Indeterminacy and interest rate rules:
The role of fiscal policy

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Abstract

The present paper analyses the stabilizing or destabilizing effects that are associated with monetary policy rules. Based on simulations of calibrated dynamical models with cash-in-advance constraints it is shown, that the specific assumptions made regarding the government sector and particularly the assumptions made about fiscal policy are important in order to assess such stabilizing or destabilizing effects. Existing analyses are extended in two respects. First, the paper considers distortionary taxation. Second, it is assumed that the government's deficit must obey specific rules regarding the deficit ratio. As a consequence we arrive at different conclusions regarding the possibly destabilizing effect of simple Taylor-like interest rate rules than earlier studies.

Key words: Interest rate rules, Monetary Policy, Indeterminacy
JEL-Classifikation: E1, E31, E4, E52, E63.

1. Introduction

The discussion about Taylor rules constitutes one of the major topics in recent monetary economics. On the one hand, this discussion focused on the empirically motivated question whether or not the actual policy followed by central banks can be described with such rules. On the other hand, there is growing interest in analyzing the consequences of Taylor rules for the real economy in comparison with alternative rules for monetary policy.

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Because there is some evidence that — at least in models with flexible prices as considered in the present paper — rules for monetary policy do not differ much with respect to their welfare effects (cf. Carlstrom and Fuerst (1995)), research in this field is mainly concerned with the possibly destabilizing effects that are associated with alternative monetary policy rules. Such destabilizing effects exist, if the particular monetary policy rule leads to an indeterminate rational expectations equilibrium. In such a case it can not be ruled out that the equilibrium is influenced by extrinsic factors — so called sunspots. So, there may exist fluctuations in economic activity without any underlying change in the economic fundamentals. Of course, such effects are important for welfare only, if the real sector of the economy is affected by the indeterminacy, meaning that the indeterminacy is not limited to the monetary sector.

With respect to the stabilizing or destabilizing effects of monetary policy rules, a first conclusion to be drawn from the relevant literature is that rules implying some kind of interest rate targeting have to be judged fundamentally different than rules that target the the money growth rate. According these results, pegging the money growth rate will not induce any indeterminacy, whereas an interest rate peg leads to indeterminacy.[2] However, this kind of indeterminacy is restricted to the monetary sector and there arise no consequences for the allocation of resources and hence no consequences regarding welfare. So, this kind of indeterminacy is of quite limited importance and relevance.

Among other things there is one assumption that appears to be responsible for the above described result regarding an interest rate peg: It is assumed that the government's only activity is to redistribute seigniorage revenues in a lump sum fashion to the private sector (cf. Carlstrom and Fuerst (1995, 2000a)). This assumption is consistent with fiscal policy playing a passive role in the sense of Leeper (1991): Monetary policy acts to target the nominal rate of interest and given the seigniorage revenues, fiscal policy then adjusts lump sum taxes in order to ensure intertemporal budget balance. Quite obviously, however, this assumption also suffers from a very restricted view of the government sector and more specifically of fiscal policy. As will be outlined below, the assumption that taxes are lump sum turns out to be very crucial with respect to the possibly stabilizing or destabilizing effects of monetary policy rules. Alternative and in the light of actual debates more realistic assumptions regarding the arrangement of fiscal policy not only imply that a formerly nominal indeterminacy becomes a real indeterminacy. Furthermore and under conditions that are described below, it might be the case that an interest rate peg will induce no indeterminacy at all.

In the present paper it is assumed that revenues of the government consist — at least partially — of distortionary taxes. A passive fiscal policy then means that the fiscal authority adjusts

[2] Of course, even money growth rules can give rise to indeterminacy. However, as argued by Carlstrom and Fuerst (2000b) this occurs only for calibrations of the relevant models that are far from being empirically plausible.

distortionary taxes or its expenditures or both in response to an active monetary policy. Both varieties of passive fiscal policy are allowed for in the models that are discussed in this paper, since the extent of the distortion caused by taxation is treated as an exogenous parameter. Indeed, the extent of distortionary taxation turns out to be the important property of fiscal policy that drives the results. Active monetary policy might now effect the government's budget and thus induce the government to change the level of distortionary taxes in order to finance its expenditures. Therefore, distortionary taxes constitute a channel through which a formerly nominal indeterminacy might grow into a real indeterminacy with corresponding consequences for the allocation of resources and welfare. However, the way in which distortionary taxes act on private decisions gives also rise to another result which says that there will be no indeterminacy at all associated with nominal interest rate pegging, whenever the extent of the distortion caused by taxation is large enough.

Although the paper is mainly concerned with monetary policy that is characterized by a simple peg of the nominal interest rate, it will be shown that the results do not only hold in this special case. The same conclusions are derived under the assumption of a monetary policy rule that — inspired by the well known Taylor rule — links the nominal rate of interest to the rate of inflation. The main part of the paper abstracts from government debt and assumes that the government's budget is balanced on a period by period basis with the help of taxes and seigniorage revenues only. While this assumption seems harmless in the presence of lump sum taxes, this is certainly not the case given distortionary taxation. Because of this, the paper also looks at a specific rule regarding fiscal policy, which explicitly allows for government debt. Following Schmitt-Grohé and Uribe (2000), it is assumed that fiscal policy follows a balanced budget rule which prescribes the deficit ratio, that is the level of the secondary budget surplus, in relation to total output. This is a slight generalization of the balanced budget rule analyzed by Schmitt-Grohé and Uribe (2000), because there the secondary budget surplus must be zero on a period by period basis. Schmitt-Grohé and Uribe (2000) show that an interest rate peg goes along with nominal indeterminacy with such a balanced budget rule. The present paper shows that this nominal indeterminacy becomes a real one in the presence of distortionary taxes. Furthermore, it is shown that more general balanced budget rules that allow the ratio of the secondary deficit to output to be greater than zero (as for instance specified in the Maastricht treaty) prevent any kind of indeterminacy, whenever some other — but not implausible — restrictions on the model's parameters hold.

The next Section describes the model with cash-in-advance restriction that forms the basis of the following analysis. This model illustrates the basic nominal indeterminacy result associated with a nominal interest rate peg and is starting point for subsequent extensions and modifications. An interest rate peg in conjunction with passive fiscal policy that is characterized by distortions exerted on the private sector is analyzed in Section 3. This Section also discusses the consequences of a more general interest rate rule. Since formal proofs are very cumbersome, numerical solutions of the models are used in order to asses the stabilizing or destabilizing effects that are associated with an interest rate peg in dependence on some of the model's parameters. Balanced budget rules that allow for government debt are introduced in Section 4. Here, for the some reason as above, the results regarding stabilizing or destabilizing effects of nominal interest rate pegs are also based on numerical solutions of the model. The final Section summarizes and discusses the main results. Detailed formal descriptions of the models, that are analyzed in the main part of the paper are given in the Appendix.

