Indeterminacy, Demand Shocks, and International Business Cycles

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Abstract

This study develops a two-country, two-good endogenous business cycle model to account for several empirical regularities of international business cycles, some of which are well-known "puzzles" that cannot be explained by the existing literature. The model has two novelties. One is mild increasing returns in production that give rise to indeterminate, multiple equilibria. The other is a persistent shock to consumer demand. The model can resolve the "cross-country correlation puzzle" while matching other key features of international business cycles. Simulation of the calibrated model shows that indeterminacy is crucial in generating hump-shaped responses of output to demand shocks and the "J-curve" property of the terms of trade.

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1 Introduction

The cornerstone of the Keynesian business cycle theory is the idea that booms and recessions are caused by fluctuations in aggregate demand. While remaining popular among policy-makers and the public, for a long time it seemed to have lost popularity in *academia*, largely because models with demand shocks cannot generate empirically-relevant business cycles within the framework of rational expectations and market-clearing. The standard real business cycle (RBC) model, for example, relies on productivity shocks to generate business fluctuations. However, recently a number of researchers have shown that with a slight modification of the environment, aggregate demand shocks can play a critical role in business cycle models (Farmer and Guo, 1994; Benhabib and Wen, 2002, among others). The key is to modify the model so that it has multiple convergent paths to the steady state equilibrium. This will allow sunspots and preference shocks to become important sources of business cycles.

Researchers find that demand shocks combined with indeterminacy can generate empirically plausible business cycles in RBC models. Farmer and Guo (1994) show that a simple RBC model driven by sunspots can replicate postwar business cycles better than a standard model driven by productivity shocks. Wen (1998) shows that when coupled with dynamic utilization rate, the model is capable of generating realistic business cycles with very mild increasing returns to scale. Harrison and Weder (2002) find that a sunspot-driven model can explain the data of the entire *Great Depression* era. Benhabib and Wen (2001) combine indeterminate equilibria with exogenous shocks to consumption and investment, and report the removal of several puzzles of the standard RBC model.

While much success has been reported for closed-economy models, works that incorporate aggregate demand shocks and indeterminacy into an open-economy general equilibrium model are rare. Guo and Sturzenegger (1998) and Xiao (2002) are the two exceptions. Both examine an international RBC (IRBC) model driven by sunspot shocks, and find that the model can generate positively correlated business cycles among countries - a fact that a standard IRBC model cannot reproduce. Both models, however, assume that there is only one consumption good. This makes it impossible for the model to shed light on certain aspects of international business cycles. For example, the dynamics of international trade and the volatility of relative prices can only be examined in a model with multiple commodities. Both models also only consider sunspot shocks, which are non-fundamental by nature and must be i.i.d. random variables that are not persistent. Recent research (Benhabib and Wen, 2001) show that sunspot-only economies do not match some important features of the data.

The goal of this paper is to enrich our understanding of international business cycles by extending the existing IRBC framework. We consider an economy with two countries and two productive sectors. The model has two essential elements. One is a persistent, exogenous shock to consumption demand. Incorporating this feature is motivated by recent empirical evidence that supports demand shocks as the main source of business cycles (Blanchard, 1989; Blanchard and Quah, 1989; Cochrane, 1994; Wen 2002), and theoretical work in this area (Benhabib and Wen, 2001; and Wen, 2001). In particular, Benhabib and Wen (2001)'s finding that demand shocks and indeterminacy combined can remove several weaknesses of a closed-economy RBC model is the major motivation.

The second element is mild increasing returns in production which are consistent with recent empirical estimates. This is the key that will give rise to indeterminate equilibria[1]. With indeterminacy, the model has an endogenous fluctuation mechanism: an economic expansion is followed by a recession, which in turn is followed by another expansion. This provides a natural amplification mechanism for demand shocks. This feature makes the paper distinct from previous studies of preference shocks in IRBC economies, such as the works of Stockman and Tesar (1995) and Wen (2002). Neither of their models considers equilibria that are indeterminate and have endogenous fluctuations.

[1] Intuitively, when equilibria are indeterminate, they are "self-fulfilling equilibria" in that the actual equilibrium path can be selected by realizations of sunspots. But this is only possible with increasing returns: from any given equilbrium, if agents have some optimistic expectations about future returns, they will increase their investment. Their expectations will be "self-fulfilling" only if future rates of returns on investment indeed increase. Constant-returns-to-scale technology does not deliver such results. Increasing returns, if strong enough, can produce a rise in the future rates of return, which justifies agents' earlier expectations. See Benhabib (1998) for a more detailed explanation.

The paper is also broadly related to other works that attempt to resolve puzzles in IRBC models. For example, the incomplete market models of Baxter and Crucini (1995) and Kollmann (1996), the endogenous incomplete market model of Kehoe and Perri (2000), and the financial autarky model of Heathcote and Perri (2002). These authors show that altering the asset market assumption can help explaining international data.

We use our model to study three topics of international business cycles. First, we try to reconcile the "cross-country correlation puzzle." As noted in the work of Backus, Kehoe, and Kydland (1992 and 1993), empirical international consumption correlations are smaller than international output correlations, whereas existing models predict consumption correlations much higher than output correlations. Furthermore, investment and employment tend to be positively correlated across countries, but the models predict a negative correlation. We believe that the two distinct features of this model, demand shocks and endogenous propagation, can go a long way towards removing the puzzle: first, demand shocks are direct shocks to consumption and naturally lower cross-country consumption correlations; Second, a rise in one country's demand increases imports from its trading partner, and therefore also increases investment, employment and output in the other country - this will generate positive international correlations of these variables.

