bretton Woods floating rate system is near or equal to unity for most of this period.

For Italy, we find that in Regimes 1 and 2 only the monetary fundamentals adjust to restore deviations from long-run equilibrium. This is what one would expect during fixed exchange rate periods, and is corroborated in Figure 4 where we see that the transition probability of being in Regime 2 during the fixed exchange rate Bretton Woods period is very high for most of the period. The probability of being in Regime 2 for the period up until the interwar period is also close or
near unity. As expected, the transition probabilities for the interwar period again exhibit a large
number of switches. The probability of being in Regime 3 during the post-Bretton Woods flexible
rate period is very high, consistent with the results in Table 3 where we find that only the exchange
rate adjusts to restore deviations from long-run equilibrium in this regime.

For Portugal we find that in Regime 1, neither the exchange rate nor the monetary fundamentals
adjust to restore long-run equilibrium (neither equilibrium correction coefficient is significant). In
Regime 2, the equilibrium correction coefficient in the exchange rate equation is significant while the
equilibrium correction coefficient in the monetary fundamentals equation is insignificant, implying
that the exchange rate adjusts to restore deviations from long-run equilibrium. This is consistent
with the transition probabilities plotted in Figure 5. We see that, with the exception of an outlier,
the probability of being in Regime 2 during the flexible exchange rate period (post-1973) is near
or close to unity. We also find that the probability of being in Regime 3 during the fixed exchange
rate Bretton Woods period is near or equal to unity. This is consistent with the results in Table
3 that show that in Regime 3 only the monetary fundamentals adjust to deviations from long-run
equilibrium.

For Switzerland we find that in Regime 1 only the monetary fundamentals adjust to devia-
tions from long-run equilibrium (as only the equilibrium correction coefficient in the fundamentals
equation is significant). In Regime 2, only the equilibrium correction term in the exchange rate
equation is significant, indicating that exchange rates adjust to restore long-run equilibrium in this
regime. This is consistent with the transition probabilities reported in Figure 6. We see that
the probability of being in Regime 1 during the period up until the interwar period and during
the Bretton Woods period (both fixed exchange rate periods) is near or equal to unity. Also,
the probability of being in Regime 2 during the floating rate period following the collapse of the
Bretton Woods system is close to or equal to unity for most of that period. As was the case for the
other countries, we find that the transition probabilities during the interwar period switch quite
often.

As a final exercise we calculate a measure designed to assess the performance of our Markov-
switching models in identifying the regimes over the sample. This regime classification measure
(RCM), recently proposed by Ang and Bekaert (2002), exploits the simple fact that the ex-post (smoothed) probabilities of observing one of the regimes ought to be close to unity at all times when regime classification is perfect. Weak regime inference implies that Markov-switching models cannot successfully distinguish among regimes from the behavior of the data and may indicate misspecification. An ideal Markov-switching model should classify sharply so that the ex-post probability to be in one specific regime is either close to zero or unity. In poorly specified models the ex-post probability to be in a specific regime may be close to \(1/M\), with \(M\) denoting the number of regimes considered. Hence, a regime classification measure may be calculated as

\[
RCM(M) = 100M^M \frac{1}{T} \sum_{t=1}^{T} \left( \prod_{j=1}^{M} p_{j,t} \right),
\]

(9)

where \(T\) is the number of observations, \(p_{j,t}\) is the smoothed probability to be in regime \(j = 1, \ldots, M\) at time \(t\), and \(RCM\) is defined between zero and one hundred. A satisfactory regime classification is associated with low \(RCM\) statistics: a value of zero, or close to zero, implies perfect regime classification, while a value of one hundred implies that virtually no information about the regime is revealed by the model.\(^{13}\)

We calculated the \(RCM\) statistic for each MSIAH-VECM estimated in Section 5.2. The \(RCM\) statistics, reported in Table 4, were calculated over the full sample and over the three different sub-samples: up to 1944, covering the classical gold standard and the world wars; 1945-1972, covering the Bretton Woods era; and 1973-2000, covering the modern float. The results in Table 4 suggest that, for all countries and for the different sub-periods considered, the values of the \(RCM\) statistics are close to zero, denoting a very satisfactory regime classification. This evidence is consistent with the visual analysis of Figures 1-6, which report the estimated transition probabilities.

