## **A Decompositional Approach to the Estimation of Technological Change**

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## **Abstract**

Technological change is a key determinant of economic growth. The estimation of technological change is a very important but troublesome task. One of the crucial problems in conventional methods for the estimation is that they do not sufficiently have micro-theoretical underpinnings. The main purpose of this paper is to suggest a new decompositional approach to evaluating technological change in deterministic AGE fashion. The multiple calibration technique is applied to assess it. This method enables us to decompose technical change into one by price substitution effects and the other by factor-biased effects.

Keywords: Technological change, Calibration, General equilibrium model

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#### 1. Introduction

Although several decades have passed since Solow's seminal papers appeared, there is still room for progress in the estimation of technological change. Although the estimation is cumbersome, it is necessary if we want to understand the contribution of factors to economic growth or the change of economic structures over time.

The purpose of this paper is to suggest a new approach to the estimation of technological change. One of the most common methods is the Total Factor Productivity measurement or the Growth Accounting method shaped by Solow (1957), which decomposes output growth into measured increases in factor inputs and technical change (see, for example, Denison, 1967; Jorgenson and Griliches, 1967). This method is of great significance with regards to the explicit integration of economic theory into such a decomposition (Griliches, 1996). This paper is motivated by Solow's theme. The "new wrinkle" we want to describe is an elementary way of segregating technological change due to price substitution effects from that due to other effects, capturing the interdependence among economic sectors. The double calibration technique (Dawkins et al., 2001) is applied to decompose technological change.

This method also takes over the inheritance of the Input Output (IO) analysis. In the IO framework, Structural Decomposition Analysis (SDA) has recently developed into a major tool for decomposition (Rose and Casler, 1996), as it overcomes the static features of the IO analysis and enables us to examine structural changes. However, as Rose and Casler (1996) point out, "a rigorous grounding in economic theory is lacking for SDA". This paper may provide some theoretical underpinnings to IO analysis.

In addition, the method has an advantage - data availability or efficiency. Although the attempt

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to conduct econometric studies often suffers from data insufficiency, our approach requires only two period datasets. It is therefore a practical alternative to econometrics.

Section 2 explains the methodology, while Section 3 applies this method to an empirical case, the oil crises in Japan. Our method can segregate price-induced technological change from other causes, and the analysis may have some implications for Japanese environmental policy, including the carbon tax that is currently being discussed.

## 2. The Methodology

In this section, our new method of evaluation is explained. The new feature of the method is the application of the double calibration technique to *ex post* decomposition analysis of technological change between two periods.<sup>[1,](#page-2-0)[2](#page-2-1)</sup> This technique enables us to disentangle the individual causes from a series of simultaneous shocks to an economy in consistent with the general equilibrium theory. In the paper, total technological change (TTC) can be decomposed exactly into two components, price-induced technological change (PITC) and factor-biased technological change (FBTC).

Let us consider the behavior of industries. Their production functions are given by constant-returns-to-scale CES functions, and they are assumed to act so as to maximize their profits in competitive markets. Hence, factor inputs per unit output (hereafter factor inputs) in the initial period

<span id="page-2-0"></span> $\frac{1}{1}$ <sup>1</sup> For the double calibration technique, see Dawkins et al. (2001). Only a few attempts have been made at the double calibration (e.g., Piggott and Whalley, 2001; Abrego and Whalley, 2002).

<span id="page-2-1"></span><sup>&</sup>lt;sup>2</sup> In the analysis, like other literature on this subject, technological change is defined as changes of factor inputs per unit output, which is identical to the changes of input coefficients in IO tables.

 $(t = t_0)$  are derived as in equation (1). Capital (K) and labor (L) are the primary factors.

$$
\frac{x_{ij}^{\circ}}{X_j^{\circ}} = \lambda_{ij}^{\circ} b_j^{\sigma_j - 1} \left( a_{ij} \frac{p_j^{\circ}}{p_i^{\circ}} \right)^{\sigma_j}, i = 1, ..., n, K, L; j = 1, ..., n,
$$
\n(1)

where  $x_{ij}^0$  is the input of *i* by sector *j* in t<sub>0</sub>,  $X_j^0$  is the output of sector *j* in t<sub>0</sub>,  $p_i^0$  is the price of *i* in t<sub>0</sub>,  $\sigma_j$  is the elasticity of substitution of sector *j*,  $\lambda_{ij}^0$  is the FBTC parameter in t<sub>0</sub>,  $a_{ij}$  is the share parameter ( $\sum_i a_{ij} = 1$ ), and  $b_i$  is the scale parameter.

