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Relative Payoffs and Evolutionary Spite — Evolutionary Equilibria in Games with Finitely Many Players

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Abstract

Evolutionary dynamics in evolutionary games as well as in evolutionary algorithms imply de–facto spiteful behavior of the players: In order to 'survive' the evolutionary process, players must perform better than their opponents. This means they maximize relative rather than absolute payoffs. The paper shows that there is a class of games resulting in different equilibria if played by maximizers of absolute or of relative payoffs, respectively. It is demonstrated that evolutionary equilibria (general ESS) can be found by formally maximizing relative payoffs. This finding makes it much easier to derive the long run behavior of evolutionary dynamics in these games. The method is analytically deduced and demonstrated at the examples of four relatively 'large' games: the Cournot oligopoly game, the public goods game, the Tullock game of rent seeking and the Van Huyck et al. (1990) coordination game.

Zusammenfassung

Evolutionäre Dynamiken sowohl in evolutionären Spielen als auch in Evolutionären Algorithmen implizieren de–facto boshaftes Verhalten der Spieler: Um den evolutionären Prozeß zu „überleben", müssen sich die Spieler besser schlagen als ihre Gegner. Das bedeutet, sie maximieren relative anstelle von absoluten Auszahlungen. Das Papier zeigt, daß es eine Klasse von Spielen gibt, die zu anderen Gleichgewichten führt, wenn sie von Maximierern absoluter beziehungsweise relativer Auszahlungen gespielt werden. Es wird gezeigt, daß evolutionäre Gleichgewichte (generelle ESS) durch die formale Maximierung relativer Auszahlungen gefunden werden können. Dieses Resultat macht es leichter, das langfristige Verhalten evolutionärer Dynamiken in diesen Spielen vorherzusagen. Die Methode wird analytisch hergeleitet und anhand der Beispiele von vier bekannten Spielen vorgeführt: dem Cournot Oligopol–Spiel, dem Öffentlich–Gut–Spiel, dem Tullock–Spiel des Rent–Seeking und dem Van Huyck et al. (1990) Koordinations–Spiel.

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1 Introduction

The idea of evolution in games is central to many contributions from the last few decades. A first step of formalizing an evolutionary equilibrium concept dates back to Maynard Smith and Price (1973) and Maynard Smith (1974, 1982), introducing the notion of an evolutionarily stable strategy (ESS). This idea implicitly relies on a class of evolutionary dynamics, some members of which have been made more explicit in form of various types of replicator dynamics (see, e.g. Weibull 1995; Samuelson 1997; Hofbauer and Sigmund 1998).

In economics, evolutionary dynamics are often seen as metaphors for processes of social learning by boundedly rational agents (Marimon, 1993; Kandori et al., 1993; Fudenberg and Levine, 1998b,a; Friedman, 1998).

The central idea of all types of evolutionary dynamics is the Darwinian theory of the 'survival of the fittest': In a population of players playing one strategy each, over time, those strategies are adopted more frequently than others that are better (in terms of payoff) than others. From a player's point of view, this means that she should use a strategy which performs better than the ones used by her opponents. If, for example, in a two–player game a player has the choice between a strategy with a high absolute payoff, which yields the same payoff to the opponent, and a strategy with a lower payoff, the use of which decreases the opponent's payoff even more, under an evolutionary regime the player should use the second strategy: Evolutionary dynamics imply maximization of relative rather than absolute payoff. This behavior is spiteful behavior (and of course incompatible with our traditional notion of the utility maximizing 'homo oeconomicus'). Nevertheless, with evolutionary dynamics at work, this type of spiteful behavior does not automatically mean that players have spiteful motives. It is simply the force of evolution that leads to this type of 'evolutionary spite'.[1]

The impact of evolutionary spite is particularly strong in games with only a finite number of players. It was Schaffer (1988, 1989) who first showed this result and consequently extended the concept of an ESS to include games with finitely many players: He introduced the concept of a general evolutionarily stable strategy (general ESS). With this, Schaffer laid the grounds for the more recent, elaborate concepts of stability in dynamic games, particularly the ideas of a 'long run stable strategy' (Young, 1993) or a 'stochastically stable equilibrium' (Kandori et al., 1993).

Still, the ideas of evolutionary equilibria, although elaborate and clear, share a common weakness: For many games, particularly in some of the 'larger' ones,

[1] An alternative interpretation suggests that agents do hold spitefull motives, which means they have 'competitive preferences'. This interpretation, though, seems to be of only limited weight: For simple decision problems, Charness and Rabin (2002) find the effect of such preferences neglectible.

the computation of evolutionary equilibria is extremely complicated. This paper aims to provide a solution to this problem. As will subsequently be shown, there exists a class of games that allow for a 'shortcut' of finding evolutionary equilibria. This is done by explicitly making use of the notion that spiteful behavior and the maximization of relative payoff is the core concept of evolutionary dynamics.