### 5.4 Summing up the empirical results

Overall, the results tell us a coherent story about the dynamics of exchange rates and its fundamentals over time. It is rare in the use of MS-VECMs that one finds such clear evidence of

\(^{13}\)Because the true variable governing the regimes is a Bernoulli random variable, the \(RCM\) statistic is essentially a sample estimate of its variance.
being in one particular regime or the other. Our results from the MS-VECM estimation and the plots of the transition probabilities provide strong evidence that adjustment towards long-run equilibrium takes different forms depending on whether one is in a fixed or floating exchange rate regime. Specifically, during fixed exchange rate periods, fundamentals bear the burden of adjustment towards the equilibrium relationship linking exchange rates and fundamentals, whereas, in periods of free float, adjustment to equilibrium occurs primarily, if not fully, via movements in the nominal exchange rate.

6 Conclusion

The linkage between nominal exchange rates and monetary fundamentals (such as money differentials and income differentials), predicted by a vast body of theories of exchange rate determination, has been extensively analyzed in the international finance literature. When employing data over the modern floating rate period, the extant literature has found little evidence supporting the existence of this linkage. Recently, some research, employing data spanning over one hundred years, has found evidence supporting the existence of a long-run relationship between exchange rates and fundamentals for a number of countries. However, the use of such long-spans of data raises concerns related to parameter stability. Long spans make it likely that different adjustment mechanisms may be at work over time within a country. In particular, the adjustment back to long-run equilibrium may take place primarily through the nominal exchange rate during floating rate periods but can only occur through fundamentals during non-floating exchange rate periods.

This paper studies in detail those countries for which there appears to be robust evidence in favor of a linkage between the nominal exchange rate and monetary fundamentals over a long span of data. In particular, we shed light on the relative importance of the exchange rate and the fundamentals in restoring the long-run equilibrium implied by the exchange rate-monetary fundamentals relationship, and identify different regimes which appear to be consistent with the historical chronology of monetary regimes characterizing the last century or so. We do this by employing a Markov-switching vector equilibrium correction model for the nominal exchange rate and a set of monetary fundamentals. Given the mounting evidence of regime switching in exchange
rate dynamics this is a natural extension of the extant literature. Our model is fairly general and allows for regime shifts in the intercept, the entire set of parameters (including autoregressive terms and the speed of adjustment parameters), as well as the variance-covariance matrix. Consistent with some previous research, conventional linear vector equilibrium correction models, often used in the extant literature, are rejected when tested against the alternative of a Markov-switching vector equilibrium correction model.

We obtained these results on annual data going back to the late 1800s or early 1900s for Belgium, Finland, France, Italy, Portugal, and Switzerland. Evidence has been found to support the monetary model for these countries in early studies. Our specification of the model finds that three regimes are appropriate for Finland, France, Italy, and Portugal, while two regimes are appropriate for Belgium and Switzerland. We find that, consistent with our conjecture, during fixed exchange rate regimes fundamentals adjust to restore deviations from long-run equilibrium. In contrast, during flexible exchange rate regimes it is primarily or solely the nominal exchange rate that adjusts to restore deviations from long-run equilibrium. Our transition probabilities are also consistent with these general results.

Overall, the evidence in this study suggests that, if using long spans of data, researchers need to be cautious when making conclusions about parameter stability and which variables adjust to restore long-run equilibrium within cointegrated systems. In particular, in the present context, our results suggest that over long spans of data it is hard to discriminate whether movements in exchange rates are determined by fundamentals or vice versa since the direction of causality between exchange rates and fundamentals appears both to vary over time and to be affected by the nominal exchange rate regime in operation.
Table 1. ‘Bottom-up’ identification procedure

<table>
<thead>
<tr>
<th>Country</th>
<th>LR1</th>
<th>LR2</th>
<th>LR3</th>
<th>LR4</th>
<th>LR5</th>
<th>LR6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>$2.98 \times 10^{-30}$</td>
<td>$3.95 \times 10^{-5}$</td>
<td>$1.04 \times 10^{-12}$</td>
<td>$1.97 \times 10^{-1}$</td>
<td>$4.20 \times 10^{-1}$</td>
<td>$8.13 \times 10^{-37}$</td>
</tr>
<tr>
<td>Finland</td>
<td>$7.20 \times 10^{-34}$</td>
<td>$1.40 \times 10^{-6}$</td>
<td>$2.79 \times 10^{-5}$</td>
<td>$3.43 \times 10^{-1}$</td>
<td>$9.56 \times 10^{-8}$</td>
<td>$5.18 \times 10^{-34}$</td>
</tr>
<tr>
<td>France</td>
<td>$1.79 \times 10^{-59}$</td>
<td>$1.14 \times 10^{-5}$</td>
<td>$5.27 \times 10^{-4}$</td>
<td>$1.32 \times 10^{-1}$</td>
<td>$1.72 \times 10^{-3}$</td>
<td>$1.08 \times 10^{-58}$</td>
</tr>
<tr>
<td>Italy</td>
<td>$8.41 \times 10^{-32}$</td>
<td>$7.96 \times 10^{-3}$</td>
<td>$2.50 \times 10^{-4}$</td>
<td>$1.38 \times 10^{-1}$</td>
<td>$5.65 \times 10^{-21}$</td>
<td>$1.57 \times 10^{-72}$</td>
</tr>
<tr>
<td>Portugal</td>
<td>$5.11 \times 10^{-5}$</td>
<td>$2.83 \times 10^{-17}$</td>
<td>$6.60 \times 10^{-4}$</td>
<td>$2.95 \times 10^{-1}$</td>
<td>$3.13 \times 10^{-6}$</td>
<td>$6.88 \times 10^{-20}$</td>
</tr>
<tr>
<td>Switzerland</td>
<td>$6.03 \times 10^{-29}$</td>
<td>$3.70 \times 10^{-23}$</td>
<td>$1.63 \times 10^{-3}$</td>
<td>$1.90 \times 10^{-1}$</td>
<td>$3.08 \times 10^{-1}$</td>
<td>$1.11 \times 10^{-31}$</td>
</tr>
</tbody>
</table>