The second issue is the so-called "price anomaly." It refers to the fact that the volatility of the terms of trade generated by existing models is too small compared with that of the data. Can this model resolve the price anomaly? Our intuition is as follows: it is well-known that with indeterminacy, there can be a continuum of convergent paths to the steady state equilibrium. Price levels can be different on each path regardless of fundamentals. When sunspot shocks shift the economy from path to path, price levels should exhibit volatile fluctuations, which will in turn render the terms of trade more volatile. In this paper we put this intuition to a test.

The third issue we consider is whether or not the model can account for the empirical regularities of open-economy business cycles. We not only examine the conventional statistics of business cycles, such as volatility and comovement, but also consider properties of the international sectors: cross-country relations of demand shocks and output, and the dynamic correlations of the terms of trade with the trade balance, often referred to as the "J-curve" property.

The central part of the paper consists of a comparison of the demand-driven business cycle model with a standard RBC model. We show that the demand-driven model is capable of resolving the cross-country correlation puzzle, while matching the remaining key quantitative predictions for international business cycles of the RBC model. Neither model, however, can generate high volatility for the terms of trade. This surprising result forces us to re-consider our earlier intuition and enhance our understanding of the puzzle. The fact that the model produces low cross-country consumption correlations but still cannot have more price volatility also shows that common understanding of the puzzle in the literature is not accurate.

The paper is organized as follows. Section 2 describes a two-country general equilibrium model, section 3 discusses the selection of parameter values and the method of computation, section 4 presents the model's simulation results and discusses major issues, and section 5 concludes.

2 A Two-Country Model

The model is an extension of Backus, Kehoe and Kydland (1993)'s IRBC model. There are two countries, domestic and foreign, each of which is populated by a large number of identical, infinitely lived agents. The two countries' preferences and technologies have the same structures and parameter values. We focus on describing the domestic economic environment; the foreign environment can be defined analogously. If mentioned, foreign country variables have stars as superscripts. All variables are in national per capita terms.

2.1 Households

The preferences of agents are characterized by an expected utility function

$$E_0 \sum_{t=0}^{\infty} \rho^t [\log(C_t - d_t) - \frac{N_t^{1+\gamma}}{1+\gamma}] \tag{1}$$

where C_t and N_t denote consumption and hours worked, d_t is a random shock to consumption that represents the urge to consume (Baxter and King, 1991), ρ is the intertemporal discount factor, and $\gamma \geq 0$ is the inverse labor supply elasticity.

Households own both factors of production, capital and labor, and sell their services in a competitive spot market to domestic firms. The firms produce a country-specific intermediate good (the domestic country produces good x, and foreign country good y). Having obtained factor income in the form of intermediate goods, households go to the international market to engage in two types of trade: commodity trade and asset trade. In the commodity market, households trade good x for good y, or vice versa. In the asset market, households trade one-period claims, each of which pays out one unit of intermediate good contingent on the states of nature. When the two markets close, households sell all their intermediate goods, both good x and y, to a domestic final-good producer in exchange for a final good which they either consume or invest. Letting the final good be the numeraire, we describe a representative household's budget constraint as[2]:

$$\begin{aligned} &C_t + K_{t+1} + p_t^x \sum_{S_{t+1}} B^x(S_{t+1}) q^x(S_{t+1}, S_t) + p_t^y \sum_{S_{t+1}} B^y(S_{t+1}) q^y(S_{t+1}, S_t) = \\ &w_t N_t + R_t K_t + p_t^x B^x(S_t) + p_t^y B^y(S_t) \end{aligned} \tag{2}$$

where p_t^x and p_t^y are the prices of domestic and foreign intermediate goods in terms of the final goods, w_t the wage rate, R_t the gross capital rental rate, and K_t the stock of physical capital. $B^x(S_t)$ and $B^y(S_t)$ are the domestic households' holdings of one-period claims whose payoffs are

[2] All quantities are state-contingent, but to simplify notation I omit the symbols for the state of nature unless the variables are directly related to the price or quantity of contingent claims.

made in good x or y. $q^x(S_{t+1}, S_t)$ and $q^y(S_{t+1}, S_t)$ are the prices of such claims.

Households maximize the objective (1) subject to the budget constraint (2) to yield:

$$\frac{1}{C_t - d_t} = \lambda_t \tag{3}$$

$$N_t^\gamma = \lambda_t w_t \tag{4}$$

$$\lambda_t = \rho E_t \lambda_{t+1} R_{t+1} \tag{5}$$

$$\lambda_t p_t^x q^x(S_{t+1}, S_t) = \rho \lambda_{t+1} p_{t+1}^x \tag{6}$$

$$\lambda_t p_t^y q^y(S_{t+1}, S_t) = \rho \lambda_{t+1} p_{t+1}^y \tag{7}$$

plus the transversality conditions. λ_t denotes the Lagrangian multiplier associated with the maximization problem. Equation (3) equates the marginal utility of consumption with with its opportunity cost (shadow price of capital). Equation (4) equates the marginal utility gained from a unit of leisure with its utility-measured cost. Equation (5) equalizes the current and future (shadow) values of capital investment. Equation (6) and (7) are the pricing equations for the state-contingent claims.