The paper starts with a simple introductory example demonstrating the impact of evolutionary spite and the finiteness of the number of players on the expected outcome of a game. At this point, the idea of formally maximizing relative payoff is introduced. The paper proceeds with a formal proof that a general ESS can be found by maximization of relative payoff. After that, the implementation of this method is demonstrated. The resulting equilibria are deduced and interpreted for the general case as well as for four exemplary games: the Cournot game of oligopolistic quantity choice, the Tullock rent seeking game, the public goods game, and the Van Huyck et al. cooperation game.

2 Relative Payoff and Evolutionary Spite

The core idea of evolutionary dynamics is to model processes similar to the process of the 'Darwinian evolution' and its corresponding principle of the 'survival of the fittest'. What matters for a long run survival is to be ahead of the others. This does not automatically mean being 'good' or successful in any other concern than pure survival.

In the analysis of evolutionary games and evolutionary dynamics, the main goal is to identify those strategies which in the long run will be played by most of, if not even by all players of the game. Evolutionary dynamics thus describe a process of changing frequencies of strategies played by a population of players. In this, the growth rate of the population share of a strategy is determined by its relative payoff: In order to spread throughout a population, a strategy has to perform better than the average, i.e. the strategy's payoff has to be higher than the population mean payoff. Consequently, players in evolutionary games try to find a strategy which leaves them better off than their opponents. (Note that this is usually *not* the primary goal of players in 'normal' games.) This might even mean that a player *does not* use a strategy which guarantees maximum *absolute* payoff, if there is a strategy available which will increase the difference between the player's payoff and the population mean payoff, i.e. the player's *relative* payoff. This type of behavior is called *spiteful behavior* (Schaffer, 1988, 1989): Players are willing to hurt themselves, if by doing so they hurt their opponents even more.

There exists a class of games which are sensitive to spiteful behavior: Games of this class will have a different outcome if agents maximize relative instead of absolute payoff.[2] This class of games will be called *spite sensitive games* in this paper. The games in focus share a common important feature: They are games with a finite number of players. For games similar to the ones analyzed here, but with infinitely many players, the results derived in this paper will be shown to coincide with the traditional findings, like e.g. for the Cournot game.

In order to provide a first, simple impression of the potential role of spiteful behavior in spite sensitive games, consider the two–player–two–strategy–game in normal form given in Table 1(a). The players are assumed to be restricted to playing pure strategies. Assuming $a > b > c > d$, the profile $(s_1,\ s_1)$ clearly is the payoff dominant equilibrium of the game. Assuming common knowledge, both players, A and B, can be expected to play s_1, as long as their goal is the maximization of absolute payoffs.

		Player B	
		s_1	s_2
Player A	s_1	$a,\ a$	$c,\ b$
	s_2	$b,\ c$	$d,\ d$

(a) absolute payoffs

		Player B	
		s_1	s_2
Player A	s_1	$0,\ 0$	$\frac{1}{2}(c-b)\,,\ \frac{1}{2}(b-c)$
	s_2	$\frac{1}{2}(b-c)\,,\ \frac{1}{2}(c-b)$	$0,\ 0$

(b) relative payoffs

Table 1: General Spite Game; $a > b > c > d$

Moreover, due to the definition of Maynard Smith (1982), the profile $(s_1,\ s_1)$ is the only evolutionarily stable equilibrium (ESS).

Payoffs in Tab. 1(a) are absolute payoffs. In evolutionary games, though, it is relative rather than absolute payoff which players try to maximize. In order to illustrate the consequences of this change in scope, the game can be re–formulated by explicitely stating relative payoffs in the normal form. Table 1(b) shows the re–formulated normal form of the exemplary game.

The relative payoff to player k, playing s_i against player $-k$'s strategy s_j, $\pi_k^r(s_i,\ s_j)$, is simply the difference between player k's payoff and the mean payoff of all players in the game (i.e. players k and $-k$ in the example):

$$\pi_k^r(s_i,\ s_j) = \pi_k(s_i,\ s_j) - \frac{1}{2}\left[\pi_k(s_i,\ s_j) + \pi_{-k}(s_i,\ s_j)\right]\ . \tag{1}$$

This concept of relative payoff is the same as the one commonly used in continuous time replicator dynamics (Samuelson 1997, p. 66; Weibull 1995, pp. 72–

[2] In the words of Bergstrom (2002, p. 10): 'There is an interesting class of games in which every player gets a higher payoff from cooperating than from defecting, but where, paradoxically, it is also true that [...] defectors receive higher payoffs than cooperators.'