**Notes:**  
LR1 is a test statistic of the null hypothesis of no regime dependent variance-covariance matrix (i.e. MSIAH(3)-VECM(p) versus MSIAH(3)-VECM(p)). LR2 is a test statistic of the null hypothesis of no regime dependent intercept (i.e. MSAH(3)-VECM(p) versus MSIAH(3)-VECM(p)). LR3 is a test statistic of the null hypothesis of no regime dependent autoregressive component (i.e. MSIH(3)-VECM(p) versus MSIAH(3)-VECM(p)). LR4 tests the null hypothesis that the model having autoregressive component of order one is equivalent to another with a higher autoregressive order (i.e. MS(3)-VECM(1) versus MS(3)-VECM(p)). LR1, LR2, LR3, LR4 are constructed as $2(\ln L^* - \ln L)$, where $L^*$ and $L$ represent the unconstrained and the constrained maximum likelihood respectively. Those tests are distributed as $\chi^2(g)$ where $g$ is the number of restrictions imposed. LR5 is the likelihood ratio test for the null hypothesis that the model with two regimes is equivalent to the model with three regimes. LR6 is a linearity test for null hypothesis that the linear VECM is equivalent to the selected MS-VECM. $p$-values relative to LR5 and LR6 tests are calculated as in Ang and Bekaert (1998). For all test statistics only $p$-values are reported.
Table 2. Relative goodness of fit

<table>
<thead>
<tr>
<th>Country</th>
<th>$\Delta s_t$</th>
<th>RV ratio</th>
<th>AIC ratio</th>
<th>SIC ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium (1880-1998)</td>
<td>1.1476</td>
<td>0.9334</td>
<td>1.1949</td>
<td>1.1216</td>
</tr>
<tr>
<td>Finland (1911-1998)</td>
<td>1.4712</td>
<td>0.8244</td>
<td>1.7093</td>
<td>1.5049</td>
</tr>
<tr>
<td>France (1880-1998)</td>
<td>1.8459</td>
<td>0.7360</td>
<td>1.8135</td>
<td>1.6670</td>
</tr>
<tr>
<td>Italy (1880-1998)</td>
<td>1.8068</td>
<td>0.7436</td>
<td>1.8261</td>
<td>1.7054</td>
</tr>
<tr>
<td>Portugal (1890-1998)</td>
<td>1.5219</td>
<td>0.6297</td>
<td>1.3096</td>
<td>1.1144</td>
</tr>
<tr>
<td>Switzerland (1880-2000)</td>
<td>1.1576</td>
<td>0.9294</td>
<td>1.3324</td>
<td>1.2834</td>
</tr>
</tbody>
</table>

Notes: $R^2$ ratio, RV ratio, AIC ratio and SIC ratio are the ratios of the $R^2$, the residual variance, the Akaike Information Criterion and the Schwartz Information Criterion respectively from each country’s preferred MSIAH($M$)-VECM($p$) model (as selected in Table 1) to the corresponding goodness-of-fit measure for the best alternative linear VECM. The AIC and SIC reported were calculated for the whole (linear or nonlinear) VECM systems.
Table 3. Regime-dependent equilibrium-correction coefficient estimates