2.2 Firms

2.2.1 Intermediate-good producers

Each country consists of a large number of identical firms that specialize in producing a country-specific intermediate good. The domestic country's production function is

$$X_t = A_t(u_t K_t)^a N_t^b Q_t \tag{8}$$

where X_t is output of good x, A_t is a stochastic shock to productivity, and u_t is the dynamic utilization rate of capital. Q is a measure of production externalities and is defined as

$$Q_t = (\overline{u}_t \overline{K}_t)^{a\theta_1} \overline{N}_t^{b\theta_2} \tag{9}$$

where $\overline{K}$, $\overline{u}$ and $\overline{N}$ are the average economy-wide levels of capital, utilization rate and labor, which are exogenous from the point of view of each firm. When $\theta_1 = 0$ and $\theta_2 = 0$, the production function collapses to the standard RBC version of Burnside and Eichenbaum (1996). In this paper we allow for the possibility that $\theta_1 > 0$ and $\theta_2 > 0$. Since all firms are identical, in equilibrium we have $K_t = \overline{K}_t$, $N_t = \overline{N}_t$, and $u_t = \overline{u}_t$. By making the parameter substitutions $\alpha = a(1+\theta_1)$, and $\beta = b(1+\theta_2)$ we obtain an aggregate production function

$$X_t = A_t (u_t K_t)^{\alpha} N_t^{\beta} \tag{10}$$

From the perspective of each firm the technology still exhibits constant returns to scale ($a+b=1$), but now the aggregate production function has increasing returns to scale ($\alpha + \beta > 1$).

Investment drives the process of capital accumulation through the dynamic constraint

$$K_{t+1} = I_t + (1 - \delta_t) K_t \tag{11}$$

where δ_t is the rate of depreciation of capital stock defined as an increasing function of capital utilization rate u_t:

$$\delta_t = \tau u_t^{\eta},\ 0 < \tau < 1,\ \eta > 1 \tag{12}$$

The speed of capital depreciation is therefore endogenously determined[3].

[3] Without loss of generality, let $\tau = 1/\eta$.

The intermediate firms' problem is

$$\max_{K_t, N_t, u_t} p_t^x X_t - w_t N_t - (R_t - 1 + \delta_t) K_t$$
$$s.t.\, X_t \geq 0, \text{ (8), (11) and (12)}$$

To see the role of dynamic utilization, note that the intermediate firms' optimal choice of utilization is governed by the first order condition

$$u_t^\eta = a \frac{p_t^x X_t}{K_t}, \tag{13}$$

which we can use to express u_t as a function of X_t. Substituting this expression into equation (10) in equilibrium, output can be expressed as

$$X_t = a^{\frac{\alpha}{\eta-\alpha}} p_t^{x\,\frac{\alpha}{\eta-\alpha}} A_t^{\frac{\eta}{\eta-\alpha}} K_t^{\frac{\alpha(\eta-1)}{\eta-\alpha}} N_t^{\frac{\eta\beta}{\eta-\alpha}} \tag{14}$$

In this equation the effective returns to labor $\frac{\eta\beta}{\eta-\alpha}$ can exceed one while the true labor-output elasticity b in (8) is still less than one. In other words, labor-output elasticity is amplified as long as $\theta_2 > 0$. Wen (1998) shows that this setup dramatically reduces the level of increasing returns required for indeterminate equilibria.

2.2.2 Final-good producers

After the intermediate goods are traded in the international market, x_t of good x are in the hands of domestic households, and x_t^* are in the hands of foreigners. Good y is allocated similarly. In each country there are a large number of final-good producers, who purchase both intermediate goods and use them to produce a consumption good using the constant-returns-to-scale production technology

$$G(x_t, y_t) = [\omega_1 x_t^{-\phi} + \omega_2 y_t^{-\phi}]^{-\frac{1}{\phi}} \tag{15}$$

$$G(x_t^*, y_t^*) = [\omega_1 y_t^{*-\phi} + \omega_2 x_t^{*-\phi}]^{-\frac{1}{\phi}} \tag{16}$$

This is the Armington Aggregator used by Backus, Kehoe and Kydland (1994). Note that when $\omega_1 > \omega_2$, there is a home bias in the composition of domestically produced final goods.

Domestic final-good producers solve a maximization problem

$$\max_{x_t, y_t} G(x_t, y_t) - p_t^x x_t - p_t^y y_t \tag{17}$$

$$s.t. \, x_t, \, y_t > 0$$

which yields the equilibrium prices of domestic and foreign intermediate goods in terms of the final good as

$$p_t^x = \frac{\partial G(x_t, y_t)}{\partial x_t}, \, p_t^y = \frac{\partial G(x_t, y_t)}{\partial y_t} \tag{18}$$

2.3 Dynamic equilibrium

A dynamic equilibrium consists of a set of prices and quantities such that the consumers and producers maximize their objectives as described above, and all markets clear.