74). This concept of relative payoff will be linked to the concept of a general evolutionarily stable strategy (Schaffer, 1988, 'general ESS')[3] later in this paper.

It is obvious that in relative payoffs, i.e. in the normal form of Tab. 1(b), the profile $(s_2,\, s_2)$ represents the dominant equilibrium. Thus, the exemplary game is a game with 'two faces': For players maximizing absolute payoff, the profile being played is $(s_1,\, s_1)$. On the other hand, players maximizing relative payoff, i.e. behaving spitefully, will play $(s_2,\, s_2)$. Note, that even though $(s_2,\, s_2)$ is not an equilibrium of the original game, in the re–formulated game it is.

It is an interesting question why the re–formulated games has its only equilibrium in $(s_2,\, s_2)$ *although* the ESS of the original game is $(s_1,\, s_1)$. The answer is simple: The canonical definition of an ESS only holds for an infinitely large number of players. The simple game in focus can be used to illustrate the importance of the number of players for the location of an evolutionary equilibrium.

Following the concept of a general ESS for finite populations introduced by Schaffer (1988), it can be found that in an n–player version of the spite game (1(a)) with n_1 of the n players playing s_1, the location of the general ESS depends on the payoffs *and* on the ratio of the total number of players to the number of players playing strategy s_1 (or s_2, respectively).

In any n–player version of the game from Tab. 1, a strategy $s^\star$ constitutes a globally stable general ESS, if in every possible population of strategies it performs better than any other strategy. In an n–player version of the game, every possible population of strategies can be completely characterized by n_1, the number of players playing s_1. Assuming players playing the field, i.e. every player playing against everybody else, the average payoffs from playing s_1 or s_2, respectively, depend on n_1 and n:

$$\pi\left(s_1 |\, \{n,\, n_1\}\right) \;=\; \frac{1}{n-1}\left(\left(n_1-1\right)a+\left(n-n_1\right)c\right)\;, \tag{2}$$

$$\pi\left(s_2 |\, \{n,\, n_1\}\right) \;=\; \frac{1}{n-1}\left(n_1\, b+\left(n-n_1-1\right)d\right)\;. \tag{3}$$

Strategy s_1 constitutes a general ESS, if in the population characterized by n and n_1, it yields a higher payoff than s_2, i.e. if $\pi\left(s_1 |\, \{n,\, n_1\}\right) > \pi\left(s_2 |\, \{n,\, n_1\}\right)$. In the opposite case s_2 is a general ESS:

$$\begin{aligned} \pi\left(s_1 |\, \{n,\, n_1\}\right) &\lesseqgtr \pi\left(s_2 |\, \{n,\, n_1\}\right) \\ &\Leftrightarrow \\ n &\lesseqgtr \frac{b-d-a+c}{c-d}\, n_1 + a - d \end{aligned} \tag{4}$$

[3] A textbook version of this concept can be found at Vega-Redondo (1996, pp. 31).

For a more intuitive notion, consider the special case with $a = 4,\ b = 3,\ c = 2,\ d = 1$. Here, the spite game has a unique general ESS in $(s_2,\ s_2)$ as long as $n < 3$ (and independent of n_1). For larger numbers of players, precisely for $n > 3$, the unique general ESS is the same as the canonical ESS in $(s_1,\ s_1)$.

The general idea which should have become clear from this section is the following: There is a class of games with finitely many players, i.e. spite sensitive games, whose results depend on the aim of players' behavior: If players are 'regular' payoff maximizers, the results differ from the results arising from the same game played by finitely many, spitefully behaving players.

3 Maximization of Relative Payoff and Evolutionary Stability

Up to this point, the notion that maximization of relative payoffs should be the central method in order to find evolutionarily stable strategies has been introduced only intuitively. This section will provide a formal derivation of the finding that maximization of relative payoffs leads to a general ESS.

Consider a game played by a population of n players. Players are restricted to playing pure strategies only. Let the strategy space of each player be $\mathbb{S} = \{s_1,\ s_2,\ \ldots\}\ ;\ \#(\mathbb{S}) \leq \infty$.

As this paper will focus on one–population evolutionary games, i.e. symmetric games only, the set of *relevant* different strategy profiles $\mathbb{V}$ can be written as a set of vectors $\mathbf{v}$ denoting the number of players playing each strategy $s_i \in \mathbb{S}$: $\mathbf{v} = (v(s_1),\ v(s_2),\ \ldots)$. Obviously, $\sum_{s\in\mathbb{S}} v(s) = n$. Further, let $\mathbb{V}_{s^\star}$ denote the set of profiles containing at least one player playing $s^\star$: $\mathbb{V}_{s^\star} := \{\mathbf{v} \in \mathbb{V} \,|\, v(s^\star) > 0\}$.