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_{\Delta s_t}(z = 1)$</th>
<th>$\alpha_{\Delta s_t}(z = 2)$</th>
<th>$\alpha_{\Delta s_t}(z = 3)$</th>
<th>$\alpha_{\Delta f_t}(z = 1)$</th>
<th>$\alpha_{\Delta f_t}(z = 2)$</th>
<th>$\alpha_{\Delta f_t}(z = 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Belgium</strong> (1880-1998)</td>
<td>-0.1617</td>
<td>0.0343</td>
<td></td>
<td>0.0377</td>
<td>0.1404</td>
<td>0.0000 (*)</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.01)</td>
<td></td>
<td>(0.06)</td>
<td>(0.03)</td>
<td>0.0000 (*)</td>
</tr>
<tr>
<td><strong>Finland</strong> (1911-1997)</td>
<td>-0.2193</td>
<td>-0.0011</td>
<td>-0.1221</td>
<td>0.0057</td>
<td>0.0775</td>
<td>0.0496 (*)</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.008)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.03)</td>
<td>(0.04) (*)</td>
</tr>
<tr>
<td><strong>France</strong> (1880-1998)</td>
<td>-0.1797</td>
<td>-0.2704</td>
<td>-0.0015</td>
<td>0.1071</td>
<td>0.1199</td>
<td>0.2233 (*)</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.002)</td>
<td>(0.05)</td>
<td>(0.03)</td>
<td>(0.06) (*)</td>
</tr>
<tr>
<td><strong>Italy</strong> (1880-1998)</td>
<td>-0.4101</td>
<td>-0.0016</td>
<td>-0.1270</td>
<td>0.3275</td>
<td>0.0836</td>
<td>0.0233 (*)</td>
</tr>
<tr>
<td></td>
<td>(0.27)</td>
<td>(0.003)</td>
<td>(0.03)</td>
<td>(0.13)</td>
<td>(0.03)</td>
<td>(0.03) (*)</td>
</tr>
<tr>
<td><strong>Portugal</strong> (1890-1998)</td>
<td>-0.0893</td>
<td>-0.0790</td>
<td>-0.0075</td>
<td>0.0545</td>
<td>0.0328</td>
<td>0.0679 (*)</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.02) (*)</td>
</tr>
<tr>
<td><strong>Switzerland</strong> (1880-2000)</td>
<td>-0.0027</td>
<td>-0.1818</td>
<td></td>
<td>0.1414</td>
<td>0.0068</td>
<td>0.0000 (*)</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.05)</td>
<td></td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>0.0000 (*)</td>
</tr>
</tbody>
</table>

**Notes:** $\alpha_{\Delta s_t}(z = i)$ for $i = 1, 2, 3$ denotes the estimated equilibrium correction coefficient in the exchange rate equation conditional on regime $i$. $\alpha_{\Delta f_t}(z = i)$ for $i = 1, 2, 3$ denotes the estimated equilibrium correction coefficient in the monetary fundamentals equation conditional on regime $i$. Figures in parentheses are asymptotic standard errors.
Table 4. Regime classification measure

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>1.54×10⁻¹</td>
<td>1.91×10⁻¹</td>
<td>4.63×10⁻¹</td>
<td>1.82×10⁻¹</td>
</tr>
<tr>
<td>Finland</td>
<td>2.62×10⁻²</td>
<td>1.45×10⁻⁴</td>
<td>7.92×10⁻²</td>
<td>2.87×10⁻⁶</td>
</tr>
<tr>
<td>France</td>
<td>4.90×10⁻¹</td>
<td>3.47×10⁻²</td>
<td>4.10×10⁻¹</td>
<td>1.75×10⁻¹</td>
</tr>
<tr>
<td>Italy</td>
<td>4.24×10⁻¹</td>
<td>2.59×10⁻¹</td>
<td>4.83×10⁻¹</td>
<td>7.82×10⁻¹</td>
</tr>
<tr>
<td>Portugal</td>
<td>9.46×10⁻⁴</td>
<td>4.53×10⁻⁵</td>
<td>3.45×10⁻³</td>
<td>8.06×10⁻⁵</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.05×10⁻¹</td>
<td>1.60×10⁻¹</td>
<td>1.07×10⁻¹</td>
<td>7.35×10⁻¹</td>
</tr>
</tbody>
</table>

Notes: The statistics are calculated as $RCM(M) = 100M^M \frac{1}{T} \sum_{i=1}^{T} (\prod_{j=1}^{M} p_{j,t})$, where $M$ is the number of regimes, $T$ is the number of observations, and $p_{j,t}$ is the smoothed (ex-post) regime probability relative to regime $j = 1,.., M$ at time $t$ (see Ang and Bekaert, 1998, 2002).
### A Appendix: MS-VECM estimation results

