McKean's Method applied to American Call Options on Jump-Diffusion Processes

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Abstract

In this paper we derive the implicit integral equation for the price of an American call option in the case where the underlying asset follows a jump-diffusion process. We extend McKean's incomplete Fourier transform approach to solve the free boundary problem under Merton's framework, with the distribution for the jump size remaining unspecified. Our results confirm those of Gukhal, who derived the same integral equation using the Geske-Johnson discretisation approach. The paper presents an iterative numerical algorithm for solving the linked integral equation system for the American call's price and early exercise boundary. This is applied to Merton's model jump-diffusion model, where the jumps are log-normally distributed.

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1 Introduction

American option contracts are a well-known derivative product within modern financial markets. Written on a range of underlying assets including stocks, futures and foreign exchange rates, the high usage of American options has made them the focus for a broad range of derivative pricing literature. There exists a significant amount of evidence that stocks and foreign exchange rates are better modeled by jump-diffusion process than pure diffusion processes, including Jarrow and Rosenfeld (1984), Ball and Torous (1985), Jorion (1988), Ahn and Thompson (1992) and Bates (1996). While Pham (1997) and Gukhal (2001) have extended the work of Merton's (1973) jump-diffusion model for European option prices to the American option case, each has done so using different techniques, and neither consider how to implement their pricing equations in a practical sense. The purpose of this paper is to firstly extend the incomplete Fourier transform method of McKean (1965) to the case of American call options written on assets with jump-diffusion price dynamics. Using Kim's (1990) simplifications, we are able to reconcile this method with the results of Gukhal, and provide a method of analysis that extends very naturally from Merton's framework. We also present an iterative numerical method that can be used to solve the resulting integral equation system for both the American call's price, and also its early exercise boundary.

The seminal works of Black and Scholes (1973) and Merton (1973) provided an arbitrage-based means for obtaining the prices of European call and put options. In their analysis, it is assumed that the underlying asset follows a diffusion process. This model was extended to include American options by McKean (1965), who used incomplete Fourier transforms to solve the Black-Scholes partial differential equation (PDE), obtaining an integral equation for the price of an American call option, as a function of the unknown early exercise boundary for the call, as well as a corresponding implicit integral equation for the free boundary itself. Kim (1990) provided an alternative derivation method, based on the finite exercise date approach of Geske and Johnson (1984). Kim showed that the limit of the Geske-Johnson technique as the number of early exercise dates tends to infinity is equivalent to the findings of McKean. In particular, Kim considered both American calls and puts, and demonstrated how to re-express

McKean's integral equations in a more economically intuitive form. Carr, Jarrow and Myneni (1992) also derive more economically intuitive forms for American put option prices, including the early exercise premium decomposition put forward by Kim and Jarrow (1991).

Merton (1976) was the first to extend the Black-Scholes model to consider European options on assets following jump-diffusion processes. He derived the jump-diffusion equivalent of the Black-Scholes formula for European calls where the arrival times of the jumps followed a Poisson distribution, and the distribution for the jump sizes was unknown. He assumed that the risk associated with the jump term could be diversified away by the holder of the call option. As a particular case, Merton considered the case where the jumps were log-normally distributed, resulting in a natural extension of Black and Scholes' results.

Pham (1997) was one of the first to expand the Merton jump-diffusion model to American options. Using probability arguments, Pham derived the integral equation for the price of an American put under jump-diffusion, along with an integral equation for the put's free boundary. Unlike Merton, he does not assume that the jump risk can be diversified away. Performing analysis on these integral equations, Pham demonstrated that value of the American put under jump-diffusion is greater than that of an American put under pure diffusion, and that the increased risk from the jump term makes the option holder more sensitive to the decision of early exercise. Mullinaci (1996) also considered the American put option under jump-diffusion. Using a discrete time model, Mullinaci finds the Snell envelope for the American put option, resulting in a numerical technique for pricing the American put.

Another exploration of American options under jump-diffusion was presented by Gukhal (2001). By extending the Geske-Johnson limiting technique of Kim to Merton's jump-diffusion model, Gukhal derived the integral equations for the prices of both American calls and puts, along with the integral equations for their free boundaries. His results were for a general jump-size distribution, and he also provided more specific equations in the case of binomial and log-normally distributed jump sizes. In particular, Gukhal provided a very intuitive decomposition for the prices of American options under jump diffusion. The first two components, namely the European value and early exercise premium, were already familiar from

Kim's pure diffusion results. The third component introduced by the presence of jumps was identified as an adjustment cost made by the holder of the option when the underlying asset jumps from the stopping region back into the continuation region. This cost is incurred because only jumps out of the continuation region will be self-financing.

While the results of Pham and Gukhal provide extensions of various American option pricing techniques to the jump-diffusion case, no one seems to have extended McKean's incomplete Fourier transform technique to the jump-diffusion case. Chiarella (2003) demonstrates how the Fourier transform method, which is able to readily solve the Black-Scholes PDE for European option prices, can be used to solve Merton's partial integro-differential equation for European option prices under jump diffusion. Kucera and Ziogas (2003) expand on McKean's paper, demonstrating in full how to apply the Fourier transform method to the American call option. In this paper we use the methods of Kucera and Ziogas to extend McKean's method to solve for the price of an American call option under Merton's jump-diffusion framework. The main advantage of the Fourier transform method is that it is broadly applicable to a wide variety of option pricing problems, and thus it provides a natural means of extending the Black-Scholes analysis to non-European payoffs.

In the existing literature, there has been little work conducted on the implementation of the integral equations for the price and free boundary of American options under jump-diffusions. Expanding on the American straddle presented by Elliot, Myneni and Viswanathan (1991), Chiarella and Ziogas (2003) derived the integral equations for an American strangle portfolio using Fourier transforms, and solved the resulting linked integral equation system for the strangle's free boundaries. Here we extend on the approach of Chiarella and Ziogas by applying a modified version of their method to solve the linked integral equation system that arises for the American call and it's free boundary in the case of jump-diffusion. While the focus of the paper is not on finding optimal numerical methods for American option prices with jumps, we are able to demonstrate that the Fourier transform technique leads to integral equation forms that are tractable for numerical implementation.

The remainder of this paper is structured as follows. Section 2 outlines the free boundary problem that arises from pricing an American call option under Merton's jump-diffusion model. Section 3 applies McKean's incomplete Fourier transform to solve the PIDE in terms of a transform variable. The transform is inverted in Section 4, providing McKean-style integral equations for the American call price, and a corresponding integral equation for the call's early exercise boundary. Section 5 outlines the numerical solution method for solving the linked integral equation system for both the free boundary and price of the American call, including the transform from McKean's equations to Kim's equations, and reconciling our findings with those of Gukhal. A selection of numerical results for the American call option and its early exercise boundary are provided in Section 6, with concluding remarks presented in Section 7.

2 Problem Statement

Let $C(S,t)$ be the price of an American call option written on the underlying asset S at time t, with time to expiry $(T-t)$, and strike price K. Let $a(t)$ denote the early exercise boundary at time t, and assume S follows the jump-diffusion process

$$dS = (\mu - \lambda k)Sdt + \sigma S dW + (Y-1)Sd\bar{q}$$

where

$$\begin{aligned}
\mu &= \text{drift of} \quad S; \\
\sigma &= \text{volatility of} \quad S; \\
dW &= \text{ Wiener process increment } \quad (dW \sim N(0,dt)); \\
d\bar{q} &= \begin{cases} 1, & \text{with probability } \lambda dt, \\ 0, & \text{with probability } (1-\lambda dt). \end{cases}
\end{aligned}$$

Let the jump size, Y, be a random variable with probability density function $G(Y)$. Thus the expected jump size, k, is given by

$$k = \mathrm{E}^{Q_Y}[Y-1] = \int_0^\infty (Y-1)G(Y)dY.$$

Following Merton's (1976) argument and assuming that the jump risk is fully diversifiable, it is known that C satisfies the following partial integro-differential equation (PIDE):

$$\frac{\partial C}{\partial t} + (r - q - \lambda k)S\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial C^2}{\partial S^2} - rC + \lambda \int_0^\infty [C(SY,t) - C(S,t)]G(Y)dY = 0, \quad (1)$$

in the region $0 \leq t \leq T$ and $0 < S < a(t)$, where

$$\begin{aligned} r &= \text{risk-free rate, and} \\ q &= \text{continuously compounded dividend yield of } S. \end{aligned}$$

The PIDE (1) is subject to the following final time and boundary conditions:

$$C(S,T) = \max(S-K,0), \quad 0 < S < \infty \quad (2)$$

$$C(0,t) = 0, \quad t \geq 0, \quad (3)$$

$$C(a(t),t) = a(t) - K, \quad t \geq 0, \quad (4)$$

$$\lim_{S \to a(t)} \frac{\partial C}{\partial S} = 1, \quad t \geq 0. \quad (5)$$

Condition (2) is the payoff function for the call at expiry, and condition (3) ensures that the option is worthless if S falls to zero. The value-matching condition (4) forces the value of the call option to be equal to its payoff on the early exercise boundary, and the smooth-pasting condition (5) makes the call's delta continuous at the free boundary to guarantee arbitrage-free prices.

Our first step is to transform the PIDE to a forward-in-time equation, with constant coefficients and a "standardised" strike of 1. Let

$$C(S,t) = KV(x,\tau) \quad (6)$$

$$\text{with} \qquad S = Ke^x, \quad \text{and} \quad t = T - \tau. \tag{7}$$

Is should be noted that

$$\begin{aligned} C(SY, t) &= KV(\ln(\frac{SY}{K}), \tau) \\ &= KV(x + \ln Y, \tau) \end{aligned}$$

The transformed PIDE for V is then

$$\frac{\partial V}{\partial \tau} = \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial x^2} + k_1 \frac{\partial V}{\partial x} - rV + \lambda \int_0^\infty [V(x + \ln Y, \tau) - V(x, \tau)] G(Y) dY,$$

where $k_1 \equiv r - q - \lambda k - \frac{\sigma^2}{2} \equiv k_1^D - \lambda k$. The transformed PIDE can be simplified to

$$\frac{\partial V}{\partial \tau} = \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial x^2} + k_1 \frac{\partial V}{\partial x} - (r + \lambda)V + \lambda \int_0^\infty [V(x + \ln Y, \tau) - V(x, \tau)] G(Y) dY, \tag{8}$$

in the region $0 \le \tau \le T$, $-\infty \le x \le \ln c(\tau)$, where $c(\tau) = \frac{a(t)}{K}$. The initial and boundary conditions are

$$\begin{aligned} V(x, 0) &= \max(e^x - 1, 0), \quad -\infty < x < \infty & (9) \\ \lim_{x \to -\infty} V(x, \tau) &= 0, \quad \tau \ge 0 & (10) \\ V(\ln c(\tau), \tau) &= c(\tau) - 1, \quad \tau \ge 0 & (11) \\ \lim_{x \to \ln c(\tau)} \frac{\partial V}{\partial x} &= c(\tau), \quad \tau \ge 0 & (12) \end{aligned}$$

For simplicity, we shall denote $c(\tau)$ by $c \equiv c(\tau)$ when it is clear at which time this function is being evaluated.