Market clearing for intermediate goods requires that

$$X_t = x_t + x_t^* \tag{19}$$

$$Y_t = y_t + y_t^* \tag{20}$$

Final goods are not traded internationally, hence we have in equilibrium

$$C_t + I_t = G(x_t, y_t) \tag{21}$$

$$C_t^* + I_t^* = G(x_t^*, y_t^*) \tag{22}$$

Asset market clearing requires that

$$B^x(S_{t+1}) + B^{x*}(S_{t+1}) = 0 \quad \forall S_{t+1} \tag{23}$$

$$B^y(S_{t+1}) + B^{y*}(S_{t+1}) = 0 \quad \forall S_{t+1} \tag{24}$$

As in Backus, Kehoe and Kydland (1993), we define the terms of trade as the ratio of import and export prices:

$$tot = p_t^y / p_t^x,$$

and net exports as

$$nx_t = \frac{x_t^* - tot \cdot y_t}{X_t}$$

3 Calibration and Computation

3.1 Calibration of parameters

Most parameter values are chosen to conform with the RBC literature. The steady-state real interest rate is set equal to 1 percent per quarter, which is close to the average rate of return on capital in the US over the past century. This implies a discount factor of 0.99. The steady-state depreciation rate is consistent with properties of quarterly data (2.5% per quarter or 10% per annum). The steady-state leisure is equal to 80% of time endowment. Following Benhabib and Wen (2001), the steady-state value of d is chosen so that the ratio $\frac{d}{C}$ is equal to 0.1. The parameters for capital and labor shares are set at $a = 0.36$ and $b = 0.64$. The variable that governs the elasticity of substitution between foreign and domestic intermediate goods is ϕ[4]. We use Backus, Kehoe and Kydland (1993)'s estimate of 1.5. We set the level of ω_1 and ω_2 such that the steady state output level can be normalized to 1, and the steady state imports to output ratio is 0.18, which is equal to its average value in OECD countries between 1970 and 2001.

In order to generate indeterminacy, a key condition is $\theta_1 > 0$ and $\theta_2 > 0$. In this model the minimum level of externality required to yield indeterminacy is 0.12. We use a value of 0.14 in the simulation ($\theta_1 = 0.14$, $\theta_2 = 0.14$). This implies a level of increasing returns that is broadly consistent with recent empirical evidence (Basu and Fernald, 1997).

Conventional measures of Solow residuals are inappropriate for this model because they are estimated based on the assumption of fixed utilization rates. In a recent empirical work, Burnside, Eichenbaum and Rebelo (1996) re-estimate the properties of productivity shocks by employing electricity use as a proxy for capacity utilization. We follow their approach to estimate a transition matrix using the Solow residuals of the US and an aggregate of European countries. The data sources are described in the appendix[5]. The VAR(1) estimation yields

$$\begin{bmatrix} \log A_t \\ \log A_t^* \end{bmatrix} = \begin{bmatrix} 0.9824 & 0.0136 \\ 0.1070 & 0.9112 \end{bmatrix} \begin{bmatrix} \log A_{t-1} \\ \log A_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_t^A \\ e_t^{A*} \end{bmatrix}, \tag{25}$$

For simulation, we use a symmetric transition matrix so that the results are not affected by country sizes. This will also facilitate exposition as we only have to report the simulation results for one country. The symmetric matrix is chosen such that its eigenvalues match those in (25). This yields a persistent parameter of 0.947 and a "spill-over" parameter of 0.052. The estimated correlation between e_t^A and e_t^{A*} is about 0.133. With the assumption of increasing returns, the

[4] The elasticity of substitution is $1/(1+\phi)$.

[5] A caveat follows. The Burnside, Christiano and Eichenbaum (1996) estimation is based on industry level data. Labor hours and electricity usage are used to proxy N_t and u_t. Since the same data is not available for European countries, I use employment indices and a general utilization rate as proxies. Therefore the estimation cannot be exactly accurate. However, it can serve as a useful approximation. In fact the estimation results are quite consistent with other authors' findings that productivity shocks are persistent and have small spill-over (Backus, Kehoe and Kydland, 1993 and Baxter and Crucini, 1995). Note also that when the economy is driven only by demand shocks, this matrix has no effect on system dynamics.

two coefficients are 0.946 and 0.052, and the correlation between $e_t^{\tilde{z}}$ and $e_t^{\tilde{z}*}$ is 0.125. Since the estimates for the two cases are very close, we use the same coefficients in all experiments. That is, we use 0.95 and 0.05 for the transition matrix, and 0.13 for the correlation of productivity shocks.

In the literature two approaches have been used to estimate the properties of preference shocks. One approach is to estimate preference shocks using the model's first order conditions. Baxter and King (1991), for example, derive preference shocks from the labor demand function of a closed-economy model, and obtain an estimate of 0.97 for the persistence parameter. The other approach is to use measures for consumer sentiment as a proxy. For example, Guo and Sturzenegger (1998) use the Consumer Sentiment Index from the US and the Harmonized Consumer Survey from Europe as proxies in a bivariate VAR regression, and find the estimated persistence parameters ranging from 0.9 to 0.5 for the US and 0.7 to 0.5 for Europe, and the correlation of innovations is 0.45.

