ASIM
A PET Analytical Simulator
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SimSET/Asim user group meeting
26 October, 2011
Valencia, Spain
a Non Monte Carlo Simulator

• Goal
  Simulator for whole-body and dynamic PET imaging

• Applications
  Test statistical methods, based on multiple realizations of a same scan

• Requirements
  • Rapid
  • Realistic Noise and Resolution Properties
Three-step simulation

I) Analytical calculation: *simul*
   - true un-scattered coincidences
   - scattered coincidences
   - random coincidences

II) Add noise to simulate the raw data: *noise*
   - prompt coincidences
   - delayed coincidences

III) Same correction procedures as used in practice: *normalize*

Accurate simulation of
   - ☹ the detection in the crystals
   - ☼ the random and scattered coincidences
   - ☀ the noise properties for the corrected data
Emission and attenuation description

• Geometric

  Analytical 3D projection
  
  prevent non-physical interactions with projections / back-projections used in reconstructions

• Voxel-based approach

  Numerical 3D projection
  
  realistic morphological geometries (Zubal, MNI, NCAT, …)
Geometric description

Collection of truncated ellipsoids:

- emission density $e(x)$ [Bq/cc]
- linear attenuation coefficient $\mu(x)$ [1/cm]
Voxel-based description

3D numerical images:

- emission density $e_i$
- linear attenuation coefficient $\mu_i$
Analytic: Input parameters

• Phantom description, its offset and orientation
• Scanner model (cylindrical scanners)
• Multi-bed acquisition
  • Initial bed position
  • Number of bed positions
  • Amount of bed overlap
• Optional
  • normalization factors
Analytic: True coincidences

normalization factors $\varepsilon$

$$t_E = \frac{AF}{\varepsilon} \sum_{\text{obj LOR}} \int e_i(x) \, dl - \sum_{\text{obj LOR}} \int \mu_i(x) \, dl$$

$$AF = e^{\sum_{\text{obj LOR}} \mu_i(x) \, dl}$$

Ray driven 3D forward-projection

$$t_E = \frac{AF}{\varepsilon} \text{FwdProj}_{\text{LOR}} \{\varepsilon_j\}$$

$$AF = e^{-\text{FwdProj}_{\text{LOR}} \{\mu_j\}}$$
Intrinsic Scanner Resolution

Two 2-D convolutions by the crystal detector intrinsic P.S.F.
Analytic: Random & scatters

- Not accurate
- Effect on the noise of emission data
- Same analytical distribution to simulate the contamination and the correction term

\[
\text{Poisson}[t_E + r_E + s_E] - \text{Poisson}[r_E] - s_E \quad t_E \quad t_E + 2r_E + s_E
\]

- Prompt coincidences
- Delayed coincidences
- True coincidences

Mean \quad Variance

\[
\text{Poisson}[t_E] \quad t_E \quad t_E
\]
Activity outside FOV

<table>
<thead>
<tr>
<th></th>
<th>Brain</th>
<th>Heart</th>
<th>Liver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scatters/Trues</td>
<td>0.3</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Randoms/Trues</td>
<td>0.2</td>
<td>1.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>
1D model

\[ e(z) = \int \int e(x, y, z) \, dx \, dy \]

\[ r_E(s, \phi, z_1 + z_2, z_1 - z_2) = r_E^\perp(s) r_E^\parallel(z_1 + z_2, z_1 - z_2) \]

\[ s_E(s, \phi, z_1 + z_2, z_1 - z_2) = s_E^\perp(s) s_E^\parallel(z_1 + z_2, z_1 - z_2) \]
Analytic Axial Random profile

\[ r_E^/\ (z_1 + z_2, z_1 - z_2) = \]

\[ \int_{z_{\min}(z_1)}^{z_{\max}(z_1)} e(z') \cdot e^{-r\sqrt{1 + \left(\frac{z'-z_1}{R}\right)^2}} \mu \]

\[ 1 + \left(\frac{z'-z_1}{R}\right)^2 \]

\[ \times \]

\[ \int_{z_{\min}(z_2)}^{z_{\max}(z_2)} e(z'') \cdot e^{-r\sqrt{1 + \left(\frac{z''-z_2}{R}\right)^2}} \mu \]

\[ 1 + \left(\frac{z''-z_2}{R}\right)^2 \]
Analytic Axial Scatter profile

\[ s_E' (z_1 + z_2, z_1 - z_2) = \]