The parameter  $\lambda_{ij}$  embodies (sector-specific) FBTC. In t<sub>0</sub>, all prices and  $\lambda_{ij}^{\circ}$  are set at unity. When the values of  $x_{ij}^{\circ}$  and  $X_j^{\circ}$  are obtained from the actual data, and the substitution parameters  $\sigma_j$  are exogenously given, all parameters of the production functions,  $a_{ij}$  and  $b_j$ , are determined to reproduce the actual economic structure in  $t_0$  as an equilibrium. This is the same procedure followed under the conventional single calibration technique.<sup>[3](#page-3-0)</sup> Then, the production functions are specified. The parameters,  $a_{ij}$ ,  $b_j$ , and  $\sigma_j$ , are assumed to be invariant over the periods.

Next, in the terminal period (t = t<sub>1</sub>), factor inputs in t<sub>1</sub> are given by equation (2).

$$
\frac{x_{ij}^1}{X_j^1} = \lambda_{ij}^1 b_j^{\sigma_{j-1}} \left( a_{ij} \frac{p_j^1}{p_i^1} \right)^{\sigma_j},\tag{2}
$$

where  $x_{ij}^1$  is the input of *i* by sector *j* in t<sub>1</sub>,  $X_j^1$  is the output of sector *j* in t<sub>1</sub>,  $p_i^1$  is the price of *i* in t<sub>1</sub>, and  $\lambda_{ij}^1$ is the FBTC parameter in  $t_1$ .

In the double calibration technique, another data period is used to specify unknown parameters. Hence, when the values of  $x_{ij}^1$ ,  $X_j^1$ , and  $p_i^1$  are obtained from the dataset, the FBTC parameters  $\lambda_{ij}^1$  are endogenously determined to replicate the economic structure in  $t_1$  as another equilibrium. In other words,

<span id="page-3-0"></span><sup>—&</sup>lt;br>3  $3$  For the single calibration technique, see Mansur and Whalley (1984) and Dawkins et al. (2001).

 $\lambda_{ij}^{\perp}$  are chosen to fill the gap between the counterfactual equilibrium associated with the price change under the specified production functions and the actual equilibrium in  $t_1$ .

From equations (1) and (2), the changes in factor inputs are

$$
\frac{\Delta(x_{ij}/X_j)}{(x_{ij}/X_j)} = \sigma_j \frac{\Delta(p_j/p_i)}{(p_j/p_i)} + \frac{\Delta\lambda_{ij}}{\lambda_{ij}}.
$$
\n
$$
\Leftrightarrow \text{TTC} = \text{PITC} + \text{FBTC}
$$
\n(3)

 $\lambda_{ij}^1$  >1, factor-augmenting technological change occurs, while when  $\lambda_{ij}^1$  <1, factor-diminishing As in equation (3), changes in factor inputs (TTC) are decomposed into PITC and FBTC. PITC, which depends on the elasticity of substitution  $\sigma_j$  and the change in relative prices over the periods, embodies the price substitution effects on the production functions. On the other hand, FBTC embodies the parts of the factor input change that cannot be explained by price substitution effects. Hence, when technological change occurs.

Figure 1 illustrates the concept of the method. From a theoretical viewpoint, PITC represents the change in factor inputs along the production functions, and FBTC represents the shift of the production functions. In contrast to IO analysis, in which technological change is measured without respect to price change, our new method can explicitly incorporate price substitution effects into the evaluation of technological change.

Further, equation (2) can also be expressed as equation (4) using matrices.

$$
\begin{pmatrix}\n\frac{x_{11}^1}{X_1^1} & \cdots & \frac{x_{1n}^1}{X_n^1} \\
\vdots & \frac{x_{ij}^1}{X_j^1} & \vdots \\
\frac{x_{n1}^1}{X_1^1} & \cdots & \frac{x_{nn}^1}{X_n^1}\n\end{pmatrix}\n\begin{pmatrix}\n\frac{1}{p_1^1}\n\end{pmatrix}^\sigma & \cdots & 0 \\
\vdots & \left(\frac{1}{p_i^1}\right)^\sigma & \vdots \\
0 & \cdots & \left(\frac{1}{p_n^1}\right)^\sigma\n\end{pmatrix}\n\begin{pmatrix}\n\lambda_{11}^1 b_1^{\sigma-1} a_{11}^{\sigma} & \cdots & \lambda_{1n}^1 b_n^{\sigma-1} a_{1n}^{\sigma} \\
\vdots & \lambda_{ij}^1 b_j^{\sigma-1} a_{ij}^{\sigma} & \vdots \\
\lambda_{n1}^1 b_1^{\sigma-1} a_{n1}^{\sigma} & \cdots & \lambda_{nn}^1 b_n^{\sigma-1} a_{nn}^{\sigma}\n\end{pmatrix}\n\begin{pmatrix}\np_1^{i\sigma} & \cdots & 0 \\
\vdots & \vdots & \vdots \\
0 & \cdots & p_n^{i\sigma}\n\end{pmatrix}
$$
\n
$$
\Leftrightarrow \mathbf{A} = \hat{\mathbf{Q}} \mathbf{L} \hat{\mathbf{P}} \tag{4}
$$