Let $\pi(s_i)$ denote the payoff to a player playing strategy s_i in a population characterized by a profile $\mathbf{v}$.

Proposition 1 *A globally stable general ESS can be computed by maximizing relative payoff:*

$$\max_s \pi^r(s) = \max_s \left[\pi(s) - \frac{1}{n} \sum_{s'\in\mathbb{S}} v(s')\,\pi(s')\right] \tag{5}$$

A sufficient (though not necessary) condition for a strategy $s^\star$ to constitute a general ESS is the definition of a globally stable general ESS: In every population of strategies containing at least one player playing $s^\star$, i.e. in every $\mathbf{v}' \in \mathbb{V}_{s^\star}$, strategy $s^\star$ yields a strictly higher payoff than any other strategy $s' \in \mathbb{S} \setminus s^\star$:

Definition 1 (globally stable general ESS (Schaffer, 1988)) *Strategy* $s^\star \in \mathbb{S}$ *is called a* globally stable general evolutionarily stable strategy (general ESS), *if for any given strategy profile* $v \in \mathbb{V}_{s^\star}$

$$\pi(s^\star) > \pi(s') \quad \forall \quad s' \in \mathbb{S} \setminus s^\star \tag{6}$$

Proof 1 *As there are* $n - v(s^\star)$ *players not using strategy* $s^\star$*, from (6) it follows that*

$$(n - v(s^\star))\ \pi(s^\star) > \sum_{s' \in \mathbb{S} \setminus s^\star} v(s')\, \pi(s') \tag{7}$$

Adding the payoff of all $s^\star$*–players to both sides of (7) results in*

$$\pi(s^\star) \quad > \quad \frac{1}{n} \sum_{s' \in \mathbb{S}} v(s')\, \pi(s') \tag{8}$$

Rearranging (8) yields the globally stable general ESS:

$$s^\star = \underset{s}{\operatorname{argmax}} \left[\pi(s) - \frac{1}{n} \sum_{s' \in \mathbb{S}} v(s')\, \pi(s') \right] \tag{9}$$

Note that the expression $\frac{1}{n} \sum_{s' \in \mathbb{S}} v(s')\, \pi(s')$ *in (9) gives the* population mean *payoff. Consequently, the entire expression to be maximized in (9) is the relative payoff to a player playing strategy* s*,* $\pi^r(s)$*.*

This means that $s^\star$*, the general ESS, can be simply computed by maximizing relative payoff as stated in the proposition.*

□

This is the shortcut method to finding general ESS in spite sensitive games. Apparently, there is an astonishingly large number of games this method can be successfully applied to. Some possible applications and results of the method are shown in the examples below.

4 Oligopolies: The Cournot Game

The Model One example of the class of spite sensitive games is the Cournot game, which is a game of simultaneous quantity choice by n firms. Every firm i produces a quantity s_i of a homogeneous good. The individual supply s_i is assumed to be of non–negligible impact on aggregate supply and thus on the market price: $\frac{\partial p}{\partial s_i} \neq 0$. Aggregate supply is the sum of the individual quantities firms choose in the model. The market equilibrium price p results from the interaction of aggregate supply and aggregate demand. Aggregate demand is given exogenously.

The only assumption needed concerning aggregate demand is the assumption of a downward sloping inverse demand function.

Absolute payoff of firm i, $\pi_i\,(s_i)$, is given by the difference between revenues and costs,

$$\pi_i\,(s_i) = p\,s_i - C\,(s_i)\ . \tag{10}$$

The cost function is assumed to be the same for every firm and increasing in the quantity. Marginal costs are assumed to be non–decreasing.

The 'standard' Cournot model is known to result in a unique symmetric Nash equilibrium with each firm supplying a quantity $s_a^\star$, implicitly given by

$$p = \frac{\partial\, C(s_a^\star)}{\partial\, s_a^\star} - \frac{\partial\, p}{\partial\, s_a^\star}\, s_a^\star\,. \tag{11}$$

The Game in Relative Payoffs The population mean payoff, $\overline{\pi}$, is the average absolute payoff of all firms,

$$\overline{\pi} = \frac{1}{n}\sum_{i=1}^{n} \pi_i\,(s_i)\ . \tag{12}$$

Consequently, relative payoff of firm i, given by the difference between the firm's absolute payoff and the average payoff of the supply side, is

$$\pi_i^r\,(s_i) = \pi_i\,(s_i) - \overline{\pi}\,. \tag{13}$$

Assuming identical equilibrium behavior of all players $-i$, i.e. all players but player i, (13) becomes

$$\pi_i^r\,(s_i) \;=\; \frac{n-1}{n}\left[p\,(s_i - s_{-i}) + C\,(s_{-i}) - C\,(s_i)\right]\ . \tag{14}$$

Individual maximization of (14) with respect to s_i yields

$$p \;=\; \frac{\partial C\,(s_i)}{\partial s_i} + (s_{-i} - s_i)\,\frac{\partial p}{\partial s_i}\,. \tag{15}$$

Note that (15) implicitly defines player i's reaction function.