2. The basic model

The model that forms the basis of the further analysis is a dynamic macroeconomic model based on individual optimization, where money plays a role because of cash-in-advance restrictions. The model is essentially identical to that of Carlstrom and Fuerst (1995, 2000a) and serves to illustrate the relevant results obtained in the literature. In this model private consumption as well as wage payments of the firms are subject to a cash-in-advance constraint. The monetary injection that takes place at the beginning of every period can not be used by households in order to finance consumption. Instead, this monetary injection together with a part of the money the households already hold are transfered to financial intermediaries. Financial intermediaries make loans to firms, because firms need to borrow cash in order to pay their wages.

The model consists of the following six equations, that are derived in detail in Appendix A.1:

$$\frac{U_{l,t}}{U_{c,t}} = \frac{w_t}{p_t}\,\frac{1}{1+i_{t-1}} \tag{1a}$$

$$\frac{U_{c,t}}{p_t} = \frac{U_{c,t+1}}{p_{t+1}}\,\beta\,(1+i_{t-1})\,\frac{1}{1+x_t} \tag{1b}$$

$$\frac{U_{c,t}}{1+i_{t-1}} = \frac{U_{c,t+1}}{1+i_t}\,\beta\,\left[F_{k,t+1}(k_{t+1},h_{t+1})+1-\delta\right] \tag{1c}$$

$$\frac{w_t}{p_t}\,(1+i_{t-1}) = F_{h,t}(k_t,h_t) \tag{1d}$$

$$c_t + k_{t+1} + g_t = F\,(k_t,h_t) + (1-\delta)\,k_t \tag{1e}$$

$$p_t\,c_t = 1 - s_t \tag{1f}$$

$$w_t\,h_t = x_t + s_t \tag{1g}$$

Here $U_{l,t}$ and $U_{c,t}$ denote marginal utility derived from leisure and consumption in period t, respectively, and $0 < \beta < 1$ is a discount factor for future utility. $F(k_t\,,h_t)$ is a standard neoclassical production function, where capital k_t and labor h_t are factors of production. $x_t = \Delta M_{t+1}/M_t$ is the growth rate of money. $w_t = W_t/M_t$ and $p_t = P_t/M_t$ are stationary versions of the wage rate W_t and the price level P_t as of period t. Equation (1a) is a standard condition for the optimal leisure-consumption choice, which — since consumption is subject to a cash-in-advance constraint — is affected by the nominal rate of interest in period t, denoted by i_{t-1}. Equation (1c) is the Euler equation specifying the optimal intertemporal consumption plan and given this equation, (1b) can be transformed into the well known Fisher equation, which relates the nominal rate of interest to the rate of inflation and the real rate of return. Optimal labor demand as described by equation (1d) is also affected by the nominal rate of interest i_{t-1}, because firms have to pay this rate of interest for their cash borrowings from the financial intermediaries. The reaming equations are equilibrium conditions: Equation (1e) is the equilibrium condition for the goods market, where g_t denotes government purchases of goods. Equation (1f) says that the share $1-s_t$ of the money stock the households hold in every period t must equal their consumption expenditures. Finally, equation (1g) says that the firms' borrowings of cash must be equal to their wage payments.

An implication of these six equations is that the government's budget is financed via lump sum taxes. Abstracting from any government debt, the budget constraint of the government is thus given as follows:

$$x_t = p_t\, g_t - p_t\, T_t \tag{2}$$

Here, the time path $\{g_t\}_{t=0}^{\infty}$ of government expenditures is taken as given and taxes T_t are assumed to adjust in a way that ensures budget balance in every period. If the monetary authority pursues to peg the nominal rate of interest, this implies that the growth rate of money x_t becomes an endogenous variable. Moreover, given such an active monetary policy, tax adjustments are the only way in which fiscal policy can achieve budget balance.

Now suppose that the steady state growth rate of money equals γ. Assume furthermore that the nominal rate of interest is pegged at its steady state level $i^* = (1+\gamma-\beta)/\beta$ and let i denote this pegged nominal rate of interest. Equations (1a), (1c) and (1e) then become:

$$\frac{U_{l,t}}{U_{c,t}} = F_{h,t}(k_t, h_t)\,\frac{1}{(1+i)^2} \tag{3a}$$

$$U_{c,t} = U_{c,t+1}\,\beta\,[F_k(k_{t+1}, h_{t+1}) + 1 - \delta] \tag{3b}$$

$$c_t + k_{t+1} + g_t = F(k_t, h_t) + (1-\delta)\,k_t \tag{3c}$$

Apart from the factor $(1+i)^{-2}$, that appears in equation (3a), we arrive at a system of equations that is well known from the basic non-monetary real business cycle model (see e.g. King et al. (1988a,b,c)). The real sector is therefore determinate irrespectively of any possible monetary indeterminacy.

With the time paths $\{c_t, h_t, k_{t+1},\}_{t=0}^{\infty}$ being determined, we are left with the following four equations:

$$\frac{U_{c,t}}{p_t} = \frac{U_{c,t+1}}{p_{t+1}} \beta (1+i) \frac{1}{1+x_t} \tag{4a}$$

$$\frac{w_t}{p_t} (1+i) = F_{h,t}(k_t, h_t) \tag{4b}$$

$$p_t\,c_t = 1 - s_t \tag{4c}$$

$$w_t\,h_t = x_t + s_t \tag{4d}$$

There is a nominal indeterminacy, whenever the five unknowns w_t, s_t, x_t as well as p_t and p_{t+1} cannot be determined with the help of these four equations. In such a case the we need an additional initial value p_0 for the price level or an initial value x_0 that fixes the initial stock of money in order to determine a unique equilibrium in the monetary sector. In the present case, the existence of a nominal indeterminacy can be established as follows (cf. Carlstrom and Fuerst (2000a)): From (4b)–(4d) it follows for the growth rate of money:

$$\begin{aligned} x_t = w_t\,h_t - s_t = w_t\,h_t - 1 + p_t\,c_t &= p_t\,\frac{F_h(k_t,h_t)}{1+i}\,h_t + p_t\,c_t - 1 \\ \Leftrightarrow 1 + x_t = p_t\left(\frac{F_h(k_t,h_t)}{1+i}\,h_t + c_t\right) &\equiv p_t\,Q(k_t,c_t,h_t) \end{aligned}$$

With respect to p_{t+1} we therefore get from (4a):

$$p_{t+1} = \beta\,(1+i)\,\frac{U_{c,t+1}}{U_{c,t}}\,\frac{1}{Q(k_t,c_t,h_t)}$$

The above derived result regarding the real sector implies that this difference equation gives rise to a converging time path for the price level, independent of the respective initial value p_0 for the price level. Thus, there is price level indeterminacy and hence nominal indeterminacy.