Interestingly, equation (4) is similar to the RAS matrices in IO analysis (e.g., Bacharach, 1970). In the RAS terminology,  $\hat{Q}$  is regarded as the  $\hat{R}$  matrix, which stands for substitution effects, and  $\hat{P}$  as the  $\hat{\mathbf{S}}$  matrix, for fabrication effects.

# 3. Empirical Results

In this section, this evaluation method is applied to an actual case, the oil crises in Japan. In the 1970s, skyrocketing oil prices greatly influenced the Japanese economy. This situation offers a typical example to apply our method.

For the analysis, 1970 and 1980 data are used. Nominal outputs (factor inputs) are obtained from Input–Output Tables (Management and Coordination Agency). Real outputs (factor inputs) are estimated by deflating nominal outputs by the following price indices. Price indices are from the Domestic Wholesale Price Index (Bank of Japan)<sup>[4](#page-5-0)</sup> or Deflators on Outputs of National Accounts (Economic Planning Agency).<sup>[5](#page-5-1)</sup> Capital and labor prices are estimated following Ito and Murota (1984).

Tables 1, 2 and 3 show FBTC in the cases where  $\sigma = 0$ ,  $\sigma = 0.5$  and  $\sigma = 1$ , respectively.<sup>[6](#page-5-2)</sup> FBTC, which is represented as a percentage change, varies depending on  $\sigma$ . First, in the case where  $\sigma = 0$ , there is no price substitution and PITC = 0. Hence, FBTC explains all the changes in factor inputs,

<span id="page-5-0"></span> $\frac{1}{4}$ <sup>4</sup> For EII, MAC, OMF, COAL, OIL, ELC and GAS.

<span id="page-5-1"></span><sup>&</sup>lt;sup>5</sup> For AGM and SER.

<span id="page-5-2"></span><sup>&</sup>lt;sup>6</sup> In this paper, elasticities of substitution are assumed to be constant in all sectors and between inputs for simplicity. However, this methodology can be applied to the case where elasticities are different in each sector and between inputs using nested production functions.

i.e., FBTC can be regarded as technological change itself. Next, as shown in the Tables, FBTC changes in line with changes in  $\sigma$ . A larger  $\sigma$  makes price substitution effects more likely. Therefore, the more  $\sigma$  increases, the larger the proportion of TTC that is explained by PITC. In the analysis, elasticities of substitution are arbitrarily changed between zero and one, since the purpose here is to explain our methodology. In practice, empirically estimated parameters should be used for substitution parameters. For the Japanese case, the existing literature (e.g., Tokutsu, 1994) shows that most elasticities of substitution are below one.

Here, the case of OIL is analyzed as an example, since FBTC for OIL is considered to be greatly affected by the oil crises. In the case where  $\sigma = 0$  (no price substitution) in Table 1, most sectors have a negative FBTC for OIL. This means that factor inputs of OIL decreased in most sectors, implying that OIL-saving technological change occurred in the 1970s.

However, price substitution effects had occurred in reality. These effects are taken into consideration in Tables 2 and 3. As has been seen, FBTC for OIL increases as  $\sigma$  becomes larger. In Table 3, all the sectors have a positive FBTC for OIL, which means factor-augmenting technological change occurred. This implies that price substitution effects were expected to induce a larger decrease in factor inputs of OIL, whereas factor inputs did not decrease to the degree that was expected from these effects. In sum, OIL-saving technological change over the periods can be explained entirely, or even more, by PITC, rather than FBTC.

# 4. Conclusion

This paper proposed a new methodology for the evaluation of technological change. This method serves as an elementary but powerful tool for empirical studies. In addition, it may give some micro-theoretical foundations to conventional methods.