#### Table A1. MSIAH(3)-VECM(1) estimation: Italy

<table>
<thead>
<tr>
<th>( \Gamma_1(z_t = 1) )</th>
<th>( \Gamma_1(z_t = 2) )</th>
<th>( \Gamma_1(z_t = 3) )</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
1.1930 & 0.3914 \\
(0.71) & (0.79) \\
-1.5377 & 2.2825 \\
(0.35) & (0.39)
\end{bmatrix}
\] | \[
\begin{bmatrix}
0.1204 & -0.0112 \\
(0.02) & (0.01) \\
0.1446 & 0.0138 \\
(0.19) & (0.14)
\end{bmatrix}
\] | \[
\begin{bmatrix}
0.2833 & -0.0683 \\
(0.05) & (0.07) \\
0.1008 & 0.1251 \\
(0.03) & (0.05)
\end{bmatrix}
\] |

<table>
<thead>
<tr>
<th>( \bar{v}(z_t = 1) )</th>
<th>( \bar{v}(z_t = 2) )</th>
<th>( \bar{v}(z_t = 3) )</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
-0.0396 \\
(0.16) \\
-0.1333 \\
(0.08)
\end{bmatrix}
\] | \[
\begin{bmatrix}
-0.0010 \\
(0.0009) \\
-0.0018 \\
(0.0008)
\end{bmatrix}
\] | \[
\begin{bmatrix}
-0.0191 \\
(0.01) \\
-0.0138 \\
(0.01)
\end{bmatrix}
\] |

<table>
<thead>
<tr>
<th>( \tilde{\alpha}(z_t = 1) )</th>
<th>( \tilde{\alpha}(z_t = 2) )</th>
<th>( \tilde{\alpha}(z_t = 3) )</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
-0.4101 \\
(0.27) \\
0.3275 \\
(0.13)
\end{bmatrix}
\] | \[
\begin{bmatrix}
-0.0016 \\
(0.003) \\
0.0836 \\
(0.03)
\end{bmatrix}
\] | \[
\begin{bmatrix}
-0.0127 \\
(0.03) \\
0.0233 \\
(0.03)
\end{bmatrix}
\] |

<table>
<thead>
<tr>
<th>( \tilde{\Sigma}(z_1) )</th>
<th>( \tilde{\Sigma}(z_2) )</th>
<th>( \tilde{\Sigma}(z_3) )</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
0.1034 & 0.0318 \\
0.0318 & 0.0240
\end{bmatrix}
\] | \[
\begin{bmatrix}
2.80 \times 10^{-5} & 1.29 \times 10^{-5} \\
1.29 \times 10^{-5} & 2.58 \times 10^{-4}
\end{bmatrix}
\] | \[
\begin{bmatrix}
0.0089 & -0.00013 \\
-0.00013 & 0.0044
\end{bmatrix}
\] |

<table>
<thead>
<tr>
<th>( \tilde{\Phi} )</th>
<th>( \tilde{\xi} )</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
0.19 & 0.02 & 0.15 \\
0.01 & 0.89 & 0.09 \\
0.80 & 0.09 & 0.76
\end{bmatrix}
\] | \[
\begin{bmatrix}
0.095 \\
0.410 \\
0.495
\end{bmatrix}
\] |

**Notes:** Tildes denote estimated values obtained using the EM algorithm for maximum likelihood. Figures in parentheses are asymptotic standard errors. \( \Phi \) and \( \xi \) denote the transition matrix and the ergodic probabilities vector respectively. \( \rho(A) \) is the spectral radius of the matrix \( A \) calculated as in Karlsen (1990), which can be thought of as a measure of stationarity of the MS-VECM. The LR linearity test is a likelihood ratio test for the null hypothesis that the true model is a linear VECM against the alternative of a MSIAH(\( M \))-VECM(\( p \)). Its \( p \)-value is calculated as in Ang and Bekaert (1998). JB is the Jarque-Bera test for normality of the standardized residuals; RESET is a RESET test calculated using a third-order polynomial (Ramsey, 1969). For each of LR, JB and RESET we only report \( p \)-values.
References


Figure 1 - MSIAH-VECM Transition Probabilities

Belgium

Regime 1

Regime 2
Figure 2 - MSIAH-VECM Transition Probabilities

Finland

Regime 1

Regime 2

Regime 3
Figure 3 - MSIAH-VECM Transition Probabilities

France

Regime 1

Regime 2

Regime 3
Figure 4 - MSIAH-VECM Transition Probabilities

Italy

Regime 1

Probability

Regime 2

Probability

Regime 3

Probability
Figure 5 - MSIAH-VECM Transition Probabilities

Portugal

Regime 1

Regime 2

Regime 3
Figure 6 - MSIAH-VECM Transition Probabilities

Switzerland

Regime 1

Regime 2