These empirical studies suggest that preference shocks are highly persistent and positively correlated across countries. Our own estimation confirms this. We use the consumer sentiment indices for the US and EU to run a bivariate VAR(1) process, and obtain the following:

$$\begin{bmatrix} \log d_t \\ \log d_t^* \end{bmatrix} = \begin{bmatrix} 0.9364 & -0.4 \\ 0.01 & 0.879 \end{bmatrix} \begin{bmatrix} \log d_{t-1} \\ \log d_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix}, \tag{26}$$

where the correlation of ε_t and ε_t^* is 0.35. The symmetric matrix that matches the eigenvalues of the above has a persistence parameter of 0.83 and a spill-over parameter of 0.07. For simulation, we use a persistence parameter of 0.9, no spill-over, and a correlation coefficient of 0.3. This calibration matches those of Wen (2002) for an IRBC economy with demand shocks.

3.2 Computation

To analyze the short-run dynamics of the model, we linearize the first order conditions around the steady state as in King, Plosser and Rebelo (1988). The dynamics of the economy depend on the system

$$\begin{bmatrix} S_t \\ \Lambda_t \end{bmatrix} = J \begin{bmatrix} S_{t-1} \\ \Lambda_{t-1} \end{bmatrix} + De_t, \tag{27}$$

where $S_t = (\widehat{K}_t, \widehat{K_t^*}, \widehat{A}_t, \widehat{A_t^*}, \widehat{d}_t, \widehat{d_t^*})'$, $\Lambda_t = (\widehat{\lambda}_t, \widehat{\lambda_t^*})'$, and the "hat" denotes log-linearized variables; e_t is a vector of shocks, including sunspots; and J and D are matrices of parameters.

Under regular parameterization of the RBC literature, the system has a unique "saddle-path" solution. In that case system dynamics depend on the evolution of S_t in response to exogenous shock e_t. Λ_t is always solved as functions of S_t since it is associated with explosive roots. With increasing returns to scale, the system can have indeterminate multiple equilibria or a "sink" solution[6]. In this case the economy can have richer dynamics for two reasons: (1) Λ_t is associated with stable roots and does not have to move one-to-one with the state and exogenous variables; and (2) With complex roots, the system fluctuates endogenously, even when the exogenous shocks are transitory. In the next section we show that indeterminacy is crucial for the model in terms of replicating the dynamic relationship between demand shocks and output.

4 Simulation Results

We run stochastic simulations by feeding the calibrated shocks processes into the model. The simulations produce artificial time series data which we use to compare with the properties of quarterly time series data of the United States (1960 - 2001). When examining cross-country relations, we also use the aggregated time series of the European Union. Simulation results are reported in the tables. Since in each simulation we calibrate the volatility of technology or demand shocks to match the volatility of US output, all the standard deviations reported in the tables are relative (to output) standard deviations.

To isolate the effects of demand shocks and indeterminacy, we run several experiments. As

[6] See Benhabib and Farmer (1994) and Wen (1998) for technical details.

a benchmark, we simulate a standard RBC economy with constant returns to scale, and allow technology shocks (25) to be the only source of uncertainty. This economy is labeled "RBC (tech-shock)" in all the tables. In the second experiment, we eliminate productivity shocks and let demand shock (26) be the impulse mechanism. We label this economy "DBC (constant return)," where "DBC" stands for "demand-driven business cycle." In the third experiment, we set the returns to scale at 1.1. This economy has increasing returns, but does not have indeterminate equilibria. We label it "DBC (increasing returns)." Finally, we simulate an economy with increasing returns ($\alpha + \beta = 1.14$) and indeterminate equilibria, and label it "DBC (indeterminacy)." All DBC models are driven by demand shocks only.

4.1 The cross-country correlation puzzle

A standard open-economy RBC model typically predicts counterfactual international comovements. Our simulation of the RBC economy (table 1) shows the puzzle in detail. In the model (row 3), cross-country correlations are much higher for consumption (0.84) than for output (-0.29), while in the data (row 1) the opposite is true. In the model, cross-country correlations of employment (-0.53), investment (-0.64) and output (-0.29) are negative, while in the data they are positive.

In the literature the most popular and widely-accepted explanation for the puzzle is that there is unrealistic risk-pooling in the model. The model assumes a complete international asset market of state-contingent claims in which agents can perfectly pool country-specific risks. Risk-pooling allows consumption to move up and down with world output and does not depend on country-specific fluctuations.

In efforts to resolve the puzzle, several authors have considered models of incomplete asset markets (Baxter and Crucini, 1995; Kollmann, 1996; Kehoe and Perri, 2000). But surprisingly, these studies could not produce very satisfactory results. For example, Baxter and Crucini (1995) find that in a single-bond economy, cross-country consumption correlation does become less than cross-country output correlation. However, the cross-country correlations of consumption, investment and labor hours all become negative. This result depends critically on the assumption of unit root technology shocks.

The fact that a complete asset market is not the *only* answer to the question can be shown with a numeric experiment. Consider two autarkic countries that do not trade any asset or commodity, and the only linkage between them is the technology shock process in (25). If a complete asset market is the problem, then this economy should surely be free from the puzzle. We show the simulation result in row 2 of table 1[7]. Strikingly the puzzle still persists.