For completely symmetric equilibrium behavior, i.e. $s_i = s\ \forall\ i$, the term $(s_{-i} - s_i)$ vanishes, such that the evolutionarily optimal quantity, $s_r^\star$, is implicitly given by

$$p = \frac{\partial C\,(s_r^\star)}{\partial s_r^\star}\,. \tag{16}$$

This is exactly the 'price equals marginal costs' condition known as the optimum condition for quantity choice in perfectly competitive markets. Consequently, $s_r^\star$ gives the Walrasian equilibrium quantity, and (16) establishes a Walrasian equilibrium.

The Walrasian equilibrium (16) is the only symmetric general ESS, i.e. the only symmetric Nash equilibrium of the re–formulated Cournot game in relative payoffs. Or, to make it sound a little more paradoxical: For relative payoffs, the Walrasian equilibrium *is* the only symmetric Cournot–Nash equilibrium.

Another feature of the Walrasian equilibrium should be stressed: In contrast to the Cournot quantity, the Walrasian quantity is independent of the number of firms in the game. Note, however, that for the number of firms approaching infinity, the elasticity of inverse supply $\frac{\partial p}{\partial s_a^\star}$ as given in (11) tends to zero and thus the regular Cournot quantity becomes the evolutionarily stable quantity, which is the Walrasian one:[4]

$$\lim_{n \to \infty} s_a^\star = s_r^\star \,. \tag{17}$$

Related Literature The result given in this section is not new. It dates back to Schaffer (1989) who showed that in a 'Darwinian Model of Economic Natural Selection' in a Cournot game, the Walrasian strategy will be the only one to survive. This result is fully in line with results by Vega-Redondo (1997), who finds the Walrasian strategy to be the unique stochastically stable strategy in the Cournot game. Although the general concept of stochastic stability in evolutionary games (Young, 1993) is a concept of finding long run stable states of noisy evolutionary dynamics, in spite sensitive games this concept results in strategies which can be found by simply maximizing relative payoffs.

Experimental Results on Spiteful Behavior As certainly any form of spiteful behavior, irrespective of the underlying motives, will lead to the result deduced above, it is an interesting question to ask, if, apart from evolutionary spite, there is evidence for different types spiteful behavior in the Cournot game. In fact, some laboratory experiments with the Cournot game give some evidence that spiteful behavior might possibly occur. In some early experiments with repeated Cournot duopoly and triopoly games and anonymous players, Fouraker and Siegel (1963) find the resulting quantities to tend to be 'more competitive' than the Cournot–Nash quantity. In a series of laboratory experiments by Holt (1985), in their personal comments, some of the players even explicitely mentioned to have been guided by spiteful motives (Holt, 1985, p. 323), though Holt himself comments that most of the players tried to maximize absolute profits. Thus, there is little,

[4]This is a textbook result. See e.g. Binmore (1992).

but more than no evidence of spiteful behavior induced by other than evolutionary forces in the Cournot game.

Another line of thought might be enlightening: Although at first sight, spiteful behavior in oligopoly games does not look too sensible to assume, there are certain situations, e.g. firms trying to maximize market share instead of absolute profits, which lead back to the idea of players' trying to perform better than the others, i.e. behaving spitefully.

5 Rent Seeking: The Tullock Game

The Model Another example of the effect of spiteful behavior is a class of rent seeking games known as 'Tullock games'. Starting with the seminal paper by Tullock (1980), there are various papers on this topic. Surveys and in–depth analysis can be found in Pérez-Castrillo and Verdier (1992) and Nitzan (1994). This paper will focus the basic form of the model: A group of n players make an investment x_i in order to participate in a lottery. The prize to be won is V. The higher a player's investment x_i the higher (ceteris paribus) is her chance of winning the prize, p_i:

$$p_i = p_i(\mathbf{x}) \quad \text{with} \quad \mathbf{x} = \{x_1, x_2, \ldots, x_n\} \, , \tag{18}$$

$$\frac{\partial p_i(\mathbf{x})}{\partial x_i} > 0 \, . \tag{19}$$

A commonly used form of the contest success function $p_i(\mathbf{x})$ is the one originally introduced by Tullock (1980):

$$p_i(\mathbf{x}) = \frac{x_i^\rho}{\sum_{j=1}^n x_j^\rho} \, . \tag{20}$$

In this function, the parameter ρ gives the main characteristic of the 'rent seeking technology', its degree of homogeneity.