Finally, to facilitate the application of integral transform methods, the x-domain shall be extended to $-\infty < x < \infty$ by expressing equation (8) as:

$$\mathrm{H}(\ln c - x) \left(\frac{\partial V}{\partial \tau} - \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial x^2} + k_1 \frac{\partial V}{\partial x} - (r + \lambda)V + \lambda \int_0^\infty V(x + \ln Y, \tau) G(Y) dY \right) = 0 \tag{13}$$

where $H(\ln c - x)$ is the Heaviside step function, defined as

$$H(x) = \begin{cases} 1, x > 0, \\ \frac{1}{2}, x = 0, \\ 0, x < 0. \end{cases} \tag{14}$$

The reason for the factor of $\frac{1}{2}$ at the discontinuity is explained by Chiarella and Ziogas (2003). The initial and boundary conditions remain unchanged following the introduction of the Heaviside function.

3 Applying the Fourier Transform

To solve the problem defined by equations (8)-(12), we shall apply the Fourier transform technique to reduce the PIDE to an OIDE whose solution is easily found. In applying the Fourier transform technique, we make the standard assumption that the function V and its first two derivatives with respect to x can be treated as zero when x tends to $\pm\infty$. This assumption is always used when solving the PIDE for European call options. This is justified by the existence and uniqueness of the solution, and the fact that the solution satisfies both the PIDE (8) and the boundary conditions (9)-(12).

Since the x-domain is not $-\infty < x < \infty$, the Fourier transform of the PIDE can be found. Define the Fourier transform of V, $\mathcal{F}\{V(x,\tau)\}$, as

$$\mathcal{F}\{V(x,\tau)\} = \int_{-\infty}^{\infty} e^{i\eta x} V(x,\tau) dx.$$

Applying this Fourier transform to the PIDE (13), we have

$$\begin{aligned} \mathcal{F}\left\{H(\ln c - x)\frac{\partial V}{\partial \tau}\right\} &= \frac{\sigma^2}{2}\mathcal{F}\left\{H(\ln c - x)\frac{\partial^2 V}{\partial x^2}\right\} + k_1 \mathcal{F}\left\{H(\ln c - x)\frac{\partial V}{\partial C}\right\} \\ &\quad -(r+\lambda)\mathcal{F}\left\{H(\ln c - x)V\right\} \\ &\quad +\lambda \mathcal{F}\left\{H(\ln c - x)\int_0^{\infty} V(x + \ln Y, \tau) G(Y) dY\right\}. \end{aligned}$$

By the definition of the Fourier transform, we have

$$
\begin{aligned}
\mathcal{F}\{H(\ln c - x)V(x,\tau)\} &= \int_{-\infty}^{\ln c} e^{i\eta x} V(x,\tau)dx \\
&\equiv \mathcal{F}^c\{V(x,\tau)\} \qquad (15)\\
&\equiv \hat{V}^c(\eta,\tau). \qquad (16)
\end{aligned}
$$

It should be noted that $\mathcal{F}^c$ is an incomplete Fourier transform, since it is a standard Fourier transform applied to $V(x,\tau)$ in the domain $-\infty < x < c(\tau)$. We shall now apply this transform to the PIDE (13).

Proposition 1:

The incomplete Fourier transform of the PIDE (13) with respect to x satisfies the ordinary integro-differential equation

$$
\frac{\partial \hat{V}}{\partial \tau} + [\frac{\sigma^2\eta^2}{2} + k_1 i\eta + (r+\lambda) - \lambda A(\eta)]\hat{V} = F(\eta,\tau) \qquad (17)
$$

where

$$
\begin{aligned}
F(\eta,\tau) &= e^{i\eta \ln x}\left[\frac{\sigma^2 c}{2} + \left(\frac{c'}{c} - \frac{\sigma^2 i\eta}{2} + k_1\right)(c-1)\right] + \lambda\Phi(\eta,\tau), \qquad (18)\\
A(\eta) &= \int_0^{\infty} e^{-i\eta \ln Y} G(Y)dY, \qquad (19)\\
\Phi(\eta,\tau) &= \int_0^{\infty} e^{-i\eta \ln Y} G(Y)[\int_{\ln c}^{\ln Y + \ln c} e^{i\eta z} V(z,\tau)dz]dY, \qquad (20)
\end{aligned}
$$

and

$$
c' \equiv \frac{dc(\tau)}{d\tau}.
$$

Furthermore, the solution to this OIDE is given by

$$
\begin{aligned}
\hat{V}(\eta,\tau) &= \hat{V}(\eta,0)e^{-(\frac{\sigma^2\eta^2}{2}k_1 i\eta + (r+\lambda) - \lambda A(\eta))(\tau - s)} \\
&+ \int_0^{\tau} e^{-(\frac{\sigma^2\eta^2}{2}k_1 i\eta + (r+\lambda) - \lambda A(\eta))(\tau - s)} F(\eta,s)ds \qquad (21)
\end{aligned}
$$

Proof: Refer to Appendix A.

□

4 Inverting the Fourier Transform

Now that $\hat{V}(\eta,\tau)$ has been found, we are required to recover $V(x,\tau)$, the American call price in the x-τ plane. By taking the inverse (complete) Fourier transform of (21), we have

$$\begin{aligned} V(x,\tau) &= (\mathcal{F}^c)^{-1}\left\{\hat{V}(\eta,0)e^{-(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta))\tau}\right\} \\ &\quad +(\mathcal{F}^c)^{-1}\left\{\int_0^\tau e^{-(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta))(\tau-s)}F(\eta,s)ds\right\} \\ &= V_1(x,\tau)+V_2(x,\tau) \\ &\equiv \frac{1}{K}\left[C_1(S,\tau)+C_2(S,\tau)\right]=\frac{1}{K}C(S,\tau) \end{aligned} \tag{22}$$

where $-\infty < x < \ln c(\tau)$. We must now determine the functions $C_1(S,\tau)$ and $C_2(S,\tau)$.

Proposition 2:

The function $C_1(S,\tau)$ is given by

$$C_1(S,\tau)=C_E-\sum_{n=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}\varepsilon_n\{\hat{C_E^D}[SX_ne^{-\lambda k\tau},K,r,q,\tau,\sigma^2]\} \tag{23}$$

where

$$\begin{aligned} C_1(S,\tau) &\equiv KV_1(x,\tau), \\ C_E &= \sum_{n=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}\varepsilon_n\{C_E^D[SX_ne^{-\lambda k\tau},K,r,q,\tau,\sigma^2]\}, \\ C_E^D[S,K,r,q,\tau,\sigma^2] &= Se^{-q\tau}N[d_1(S,K,r,q,\tau,\sigma^2)]-Ke^{-r\tau}N[d_2(S,K,r,q,\tau,\sigma^2)], \\ d_1(S,K,r,q,\tau,\sigma^2) &= \frac{\ln\frac{S}{K}+\left(r-q+\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \\ d_2(S,K,r,q,\tau,\sigma^2) &= d_1(S,K,r,q,\tau,\sigma^2)-\sigma\sqrt{\tau}, \\ N[\alpha] &= \frac{1}{\sqrt{2\tau}}\int_{-\infty}^{\alpha}e^{-\frac{\beta^2}{2}}d\beta, \end{aligned}$$

$$
\begin{aligned}
\hat{C_E^D}[S,K,r),q,\tau,\sigma^2] &= Se^{-q\tau}N[d_1(S,Kc(0),r,q,\tau,\sigma^2)] - Ke^{-r\tau}N[d_2(S,Kc(0),r,q,\tau,\sigma^2)],\\
X_n &= Y_1Y_2...Y_n; X_0 \equiv 1,\\
and \quad \varepsilon_n\{f(X_n)\} &\equiv \int_0^\infty \int_0^\infty ... \int_0^\infty G(Y_1)G(Y_2)...G(Y_n)f(X_n)dY_1dY_2...dY_n.
\end{aligned}
$$

Proof: Refer to Appendix B.1.

□

Next we consider the more complicated function $V_2(x,\tau)$. The first step is to break the function down into two linear components:

$$
\begin{aligned}
V_2(x,\tau) &= (\mathcal{F}^c)^{-1}\left\{\int_0^\tau e^{-(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta))(\tau-s)}F(\eta,s)ds\right\}\\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\eta x}\int_0^\tau e^{-(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta))(\tau-s)}F(\eta,s)dsd\eta\\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\eta x}\int_0^\tau e^{-(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta))(\tau-s)}\\
&\qquad \times e^{i\eta\ln c(s)}\left[\frac{\sigma^2 c(s)}{2}+\left(\frac{c'(s)}{c(s)}-\frac{\sigma^2 i\eta}{2}+k_1\right)(c(s)-1)\right]dsd\eta\\
&\quad +\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\eta x}\int_0^\tau e^{-(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta))(\tau-s)}\lambda\Phi(\eta,s)dsd\eta\\
&\equiv V_2^{(1)}(x,\tau)+V_2^{(2)}(x,\tau)\\
&\equiv \frac{1}{K}\left[C_2^{(1)}(S,\tau)+C_2^{(2)}(S,\tau)\right]\\
&= \frac{1}{K}C_2(S,\tau).
\end{aligned}
$$

We start by considering the function $C_2^{(1)}(S,\tau)$.

Proposition 3:

The term $C_2^{(1)}(S,\tau)$ *is given by*

$$
\begin{aligned}
C_2^{(1)}(S,\tau) &= \sum_{n=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}\varepsilon_n\{\hat{C_E^D}[SX_ne^{-\lambda k\tau},K,r,q,\tau,\sigma^2]\}\\
&\quad +\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon\Big\{\int_0^\tau(\tau-\xi)^{n-1}\Big[SX_ne^{-\lambda k(\tau-\xi)}e^{-(q+\lambda)(\tau-\xi)}[(\lambda[k+1]+q)(\tau-\xi)-n]\\
&\qquad \times N[d_1(SX_ne^{-\lambda k(\tau-\xi)},Kc(\xi),r,q,\tau-\xi,\sigma^2)]
\end{aligned}
$$

$$
\begin{aligned}
&-Ke^{-(r+\lambda)(\tau-\xi)}[(\tau-\xi)(r+\lambda)-n] \\
&\times N[d_2(SX_ne^{-\lambda k(\tau-\xi)},Kc(\xi),r,q,\tau-\xi,\sigma^2)]\Big]\,d\xi\Big\}, \quad (24)
\end{aligned}
$$

where

$$C_2^{(1)}(S,\tau)\equiv KV_2^{(1)}(x,\tau).$$

Proof: Refer to Appendix B.2.

□

Before proceeding further, it is worth noting that if we now combine $C_1(S,\tau)$ with $C_1^{(1)}(S,\tau)$, some of the terms will cancel, leaving us with

$$
\begin{aligned}
C_1(S,\tau)+C_1^{(1)}(S,\tau) &= \sum_{n=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}\varepsilon_n\left\{C_E^D\left[SX_ne^{-\lambda k\tau},K,r,q,\tau,\sigma^2\right]\right\} \\
&+\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{\int_0^\tau(\tau-\xi)^{n-1}SX_ne^{-\lambda k(\tau-\xi)}e^{-(q+x\lambda)(\tau-\xi)} \qquad (25)\\
&\qquad\times[(\lambda[k+1]+q)(\tau-\xi)-n] \\
&\qquad\times N[d_1(SX_ne^{-\lambda k(\tau-\xi)},Kc(\xi),r,q,\tau-\xi,\sigma^2)]d\xi\Big\} \\
&-\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{\int_0^\tau(\tau-\xi)^{n-1}Ke^{-(r+\lambda)(\tau-\xi)}[(\tau-\xi)(r+\lambda)-n] \\
&\qquad\times N\left[d_2(SX_ne^{-\lambda k(\tau-\xi)},,Kc(\xi),r,q,\tau-\xi,\sigma^2)\right]\Big]\,d\xi\Big\}.
\end{aligned}
$$

The last remaining term to be evaluated is $C_1^{(2)}(S,\tau)$, which is the extra term introduced by the presence of jumps in the stochastic process for S.