3. Distortionary taxes and a simple fiscal rule

The just described nominal indeterminacy grows into a real indeterminacy, if the indeterminacy of the price level operates on the real sector of the economy through fiscal policy. Suppose that distortionary taxes on income are the only form of government's tax revenues and that besides this, equation (2) continues to be the government's budget constraint. With τ_t denoting the proportional tax rate on factor incomes in period t, we thus have:

$$x_t = p_t\,g_t - p_t\,\tau_t\,F(k_t,h_t) \tag{5}$$

The above analyzed equations (3a)–(3c) then become:

$$\frac{U_{l,t}}{U_{c,t}} = F_{h,t}(k_t,h_t)\,\frac{1-\tau_t}{(1+i)^2} \tag{6a}$$

$$U_{c,t} = U_{c,t+1}\,\beta\,[(1-\tau_{t+1})\,F_k(k_{t+1},h_{t+1}) + 1 - \delta] \tag{6b}$$

$$c_t + k_{t+1} + g_t = F(k_t,h_t) + (1-\delta)\,k_t \tag{6c}$$

Apparently, any change of the tax rate τ_t affects the allocation of resources and such changes of the tax rate can be caused by changes of the price level: Since government expenditures

g_t are assumed to be exogenous and the growth rate of money is determined by the interest rate peg, this must be the case according to equation (5). In this way, there arises a real indeterminacy, where it was formerly only a nominal one. However, the appearance of any kind of indeterminacy becomes the more unlikely, the greater is the distortionary effect of taxation is. These two topics will now be analyzed within the framework of the simple model discussed so far. Afterwards we will proceed to look at more realistic formulations of fiscal policy rules.

3.1. *Consequences of distortionary taxation*

In order to allow for a varying extent of the distortion causes by taxes, it is assumed that the fraction $0 \leq \mu \leq 1$ of the government expenditures that remains after seigniorage revenues have been subtracted are financed via distortionary taxes. The remaining fraction is then financed via lump sum taxes. Likewise it can be assumed that government expenditures are reduced in such a way that only this fraction μ of the originally planned expenditures actually takes place. Therefore, taxes are determined according to the following equation:

$$\mu\,[p_t\,g_t - x_t] = p_t\,\tau_t\,F(k_t, h_t)$$

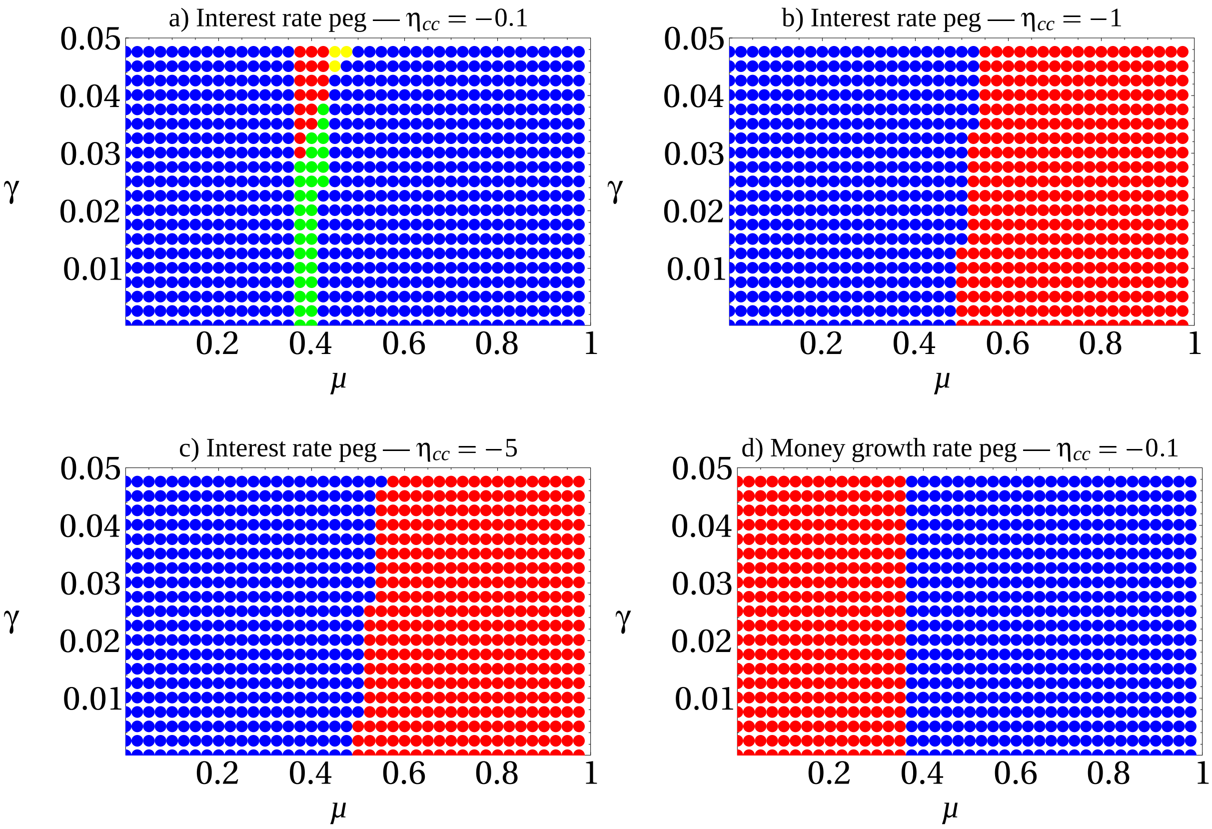
Whether or not an interest rate peg gives rise to stabilizing or destabilizing effects in such an environment can now be studied looking at the dynamic properties of the respective steady state of the model. In the present paper, the dynamic properties of a linearized version of the model's equations are analyzed.[3] Unfortunately, a formal analysis even of this linear system turns out to be very cumbersome. Because of this, a numerical analysis is performed that sheds some light on the dynamic properties in question. Given the linearized version of the model's dynamic equations, the eigenvalues of the coefficient matrix of the resulting system of linear difference equations are of interest. This system of difference equations describes the deterministic dynamics of the underlying nonlinear economic model in the neighborhood of the steady state. The rational expectations equilibrium is determinate only if this steady state is a saddlepoint.[4] Since — as is shown in appendix A.2 — the whole system can be reduced to a system of three difference equations in the three variables k_t, c_t and p_t, this matrix possesses three eigenvalues. Hence, two of these eigenvalues have to be greater than one in absolute value in order for existence of a determinate rational expectations equilibrium.

Figures 1.a–1.c summarize what can be said regarding the dynamic properties of the linearized model in dependence on some of the model's parameters. The figures allow to identify regions for the respective parameters of the model, where specific restrictions on the absolute

[3] Appendix A.2 describes the linearized version of the model that forms the basis of the simulations that are presented below.