- Annihilation location
- Compton scatter location
- Compton scatter angle

⇒ Monte Carlo integration technique
Analytic scatter & random radial profiles

\[ {r_E^\perp}(s), {s_E^\perp}(s) \]

- Not calculated
- Input to the simulation
  - 20-cm diameter cylinder scan
    - random: uniform profile
    - scatter: bell-shaped profile
Noise model

Analytical sinograms (true, random, and scattered coincidences)

- Arbitrary unit, does NOT predict the number of coincidences

Noise level

- Total numbers of true unscattered ($N_{tE}$), random ($N_{rE}$), and scattered ($N_{sE}$) coincidences for some range of bed positions $[b_1, b_2]$ or frames $[f_1, f_2]$
- Half-life of isotope $T_{1/2}$ ($\lambda = \ln 2 / T_{1/2}$)
- Scan duration $\Delta t$ and start time $t_b$

$$DF_b = \frac{1 - e^{-\lambda \Delta t}}{e^{\lambda t_b} \cdot \lambda}$$

$\Rightarrow$ calibration factors $\alpha_{tE}$, $\alpha_{rE}$, and $\alpha_{sE}$

$$\alpha_{tE} \cdot \sum_{b=b_1}^{b_2} DF_b \sum_{s, \varphi, z_1, z_2} N_E(s, \varphi, z_1, z_2) = N_{tE}$$
Noise model

Alternatively

Noise level

• Calibration factors $\alpha_{tE}$, $\alpha_{rE}$, and $SF$
• Half-life of isotope $T_{1/2}$ ($\lambda = \ln2 / T_{1/2}$)
• Scan duration $\Delta t$ and start time $t_b$

$$\alpha_{tE} \cdot \sum_{b=b_1}^{b_2} DF_b \sum_{s, \varphi, z_1, z_2} t_E(s, \varphi, z_1, z_2) = N_{tE}$$
Full noise simulation

1) Acquired data

\[
\tilde{p}_E = \text{Poisson}[\alpha_{tE} \cdot DF \cdot t_E + \alpha_{rE} \cdot DF' \cdot r_E + \alpha_{sE} \cdot DF \cdot s_E ]
\]

\[
\tilde{d}_E = \text{Poisson}[\alpha_{rE} \cdot DF' \cdot r_E ]
\]

2) Corrections

\[
\tilde{c}_E = \frac{\varepsilon}{\alpha_{tE} \cdot DF \cdot AF} (\tilde{p}_E - \tilde{d}_E - \alpha_{sE} \cdot s_E )
\]
Use

Uses

- Multiple realizations of a same acquisition
  - noise and SNR
  - human observer studies

Don’t use:

- To evaluate Scatter correction techniques
- To predict Detector response
Scatter & random: comparison with cylinders
Scatter & random: comparison with patients
Validation

- Statistical distribution of the simulated events ($c_E$)

<table>
<thead>
<tr>
<th>Bed position</th>
<th>Measured</th>
<th>Simulated</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>3</td>
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</tbody>
</table>

- Gated acquisitions

ASIM, an Analytical Simulator

Valencia, 26 October, 2011
Programs

1) simul
   • analytic emission scan \((t_E, s_E, r_E)\)
   • voxelized image (to check geometric phantom)
   • attenuation correction factors (ACF)
   • run also under MPI

2) noise
   • prompt and delayed emission data

3) normalize
   • normalization factors
   • scatter and random correction
   • attenuation correction