Proposition 4:

The term $C_2^{(2)}(S,\tau)$ *is given by*

$$
\begin{aligned}
C_2^{(2)}(S,\tau) &= -\lambda\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{\int_o^\tau(\tau-\xi)^ne^{-(r+x)(\tau-\xi)} \\
&\times\left[\int_0^1 G(Y)\int_{Kc(\xi)}^{\frac{Kc(\xi)}{Y}}C(\omega Y,\xi)J(\omega,\xi,SX_n,\tau)d\omega dY \qquad (26)\right.
\end{aligned}
$$

$$- \int_{1}^{\infty} G(Y) \int_{\max\left(\frac{S}{c(\xi)}, \frac{Kc(\xi)}{Y}\right)}^{Kc(\xi)} (\omega Y - K) J(\omega, \xi, SX_n, \tau) d\omega dY \Bigg] d\xi \Bigg\}$$

where

$$C_2^{(2)}(S, \tau) \equiv K V_2^{(2)}(x, \tau),$$

and

$$J(\omega, \xi, SX_n, \tau) = \frac{1}{\omega\sigma\sqrt{2\pi(\tau - \xi)}} \exp\left\{ \frac{-\left[\left(r - q - \lambda k - \frac{\sigma^2}{2}\right)(\tau - \xi) + \ln \frac{SX_n}{\omega}\right]^2}{2\sigma^2(\tau - \xi)} \right\}. \tag{27}$$

Proof: Refer to Appendix B.3.

□

Now that we have derived the function $C_1(S, \tau)$ and $C_2(S, \tau)$, we can provide an integral equation for the price of the American call $C(S, \tau)$.

Proposition 5:

The integral equation for the price of the American call option $C(S, \tau)$ is

$$\begin{aligned} C(S, \tau) = {} & C_E + \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \varepsilon_n \Bigg\{ \int_0^{\tau} (\tau - \xi)^{n-1} SX_n e^{-\lambda k(\tau - \xi)} e^{-(q+\lambda)(\tau - \xi)} \\ & \times [(\lambda[k+1] + q)(\tau - \xi) - n] \\ & \times N\left[d_1\left(SX_n e^{-\lambda k(\tau-\xi)}, Kc(\xi), r, q, \tau - \xi, \sigma^2\right)\right] d\xi \Bigg\} \\ & - \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \varepsilon_n \Bigg\{ \int_0^{\tau} (\tau - \xi)^{n-1} K e^{-(r+\lambda)(\tau-\xi)} [(\tau - \xi)(r + \lambda) - n] \\ & \times N\left[d_2\left(SX_n e^{-\lambda k(\tau-\xi)}, Kc(\xi), r, q, \tau - \xi, \sigma^2\right)\right] d\xi \Bigg\} \\ & - \lambda \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \varepsilon_n \Bigg\{ \int_0^{\tau} (\tau - \xi)^n e^{-(r+\lambda)(\tau-\xi)} \\ & \times \Bigg[\int_0^1 G(Y) \int_{Kc(\xi)}^{\frac{Kc(\xi)}{Y}} C(\omega Y, \xi) J(\omega, \xi, SX_n, \tau) d\omega dY \\ & - \int_1^{\infty} G(Y) \int_{\max\left(\frac{S}{c(\xi)}, \frac{Kc(\xi)}{Y}\right)}^{Kc(\xi)} (\omega Y - K) J(\omega, \xi, SX_n, \tau) d\omega dY \Bigg] d\xi \Bigg\}, \end{aligned} \tag{28}$$

where

$$\begin{aligned}
C_E &= \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \varepsilon_n\{C_E^D[SX_n e^{-\lambda k\tau}, K, r, q, \tau, \sigma^2]\} \\
C_E^D[S,K,r,q,\tau,\sigma^2] &= Se^{-q\tau}N[d_1(S,K,r,q,\tau,\sigma^2)] - Ke^{-r\tau}N[d_2(S,K,r,q,\tau,\sigma^2)], \\
d_1(S,K,r,q,\tau,\sigma^2) &= \frac{\ln\frac{S}{K} + \left(r - q + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \\
d_2(S,K,r,q,\tau,\sigma^2) &= d_1(S,K,r,q,\tau,\sigma^2) - \sigma\sqrt{\tau}, \\
N[\alpha] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} e^{\frac{\beta^2}{2}} d\beta, \\
X_n &= Y_1 Y_2 \ldots Y_n, \quad \textit{with} \quad X_0 \equiv 1, \\
\textit{and} \quad J(\omega,\xi,SX_n,\tau) &= \frac{1}{\omega\sigma\sqrt{2\pi(\tau-\xi)}} \exp\left\{ \frac{-\left[\left(r - q - \lambda k - \frac{\sigma^2}{2}\right)(\tau - \xi) + \ln\frac{SX_n}{\omega}\right]^2}{2\sigma^2(\tau-\xi)} \right\}.
\end{aligned}$$

Proof: Equation (28) follows from substituting equations (23), (24) and (26) into equation (22).

□

In equation (28), the value of the American call option is expressed as a function of the original underlying variable S, and the new time variable τ, which is a measure of time to maturity. It is important to note that equation (28) also depends upon the unknown early exercise boundary, now defined as $c(\tau) = a(t)/K$. By requiring the expression for $C(x,\tau)$ to satisfy the boundary conditions (4) and (5), we can derive a similar integral equation for the value of $c(\tau)$. This integral equation is given by

$$K(c(\tau) - 1) = C(Kc(\tau), \tau). \tag{29}$$

It is important to note that the integral equation (29) will depend upon the unknown call value $C(S,\tau)$, and this dependance arises from integral terms that have been introduced by the presence of jumps in the dynamics for S. In order to implement these integral equations for the free boundary and call price, we must therefore use numerical techniques to solve the linked integral equation system consisting of (28) and (29).

5 Numerical Implementation

To help facilitate the numerical implementation of the integral equation system formed by (28) and (29), we shall first carry out some algebraic manipulations in an effort to achieve the form presented by Gukhal (2001) for American call options under jump-diffusion. In the final version of the paper we will present the full details for these manipulations, resulting in expressions which are equivalent to Gukhal's equation (4.3).

The primary difficulty in solving for the American call price under jump-diffusion dynamics is caused by the dependance of equation (28), and subsequently equation (29), on the unknown American call price $C(S, \tau)$. To overcome this complication, we propose an iterative numerical scheme similar to the one employed in the American strangle analysis of Chiarella and Ziogas (2003). We begin by finding a satisfactory initial approximation for the American call option price, denoted here by $C_0(S, \tau)$. A natural starting approximation can be readily obtained from the American call in the case of pure-diffusion. Using this initial approximation, we will solve equation (29) numerically for our first approximation of the free boundary c, denoted by $c_1(\tau)$. This free boundary is then used, along with $C_0(S, \tau)$ to numerically solve equation (28) for the new American call price approximation $C_1(S, \tau)$. This process is then repeated until the difference between two successive call price approximations $C_i(S, \tau)$ and $C_{i+1}(S, \tau)$, and also two successive free boundary approximations $c_i(\tau)$ and $c_{i+1}(\tau)$ are both within an arbitrary tolerance level, TOL. This difference will be measured using an L_2 norm applied to $C_i(S, \tau)$ iterates for a suitable range of S values, and to the free boundary iterates $c_i(\tau)$ for a range of τ values.

The fundamental numerical calculations, such as root-finding and numerical integration, shall be conducted along the same lines as presented in Chiarella and Ziogas (2003). A log-normal distribution is selected for the jump sizes Y, in accordance with a model suggested by Merton (1976). Thus the probability density function for Y is given by

$$G(Y) = \frac{1}{Y\delta\sqrt{2\pi}} e^{\frac{(\ln Y - (\gamma - \delta^2/2))^2}{2\delta^2}}, \tag{30}$$

where we set $\gamma \equiv \ln(1+k)$, and δ^2 is the variance of $\ln Y$ respectively. Gukhal (2001) assumes that $\gamma = -\delta^2/2$ when deriving his equation (5.1) for the American call option price. In the final version of this paper, we will forego this assumption, and provide a more general form of Gukhal's results, along with a means of numerical implementation for the resulting linked integral equation system.

6 Results

The final version of the paper will present a range of American call price profiles, along with their corresponding early exercise boundaries. We will also compare these results to the pure-diffusion model, so as to demonstrate the impact of jumps in the features of American call options.

7 Conclusion

Concluding remarks will be presented in the final version of the paper, once the numerical results have been completed.

Appendix A. Properties of the Incomplete Fourier Transform

According to Kucera and Ziogas (2003), from the "pure" diffusion case (i.e. the model with no jumps) we know that:

$$\mathcal{F}^c\left\{\frac{\partial V}{\partial x}\right\} = (c-1)e^{i\eta \ln c} - i\eta\hat{V}, \tag{31}$$

$$\mathcal{F}^c\left\{\frac{\partial^2 V}{\partial x^2}\right\} = e^{i\eta \ln c}(c - i\eta(c-1)) - \eta^2\hat{V}, \tag{32}$$

$$\text{and} \quad \mathcal{F}^c\left\{\frac{\partial V}{\partial \tau}\right\} = \frac{\partial \hat{V}}{\partial \tau} - \frac{c'}{c}e^{i\eta \ln c}(c-1), \tag{33}$$

where $c' \equiv \frac{\partial c(\tau)}{\partial \tau}$. This leaves one term to be evaluated, namely:

$$\mathcal{F}\left\{H(\ln c - x)\int_0^\infty V(x+\ln Y,\tau)G(Y)d(y)\right\} = \int_{-\infty}^{\ln c} e^{i\eta x}\int_0^\infty V(x+\ln Y,\tau)G(Y)dYdx. \tag{34}$$