[4] This is also true for a stochastic version of the same model, which includes, for example, exogenous productivity shocks or shocks to government expenditures.

Figure 1. *Dynamic properties of the model with distortionary taxation: a)–c) under an interest rate peg , d) under a money growth rate peg* ($\alpha = 0.4$, $\beta = 0.99$, $\eta_{ll} = 0$, $\delta = 0.02$, $h^* = 0.25$ *and* $s_g = 0.2$*).*



Note: red: Determinacy (1 eigenvalue <1, 2 eigenvalues > 1), blue: Indeterminacy (2 eigenvalues <1, 1 eigenvalue > 1), yellow: Instability/Nonexistence (3 eigenvalues > 1), green: Indeterminacy (3 eigenvalues < 1).

values of the eigenvalues are satisfied. The numerical values of the parameters of the model are specified such that they conform by and large to values used elsewhere in the literature for calibrations of such models and that are conceived to be empirically plausible for the US economy (cf. for instance Burnside et al. (1993) or Christiano (1991)). The relevant time period for the calibration of the model that underlies these figures is a quarter. The sensitivity of the results with respect to the steady state growth rate of money γ as well as the intertemporal elasticity of substitution of consumption $-1/\eta_{cc}$ is analyzed, because other studies (cf. Meng (2002)) show, that the magnitudes of these two parameters have lasting effects for the dynamic properties of a cash-in-advance model.

First of all, these figures show the well known result regarding an interest rate peg that was already discussed earlier: Whenever the tax revenues of the government consist only of lump sum taxes, an interest rate peg leads to nominal indeterminacy. This indeterminacy is preserved if we allow for distortionary taxation, whenever the extent of this distortion μ remains low. It should be noticed that — because of the reasons already given above — this indeterminacy is no longer restricted to the monetary sector but also affects the real sector of the economy.

The most important result displayed in the figures, however, is that the equilibrium is determinate under an interest rate peg, whenever the extent of the distortion μ is sufficiently large and the value of η_{cc} — the inverse of the intertemporal elasticity of substitution of consumption — is sufficiently large. Only in case of an empirically implausible value of this parameter (cf. the case $\eta_{cc} = -0.1$ in figure 1.a) we get a complex picture of the dynamic properties of the model. For a numerical value of ρ within the empirically plausible range, the resulting picture always looks as depicted in figures 1.b and 1.c: There always exists a lower bound for the parameter μ above of which the equilibrium becomes determinate: As can be seen, this is the case, whenever about 50% of tax revenues of the government consist of distortionary taxes.

Only for completeness, figure 1.d depicts the stability properties of the same model if the interest rate peg is replaced by a money growth rate peg. As can be inferred from equations (1a), (1c) and (1e), in view of such a monetary policy rule any indeterminacy will hit the monetary as well as the real sector of the economy, even if $\mu = 0$. Figure 1.d then illustrates a well known result (cf. Carlstrom and Fuerst (2000b)), according to which a money growth rate peg leads to indeterminacy only for empirically implausible numerical values of the model's parameters: An indeterminate equilibrium exists only if the value of the parameter η_{cc} is very low. As in case of an interest rate peg, the extent of the distortion caused by tax financed government expenditures is still responsible for the existence of an indeterminacy. However, contrary to the case of an interest rate peg, a large value of μ now favors the existence of an indeterminate equilibrium. [5]

The explanation of this result is not a simple task, because it relies upon a combination of different effects that determine the properties of the steady state equilibrium of the model. A possible starting point for such an explanation is a paper by Schmitt-Grohé and Uribe (1997), where it is shown that distortionary taxation of labor income when combined with a balanced budget rule leads to indeterminacy because labor income taxes are after all regressive. Reversing the underlying argument, Guo and Lansing (1998) as well as Guo (1999), have shown that progressive income taxes can stabilize an economy against sunspot fluctuations ensuring existence of a determinate equilibrium. Now, the government's budget in the present model is financed via distortionary income taxes and seignorage revenues. Without such distortionary taxes, any expectation regarding the time path for future prices is self-fulfilling. This, however, needs not to be true in face of distortionary income taxes. The reason is that taxes in the present model are progressive as in Guo and Lansing (1998). Dependent on the value of the parameter

[5] Whenever plausible values for η_{cc} are assumed, the equilibrium is determinate irrespectively of the value μ takes.

μ, there might result an overall stabilizing effect from progressive taxation, whenever this effect outweighs the destabilizing effect that is associated with an interest rate peg.

3.2. *Interest rate rules and distortionary taxation*

As demonstrated by Carlstrom and Fuerst (2000a), the above described indeterminacy result regarding an interest rate peg does carry over to more general interest rate rules: If interest rate rules are analyzed, which link the current rate of inflation to the nominal rate of interest, there will always result a real indeterminacy.[6] However, this result also depends on the specific assumptions made about fiscal policy. If it is assumed that government expenditures are exogenously determined and financed via seigniorage revenues and distortionary taxes, then again the extend of the distortion is crucial for the existence of an indeterminacy.

Let Π_{t+1} denote the inflation factor between period t and $t+1$ and assume that monetary policy can be described by the following simple rule:

$$i_t = H(P_{t+1}/P_t) = H(\Pi_{t+1}), \quad \text{where:} \quad H'(\Pi) > 0, \quad H(\Pi^*) = i^*$$

Since i_{t-1} is the nominal rate of interest to be paid by firms for their loans in period t, this rule indeed links current inflation to the current nominal rate of interest, even though the time subscripts of the relevant variables differ. The linearized version of this rule is given by:

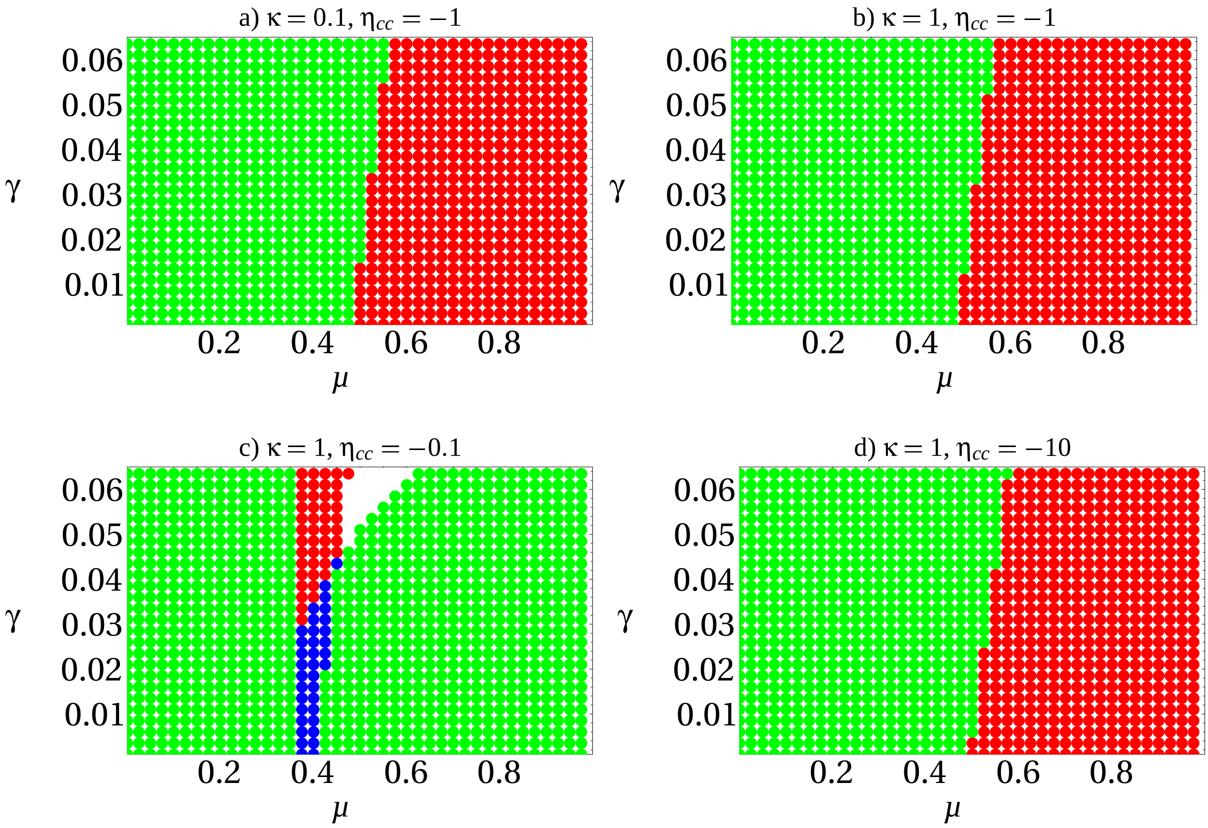
$$\hat{i}_t = \kappa \left[\hat{p}_{t+1} - \hat{p}_t + \frac{\gamma}{1+\gamma} \hat{x}_t \right] \tag{7}$$

Here the elasticity $\kappa > 0 = H'(\Pi^*)\,\Pi^*/H(\Pi^*)$ measures, how strong the nominal interest rate reacts to changes in inflation. Integration of equation (7) into the system of equations given by equations (A.2a)-(A.2g) then allows to analyze the stability properties of the steady state. The only difference to the model with an interest peg is that we now end up with a system of four equations, because the interest rate as specified by equation (7) is now included in the system of difference equations. Hence the coefficient matrix of the system of linear difference equations possesses four eigenvalues. Since there is still one predetermined variable with initial value k_0, three of the eigenvalues must be smaller than one in absolute value for a determinate equilibrium to exist.

Figures 2.a–2.d summarizes the results on the dynamic properties of the model obtained from numerical simulations. At first, the formal results of Carlstrom and Fuerst (2000a) are reproduced, according to which the interest rate rule under consideration implies indeterminacy in case of pure lump sum taxes ($\mu = 0$). Apart from this, however, the figures reveal that dropping this assumption leads to lasting changes in the dynamic properties of the model. As in case of an interest rate peg, there exists a critical value for the extent μ of the tax distortion, beyond

[6] Only in the special case of an interest rate peg, this indeterminacy is limited to the monetary sector.

Figure 2. *Dynamic properties of the model with interest rate rule* $i_t = H(\Pi_{t+1})$ *and distortionary taxation* ($\alpha = 0.4$, $\beta = 0.99$, $\eta_{ll} = 0$, $\delta = 0.02$, $h^* = 0.25$ *and* $s_g = 0.2$).



Note: white: Instability/Nonexistence (4 eigenvalues > 1), red: Determinacy (3 eigenvalues >1, 1 eigenvalue < 1), blue: Indeterminacy (3 eigenvalues < 1), green: Indeterminacy (2 eigenvalues >1, 2 eigenvalues < 1), yellow: Indeterminacy (4 eigenvalues < 1).

which there exists a determinate equilibrium. Furthermore, the specific property of the interest rate rule — the elasticity κ — appears to be not important for this result. As shown in figures 2.a–2.c, it is rather the intertemporal elasticity of substitution of consumption $-1/\eta_{cc}$ as well as the steady state growth rate of money γ that matter in this respect.

4. More general fiscal rules

A central result of the recent debate on the fiscal theory of the price level is that it is possible to have a determined price level even under an interest rate peg: The price level will be determinate, whenever the time path of the primary deficit of the government is exogenously given (cf. Sims (1994), Woodford (1994)): Under this premise, the price level can be determined with the help of the condition of intertemporal budget balance which requires the equality of the present value of future primary surpluses and the initial liabilities of the government. This result, however, does not hold, if instead of the primary surplus, the secondary surplus is given exogenously: Schmitt-Grohé and Uribe (2000) establish within the framework of a cash-in-advance model, that a nominal indeterminacy results, if the government follows a fiscal rule according to which the secondary surplus of the government is zero on a period by period basis. In this section, a generalized version of such a fiscal rule is combined with distortionary taxation as described above. Moreover, the model now allows for government debt. It is assumed that the government issues a riskless discounted bond. B_t denotes bonds maturing in period t. Defining $b_t = B_t/M_t$, the budget constraint of the government is given by:

$$x_t + (1+x_t)\frac{b_{t+1}}{1+i_t} = p_t(g_t - T_t) + b_t \tag{8}$$

The balanced budget rule analyzed by Schmitt-Grohé and Uribe (2000) is generalized in the following way: It is assumed that there exists a criterion or rule according to which the secondary surplus of the government must be proportional to output. With ϕ denoting the deficit ratio, we then get: [7]

$$p_t(g_t - T_t) + b_t \frac{i_{t-1}}{1+i_{t-1}} = \phi\, p_t\, F(k_t, h_t)\,, \quad 0 < \phi < 1 \tag{9}$$

If we restrict the analysis to distortionary taxes, we have $p_t\, T_t = \tau_t\, p_t\, F(k_t, h_t)$ and from (8) and (9) it follows:

$$p_t\,(g_t - (\tau_t + \phi)\,F(k_t, h_t)) = -b_t \frac{i_{t-1}}{1+i_{t-1}} \tag{10a}$$

$$x_t + (1+x_t)\,\frac{b_{t+1}}{1+i_t} = \phi\, p_t\, F(k_t, h_t) + \frac{b_t}{1+i_{t-1}} \tag{10b}$$