In figure 1, we plot the impulse responses of the RBC economy to a productivity shock in the home country. The dotted lines represent impulse response functions of the autarkic economy, and the solid lines represent those of the complete asset-market economy. In the latter economy, on impact the home country's consumption, hours, investment and output all increase. But in the foreign country, all variables but consumption decrease. This is precisely why cross-country correlations are positive for consumption but negative for the other variables. What is surprising is how these variables respond under autarky. In terms of the *direction* of response, the autarkic economies behave virtually the same as the complete-market version. That is, consumption still "moves together" and output "moves apart." What's different are the sizes of responses. For example, the size becomes larger for consumption and smaller for other variables. This is no doubt due to the elimination of asset markets which reduces the level of consumption smoothing and the size of cross-border resource flows. But these changes do not alter the pattern of international correlations (table 1). Therefore, we conclude that asset-market restrictions cannot completely resolve the puzzle.

The key is to understand what mechanisms other than risk-pooling are in play here. Consider a productivity shock in the home country. The home households' response should be the same as in a closed-economy RBC model. The productivity shocks are propagated through two distinct

[7] To run the experiment, I slightly modify the model as follows. Without international trade, the final-good sector is redundant. Each country's aggregate demand equals its output. The model essentially collapses to two closed-economies with the only international linkage of technology spill-over.

effects: a wealth effect that tends to increase both consumption and leisure, and an intertemporal substitution effect that tends to decrease leisure and increase employment and investment (The latter effect can be further decomposed into a wage effect and an interest rate effect.). By setup, the RBC model always has an intertemporal substitution effect that is stronger than the wealth effect, so that employment would increase during an economic expansion.

The same conclusion, however, cannot be drawn for the foreign country, because the ranking of the two effects are determined by the size of productivity changes[8]. In this model the only channel through which the foreign productivity can be changed is the "spill-over" of shocks, which is only equal to 5 percent of the home productivity change on initial impact. The small rise in wage and interest rate cannot create a strong substitution effect to dominate the wealth effect, so leisure goes up and labor hours and investment go down, creating an economic recession in the foreign country. This is true regardless of the structure of the asset markets. Hence, besides complete asset markets, the propagation of shocks is also responsible for the puzzle.

This paper considers an alternative source of fluctuations: shocks to consumer demand. A demand shock differs from a supply shock in its propagation mechanism. A positive demand shock at the home country raises people's desire to consume and causes an upward shift of aggregate demand. In an open economy this increases productive activities in both countries, because the final-good firms need both home and foreign intermediate goods to produce the consumption goods. The demand for imports increases foreign country's employment, investment and output. The concerted responses of these variables in the two countries should generate positive cross-country correlations. What is more, demand shocks enter the consumption Euler equation rather than the production function. These fluctuations naturally lower consumption correlation between the two countries.

This intuition can be numerically tested. The rest of table 1 (row 4 to row 6) reports the simulation results. The model's predictions are dramatically different from previous versions. The

[8] One way to see this is to note that wage rate is equal to $bA_t p_t^x K_t^a N_t^{b-1} Q$; the size of A_t determines the size of wage rate change and therefore the wage effect. The same applies to the interest rate.

constant-returns-to-scale economy predicts positive correlations of all variables but investment. The two increasing-returns-to-scale economies predict positive international correlations for all four variables, and a higher correlation for output than for consumption. Figure 2 - 4 plot the impulse response functions of the DBC economies, so that a direct comparison with the RBC economy can be made.

A comparison of row 4 and 5 of table 1, or equivalently, figure 2 and 3, highlights the importance of increasing returns to scale. With constant returns, a demand shock can initiate an economic boom in the home country, but only to a mild degree. Investment increases for only 0.15 percent, and output for about 0.4 percent. Its impact on foreign investment is even negative. This result has been extensively analyzed by Baxter and King (1991). Essentially, when there are constant returns to scale, a demand shock cannot sufficiently raise the wage rate and the interest rate. Therefore investment and employment only increase mildly. With increasing returns to scale, the effect of demand shock is much stronger: higher rates of returns helps to boost both domestic and foreign investments. As figure 3 shows, with increasing returns, a demand shock gives rise to more than 1.5 percent increase in domestic investment and about 0.4 percent increase in foreign investment, resulting in economic booms in both countries.

A comparison of figure 3 and 4 highlights the importance of endogenous fluctuations under indeterminacy. With indeterminacy, a demand shock not only causes economic booms in both countries, but also triggers the type of hump-shaped responses of hours, investment and output that we typically see in the data. Economic expansions are followed recessions, which are in turn followed by expansions. Essentially, there is a multiplier-accelerator mechanism in play: a rise in consumption demand stimulates capital accumulation, which reduces its marginal product. As a result employment goes down and output shrinks, which drives up the marginal products and triggers another round of expansion (see Farmer and Guo, 1994).

4.2 The price anomaly

Several authors have reported the inability of theoretical IRBC models to generate volatile terms of trade (Backus, Kehoe and Kydland, 1994; Zimmermann, 1991; and Stockman and Tesar, 1995). At the outset, we had expected indeterminacy models to remove this anomaly. Our intuition is as follows: with indeterminacy, there can be a continuum of convergent paths to the steady state equilibrium regardless of the fundamentals. Also, there can be sunspot shocks that shift the economy from one path to another (Farmer and Guo, 1994). Imagine that on some paths the price levels are high, but on others the price levels are low. When sunspot shocks shift the economy from path to path, price levels will fluctuate dramatically. Therefore the relative prices, that is, the terms of trade, should be more volatile.