The expected payoff of player i results as

$$\pi_i = \frac{x_i^\rho}{\sum_{j=1}^n x_j^\rho} V - x_i \, . \tag{21}$$

It is widely known that for the contest success function (20) the unique symmetric pure Nash strategy (for absolute payoffs), $x_a^\star$, is

$$x_a^\star = \frac{n-1}{n^2} \rho V \, . \tag{22}$$

The interesting point of this finding is the fact that there is underdissipation of the prize V as long as $\rho < \frac{n-1}{n}$. This means that in these cases players' aggregate investment $n\, x_a^\star$ falls short of the prize V:

$$n\, x_a^\star < V \quad \text{for} \quad \rho < \frac{n-1}{n}\,. \tag{23}$$

Whether there is underdissipation depends on the rent seeking technology, which is characterized by the parameter ρ, and — which is important here — by the number of players.

The Game in Relative Payoffs The re–formulation of the Tullock game into relative payoffs is straightforward. Let again relative payoff be

$$\pi_i^r = \pi_i - \overline{\pi}\,, \tag{24}$$

with population mean payoff $\overline{\pi}$

$$\overline{\pi} = \frac{1}{n}\sum_{k=1}^{n} \pi_k = \frac{1}{n}\left(V - \sum_{i=1}^{n} x_i\right)\,, \tag{25}$$

such that relative playoff results as

$$\pi_i^r \;=\; \frac{x_i^\rho}{\sum_{j=1}^n x_j^\rho}\, V - x_i - \frac{V}{n} + \frac{\sum_{j=1}^n x_j}{n} \tag{26}$$

Maximization of relative payoff with respect to x_i and application of the symmetry condition $x_i = x\,\forall\, i$ yields

$$\frac{\rho\, x^{2\,\rho-1}(n-1)}{(n\, x^\rho)^2}\, V - 1 + \frac{1}{n} \;=\; 0 \tag{27}$$

$$\Leftrightarrow \quad x_r^\star \;=\; \frac{\rho}{n}\, V \tag{28}$$

$x_r^\star$ (28) gives the unique symmetric pure Nash strategy for relative payoffs, i.e. characterizes the unique symmetric general ESS.

The interesting fact concerning this result follows from again considering the question of over– or underdissipation. Considering the degree of dissipation of the prize V, it follows that

$$n\, x_r^\star < V \quad \Leftrightarrow \quad \rho < 1\,. \tag{29}$$

This means that for the given rent seeking technology, decreasing marginal returns to x_i will under all circumstances lead to underdissipation of V. This result holds irrespective and independent of the number of players involved in the game.

Moreover, it becomes clear that the evolutionary optimal level of investment, $x_r^\star$, is larger than the 'regular' one, $x_a^\star$, as long as the number of players is finite:

$$x_a^\star = \frac{n-1}{n}\, x_r^\star \quad \Rightarrow \quad x_a^\star < x_r^\star \;\; \forall \;\; n < \infty \,. \tag{30}$$

For the number of players approaching infinity, evolutionarily optimal and regular optimal investment levels coincide:

$$\lim_{n\to\infty} x_a^\star = x_r^\star \,. \tag{31}$$

This notion is similar to the case of the Cournot game (see equation (17)), for which the regular Cournot–Nash quantity is known to approach the Walrasian one, i.e. the evolutionarily stable strategy, for the number of firms approaching infinity.

Experimental Results on Spiteful Behavior In cases where agents compete for a rent, the assumption of spiteful behavior might look quite sensible: What counts in order to win the prize in the Tullock competition is to be better than the others. Thus, apart from evolutionary forces, there could be other motives for spiteful behavior in this game.

Nevertheless, results from laboratory experiments with the Tullock rent seeking games give very little evidence for spiteful behavior. For the case of a linear rent seeking technology ($r = 1$), Shogren and Baik (1991) find their laboratory players to significantly play the Nash strategy in absolute payoffs. Millner and Pratt (1989) and Potters et al. (1998) find similar results with at least some evidence that some players tend to invest more than the regular Nash amount, but still clearly less than the evolutionarily optimal one. Thus, to the experimental results, evolutionary forces can at best serve as a reason for deviations from the canonical Nash solution, but not as a behavioral attractor towards a general ESS.

6 Public Goods Games

The Model For another example of the role of the spite effect in larger games, consider the continuous public goods game. In order to provide a simple example, a very basic variant of the model will be considered. The model in focus is a model of quasilinear preferences, which helps to identify the optimal provision level of a public good regardless of income effects. For more details on public goods games, Cornes and Sandler (1996) is an excellent reference.