The steady state level of government debt therefore satisfies:

$$b^* = (1+i^*)\left[\frac{\phi}{\gamma}\, p^*\, F(k^*, h^*) - 1\right]$$

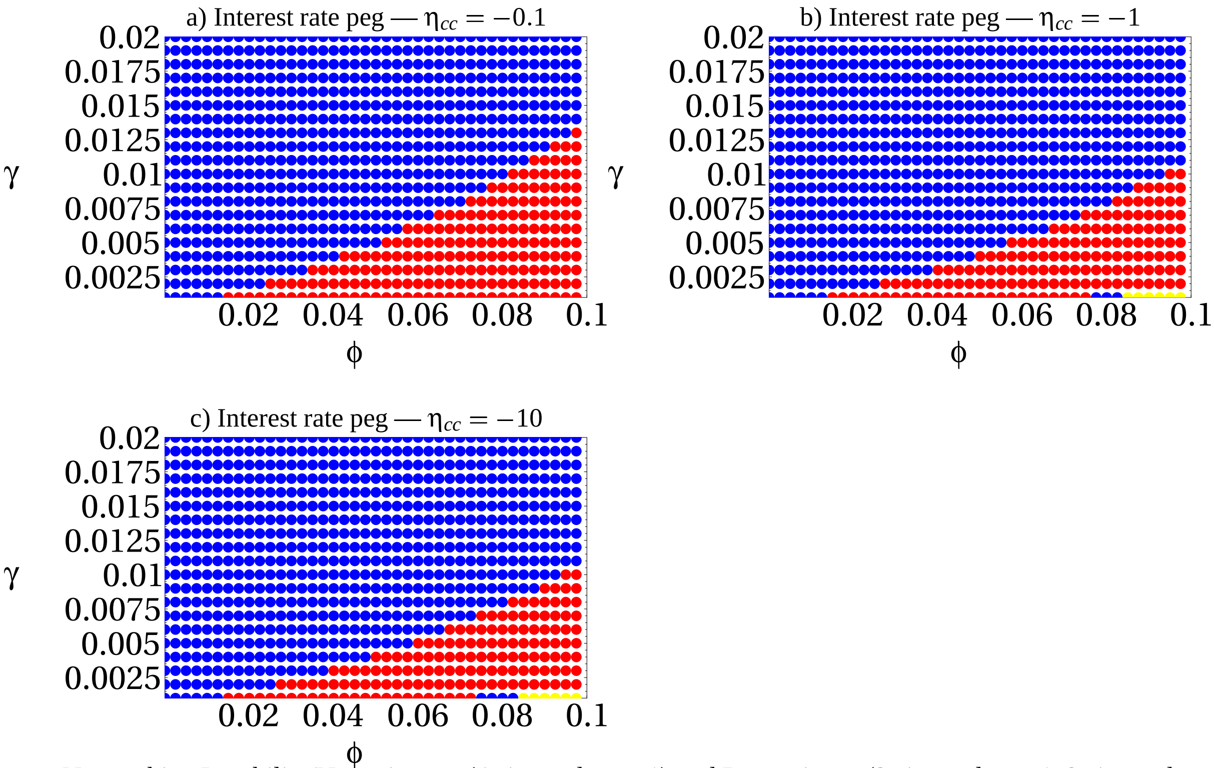
From this, the steady state tax rate can be derived as follows:

$$p^*\, F(k^*, h^*)\,\left[s_g - \tau^* - \phi + i^* \frac{\phi}{\gamma}\right] = i^*$$

The linearized version of the model is described in appendix A.3. The resulting dynamic system consists of four equations such that there are again four eigenvalues that determine the dynamic properties of the model. Based on numerical simulations of the model, figures 3.a–3.c illustrate, how these dynamic properties depend on the deficit ratio ϕ as well as on the intertemporal elasticity of substitution of consumption $-1/\eta_{cc}$ and the steady state rate of money growth γ. Since there are now two predetermined variables in the model — initial

[7] In the present model it is assumed that fiscal policy must exactly attain a deficit ratio of ϕ, whereas for instance the respective deficit criterion of the Maastricht treaty only specifies an upper bound for the deficit.

Figure 3. *Dynamic properties of the model with distortionary taxation and deficit rule under an interest rate peg* ($\alpha = 0.4$, $\beta = 0.99$, $\eta_{ll} = 0$, $\delta = 0.02$, $h^{*} = 0.25$ *and* $s_g = 0.2$*).*



Note: white: Instability/Nonexistence (4 eigenvalues > 1), red Determinacy (2 eigenvalues >1, 2 eigenvalues < 1), blue: Indeterminacy (3 eigenvalues < 1), green: Indeterminacy (3 eigenvalues >1, 1 eigenvalue < 1), yellow: Indeterminacy (4 eigenvalues < 1).

government debt as well as the initial stock of capital —, a determinate equilibrium exists, whenever two of the four eigenvalues are smaller than one in absolute value. As in the model of the previous section, the presence of distortionary taxation implies that any resulting indeterminacy is a real indeterminacy.

First of all, figures 3.a–3.c reproduce the result derived by Schmitt-Grohé and Uribe (2000), according to which a rule that prescribes a zero secondary surplus on a period by period basis always results in nominal indeterminacy. However, as the figures also reveal, this is not necessarily the case if we look at more general deficit rules: There exist values for ϕ, where a determinate equilibrium exists, as long as the money growth rate γ is not too large.

This result is interesting in the following respects: On the one hand, Schmitt-Grohé and Uribe (1997) show that a balanced budget rule may cause an indeterminacy in a non-monetary model, when combined with distortionary taxation. In the monetary model considered here, the opposite is true, since here distortionary taxation favors the existence of a determinate equilibrium. On the other hand, von Thadden (2002) analyses the above described deficit rule within the framework of a OLG model. However, the conclusion derived by von Thadden (2002) is completely contradictory to the one derived in the present paper based on a dynamic model with households that optimize over an infinite horizon, because in his OLG model, a deficit rule always results in indeterminacy.

5. Conclusion

The views on the stabilizing or destabilizing effects generated by interest rate rules that are expressed in the literature are particularly dependent on the underlying assumptions regarding the government sector. In the present paper it was shown which conclusions can be drawn, if it is assumed that the government levies distortionary taxes in oder to finance its expenditures and if there are specific rules the government's budget deficit must fulfill. A consequence of distortionary taxation is, that any indeterminacy caused by a specific monetary policy rule will be a real indeterminacy. Moreover, a sufficiently large extent of the tax distortion may give rise to a situation, where no indeterminacy at all will occur.

The results derived in this paper altogether rely upon numerical simulations of a linearized version of the model. Because of this, only the dynamic properties in the neighborhood of the steady state are analyzed and because of this, all the results are only locally valid. To draw general conclusions based on such local stability analyses can be problematic as is demonstrated by Benhabib et al. (2001) in their global stability analysis of the dynamics affected by Taylor rules. One of the next steps in this line of research therefore is to generalize the results derived in this paper in this regard as well as with respect to the interest rate rules under consideration. Furthermore, a formalization of the results is desired in order to dispense with numerical simulations.