Using the change of variable $z = x + \ln Y$, equation (34) becomes

$$\begin{aligned}
&\mathcal{F}\left\{H(\ln c - x)\int_0^\infty V(x+\ln Y,\tau)G(Y)d(Y)\right\} \\
&= \int_0^\infty \int_{-\infty}^{\ln c} e^{i\eta x}V(x+\ln Y,\tau)G(Y)dxdY \\
&= \int_0^\infty \int_{-\infty}^{\ln c+\ln Y} e^{i\eta(z-\ln Y)}V(z,\tau)G(Y)dzdY \\
&= \int_0^\infty \left[\int_{-\infty}^{\ln c} e^{i\eta(z-\ln Y)}V(z,\tau)G(Y)dz \right.\\
&\quad \left. + \int_{\ln c}^{\ln y+\ln c} e^{i\eta(z-\ln Y)}V(z,\tau)G(Y)dz\right]dY \\
&= \int_0^\infty e^{-i\eta \ln Y}G(Y)dY\int_{-\infty}^{\ln c} V(z,\tau)e^{i\eta z}dz \\
&\quad + \int_0^\infty e^{-i\eta \ln Y}G(Y)\left[\int_{\ln c}^{\ln Y+\ln c} e^{i\eta z}V(z,\tau)dz\right]dY \\
&= A(\eta)\hat{V}(\eta,\tau) + \Phi(\eta,\tau),
\end{aligned}$$

where

$$A(\eta) = \int_0^\infty e^{i\eta \ln Y}G(Y)dY,$$

and

$$\Phi(\eta,\tau) = \int_0^\infty e^{-i\eta \ln Y}G(Y)\left[\int_{\ln c}^{\ln Y+\ln C} e^{i\eta \ln z}V(z,\tau)dz\right]dY.$$

Hence, our PIDE is transformed into the following OIDE:

$$\begin{aligned}
\frac{\partial \hat{V}}{\partial \tau} - \frac{c'}{c}e^{i\eta \ln c}(c-1) &= \frac{\sigma^2}{2}\left(e^{i\eta \ln c}(c - i\eta(c-1)) - \eta^2\hat{V}\right) \\
&\quad + k_1\left((c-1)e^{i\eta \ln c} - i\eta\hat{V}\right) - (r+\lambda)\hat{V} \\
&\quad + \lambda[A(\eta)\hat{V}(\eta,\tau) + \Phi(\eta,\tau)],
\end{aligned}$$

which is readily simplified to

$$\frac{\partial \hat{V}}{\partial \tau} + \left[\frac{\sigma^2\eta^2}{2} + k_1 i\eta + (r+\lambda) - \lambda A(\eta)\right]\hat{V} = F(\eta,\tau),$$

where

$$F(\eta,\tau) = e^{i\eta \ln c}\left[\frac{\sigma^2 c}{2} + \left(\frac{c'}{c} - \frac{\sigma^2 i\eta}{2} + k_1\right)(c-1)\right] + \lambda\Phi(\eta,\tau).$$

The solution to this OIDE is given by

$$\begin{aligned}\hat{V}(\eta,\tau) &= \hat{V}(\eta,0)e^{-(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta))\tau} \\ &\quad + \int_0^\tau e^{-(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta))(\tau-S)}F(\eta,S)dS,\end{aligned}$$

where $\mathcal{F}^c\{V(x,0)\} \equiv \hat{V}(\eta,0)$.

Appendix B. Derivation of the American Call Integral Equations

B.1. Proof of Proposition 2

Consider the function $V_1(x,\tau)$, given by

$$V_1(x,\tau) = (\mathcal{F}^c)^{-1}\left\{\hat{V}(\eta,0)e^{-\left(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta)\right)\tau}\right\}.$$

To evaluate this inversion, recall the following convolution result for Fourier transforms:

$$\hat{F}(\eta,\tau_1)\hat{G}(\eta,\tau_2) = \mathcal{F}\left\{\int_{-\infty}^{\infty} f((x-u),\tau_1)g(u,\tau_2)du\right\}.$$

If we let

$$\hat{F}(\eta,\tau_1) = e^{-\left(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta)\right)\tau},$$

then $f(x,\tau_1)$ is given by

$$\begin{aligned} f(x,\tau_1) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\left(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta)\right)\tau} e^{-i\eta x} d\eta \\ &= \frac{e^{-(\lambda+r)\tau}}{2\pi}\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}-\lambda A(\eta)\right]\tau - i\eta[k_1\tau+x]} d\eta. \end{aligned}$$

Furthermore, let

$$\hat{G}^{b_2}(\eta,\tau_2) = \hat{V}(\eta,0).$$

Hence $g(x,\tau_2)$ will simply be the payoff function in the continuation region, given by

$$g(x,\tau_2) = H(\ln c(\tau)-x)\max(e^x-1,0) = H(\ln c(\tau)-x)H(x)(e^x-1).$$

Thus $V_1(x,\tau)$ becomes

$$V_1(x,\tau) = \int_{-\infty}^{\infty}\left[H(\ln c(0)-u)H(u)(e^u-1)\frac{e^{-(\lambda+r)\tau}}{2\pi}\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}-\lambda A(\eta)\right]\tau - i\eta[k_1\tau+x-u]} d\eta\right] du.$$

The expression for $V_1(x,\tau)$ can now be further simplified:

$$\begin{aligned} V_1(x,\tau) &= \frac{e^{-(\lambda+r)\tau}}{2\pi}\int_{-\infty}^{\infty}\left[H(u)H(\ln c(0)-u)(e^u-1)\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}-\lambda A(\eta)\right]\tau - i\eta[k_1\tau+x-u]} d\eta\right] du \\ &= \frac{e^{-(\lambda+r)\tau}}{2\pi}\int_{-\infty}^{\ln c(0)}\left[H(u)(e^u-1)\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}-\lambda A(\eta)\right]\tau - i\eta[k_1\tau+x-u]} d\eta\right] du \\ &= \frac{e^{-(\lambda+r)\tau}}{2\pi}\int_{-\infty}^{\infty}\left[H(u)(e^u-1)\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}-\lambda A(\eta)\right]\tau - i\eta[k_1\tau+x-u]} d\eta\right] du \\ &\quad -\frac{e^{-(\lambda+r)\tau}}{2\pi}\int_{\ln c(0)}^{\infty}\left[H(u)(e^u-1)\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}-\lambda A(\eta)\right]\tau - i\eta[k_1\tau+x-u]} d\eta\right] du. \end{aligned}$$

Letting $C_1(S,\tau) = KV_1(x,\tau)$, the problem can be re-expressed in terms of the original space variable S as follows:

$$\begin{aligned} C_1(S,\tau) &= e^{-(\lambda+r)\tau}\int_{-\infty}^{\infty} KH(u)(e^u-1)\hat{K}(u,S,\tau)du \\ &\quad -e^{-(\lambda+r)\tau}\int_{\ln c(0)}^{\infty} KH(u)(e^u-1)\hat{K}(u,S,\tau)du. \end{aligned}$$

where

$$\hat{K}(u,S,\tau) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}-\lambda A(\eta)\right]\tau - i\eta[k_1\tau+\ln\frac{S}{K}-u]} d\eta$$

We shall now consider further the function $\hat{K}(u,S,\tau)$ From Taylor series expansions we know that

$$e^{\alpha} = \sum_{n=0}^{\infty}\frac{\alpha^n}{n!}.$$

Using this result, the expression for $\hat{K}(u,S,\tau)$ becomes

$$\begin{aligned}
\hat{K}(u,S,\tau) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}\tau\right] - i\eta[k_1\tau+\ln\frac{S}{K}-u]} \sum_{n=0}^{\infty}\frac{(\lambda\tau)^n}{n!}A(\eta)^n d\eta \\
&= \frac{1}{2\pi}\sum_{n=0}^{\infty}\frac{(\lambda\tau)^n}{n!}\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}\tau\right] - i\eta[k_1\tau+\ln\frac{S}{K}-u]} A(\eta)^n d\eta.
\end{aligned}$$

Note that by definition

$$\begin{aligned}
A(\eta)^n &= \left\{\int_0^{\infty} e^{-i\eta\ln Y}G(Y)dY\right\}^n \\
&= \int_0^{\infty} e^{-i\eta\ln Y_1}G(Y_1)dY_1 ... \int_0^{\infty} e^{-i\eta\ln Y_n}G(Y_n)dY_n.
\end{aligned}$$

Using this definition in the expression for $\hat{K}(u,S,\tau)$, we have

$$\begin{aligned}
\hat{K}(u,S,\tau) &= \frac{1}{2\pi}\sum_{n=0}^{\infty}\frac{(\lambda\tau)^n}{n!}\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}\tau\right] - i\eta[k_1\tau+\ln\frac{S}{K}-u]} \\
&\quad \times \left\{\int_0^{\infty} e^{-i\eta\ln Y_1}G(Y_1)dY_1 ... \int_0^{\infty} e^{-i\eta\ln Y_n}G(Y_n)dY_n\right\} d\eta \\
&= \frac{1}{2\pi}\sum_{n=0}^{\infty}\frac{(\lambda\tau)^n}{n!}\int_0^{\infty}\int_0^{\infty} ... \int_0^{\infty} G(Y_1)G(Y_2)...G(Y_n) \\
&\quad \times \left\{\int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}\tau\right] - i\eta[k_1\tau+\ln\frac{S}{K}-u] - i\eta\ln(Y_1Y_2...Y_n)} d\eta\right\} dY_1dY_2...dY_n \\
&= \frac{1}{2\pi}\sum_{n=0}^{\infty}\frac{(\lambda\tau)^n}{n!}\int_0^{\infty}\int_0^{\infty} ... \int_0^{\infty} G(Y_1)G(Y_2)...G(Y_n)I(\theta_n,\tau)dY_1dY_2...dY_n
\end{aligned}$$

where

$$I(\theta_n,\tau) \equiv \int_{-\infty}^{\infty} e^{-\left[\frac{\sigma^2\eta^2}{2}\tau\right] - i([k_1\tau+\ln\frac{\theta_n}{K}-u]\eta} d\eta,$$

$$\text{and} \quad \theta_n \equiv SY_1Y_2...Y_n.$$

Next consider the function $I(\theta_n, \tau)$, which can be rewritten as

$$I(\theta_n, \tau) = \int_{-\infty}^{\infty} e^{-p\eta^2 - q\eta} d\eta,$$

where

$$p = \frac{1}{2}\sigma^2\tau$$

and

$$q = i[k_1\tau + \ln\frac{\theta n}{K} - u].$$

Using the result that

$$\int_{-\infty}^{\infty} e^{-p\eta^2 - q\eta} d\eta = \sqrt{\frac{\pi}{p}} e^{\frac{q^2}{4p}},$$

we have

$$I(\theta_n, \tau) = \sqrt{\frac{2\pi}{\sigma^2\tau}} \exp\left\{\frac{-[u - \ln\frac{\theta n}{K} - k_1\tau]^2}{2\sigma^2\tau}\right\}.$$

Therefore the expression for $\hat{K}(u, S, \tau)$ becomes

$$\hat{K}(u, S, \tau) = \frac{1}{\sigma\sqrt{2\pi\tau}} \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} \varepsilon_n \left\{\exp\left\{\frac{-[u - \ln\frac{\theta n}{K} - k_1\tau]^2}{2\sigma^2\tau}\right\}\right\}$$

where we note that

$$\varepsilon_n\{(\cdot)\} = \int_0^{\infty}\int_0^{\infty} ... \int_0^{\infty} (\cdot) G(Y_1)G(Y_2)...G(Y_n) dY_1 dY_2 ... dY_n,$$

$$\text{and} \quad \varepsilon_0\{(\cdot)\} \equiv (\cdot) \quad \text{with} \quad \theta_0 = S.$$

To further simplify the expression for $C_1(S, \tau)$, let

$$C_1^{(1)}(S, \tau) \equiv e^{-(\lambda + r)\tau} \int_{-\infty}^{\infty} KH(u)(e^u - 1)\hat{K}(u, S, \tau) du$$

and

$$C_1^{(2)}(S,\tau) \equiv e^{-(\lambda+r)\tau} \int_{\ln c(0)}^{\infty} KH(u)(e^u-1)\hat{K}(u,S,\tau)du.$$