Surprisingly, the model predictions are quite different from what our intuition suggests. The volatility of the terms of trade is significantly smaller in all versions of the theoretical economy than it is in the data. In table 2, the standard deviation of the terms of trade ranges from 5 to 33 percent in the theoretical economies, but is well above 170 percent in the US data (relative to output). The anomaly is robust under different model specifications. In the row labeled "large import share," we let imports be 40% of output in the steady state. In the row labeled "small elasticity," we change the elasticity of substitution of the final-good production from 1.5 to 0.5. In the last experiment labeled "multiple shocks," we let the economy be disturbed by demand and productivity shocks at the same time. For the indeterminacy economy, we also consider a version that is driven by sunspot shocks alone. None of these changes produces realistic volatility for the terms of trade.

So what went wrong? It seems that we neglected the real effect of sunspot disturbances. Recall that with increasing returns, the effective elasticity of labor supply is very large. When the economy is subject to sunspot shocks, the labor market responds strongly, and hence creates large real effect. In our simulations, it is true that when we add sunspot shocks to an existing model, the *absolute volatility* of the terms of trade will increase. But the real effect of sunspot shocks is so large that the absolute volatility of hours, investment and output also increase dramatically, rendering the *relative volatility* of the terms of trade unchanged. Note that the empirical puzzle is more related to relative volatilities: when models are calibrated to match output volatility, they cannot simultaneously account for the (relative) large volatility of the terms of trade. Our model increases the volatilities of output and the terms of trade proportionally, and hence cannot be considered as a successful resolution of the puzzle.

In the literature, the price anomaly is often understood as being closely related to the size of cross-country consumption correlations. For example, Heathcote and perri (2002) state that "The equilibrium real exchange rate in complete market models is closely related to the ratio of consumptions across the two countries. Since consumption is highly correlated across countries in the models, this ratio displays low volatility, and the real exchange rate is consequently less volatile than in the data[9]." Our numerical experiment shows that this is not necessarily true. For example, the demand-driven models can reduce cross-country consumption correlations to 0.34, yet the relative price volatility does not increase, but decreases from the predictions of the RBC model.

The terms of trade is defined as

$$tot = \frac{p_t^y}{p_t^x} = \frac{\partial G(x_t, y_t)/\partial y_t}{\partial G(x_t, y_t)/\partial x_t} = \frac{\omega_2}{\omega_1}(\frac{y_t}{x_t^*})^{1+\phi}, \tag{28}$$

which implies that its volatility is closely related to the volatility and correlation of imports and exports. In the data, neither of the two statistics is large enough to support the size of the volatility of the terms of trade. In real life, the terms of trade are calculated as the ratio of export and import prices, which are denominated in money terms. Such prices are strongly influenced by fluctuations in the foreign exchange markets. The current definition is unable to capture these fluctuations. It is possible that a modification of the economic environment is necessary for the relative price

[9] Although this statement is about real exchange rates, its implications are the same for the terms of trade, because the two relative prices have very similar definitions in the model.

volatility to be accounted for. As Backus, Kehoe and Kydland (1993, page 26) suggest, "at the very least, the tight connection between prices and quantities implied by first order conditions (like 28)must be abandoned." In our context, we need a model in which sunspots have separate effects on prices and real variables. That is, sunspots need to cause large fluctuations in prices, but not in real variables such as output and investment. We intend to pursue this project in the future.

4.3 Open-economy business cycles and the role of indeterminacy

Next we examine business cycle properties of the theoretical economy. We not only examine the conventional statistics of business cycles, such as volatility, comovement and persistence, but also consider properties of the international sectors: relationships between demand shocks and output, and the dynamic correlations of the terms of trade with the trade balance, often referred to as the "J-curve" property.

4.3.1 Measures of volatility and comovement

We examine a set of unconditional first and second moments commonly used in the literature for evaluating the empirical success of RBC models. For each variable in table 3, the numbers outside the parentheses are the model's predictions for volatilities. All versions of the model predict moderate volatility of consumption and hours, and high volatility of investment and utilization rate. All versions correctly predict the high volatility of exports and imports, and the moderate volatility of net exports.

The numbers inside the parentheses are each variable's contemporaneous correlation with output. All versions of the model predict procyclical consumption, hours, investment and utilization rate. The data suggests that net exports are counter-cyclical, imports are strongly pro-cyclical, and exports are pro-cyclical in most countries. The demand-driven economy correctly predicts all of these features.

When the constant-return economy is driven by demand shocks, however, the model tends to over-predict consumption volatility (1.13) and under-predicts investment volatility (0.6). This is easy to understand. Consider the first order condition

$$\frac{C_t - d_t}{p_t^x} = \frac{C_t^* - d_t^*}{p_t^{x*}} \tag{29}$$

Consumption is forced to respond to country-specific demand shocks, and will naturally become more volatile. Investment is less volatile because much of the demand shocks have been absorbed by changes in consumption. With constant returns (and therefore decreasing marginal products), there is little incentive for investment to respond to the shocks. Increasing returns entail higher payoffs for investment and production, and will enhance the responses of investment and correspondingly dampen the response of consumption. As Columns 4 and 5 of table 3 report, the predictions of the increasing-returns-to-scale DBC economies generally match those in the data.