Consider a game of n players. Each player i is endowed with a budget of w, which she can spend for an amount of a private good or for a contribution to the public good, x_i. Prices of both goods are assumed to be unity. The payoff π_i is given as the player's utility, which is quasilinear in the consumption of the private good, $w - x_i$, and the total amount of the public good provided, G:

$$\pi_i = w - x_i + \beta \, \ln G \,, \tag{32}$$

where the total amount of the public good, G, is given as the sum of individual contributions

$$G = \sum_{j=1}^{n} x_j \,. \tag{33}$$

Maximization of absolute payoff π_i with respect to the individual contribution x_i leads to a unique individual Nash strategy given by

$$x^{\star}_{i,\,a} = \max \left[\beta - \sum_{\substack{j=1 \\ j \neq i}}^{n} x_j ; \; 0 \right] \,. \tag{34}$$

For symmetric behavior $x_i = x \, \forall \, i$ this leads to a symmetric Nash strategy of

$$x^{\star}_a = \frac{\beta}{n} \,. \tag{35}$$

It is worth mentioning two important features of the variant model presented here: First, the Nash contribution level is strictly positive: $x^{\star}_a > 0$. This is different from many public goods games formulated throughout game theoretic literature. For the purpose of this paper, however, it is a useful feature in that it helps to point out the difference between a 'regular' optimal contribution level and an evolutionarily optimal one. The second important feature of the model is the fact that the optimal symmetric contribution level (35) depends on the number of players in the game: The more players take part in the game, the smaller is the symmetric optimal contribution to the public good: Free riding increases with group size.

The Game in Relative Payoffs In order to find the evolutionarily optimal strategy, i.e. the general ESS, the population mean payoff is computed as

$$\overline{\pi} = \frac{1}{n} \sum_{j=1}^{n} \pi_j = w + \beta \, \ln G - \frac{G}{n} \,. \tag{36}$$

Consequently, relative payoff equals

$$\pi^r_i = \pi_i - \overline{\pi} = \frac{1}{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} x_j - \frac{n-1}{n} \, x_i \,. \tag{37}$$

Maximization of π_i^r for plausible contributions of $x_i \geq 0$ results in a corner solution. The optimal contribution in relative payoffs is

$$x_{i,r}^{\star} = x_r^{\star} = 0\,. \tag{38}$$

This means that in the public goods game presented here, there is a positive symmetric Nash contribution of $x_a^{\star} = \beta/n$. This, however, is not the case for the maximization of relative payoffs: The evolutionarily optimal contribution (the symmetric general ESS) is $x_r^{\star} = 0$. Moreover, this contribution is independent of the number of players involved.

Note, that for the number of players approaching infinity, the optimal contribution for absolute payoffs approaches the evolutionary one:

$$\lim_{n\to\infty} x_a^{\star} = x_r^{\star}\,. \tag{39}$$

This result is similar to the outcomes derived for the Cournot game (equation (17)) and the Tullock game (equation (31)).

Related Literature The results presented in this section give a theoretical foundation of the findings by Miller and Andreoni (1991). Miller and Andreoni conducted a number of numerical simulations of replicator dynamics (i.e. evolutionary dynamics) in a public goods game. Their most prominent result is the observation that over time, the populations of players tended to converge to playing a common contribution of zero. This long run result is found to be independent of the size of the population, i.e. the number of players. Unfortunately for the purpose of this paper, Miller and Andreoni based their analysis on a model with the 'regular' symmetric equilibrium equal to the general ESS equal to zero. Thus, long run results generated by maximization of absolute payoffs and those generated by spiteful behavior coincide in their model.

Experimental Results on Spiteful Behavior Most of the experiment conducted with public goods models share the same problem: The settings are such that the individually optimal solution is a contribution of zero, the strategy of so called 'complete free riding'. The common finding to public goods experiments (to be more precise: to experiments with continuous public goods) is the fact that in the long run, contributions decline, but that at the same time, complete free riding is never achieved (Ledyard, 1995). This gives at least some evidence against strong forces of spiteful behavior in public goods games.

7 Coordination Games: The Van Huyck/Battalio/Beil Game

The Model Another example of the role of spiteful behavior is the model by Van Huyck et al. (1987, 1990). The authors motivate the game by using a case from labor economics: A group of n players in a work group produce goods by means of a Leontieff technology. Each player i's payoff increases in the output $\min_j \{s_j\}$ and decreases in personal effort s_i.