Firstly, for $C_1^{(1)}(S,\tau)$, we have:

$$\begin{aligned}
C_1^{(1)}(S,\tau) &= \frac{e^{-(\lambda+r)\tau}K}{\sigma\sqrt{2\pi\tau}} \int_0^{\infty} (e^u-1) \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} \varepsilon_n \left\{ \exp\left\{ \frac{-[u-\ln\frac{\theta n}{K}-k_1\tau]^2}{2\sigma^2\tau} \right\} \right\} du \\
&= \sum_{n=0}^{\infty} e^{-\lambda\tau} \frac{(\lambda\tau)^n}{n!} \varepsilon_n \left\{ \frac{e^{-r\tau}}{\sigma\sqrt{2\pi\tau}} \int_0^{\infty} (Ke^u-K) \exp\left[\frac{-[u-\ln\frac{\theta n}{K}-k_1\tau]^2}{2\sigma^2\tau} \right] du \right\}.
\end{aligned}$$

Using the change of variable $e^z = Ke^u$, we obtain

$$\begin{aligned}
C_1^{(1)}(S,\tau) &= \sum_{n=0}^{\infty} e^{-\lambda\tau} \frac{(\lambda\tau)^n}{n!} \varepsilon_n \left\{ \frac{e^{-r\tau}}{\sigma\sqrt{2\pi\tau}} \int_{\ln k}^{\infty} (e^z-K) \exp\left[\frac{-[z-\ln\theta n-k_1\tau]^2}{2\sigma^2\tau} \right] dz \right\} \\
&= \sum_{n=0}^{\infty} e^{-\lambda\tau} \frac{(\lambda\tau)^n}{n!} \varepsilon_n \left\{ C_E^D[SX_n e^{-\lambda k\tau}, K, r, q, \tau, \sigma^2] \right\},
\end{aligned}$$

where

$$\begin{aligned}
C_E^D[S,K,r,q,\tau,\sigma^2] &= Se^{-r\tau}N[d_1(S,K,r,q,\tau,\sigma^2)] - Ke^{-r\tau}N[d_2(S,K,r,q,\tau,\sigma^2)] \\
\text{with} \quad d_1(S,K,r,q,\tau,\sigma^2) &= \frac{\ln\frac{S}{K}+(r-q+\frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}, \\
d_2(S,K,r,q,\tau,\sigma^2) &= d_1(S,K,r,q,\tau,\sigma^2) - \sigma\sqrt{\tau}, \\
N[\alpha] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{\beta^2}{2}} d\beta, \\
\text{and} \quad X_n &= Y_1Y_2...Y_n \quad \text{and} \quad X_0 \equiv 1.
\end{aligned}$$

The details for this conclusion can be found in Chiarella (2003).

Next, for $C_1^{(2)}(S,\tau)$ we have

$$\begin{aligned}
C_1^{(2)}(S,\tau) &= \frac{e^{-(\lambda+r)\tau}K}{\sigma\sqrt{2\pi\tau}} \int_{\ln c(0)}^{\infty} H(u)(e^u-1) \sum_{n=0}^{\infty} \frac{(\lambda\tau)^n}{n!} \varepsilon_n \left\{ \exp\left[\frac{-[u-\ln\frac{\theta n}{K}-k_1\tau]^2}{2\sigma^2\tau} \right] du \right\} \\
&= \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \varepsilon_n \left\{ \frac{e^{-r\tau}}{\sigma\sqrt{2\pi\tau}} \int_{\ln c(0)}^{\infty} (Ke^u-K) \exp\left[\frac{-[u-\ln\frac{\theta n}{K}-k_1\tau]^2}{2\sigma^2\tau} \right] du \right\}.
\end{aligned}$$

Note that since $a(T) \geq K$, we know that $\ln c(0) \geq 0$. Thus using the change of variable

$e^z = Ke^u$, we obtain

$$C_1^{(2)}(S,\tau) = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \varepsilon_n \left\{ \frac{e^{-r\tau}}{\sigma\sqrt{2\pi\tau}} \int_{\ln[Kc(0)]}^{\infty} (e^z - K) \exp\left[\frac{-[z - \ln\theta n - k_1\tau]^2}{2\sigma^2\tau} \right] dz \right\}.$$

Hence it is readily shown that

$$C_1^{(2)}(S,\tau) = \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \varepsilon_n \left\{ \hat{C_E^D}[SX_n e^{-\lambda k\tau}, K, r, q, \tau, \sigma^2] \right\}$$

where

$$\begin{aligned} \hat{C_E^D}[S,K,r,q,\tau,\sigma^2] &= Se^{-r\tau}N[d_1(S,Kc(0),r,q,\tau,\sigma^2)] - Ke^{-r\tau}N[d_2(S,Kc(0),r,q,\tau,\sigma^2)], \\ \text{with} \quad \hat{d}_1(S,K,r,q,\tau,\sigma^2) &= \frac{\ln\frac{S}{Kc(0)} + (r - q + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}}, \\ \text{and} \quad \hat{d}_2(S,K,r,q,\tau,\sigma^2) &= \hat{d}_1(S,K,r,q,\tau,\sigma^2) - \sigma\sqrt{\tau}. \end{aligned}$$

Thus the final expression for $C_1(S,\tau)$ is given by

$$\begin{aligned} C_1(S,\tau) &= \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \varepsilon_n \left\{ C_E^D[SX_n e^{-\lambda k\tau}, K, r, q, \tau, \sigma^2] \right\} \\ &\quad - \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \varepsilon_n \left\{ \hat{C_E^D}[SX_n e^{-\lambda k\tau}, K, r, q, \tau, \sigma^2] \right\} \end{aligned}$$

B.2. Proof of Proposition 3

We begin this proof by examining the function $V_2^{(1)}(x,\tau)$.

$$\begin{aligned} V_2^{(1)}(x,\tau) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_0^{\tau} e^{-\left(\frac{\sigma^2\eta^2}{2} + k_1 i\eta + (r+\lambda) - \lambda A(\eta)\right)(\tau - s)} e^{-i\eta x} \\ &\quad \times e^{-i\eta \ln c(s)} \left[\frac{\sigma^2 c(s)}{2} + \left(\frac{c'(s)}{c(s)} - \frac{\sigma^2 i\eta}{2} + k_1 \right)(c(s) - 1) \right] ds d\eta \\ &= \frac{1}{2\pi} \int_0^{\tau} e^{-(r+\lambda)(\tau - s)} \int_{-\infty}^{\infty} e^{-\frac{\sigma^2\eta^2}{2}(\tau - s) - i\eta[k_1(\tau - s) + x - \ln c(s)]} e^{\lambda A(\eta)(\tau - s)} \\ &\quad \times \left[\frac{\sigma^2 c(s)}{2} + \left(\frac{c'(s)}{c(s)} - \frac{\sigma^2 i\eta}{2} + k_1 \right)(c(s) - 1) \right] ds d\eta. \end{aligned}$$

By Taylor series expansion, we know that

$$e^{\lambda A(\eta)(\tau-s)} = \sum_{n=0}^{\infty} \frac{\lambda^n(\tau-s)^n}{n!} A(\eta)^n,$$

and if we let

$$\begin{aligned}
p &= \frac{\sigma^2}{2}(\tau-s), \\
f_1(s) &= \frac{\sigma^2 c(s)}{2} + \left(\frac{c'(s)}{c(s)} + k_1\right)(c(s)-1) \\
\text{and} \quad f_2(s) &= \frac{\sigma^2 i}{2}(c(s)-1),
\end{aligned}$$

then we can rewrite $V_2^{(1)}(x,\tau)$ as

$$\begin{aligned}
V_2^{(1)}(x,\tau) &= \frac{1}{2\pi}\int_0^{\tau} e^{-(r+\lambda)(\tau-s)} \left[\int_{-\infty}^{\infty} e^{-p\eta^2 - i\eta[k_1(\tau-s)+x-\ln c(s)]} \sum_{n=0}^{\infty} \frac{\lambda(\tau-s)^n A(\eta)^n}{n!} \right. \\
&\qquad \left. \times \{f_1(s) - \eta f_2(s)\} d\eta\right] ds \\
&= \sum_{n=0}^{\infty} \frac{1}{2\pi}\frac{1}{n!}\int_0^{\tau} e^{-(r+\lambda)(\tau-s)}(\lambda[\tau-s])^n \\
&\qquad \times \left[\int_{-\infty}^{\infty} e^{-p\eta^2 - i\eta[k_1(\tau-s)+x-\ln c(s)]}\left\{\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} e^{-i\eta(\ln Y_1Y_2\ldots Y_n)}\right.\right. \\
&\qquad \left.\left. \times G(Y_1)G(Y_2)\ldots G(Y_n)dY_1dY_2\ldots dY_n\right\}\{f_1(s)-\eta f_2(s)\}d\eta\right] ds \\
&= \sum_{n=0}^{\infty} \frac{1}{2\pi}\frac{1}{n!}\int_0^{\infty}\cdots\int_0^{\infty}\int_0^{\tau} e^{-(r+\lambda)(\tau-s)}(\lambda[\tau-s])^n \\
&\qquad \times \left[\int_{-\infty}^{\infty} e^{-p\eta^2 - i\eta[k_1(\tau-s)+x+\ln(Y_1Y_2\ldots Y_n)-\ln c(s)]}\{f_1(s)-\eta f_2(s)\}d\eta\right] \\
&\qquad \times ds G(Y_1)G(Y_2)\ldots G(Y_n)dY_1dY_2\ldots dY_n \\
&= \sum_{n=0}^{\infty} \frac{1}{2\pi}\frac{\lambda^n}{n!}\varepsilon_n \left\{\int_0^{\tau} e^{-(r+\lambda)(\tau-s)}(\tau-s)^n \left[\int_{-\infty}^{\infty} e^{-p\eta^2-q\eta}\{f_1(s)-\eta f_2(s)\}d\eta\right] ds\right\},
\end{aligned}$$

where ε_n is as stated in Appendix B.1, and

$$q = i[x + \ln(Y_1Y_2\ldots Y_n) + k_1(\tau-s) - \ln c(s)].$$

Following Kucera and Ziogas (2003) we can readily show that

$$\begin{aligned} V_2^{(1)}(x,\tau) &= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Big\{ \int_0^{\tau} \frac{e^{-g(x,s;n)}(\tau-s)}{\sigma\sqrt{2\pi(\tau-s)}} \left[\frac{\sigma^2 c(s)}{2} \right. \\ & \left. + \left(\frac{c'(s)}{c(s)} + \frac{1}{2}\left[k_1 - \frac{x+\ln(Y_1...Y_n) - \ln c(s)}{\tau - s} \right] \right) \right] (c(s)-1)ds \Big\}, \end{aligned}$$

where

$$g(x,s;n) \equiv \frac{(x+\ln(Y_1Y_2...Y_n)+k_1(\tau-s)-\ln c(s))^2}{2\sigma^2(\tau-s)} + (r+\lambda)(\tau-s).$$