4.3.2 Dynamic correlations between demand shocks and output

We next focus on the relationship between demand shocks and output. The last row of table 3 shows that the relative volatility of consumer sentiment index is 1.8, and its contemporaneous correlation with output is 0.3. Figure 5 (left panel) provides further evidence of the dynamic relationship between the two variables in the US and the EU. The figure shows correlations between the log of consumer sentiment index and the log of real GDP at different leads and lags up to 8 quarters (More precisely, the figure plots the correlation between $senti_t$ and Y_{t+j} against j where j represents the number of leads or lags in quarters). The plot displays an asymmetric shape: consumer sentiment tends to be positively correlated with future output, but negatively correlated with past output. In other words, high GDP tends to be preceded by high values of consumer sentiment and followed by low values. The positive correlation between $senti_t$ and Y_{t+j} for $j > 0$ indicates that movement in consumer sentiment leads movement in output. This finding is consistent with the evidence of "consumption-led fluctuations" in the literature (Cochrane, 1994, and Blanchard, 1993).

The relationship between demand shocks and output can be more rigorously described with a

VAR analysis. Using the data of the US and the EU, we calculate the impulse response functions of output to one unit of exogenous shock to aggregate demand. The upper panel of figure 5 plots the response of output in both economies to a shock in the US, and the lower panel plots the response to a shock in the EU. The plot displays two distinct features. One is that output in both economies responds positively to a demand shock in either country. The other is that the impulse response curves display the well-known trend-reverting (hump) shape[10].

These empirical regularities suggest the possibility of causal relationship between demand shocks and output. If this relationship exists, it would be consistent with the theoretical construct of this paper. In what follows, we examine whether or not the simulated theoretical model can replicate these empirical regularities as a way of deductive inference.

We first look at unconditional volatility and contemporaneous correlations. The last row of table 3 shows that the two demand-driven economies with a unique equilibrium over-predicts the sizes of these statistics. The constant-return economy requires a relative standard deviation of 12 for demand shocks to match the US output volatility, and predicts a correlation between output and demand shocks as high as 0.95. The indeterminacy economy, on the other hand, can predict more realistic statistics for demand shocks (3.4 for volatility and 0.27 for correlation).

The right panel of figure 3 plots the dynamic correlations between output and demand shocks using artificial data from three different versions of the demand-driven economies. Again when the model has a unique equilibrium, it fails to display an asymmetric correlation curve. The indeterminacy economy is the only one that can replicate the pattern of correlations observed in the data. The peak of positive correlations appears at leads of 2-4 quarters, which generally matches what it is in the US and EU data.

In figure 5, we plot the impulse response functions of outputs in the theoretical economies. Since the RBC economy has no demand shocks, we show the response functions to productivity

[10] The impulse response functions are estimated with VAR of 4 lags. The variables are real GDP and consumer sentiment indices in the US and the EU. I use the latter as a proxy of changes in aggregate demand. Other proxies may also be used. For example, Benhabib and Wen (2001) use investment to output ratio. Cochrane (1994) uses consumption to output ratio.

shocks instead. As discussed, the RBC economy fails to deliver the correct response patterns. The demand-driven economies with a determinate equilibrium are capable of generating positive responses in output for both countries, but fail to replicate the hump-shape of the response function. Once again the only model that can produce positive and hump-shaped responses in both countries is the DBC model with indeterminate equilibria.

4.3.3 "J-curve" property of the terms of trade

Another area in which the indeterminacy model outperforms the others is in replicating the dynamic relationship between the terms of trade and trade balance.

Figure 7 plots the correlations of the terms of trade with net exports over different leads and lags for a number of OECD countries. The relationship mimics the plot between output and demand shocks: the terms of trade is less correlated with current and past net exports, but more correlated with future net exports. This resembles the "J-curve" properties in trade theories that unfavorable movements in the terms of trade usually result in a decline in exports first, and then a boom of exports after several quarters.

Figure 8 reports the predictions of different theoretical economies. While the constant-return DBC economy fails, the two increasing-return DBC economies perform as well as the RBC model in replicating the J-curve properties.

To sum up, if we simply replace the productivity shock in an RBC economy with a demand shock, the model cannot predict (1) the relative volatility of consumption, (2) the dynamic relationship between demand shocks and output, and (3) the "J-curve" property of the terms of trade. However, if the economy has small increasing returns to scale and indeterminate equilibria, the model can predict these features.

5 Conclusion

In this paper, we extend the analytical basis for understanding international trade and business cycles by considering the effect of indeterminacy and shocks to consumer demand. The framework for this analysis is a two-country, two-sector general equilibrium model. The model can resolve the cross-country correlation puzzle while matching other key features of international business cycles captured by existing models.

Demand shocks are distinct from productivity shocks in their propagation mechanism: a rise in domestic consumer demand will increase foreign imports as well as domestic output. An increase in imports lead to a foreign economic boom. This is contrary to the prediction of an RBC model driven by productivity shocks. Indeterminacy is crucial in that it creates endogenous fluctuations in the economy that other models fail to deliver. Only the indeterminacy model can match the hump-shaped responses of output to consumer demand that we observe in the data.

Now that the cross-country correlation puzzle has been resolved, a direction for future research concerns the volatility of the terms of trade. Such volatility remains excessively low in the model when compared with the data. In this paper we showed that there is no obvious connection between this “price anomaly” and the level of consumption correlations across countries, as other researchers have previously suggested. Instead, a change of economic environment that specifically introduces price fluctuations might be necessary.

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