Appendix A.

A.1. *The baseline model*

The intertemporal optimization problem of the representative household is given as follows:

$$
\begin{aligned}
&\max_{\{c_t, h_t, s_t, k_{t+1}, M_{t+1}, V_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t\, u(c_t,\, 1-h_t) \\
\text{s.t.}\quad & Q_t\, k_t + W_t\, h_t + P_t\,(1-\delta)\, k_t + (1+z_t)\, s_t\, M_t + z_t\, \Delta M_{t+1} \\
&\qquad + (1-s_t)\, M_t + V_t - P_t\, T_t - M_{t+1} - \frac{V_{t+1}}{1+i_t} - P_t\, k_{t+1} - P_t\, c_t \geq 0 \\
& (1-s_t)\, M_t - P_t\, c_t \geq 0 \\
& k_0 > 0\,, \quad M_0 > 0
\end{aligned}
$$

Here Q_t and W_t denote the factor prices of capital and labor, respectively. $1-s_t$ is the fraction of the money stock M_t the household holds at the beginning of period t that is used for consumption. The term T_t represents lump sum taxes. The monetary injection is received by the financial intermediaries and cannot be used for consumption in this period. Deposits at the intermediaries pay the nominal interest rate z_t. V_t denotes bond holdings of the household, where i_t denotes the nominal rate of interest. With λ_t and μ_t denoting the multipliers associated with the two constraints, the following first order conditions must hold:

$$
\begin{aligned}
\beta^t U_c(c_t, 1-h_t) &= P_t\,(\lambda_t + \mu_t) \\
\beta^t U_l(c_t, 1-h_t) &= W_t\,\lambda_t \\
\lambda_t\, z_t\, M_t &= \mu_t\, M_t \\
\lambda_t &= \lambda_{t+1}\,(1+z_{t+1})\,s_{t+1} + (\lambda_{t+1}+\mu_{t+1})\,(1-s_{t+1}) \\
\frac{\lambda_t}{1+i_t} &= \lambda_{t+1} \\
P_t\,\lambda_t &= \lambda_{t+1}\,[Q_{t+1} + P_{t+1}\,(1-\delta)]
\end{aligned}
$$

The intermediaries make loans to the firms, at the interest z_t. These loans are used for wage payments. Profit maximization and perfect competition on factor markets then implies that for all t we have $f_h(k_t\,h_t) = \frac{W_t}{P_t}\,(1+z_t)$ as well as $f_k(k_t, h_t) = \frac{Q_t}{P_t}$. Defining the stationary variables $x_t = \Delta M_{t+1}/M_t$, $p_t = P_t/M_t$, $q_t = Q_t/M_t$ and $w_t = W_t/m_t$, the following set of equations results:

$$
\begin{aligned}
\frac{U_l(c_t, 1-h_t)}{U_c(c_t, 1-h_t)} &= \frac{w_t}{p_t}\,\frac{1}{1+i_{t-1}} \\
\frac{U_c(c_t, 1-h_t)}{p_t} &= \beta\,(1+i_{t-1})\,\frac{U_c(c_{t+1}, 1-h_{t+1})}{p_{t+1}}\,\frac{1}{1+x_t} \\
\frac{U_c(c_t, 1-h_t)}{1+i_{t-1}} &= \beta\,\frac{U_c(c_{t+1}, 1-h_{t+1})}{1+i_t}\,[f_k(k_{t+1}, h_{t+1}) + 1 - \delta] \\
c_t + k_{t+1} + g_t &= f(k_t, h_t) + (1-\delta)\,k_t \\
p_t\,c_t &= (1-s_t) \\
w_t\,h_t &= (s_t + x_t) \\
1 + z_{t+1} &= 1 + i_t \\
\frac{w_t}{p_t}\,(1+i_{t-1}) &= f_h(k_t\,h_t)
\end{aligned}
$$

The following analysis of the model is based on a linear approximation in the neighborhood of the model's steady state. In such a steady state we have constant money growth at the rate $x^* = \gamma$ and the nominal rate of interest is given as $i^* = \frac{1+\gamma-\beta}{\beta}$. The production function is of the Cobb Douglas type, such that $F(k,h) = k^{\alpha}\,h^{1-\alpha}$. With respect to consumption, the resource constraint implies $\frac{c^*}{y^*} \equiv s_c = 1 - s_g - \delta\,\frac{\alpha\beta}{1-\beta\,(1-\delta)}$. Utility is assumed to be additively separable in consumption and leisure, such that $u(c,l) = \frac{c^{1-\rho}}{1-\rho} + B\,\frac{l^{1-\nu}}{1-\nu}$. In the steady state we therefore get that h^* solves the equation $B s_c = (1-\alpha)\,\left(\frac{\beta}{1+\gamma}\right)^2\,\frac{(1-h^*)^{\nu}}{h^*}$. The elasticities of the marginal utility of consumption and leisure with respect to consumption and leisure are denoted by $n_{cc} = -\rho$ and $n_{ll} = -\nu = -n_{hh}\,l^*/h^*$, respectively. Furthermore, we have $\varepsilon_k = \varepsilon_{hk} = \alpha$, $\varepsilon_h = \varepsilon_{kh} = 1-\alpha$, $\varepsilon_{kk} = \alpha - 1$ as well as $\varepsilon_{hh} = -\alpha$. Bearing in mind that $\hat{g}_t = 0$, since $\{g_t\}_{t=0}^{\infty}$ is exogenous, it is possible to derive the following set of linear equations:

$$s_c\,\hat{c}_t + \frac{1-s_c}{\delta}\,\hat{k}_{t+1} = \left(\varepsilon_k + \frac{1-s_c}{\delta}(1-\delta)\right)\,\hat{k}_t + \varepsilon_h\,\hat{h}_t \tag{A.1a}$$

$$\eta_{cc}\,\hat{c}_t - \hat{p}_t = \eta_{cc}\,\hat{c}_{t+1} - \hat{p}_{t+1} + \frac{i^*}{1+i^*}\,\hat{i}_{t-1} - \frac{\gamma}{1+\gamma}\,\hat{x}_t \tag{A.1b}$$

$$\eta_{cc}\,\hat{c}_t - \frac{i^*}{1+i^*}\,\hat{i}_{t-1} = \eta_{cc}\,\hat{c}_{t+1} - \frac{i^*}{1+i^*}\,\hat{i}_t + [1-\beta\,(1-\delta)]\left[\varepsilon_{kk}\,\hat{k}_{t+1} + \varepsilon_{kh}\,\hat{h}_{t+1}\right] \tag{A.1c}$$

$$\eta_{hh}\,\hat{h}_t - \eta_{cc}\,\hat{c}_t = \hat{w}_t - \hat{p}_t - \frac{i^*}{1+i^*}\,\hat{i}_{t-1} \tag{A.1d}$$

$$\hat{w}_t - \hat{p}_t + \frac{i^*}{1+i^*}\,\hat{i}_{t-1} = \hat{\theta}_t + \varepsilon_{hk}\,\hat{k}_t + \varepsilon_{hh}\,\hat{h}_t \tag{A.1e}$$

$$\hat{p}_t + \hat{c}_t = -\frac{s^*}{1-s^*}\,\hat{s}_t \tag{A.1f}$$

$$\hat{w}_t + \hat{h}_t = \frac{s^*}{s^*+\gamma}\,\hat{s}_t + \frac{\gamma}{s^*+\gamma}\,\hat{x}_t \tag{A.1g}$$