The game is an n–player coordination game. Every player chooses from an action space consisting of actions (effort levels) named 1, 2, 3, 4, 5, 6, and 7: $s_i \in \mathbb{S} = \{1, 2, 3, 4, 5, 6, 7\}$. Each player's payoff is given by

$$\pi_i = a \min_j \{s_j\} - b\, s_i, \quad a > b > 0\,. \tag{40}$$

Table 2 gives an example of the game for $a = 10$ and $b = 1$. Note that there are no entries below the main diagonal of the matrix, because the respective cases are simply impossible: If, for example, player i chooses action 2, the minimum action of all players cannot be larger than 2.

		$\min_j \{s_j\}$						
		7	6	5	4	3	2	1
s_i	7	63	53	43	33	23	13	3
	6	–	54	44	34	24	14	4
	5	–	–	45	35	25	15	5
	4	–	–	–	36	26	16	6
	3	–	–	–	–	27	17	7
	2	–	–	–	–	–	18	8
	1	–	–	–	–	–	–	9

(a) Absolute Payoffs to Player i

		$\min_{j\neq i} \{s_j\}$						
		7	6	5	4	3	2	1
s_i	7	0	-1	-2	-3	-4	-5	-6
	6	1	0	-1	-2	-3	-4	-5
	5	2	1	0	-1	-2	-3	-4
	4	3	2	1	0	-1	-2	-3
	3	4	3	2	1	0	-1	-2
	2	5	4	3	2	1	0	-1
	1	6	5	4	3	2	1	0

(b) Relative Payoffs to Player i

Table 2: Coordination Game.

The game has seven symmetric Nash equilibria in pure strategies: Every strategy profile with all players playing the same strategy, i.e. $s = (s_k)^n$, $s_k \in \mathbb{S}$, constitutes an equilibrium. It is easy to recognize that the equilibrium $s = (7)^n$ is Pareto efficient and that $s = (1)^n$ is the risk dominant equilibrium.

Empirical Findings The model has been the center of interest for a large number experimental investigations.[5] The common outcome of these experiments is the

[5]A survey can be found in Ochs (1995).

finding that players tend to coordinate on the equilibrium with all players playing strategy '1' as the game is repeated. This is commonly applied to the fact that this equilibrium represents the risk dominant one. It was Young (1993) who first showed that at least in 2 by 2 coordination game the risk dominant equilibrium coincides with the long run evolutionary one, i.e. with the equilibrium reached in the long run by evolutionary processes with a minimal degree of noise. In this case, the long run evolutionary equilibrium is the same as the general ESS, which will be computed in the following.

The Game in Relative Payoffs The model in focus is different from the other models introduced above: The strategy space is finite, the payoff function is non differentiable. Consequently, re–formulating this game into a game of relative payoff requires some consideration about the appropriate concept of relative payoff in this game. For each player i, the only determinant of her payoff apart from her own strategy is the member of the population with the smallest strategy apart from player i's one, $\min_{j \neq i} \{s_j\}$. This player is the one with the highest payoff in the population without player i: $\max_{j;\, j \neq i} \{\pi_j\}$. Thus, an appropriate measure of relative payoff is

$$\pi_i^r = \pi_i - \max_{j;\, j \neq i} \{\pi_j\} \ . \tag{41}$$

With the help of (41), Table 2 can be re–written into relative payoffs, resulting in Table 2(b). From Table 2(b), it is easy to recognize, that in relative payoffs, i.e. under the regime of spiteful behavior, the only remaining equilibrium is the risk dominant equilibrium $s = (1)^n$.

Experimental Results on Spiteful Behavior Although the behavior leading to a convergence of players' strategies to the '1' strategy is spiteful behavior in the meaning of the term used in this paper, in this game players need not have spiteful motives in order to behave spitefully: If in each round of the game players are confronted with the respective column of the payoff table 2(a), maximization of *absolute* payoffs (in this row) will implicitly lead to maximization of relative payoffs in the game as a whole. It is helpful to consider the following example: Let us assume players are completely myopic and hold no memory of previous periods. Then, let us assume the minimum strategy played in period $t - 1$ was 3. If in period t players get to see the 3–column of table 2(a) only, even players planning to maximize *absolute* payoff will play 3 (or even less) in the next period. Thus, in this game, there is room for spiteful behavior which is solely induced by the structure of the information available to the players.

8 Summary

Spiteful behavior is the core ingredient of evolutionary dynamics: Players maximize relative rather than absolute payoffs. This notion is the basis of a method for finding general evolutionarily stable strategies in a class of games with finitely many players as discussed in this paper. The method simply consists of computing the strategy that maximizes relative payoff. The class of games this method can be applied to includes the Cournot game, the public goods game, the Tullock game of rent seeking, and the Van Huyck et al. coordination game. For all of these games it can be shown that for finitely many players the evolutionarily stable equilibrium is independent of the number of players and differs from the 'regular' equilibrium, although these equilibria coincide if the number of players approaches infinity.

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