Next we return to the original space variable S by setting $C_2^{(1)}(S,\tau) = KV_2^{(1)}(x,\tau)$, with $S = Ke^x$. This results in

$$\begin{aligned} C_2^{(1)}(S,\tau) &= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Big\{ \int_0^{\tau} \frac{e^{-h_n(S,\xi)}(\tau-\xi)^n}{\sigma\sqrt{2\pi(\tau-\xi)}} \\ & \times \left[\frac{K\sigma^2 c(\xi)}{2} + \left(\frac{c'(\xi)}{c(\xi)} + \frac{1}{2}\left[k_1 - \frac{\ln\frac{S}{K} + \ln(Y_1...Y_n) - \ln c(\xi)}{\tau-\xi} \right] \right) \right] \\ & \times (Kc(\xi) - K)d\xi \Big\}, \end{aligned}$$

where

$$\begin{aligned} h_n(S,\xi) &\equiv \frac{\left(\ln\frac{S}{K} + \ln(Y_1Y_2...Y_n) + k_1(\tau-\xi) - \ln c(\xi)\right)^2}{2\sigma^2(\tau-\xi)} + (r+\lambda)(\tau-\xi) \\ &= \frac{\left[\ln\frac{SX_n}{Kc(\xi)} + k_1(\tau-\xi)\right]^2}{2\sigma^2(\tau-\xi)} + (r+\lambda)(\tau-\xi) \end{aligned}$$

with X_n as defined in Appendix B.1. Therefore $C_2^{(1)}(S,\tau)$ is given by

$$\begin{aligned} C_2^{(1)}(S,\tau) &= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Big\{ \int_0^{\tau} \frac{e^{-h_n(S,\xi)}(\tau-\xi)^n}{\sigma\sqrt{2\pi(\tau-\xi)}} \\ & \times \left[\frac{K\sigma^2 c(\xi)}{2} + \left(\frac{c'(\xi)}{c(\xi)} + \frac{1}{2}\left[k_1 - \frac{\ln\frac{SX_n}{Kc(\xi)}}{\tau-\xi} \right] \right) \right] \\ & \times (Kc(\xi) - K)d\xi \Big\}. \end{aligned}$$

Next we will aim to simplify the expression for $C_2^{(1)}(S,\tau)$ using the methods of Kim (1990).

For simplicity of notation, we define $G(\xi) \equiv Kc(\xi)$. The first step is to rewrite $h_n(S,\xi)$ as

$$\begin{aligned} h_n(S,\xi) &= \frac{\left[\ln(SX_n) - \ln G(\xi) + \left(r-q-\lambda k - \frac{\sigma^2}{2}\right)(\tau-\xi)\right]^2}{2\sigma^2(\tau-\xi)} + (r+\lambda)(\tau-\xi) \\ &= \frac{1}{2(\tau-\xi)}\left(\frac{\ln SX_n + \left(r-q-\lambda k-\frac{\sigma^2}{2}\right)\tau}{\sigma} - \frac{\ln G(\xi) + \left(r-q-\lambda k - \frac{\sigma^2}{2}\right)\xi}{\sigma}\right)^2 \\ &\quad +(r+\lambda)(\tau-\xi) \\ &= \frac{[y_n - H(\xi)]^2}{2(\tau-\xi)} + (r+\lambda)(\tau-\xi), \end{aligned}$$

where

$$\begin{aligned} y_n &\equiv \frac{\ln SX_n + \left(r-q-\lambda k - \frac{\sigma^2}{2}\right)\tau}{\sigma}, \\ \text{and } \; H(\xi) &\equiv \frac{\ln SX_n + \left(r-q-\lambda k-\frac{\sigma^2}{2}\right)\xi}{\sigma}. \end{aligned}$$

It is important to note that the derivative of $H(\xi)$ with respect to ξ is given by

$$H'(\xi) = \frac{1}{\sigma}\left(\frac{G'(\xi)}{G(\xi)} + \left(r-q-\lambda k - \frac{\sigma^2}{2}\right)\right).$$

Using these results, $C_2^{(1)}(S,\tau)$ becomes

$$\begin{aligned} C_2^{(1)}(S,\tau) &= \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{\int_0^\tau \frac{(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}}{\sqrt{2\pi(\tau-\xi)}} \\ &\qquad \times\left[\frac{\sigma G(\xi)}{2} + \frac{1}{\sigma}\left(\frac{G'(\xi)}{G(\xi)} + \left(r-q-\lambda k-\frac{\sigma^2}{2}\right) - \left(r-q-\lambda k - \frac{\sigma^2}{2}\right)\right.\right. \\ &\qquad \left.\left. +\frac{1}{2}\left[\left(r-q-\lambda k-\frac{\sigma^2}{2}\right) - \frac{\ln\frac{SX_n}{G(\xi)}}{\tau-\xi}\right]\right)(G(\xi)-K)\right]d\xi\Big\} \\ &= \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{\int_0^\tau e^{-(r+\lambda)(\tau-\xi)}(\tau-\xi)^n\frac{e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}}{\sqrt{2\pi(\tau-\xi)}} \\ &\quad \times\left[\frac{\sigma G(\xi)}{2} + \left(H'(\xi) - \frac{1}{\sigma}\left(r-q-\lambda k - \frac{\sigma^2}{2}\right)\right.\right. \\ &\quad \left.\left. - \frac{1}{2\sigma}\left[\frac{\ln SX_n - \ln G(\xi) - \left(r-q-\lambda k-\frac{\sigma^2}{2}\right)(\tau-\xi)}{\tau-\xi}\right]\right)(G(\xi)-K)\right]d\xi \end{aligned}$$

$$= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Big\{ \int_0^\tau e^{-(r+\lambda)(\tau-\xi)}(\tau-\xi)^n \frac{e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}}{\sqrt{2\pi(\tau-\xi)}} \times \left[\frac{\sigma G(\xi)}{2} + \left(H'(\xi) - \frac{y_n - H(\xi)}{2(\tau-\xi)}\right)(G(\xi)-K)\right] d\xi \Big\}.$$

Thus we arrive at the following expression for $C_2^{(1)}(S,\tau)$:

$$\begin{aligned} C_2^{(1)}(S,\tau) &= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Big\{ \int_0^\tau G(\xi) e^{-(r+\lambda)(\tau-\xi)}(\tau-\xi)^n \frac{e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}}{\sqrt{2\pi(\tau-\xi)}} \\ &\quad \times \left[\frac{\sigma}{2} + H'(\xi) - \frac{y_n - H(\xi)}{2(\tau-\xi)}\right] d\xi \\ &\quad -K \int_0^\tau e^{-(r+\lambda)(\tau-\xi)}(\tau-\xi)^n \frac{e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}}{\sqrt{2\pi(\tau-\xi)}} \\ &\quad \times \left[H'(\xi) - \frac{y_n - H(\xi)}{2(\tau-\xi)}\right] d\xi \Big\}. \end{aligned} \tag{35}$$

In order to simplify equation (35), we must derive two results. For the first result, we have

$$\begin{aligned} &(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)} G(\xi) \frac{e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}}{\sqrt{2\pi(\tau-\xi)}} \left[\frac{\sigma}{2} + H'(\xi) - \frac{[y_n - H(\xi)]}{2(\tau-\xi)}\right] \\ &= (\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)} \frac{G(\xi)}{\sqrt{\tau-\xi}} \left[\frac{\sigma(\tau-\xi) + 2H'(\xi)(\tau-\xi) - y_n + H(\xi)}{2(\tau-\xi)}\right] \\ &\quad \times \frac{1}{\sqrt{2\pi}} e^{-\frac{[y_n-H(\xi)+\sigma(\tau-\xi)]^2}{2(\tau-\xi)}} e^{\frac{\sigma^2}{2}(\tau-\xi)+\sigma(y_n-H(\xi))} \\ &= (\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)+\ln\frac{SX_n}{c(\xi)}+(r-q-\lambda k-\frac{1}{2}\sigma^2)(\tau-\xi)+\frac{\sigma^2}{2}(\tau-\xi)} \frac{1}{\sqrt{2\pi}} e^{-\frac{[y_n-H(\xi)+\sigma(\tau-\xi)]^2}{2(\tau-\xi)}} \\ &\quad \times G(\xi) \left[\frac{\frac{1}{2}\sigma(\tau-\xi) + H'(\xi)(\tau-\xi) - \frac{1}{2}y_n + \frac{1}{2}H(\xi)}{(\tau-\xi)\sqrt{\tau-\xi}}\right] \\ &= -(\tau-\xi)^n \frac{SX_n}{G(\xi)} G(\xi) e^{-q(\tau-\xi)-(\lambda+\lambda k)(\tau-\xi)} \frac{1}{\sqrt{2\pi}} e^{-\frac{[y_n-H(\xi)+\sigma(\tau-\xi)]^2}{2(\tau-\xi)}} \\ &\quad \times \left[\frac{\frac{1}{2}(y_n - H(\xi) + \sigma(\tau-\xi)) - (H'(\xi)+\sigma)(\tau-\xi))}{(\tau-\xi)\sqrt{\tau-\xi}}\right] \\ &= -(\tau-\xi)^n SX_n e^{-\lambda k(\tau-\xi)} e^{-(q+\lambda)(\tau-\xi)} \frac{1}{\sqrt{2\pi}} e^{-\frac{[y_n-H(\xi)+\sigma(\tau-\xi)]^2}{2(\tau-\xi)}} \\ &\quad \times \left[\frac{\frac{1}{2}\frac{1}{\sqrt{\tau-\xi}}(y_n - H(\xi) + \sigma(\tau-\xi)) - (H'(\xi)+\sigma)\sqrt{\tau-\xi}}{(\sqrt{\tau-\xi})^2}\right] \\ &= -(\tau-\xi)^n SX_n e^{-\lambda k(\tau-\xi)} e^{-(q+\lambda)(\tau-\xi)} \frac{\partial}{\partial\xi} N\left(\frac{y_n - H(\xi) + \sigma(\tau-\xi)}{\sqrt{\tau-\xi}}\right). \end{aligned} \tag{36}$$

For the second result, consider

$$
\begin{aligned}
&(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)}\frac{e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}}{\sqrt{2\pi(\tau-\xi)}}\left[H'(\xi)-\frac{[y_n-H(\xi)]}{2(\tau-\xi)}\right]\\
&=(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)}\frac{1}{\sqrt{2\pi}}e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}\left[\frac{2H'(\xi)\sqrt{\tau-\xi}-y_n+H(\xi)}{2(\tau-\xi)(\sqrt{\tau-\xi})}\right]\\
&=(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)}\frac{1}{\sqrt{2\pi}}e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}\left[\frac{H'(\xi)\sqrt{\tau-\xi}-\frac{1}{2}(y_n-H(\xi))\frac{1}{\sqrt{\tau-\xi}}}{(\sqrt{\tau-\xi})^2}\right]\\
&=-(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)}\frac{1}{\sqrt{2\pi}}e^{-\frac{[y_n-H(\xi)]^2}{2(\tau-\xi)}}\left[\frac{-H'(\xi)\sqrt{\tau-\xi}+\frac{1}{2}(y_n-H(\xi))\frac{1}{\sqrt{\tau-\xi}}}{(\sqrt{\tau-\xi})^2}\right]\\
&=-(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)}\frac{\partial}{\partial\xi}N\left(\frac{y_n-H(\xi)}{\sqrt{\tau-\xi}}\right). \qquad (37)
\end{aligned}
$$