A.2. *The model with an interest rate peg and distortionary taxation*

The budget restriction of the government $\mu\,[p_t\,g_t - x_t] = p_t\,\tau_t\,F(k_t,h_t)$ supplements now the model analyzed so far. Since $1+x_t = p_t\,\frac{F_h(k_t,h_t)}{1+i}\,h_t + p_t\,c_t$, we have in the steady state that $p^*\,F(k^*,h^*) = \frac{(1+\gamma)^2}{(1-\alpha)\,\beta+(1+\gamma)\,s_c}$. The steady state share of consumption s_c is now given by $s_c = s_c(\tau^*) = 1 - s_g - \delta\,\frac{\alpha\beta\,(1-\tau^*)}{1-\beta\,(1-\delta)}$. Therefore, the steady state tax rate τ^* can then be derived from the budget restriction of the government as the solution of the following equation:

$$\mu\gamma = p^*\,F(k^*,h^*)\,(s_g\,\mu - \tau^*) = \frac{(1+\gamma)^2}{(1-\alpha)\,\beta + (1+\gamma)\,s_c(\tau^*)}\,(s_g\,\mu - \tau^*)$$

Finally, h^* is now given as the solution of the equation $B\,s_c(\tau^*) = (1-\alpha)(1-\tau^*)\left(\frac{\beta}{1+\gamma}\right)^2 \frac{(1-h^*)^{\nu}}{h^*}$.

After a linearization in the neighborhood of the steady state, model is then characterized by the following set of linear equations:

$$s_c\,\hat{c}_t + s_g\,\hat{g}_t + \frac{1-s_c-s_g}{\delta}\,\hat{k}_{t+1} = \left(\varepsilon_k + \frac{1-s_c-s_g}{\delta}(1-\delta)\right)\hat{k}_t + \varepsilon_h\,\hat{h}_t \tag{A.2a}$$

$$\eta_{cc}\,\hat{c}_t - \hat{p}_t = \eta_{cc}\,\hat{c}_{t+1} - \hat{p}_{t+1} + \frac{i^*}{1+i^*}\,\hat{i}_{t-1} - \frac{\gamma}{1+\gamma}\,\hat{x}_t \tag{A.2b}$$

$$\eta_{cc}\,\hat{c}_t - \frac{i^*}{1+i^*}\,\hat{i}_{t-1} = \eta_{cc}\,\hat{c}_{t+1} - \frac{i^*}{1+i^*}\,\hat{i}_t + \frac{1-\beta\,(1-\delta)}{1-\tau^*}\left[\varepsilon_{kk}\,\hat{k}_{t+1} + \varepsilon_{kh}\,\hat{h}_{t+1} - \frac{\tau^*}{1-\tau^*}\,\hat{\tau}_{t+1}\right] \tag{A.2c}$$

$$\eta_{hh}\,\hat{h}_t - \eta_{cc}\,\hat{c}_t = \hat{w}_t - \hat{p}_t - \frac{i^*}{1+i^*}\,\hat{i}_{t-1} - \frac{\tau^*}{1-\tau^*}\,\hat{\tau}_t \tag{A.2d}$$

$$\hat{w}_t - \hat{p}_t + \frac{i^*}{1+i^*}\,\hat{i}_{t-1} = \hat{\theta}_t + \varepsilon_{hk}\,\hat{k}_t + \varepsilon_{hh}\,\hat{h}_t \tag{A.2e}$$

$$\mu\,\frac{s_g}{\tau^*}\,\hat{g}_t + \left(\mu\,\frac{s_g}{\tau^*} - 1\right)\hat{p}_t + \left(1 - \mu\,\frac{s_g}{\tau^*}\right)\hat{x}_t = \hat{\tau}_t + \varepsilon_k\,\hat{k}_t + \varepsilon_h\,\hat{h}_t \tag{A.2f}$$

$$\hat{p}_t + \hat{c}_t = -\frac{s^*}{1-s^*}\,\hat{s}_t$$

$$\hat{w}_t + \hat{h}_t = \frac{s^*}{s^*+\gamma}\,\hat{s}_t + \frac{\gamma}{s^*+\gamma}\,\hat{x}_t \tag{A.2g}$$

Pegging the nominal rate of interest implies that $\hat{i}_t = 0$ for all t. Moreover, the assumption of a given time path for government expenditures implies $\hat{g}_t = 0$ for all t. Equations (A.2d)–(A.2g) can then be used to eliminate the variables x_t, h_t, w_t and s_t from equations (A.2a) to (A.2c). The remaining dynamical system can be written as $z_{t+1} = P\,z_t$, where $z_t' = (k_t,\,c_t,\,p_t)$. Since capital is the only predetermined variable with initial value k_0, two eigenvalues of the matrix P have to be smaller than one in absolute value in order for a determinate equilibrium to exist.

A.3. *The model with a more general fiscal rule*

Starting point is the system of equations (A.2a)–(A.2g). Equation (A.2f) is replaced by the following two equations (here $d_0 = s_g/(s_g - \tau^* - \phi)$):

$$\hat{b}_t + \frac{1}{1+i^*}\,\hat{i}_{t-1} = d_0\,\hat{g}_t + \hat{p}_t + (1-d_0)\left[\frac{\tau^*}{\tau^*+\phi}\,\hat{\tau}_t + \varepsilon_k\,\hat{k}_t + \varepsilon_h\,\hat{h}_t\right] \tag{A.3a}$$

$$\phi\,p^*\,y^*\left[\hat{x}_t - \hat{p}_t - \varepsilon_k\,\hat{k}_t - \varepsilon_h\,\hat{h}_t\right] = \left(\frac{\phi\,p^*\,y^*}{\gamma} - 1\right)\left[\hat{b}_t - \frac{i^*}{1+i^*}\,\hat{i}_{t-1} - (1+\gamma)\,\hat{b}_{t+1} + (1+\gamma)\,\frac{i^*}{1+i^*}\,\hat{i}_t\right] \tag{A.3b}$$

The remaining analysis is performed in the same way as in case of the above discussed models.

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