Griliches (1996) has mentioned that all the pioneers of this subject were clear about the tenuousness of the estimation of technological change. This caution holds true for our method as well - for example, one limitation of the method is that it employs a deterministic procedure. The method could be more fruitful if used complementarily with other conventional methods such as IO-SDA or econometric methods.

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Input	Sector				
	<b>AGM</b>	EП	MAC	<b>OMF</b>	<b>SER</b>
AGM	$-4.7\%$	$-32.4%$	$-53.1%$	$-0.4%$	$-1.8%$
EII	15.4%	$-3.1\%$	$-69.3%$	18.1%	2.1%
<b>MAC</b>	105.5%	23.0%	4.9%	109.0%	36.0%
<b>OMF</b>	$0.1\%$	$-5.7\%$	$-58.7\%$	$-11.1%$	$-32.4%$
<b>SER</b>	32.4%	$-2.0\%$	$-36.9%$	29.1%	5.3%
<b>COAL</b>	$-117.3%$		$-12.2\% -142.1\%$	$-72.5%$	$-5.7\%$
OIL	$-11.2%$		$-8.0\% -117.3\%$	0.6%	$-51.2%$
ELC	28.2%	2.6%	$-37.6%$	36.5%	20.4%
<b>GAS</b>	40.0%	32.0%	$-59.4%$	34.0%	54.3%
K	29.3%	11.2%	$-33.0%$	52.9%	37.0%
L	$-57.8%$	$-31.6%$	$-85.1%$	$-19.1%$	$-25.9%$

**Table 1 Factor-biased technological change (percentage changes) when**  $\sigma = 0$ 

Classifications are as follows.

AGM: Agriculture, forestry, fishery and mining, EII: Energy intensive industry (paper and pulp, chemical, ceramics, and iron and steel), MAC: Machinery, OMF: Other manufacturing, SER: Services and others, COAL: Coal and coal products, OIL: Oil and oil products, ELC: Electricity, GAS: Gas, K: Capital, L: Labor.

**Table 2 Factor-biased technological change (percentage changes) when** σ **= 0.5** 

Input	Sector				
	AGM	EП	<b>MAC</b>	OMF	<b>SER</b>
AGM	$-4.7\%$	$-32.0%$	$-29.6%$	$-8.7\%$	$-5.2\%$
EH	15.1%	$-3.1\%$	$-46.2%$	9.4%	$-1.7\%$
<b>MAC</b>	82.0%	$-0.1\%$	4.9%	77.1%	9.1%
<b>OMF</b>	8.5%	3.0%	$-26.8%$	$-11.1%$	$-27.5%$
<b>SER</b>	35.9%	1.8%	$-9.9\%$	24.1%	5.3%
<b>COAL</b>	$-103.2%$		2.2% -104.5%	$-66.8%$	5.0%
OIL	36.8%	40.4%	$-45.7%$	40.2%	$-6.6%$
<b>ELC</b>	51.5%	26.3%	9.2%	51.5%	40.3%
GAS	53.8%	46.2%	$-22.2%$	39.4%	64.6%
K	4.7%	$-13.0%$	$-34.0%$	20.0%	9.0%
L	$-32.2%$	$-5.5\%$	$-35.9%$	$-1.8%$	$-3.7\%$

Classifications are the same as in Table 1.

**Table 3 Factor-biased technological change (percentage changes) when**  $\sigma = 1$ 

Input	Sector				
	<b>AGM</b>	EП	MAC	OMF	<b>SER</b>
AGM	$-4.7%$	$-31.7%$	$-6.1\%$	$-17.1%$	$-8.7\%$
EП	14.7%	$-3.1\%$	$-23.0%$	$0.7\%$	$-5.5\%$
<b>MAC</b>	58.5%	$-23.3%$	4.9%	45.2%	$-17.9%$
<b>OMF</b>	16.8%	11.8%	5.0%	$-11.1%$	$-22.6%$
<b>SER</b>	39.3%	5.6%	17.0%	19.2%	5.3%
<b>COAL</b>	$-89.1%$	16.7%	$-66.9%$	$-61.1%$	15.6%
OIL	84.9%	88.7%	25.8%	79.9%	38.0%
ELC	74.9%	50.0%	56.0%	66.5%	60.2%
GAS	67.6%	60.3%	15.1%	44.9%	75.0%
K	$-19.8%$	$-37.2%$	$-35.1%$	$-12.9%$	$-19.0%$
L	$-6.5\%$	20.5%	13.3%	15.5%	18.5%

Classifications are the same as in Table 1.



**Figure 1 The method**