Using equations (36) and (37) in equation (35), $C_2^{(1)}(S,\tau)$ becomes

$$
\begin{aligned}
C_2^{(1)}(S,\tau) &= \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{-\int_0^\tau(\tau-\xi)^n SX_n e^{-\lambda k(\tau-\xi)}e^{-(q+\lambda)(\tau-\xi)}\\
&\qquad\times\frac{\partial}{\partial\xi}N\left(\frac{y_n-H(\xi)+\sigma(\tau-\xi)}{\sqrt{\tau-\xi}}\right)d\xi\\
&\qquad+K\int_0^\tau(\tau-\xi)^n e^{-(\lambda+r)(\tau-\xi)}\frac{\partial}{\partial\xi}N\left(\frac{y_n-H(\xi)}{\sqrt{\tau-\xi}}\right)d\xi. \qquad (38)
\end{aligned}
$$

By applying integration by, we can evaluate equation (38) as follows::

$$
\begin{aligned}
C_2^{(1)}(S,\tau) &= \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{-SX_n\left[(\tau-\xi)^n e^{-(\lambda k+q+\lambda)(\tau-\xi)}N\left(\frac{y_n-H(\xi)+\sigma(\tau-\xi)}{\sqrt{\tau-\xi}}\right)\right]_0^\tau\\
&\qquad-\int_0^\tau(\tau-\xi)^{n-1}e^{-(\lambda k)(\tau-\xi)}e^{-(q+\lambda)(\tau-\xi)}\\
&\qquad\qquad\times[(\lambda[k+1]+q)(\tau-\xi)-n]\,N\left(\frac{y_n-H(\xi)+\sigma(\tau-\xi)}{\sqrt{\tau-\xi}}\right)d\xi\\
&\quad+K\Big\{\left[(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)}N\left(\frac{y_n-H(\xi)}{\sqrt{\tau-\xi}}\right)\right]_0^\tau\\
&\qquad-\int_0^\tau(\tau-\xi)^{n-1}e^{-(r+\lambda)(\tau-\xi)}[(\tau-\xi)(r+\lambda)-n]\\
&\qquad\qquad\times N\left(\frac{y_n-H(\xi)}{\sqrt{\tau-\xi}}\right)d\xi\Big\}\Big\}\\
&= \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{SX_n e^{-\lambda k\tau}e^{-(q+\lambda)\tau}\tau^n N\left(\frac{y_n-H(0)+\sigma\tau}{\sqrt{\tau}}\right)
\end{aligned}
$$

$$\int_0^\tau SX_n e^{-\lambda k(\tau-\xi)} e^{-(q+\lambda)(\tau-\xi)} (\tau-\xi)^{n-1}[(\lambda[k+1]+q)(\tau-\xi)-n]$$

$$\times N\left(\frac{y_n - H(\xi)+\sigma(\tau-\xi)}{\sqrt{\tau-\xi}}\right) d\xi$$

$$-Ke^{-(r+\lambda)\tau}\tau^n N\left(\frac{y_n-H(0)}{\sqrt{\tau}}\right)$$

$$-\int_0^\tau K(\tau-\xi)^{n-1}e^{-(r+\lambda)(\tau-\xi)}\left[(\tau-\xi)(r+\lambda)-n\right] N\left(\frac{y_n-H(\xi)}{\sqrt{\tau-\xi}}\right) d\xi \Bigg\}$$

$$= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Bigg\{ SX_n e^{-\lambda k\tau} e^{-(q+\lambda)\tau}\tau^n N\left(\frac{\ln\frac{SX_n}{G(0)} + \left(r-q-\lambda k+\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right)$$

$$-Ke^{-(r+\lambda)\tau}\tau^n N\left(\frac{\ln\frac{SX_n}{G(0)} + \left(r-q-\lambda k+\frac{\sigma^2}{2}\right)}{\sigma\sqrt{\tau}}\right)$$

$$+\int_0^\tau SX_n e^{-\lambda k(\tau-\xi)} e^{-(q+\lambda)(\tau-\xi)}(\tau-\xi)^{n-1}$$

$$\times[(\lambda[k+1]+q)(\tau-\xi)-n] N\left(\frac{\ln\frac{SX_n}{G(\xi)} + \left(r-q-\lambda k+\frac{\sigma^2}{2}\right)(\tau-\xi)}{\sigma\sqrt{\tau-\xi}}\right) d\xi$$

$$-\int_0^\tau Ke^{-(r+\lambda)(\tau-\xi)}[(\tau-\xi)(r+\lambda)-n]$$

$$\times N\left(\frac{\ln\frac{SX_n}{G(\xi)} + \left(r-q-\lambda k+\frac{\sigma^2}{2}\right)(\tau-\xi)}{\sigma\sqrt{\tau-\xi}}\right) d\xi \Bigg\}$$

$$= \sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}\varepsilon_n \left\{\hat{C_E^D}\left[SX_n e^{-\lambda k\tau}, K, r, q, \tau, \sigma^2\right]\right\}$$

$$+\sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Bigg\{ \int_0^\tau (\tau-\xi)^{n-1}\left[SX_n e^{-\lambda k(\tau-\xi)} e^{-(q+\lambda)(\tau-\xi)}[(\lambda[k+1]+q)(\tau-\xi)-n]\right.$$

$$\times N\left[d_1\left(SX_n e^{-\lambda k(\tau-\xi)}, Kc(\xi), r, q, \tau-\xi, \sigma^2\right)\right]$$

$$-Ke^{-(r+\lambda)(\tau-\xi)}[(\tau-\xi)(r+\lambda)-n]$$

$$\left.\times\ N\left[d_2\left(SX_n e^{-\lambda k(\tau-\xi)}, Kc(\xi), r, q, \tau-\xi, \sigma^2\right)\right]\right] d\xi \Bigg\},$$

which is the final result stated in Proposition 3.

B.3. Proof of Proposition 4

The term $V_2^{(2)}(x,\tau)$ is given by

$$V_2^{(2)}(x,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_0^\tau e^{-\left(\frac{\sigma^2\eta^2}{2}+k_1 i\eta+(r+\lambda)-\lambda A(\eta)\right)(\tau-s)} e^{-i\eta x}\lambda\Phi(\eta,s)ds d\eta,$$

where we recall that

$$\Phi(\eta, s) = \int_0^\infty e^{-i\eta \ln Y} G(Y) \left[\int_{\ln c(s)}^{\ln Y + \ln c(s)} e^{i\eta z} V(z, s) dz \right] dY.$$

We begin by changing the order of integration within $V_2^{(2)}(x, \tau)$:

$$\begin{aligned}
V_2^{(2)}(x,\tau) &= \frac{\lambda}{2\pi} \int_{-\infty}^{\infty} \int_0^\tau e^{-\left(\frac{\sigma^2\eta^2}{2} + k_1 i\eta + (r+\lambda) - \lambda A(\eta)\right)(\tau-\xi) - i\eta x} \\
&\quad \times \int_0^\infty e^{-i\eta \ln Y} G(Y) \int_{\ln c(\xi)}^{\ln Y + \ln c(\xi)} e^{i\eta z} V(z,\xi) dz dY d\xi d\eta \\
&= \lambda \int_0^\tau \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\left(\frac{\sigma^2\eta^2}{2} + k_1 i\eta + (r+\lambda) - \lambda A(\eta)\right)(\tau-\xi) - i\eta x} \\
&\quad \times \int_0^\infty e^{-i\eta \ln Y} G(Y) \int_{\ln c(\xi)}^{\ln Y c(\xi)} e^{i\eta z} V(z,\xi) dz dY d\xi d\eta \\
&= \lambda \int_0^\tau \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\lambda A(\eta)(\tau-\xi)} e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+x)} e^{-(r+\lambda)(\tau-\xi)} \\
&\quad \times \int_0^\infty e^{-i\eta \ln Y} G(Y) \int_{\ln c(\xi)}^{\ln Y c(\xi)} e^{i\eta z} V(z,\xi) dz dY d\eta d\xi \\
&= \lambda \int_0^\tau e^{-(r+\lambda)(\tau-\xi)} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\lambda A(\eta)(\tau-\xi)} e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+x)} \\
&\quad \times \int_0^\infty e^{-i\eta \ln Y} G(Y) \int_{\ln c(\xi)}^{\ln Y c(\xi)} e^{i\eta z} V(z,\xi) dz dY d\eta d\xi
\end{aligned}$$

Note that according to Taylor series expansions for e^x we have

$$e^{\lambda A(\eta)(\tau-\xi)} = \sum_{n=0}^{\infty} \frac{\lambda^n (\tau-\xi)^n}{n!} [A(\eta)]^n,$$

and note also that by the definition of the Fourier transform, we have

$$\begin{aligned}
\int_{\ln c(\xi)}^{\ln Y c(\xi)} e^{i\eta z} V(z,\xi) dz &= \int_{-\infty}^{\infty} H(\ln Y c(\xi) - z) H(z - \ln c(\xi)) V(z,\xi) e^{i\eta z} dz \\
&\equiv \hat{V}^{c(\xi), Y c(\xi)}(\eta, \xi).
\end{aligned}$$

Using these results, we can rewrite $V_2^{(2)}(x, \tau)$ as follows:

$$\begin{aligned}
V_2^{(2)}(x,\tau) &= \sum_{n=0}^{\infty} \frac{\lambda^{n+1}}{n!} \int_0^\tau (\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+x)} \\
&\quad \times \int_0^\infty \ldots \int_0^\infty e^{-i\eta \ln Y_1 \ldots Y_n} G(Y_1) \ldots G(Y_n) dY_1 \ldots dY_n
\end{aligned}$$

$$
\begin{aligned}
& \times \int_0^\infty e^{-i\eta \ln Y} G(Y) \hat{V}^{c(\xi),Yc(\xi)}(\eta,\xi) dY d\eta d\xi \\
= & \sum_{n=0}^{\infty} \frac{\lambda^{n+1}}{n!} \varepsilon_n \Big\{ \int_0^\tau (\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)} \int_0^\infty G(Y) \\
& \times \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+x+\ln X_n Y)} \hat{V}^{c(\xi),Yc(\xi)}(\eta,\xi) d\eta dY d\xi \Big\}.
\end{aligned}
$$

Consider the two innermost integrals, $I(x,\tau,Y,\xi)$, defined as

$$
\begin{aligned}
I(x,\tau,Y,\xi) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+x+\ln Y \ln X_n)} \\
& \times \int_{\ln c(\xi)-\ln Y}^{\ln c(\xi)} e^{i\eta(x+\ln Y)} V(x+\ln Y,\xi) dx d\eta,
\end{aligned}
$$

where the integral with respect to x has been derived using the change of variable $z = x + \ln Y$. To evaluate I, we shall express it as an inverse Fourier transform:

$$
\begin{aligned}
I(x,\tau,Y,\xi) &= -\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\eta x} e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+\ln X_n)} \\
& \times \int_{\ln c(\xi)}^{\ln c(\xi)-\ln Y} e^{i\eta x} V(x+\ln Y,\xi) dx d\eta \\
&= -\mathcal{F}^{-1} \left\{ e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+\ln X_n)} \int_{\ln c(\xi)}^{\ln c(\xi)-\ln Y} e^{i\eta x} V(x+\ln Y,\xi) dx \right\}. \quad (39)
\end{aligned}
$$

Since we know that $0 < Y < \infty$, we must now consider two separate cases to evaluate the inverse Fourier transform in equation (39). The first case to consider is when $0 < Y < 1$, which means we can rewrite equation (39) as

$$
\begin{aligned}
I(x,\tau,Y,\xi) &= -\mathcal{F}^{-1} \Big\{ e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+\ln X_n)} \\
& \times \int_{-\infty}^{\infty} H(\ln c(\xi) - \ln Y - x) H(x - \ln c(\xi)) e^{i\eta x} V(x+\ln Y,\xi) dx \Big\}.
\end{aligned}
$$

To evaluate this inversion, we again refer to the standard convolution result for Fourier transforms (see Section B.1). Let

$$
\hat{F}(\eta,\xi_1) = e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi) - i\eta(k_1(\tau-\xi)+\ln X_n)}.
$$

Therefore, $f(x,\xi)$ is given as

$$\begin{aligned} f(x,\xi_1) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\eta x} e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi)-i\eta(k_1(\tau-\xi)+\ln X_n)} d\eta \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-p\eta^2-q\eta} d\eta \end{aligned}$$

where $p=\frac{\sigma^2(\tau-\xi)}{2}$ and $q=i(k_1(\tau-\xi)+\ln X_N+x)$. Hence we have

$$\begin{aligned} f(x,\xi_1) &= \frac{1}{2\pi}\sqrt{\frac{\pi}{p}}e^{\frac{q^2}{4p}} \\ &= \frac{1}{2\pi}\sqrt{\frac{2\pi}{\sigma^2(\tau-\xi)}}\exp\left\{-\frac{[k_1(\tau-\xi)+\ln X_n+x]^2}{2\sigma^2(\tau-\xi)}\right\} \\ &= \frac{1}{\sigma\sqrt{2\pi(\tau-\xi)}}\exp\left\{-\frac{[k_1(\tau-\xi)+\ln X_n+x]^2}{2\sigma^2(\tau-\xi)}\right\}. \end{aligned}$$

For the second part of the convolution, let

$$\hat{G}^{b_2,c_2}(\eta,\xi_2)=\int_{-\infty}^{\infty} H(\ln c(\xi)-\ln Y-x)H(x-\ln c(\xi))e^{i\eta x}V(x+\ln Y,\xi)dx.$$

Therefore $g(x,\xi)$ is simply

$$g(x,\xi_2)=H(\ln c(\xi)-\ln Y-x)H(x-\ln c(\xi))V(x+\ln Y,\xi).$$

Combining f and g, the inverse Fourier transform I becomes

$$\begin{aligned} I(x,\tau,Y,\xi) &= -\int_{-\infty}^{\infty}\frac{1}{\sigma\sqrt{2\pi(\tau-\xi)}}\exp\left\{-\frac{[k_1(\tau-\xi)+(x-u)+\ln X_n]^2}{2\sigma^2(\tau-\xi)}\right\} \\ &\qquad \times H(\ln c(\xi)-\ln Y-u)H(u-\ln c(\xi))V(u+\ln Y,\xi)du \\ &= -\int_{-\infty}^{\infty}\frac{1}{\sigma\sqrt{2\pi(\tau-\xi)}}\exp\left\{-\frac{[k_1(\tau-\xi)+(x-u)\ln X_n]^2}{2\sigma^2(\tau-\xi)}\right\} \\ &\qquad \times H(\ln c(\xi)-\ln Y-u)H(u-\ln c(\xi))V(u+\ln Y,\xi)du \\ &= -\int_{\ln c(\xi)}^{\ln c(\xi)-\ln Y}\frac{V(u+\ln Y,\xi)}{\sigma\sqrt{2\pi(\tau-\xi)}}\exp\left\{-\frac{[k_1(\tau-\xi)+(x-u)\ln X_n]^2}{2\sigma^2(\tau-\xi)}\right\}du. \quad (40) \end{aligned}$$

In the second case, we have $1 < Y < \infty$, which means we can now rewrite equation (39) as

$$\begin{aligned} I(x,\tau,Y,\xi) &= \mathcal{F}^{-1}\Big\{ e^{-\frac{\sigma^2\eta^2}{2}(\tau-\xi)-i\eta(k_1(\tau-\xi)+\ln X_n)} \\ &\qquad \times \int_{-\infty}^{\infty} H(\ln c(\xi)-x)H(x-\ln c(\xi)+\ln Y)e^{i\eta x}V(x+\ln Y,\xi)dx \Big\}. \end{aligned}$$

Following the same method as used in the first case, we find that

$$\begin{aligned} I(x,\tau,Y,\xi) &= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi(\tau-\xi)}} \exp\left\{ -\frac{[k_1(\tau-\xi)+(x-u)+\ln X_n]^2}{2\sigma^2(\tau-\xi)} \right\} \\ &\qquad \times H(\ln c(\xi)-u)H(u-\ln c(\xi)+\ln Y)V(u+\ln Y,\xi)du \\ &= \int_{(\ln c(\xi)-\ln Y)}^{\ln c(\xi)} \frac{V(u+\ln Y,\xi)}{\sigma\sqrt{2\pi(\tau-\xi)}} \exp\left\{ -\frac{[k_1(\tau-\xi)+(x-u)+\ln X_n]^2}{2\sigma^2(\tau-\xi)} \right\} du. \quad (41) \end{aligned}$$

Since results (40) and (41) depend entirely upon the relevant value of Y, we can integrate piecewise over the Y-domain, and thereby express $V_2^{(2)}(x,\tau)$ as

$$\begin{aligned} V_2^{(2)}(x,\tau) &= \lambda \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \left\{ \int_0^{\tau} (\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)} \int_0^{\infty} G(Y)I(x,\tau,Y,\xi)dYd\xi \right\} \\ &= \lambda \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Big\{ \int_0^{\tau} (\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)} \Big[\int_0^1 G(Y) \int_{\ln c(\xi)}^{\ln c(\xi)-\ln Y} -\frac{V(u+\ln Y,\xi)}{\sigma\sqrt{2\pi(\tau-\xi)}} \\ &\qquad \times \exp\left\{ -\frac{[k_1(\tau-\xi)+(x-u)+\ln X_n]^2}{2\sigma^2(\tau-\xi)} \right\} dudY \\ &\qquad + \int_1^{\infty} G(Y) \int_{\ln c(\xi)-\ln Y}^{\ln c(\xi)} \frac{V(u+\ln Y,\xi)}{\sigma\sqrt{2\pi(\tau-\xi)}} \\ &\qquad \times \exp\left\{ -\frac{[k_1(\tau-\xi)+(x-u)+\ln X_n]^2}{2\sigma^2(\tau-\xi)} \right\} dudY \Big] d\xi \Big\}. \end{aligned}$$

Setting $C_2^{(2)}(S,\tau) = KV_2^{(2)}(x,\tau)$, we have

$$\begin{aligned} C_2^{(2)}(S,\tau) &= -\lambda \sum_{n=0}^{\infty} \frac{\lambda^n}{n!}\varepsilon_n \Big\{ \int_0^{1} (\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)} \Big[\int_0^1 G(Y) \int_{\ln c(\xi)}^{\ln c(\xi)-\ln Y} \frac{C(Ke^uY,\xi)}{\sigma\sqrt{2\pi(\tau-\xi)}} \\ &\qquad \times \exp\left\{ -\frac{[k_1(\tau-\xi)+(x-u)+\ln X_n]^2}{2\sigma^2(\tau-\xi)} \right\} dudY \\ &\qquad - \int_1^{\infty} G(Y) \int_{\ln\frac{c(\xi)}{Y}}^{\ln c(\xi)} \frac{C(Ke^uY,\xi)}{\sigma\sqrt{2\pi(\tau-\xi)}} \end{aligned}$$

$$\times \exp\left\{-\frac{[k_1(\tau-\xi)+(x-u)+\ln X_n]^2}{2\sigma^2(\tau-\xi)}\right\} du dY\Bigg] d\xi\Bigg\}.$$

Finally, we shall introduce some additional notation and a change of variable to simplify the expression for $C_2^{(2)}(S,\tau)$. Letting $\omega = Ke^u$,we have

$$\begin{aligned} C_2^{(2)}(S,\tau) &= -\lambda\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{\int_0^{\tau}(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)}\Bigg[\int_0^1 G(Y)\int_{Kc(\xi)}^{K\frac{c(\xi)}{Y}}\frac{C(\omega Y,\xi)}{\sigma\sqrt{2\pi(\tau-\xi)}} \\ &\qquad \times \exp\left\{-\frac{[k_1(\tau-\xi)+\ln\frac{SX_n}{\omega}]^2}{2\sigma^2(\tau-\xi)}\right\}\frac{1}{\omega}d\omega dY \\ &\qquad -\int_1^{\infty} G(Y)\int_{\frac{Kc(\xi)}{Y}}^{Kc(\xi)}\frac{C(\omega Y,\xi)}{\sigma\sqrt{2\pi(\tau-\xi)}} \\ &\qquad \times \exp\left\{-\frac{[k_1(\tau-\xi)+\ln\frac{SX_n}{\omega}]^2}{2\sigma^2(\tau-\xi)}\right\}\frac{1}{\omega}d\omega dY\Bigg] d\xi\Big\} \end{aligned}$$

Next, we consider carefully the domains for the integrals with respect to ω. For the first integral, the domain for ωY is

$$YKc(\xi) < \omega Y < Kc(\xi).$$

Thus ωY lies in the continuation region, and the value of $C(\omega Y,\xi)$ is unknown.

For the second integral, the domain for ωY is

$$Kc(\xi) < \omega Y < YKc(\xi).$$

Thus ωY lies in the stopping region, and therefore the value of $C(\omega Y,\xi)$ is known to be

$$C(\omega Y,\xi) = \omega Y - K, \quad \text{where} \quad \omega > \frac{K}{Y}.$$

Thus $C_2^{(2)}(S,\tau)$ can be written more simply as

$$C_2^{(2)}(S,\tau) = -\lambda\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\varepsilon_n\Big\{\int_0^{\tau}(\tau-\xi)^n e^{-(r+\lambda)(\tau-\xi)}$$

$$\times \left[\int_0^1 G(Y) \int_{Kc(\xi)}^{K\frac{c(\xi)}{Y}} C(\omega Y, \xi) J(\omega, \xi, SX_n, \tau) d\omega dY \right.$$
$$\left. - \int_1^\infty G(Y) \int_{\frac{Kc(\xi)}{Y}}^{Kc(\xi)} (\omega Y - K) J(\omega, \xi, SX_n, \tau) d\omega dY \right] d\xi \Big\},$$

where

$$J(\omega, \xi, SX_n, \tau) \equiv \frac{1}{\omega\sigma\sqrt{2\pi(\tau-\xi)}} \exp\left\{ -\frac{[k_1(\tau-\xi) + \ln\frac{SX_n}{\omega}]^2}{2\sigma^2(\tau-\xi)} \right\}$$
$$\text{and} \quad k_1 = r - q - \lambda k - \frac{\sigma^2}{2}.$$

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