ASIM
A PET Analytical Simulator
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SimSET/Asim user group meeting

26 October, 2011

Valencia, Spain

a Non Monte Carlo Simulator

- Goal

  Simulator for whole-body and dynamic PET imaging

- Applications

  Test statistical methods, based on multiple realizations of a same scan

- Requirements

  - Rapid
  - Realistic Noise and Resolution Properties

Three-step simulation

I) Analytical calculation: ***simul***
- true un-scattered coincidences
- scattered coincidences
- random coincidences

II) Add noise to simulate the raw data: ***noise***
- prompt coincidences
- delayed coincidences

III) Same correction procedures as used in practice: ***normalize***

Accurate simulation of
- ⃠ the detection in the crystals
- ⃠ the random and scattered coincidences
- ☺ the noise properties for the corrected data

Emission and attenuation description

- Geometric

  Analytical 3D projection

  prevent non-physical interactions with projections / back-projections used in reconstructions

- Voxel-based approach

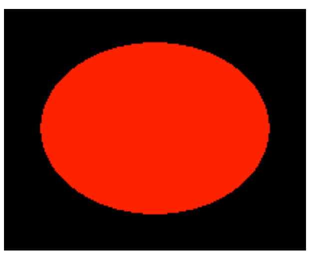
  Numerical 3D projection

  realistic morphological geometries (Zubal, MNI, NCAT, …)

Geometric description

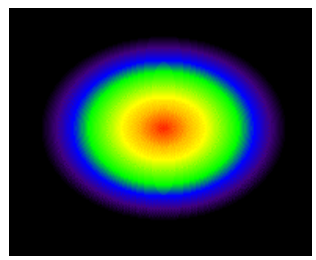
Collection of truncated ellipsoids:

- emission density $e(\mathbf{x})$ [Bq/cc]
- linear attenuation coefficient $\mu(\mathbf{x})$ [1/cm]

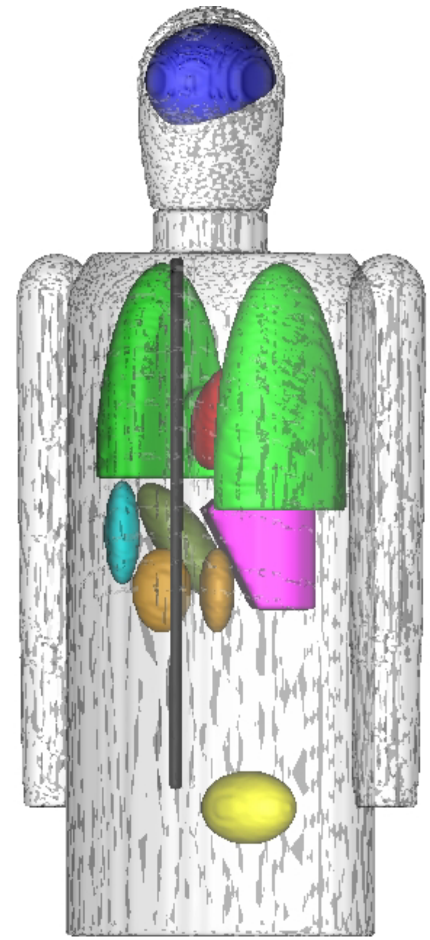


$e(\mathbf{x})$, $\mu(\mathbf{x})$

$e(\mathbf{x})$



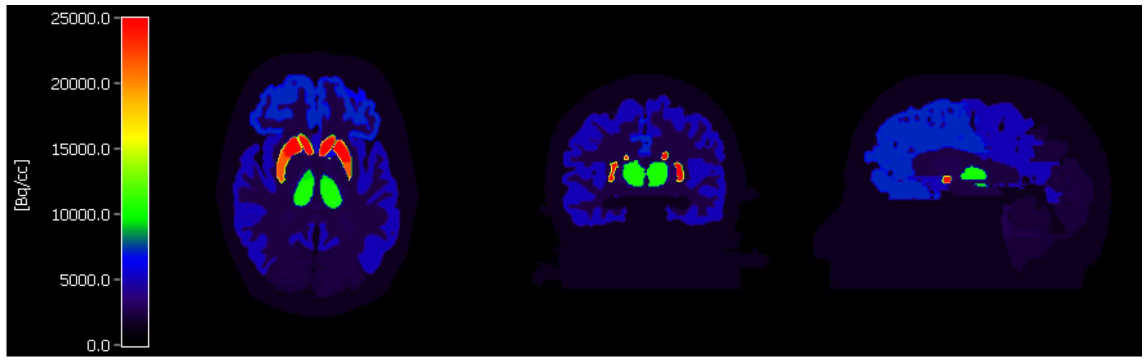
$e(\mathbf{x})$



Voxel-based description

3D numerical images:

- emission density e_i



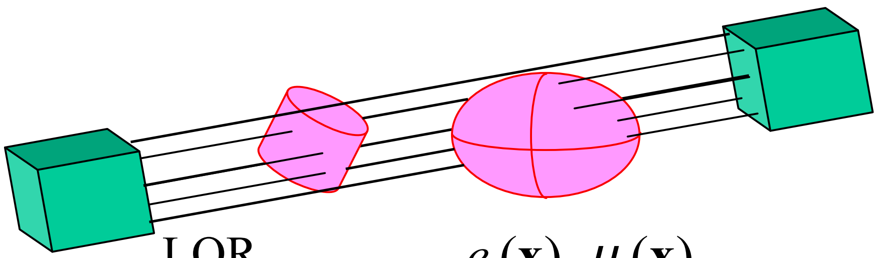
- linear attenuation coefficient μ_i

Analytic: Input parameters

- Phantom description, its offset and orientation
- Scanner model (cylindrical scanners)
- Multi-bed acquisition
    - Initial bed position
    - Number of bed positions
    - Amount of bed overlap
- Optional
    - normalization factors

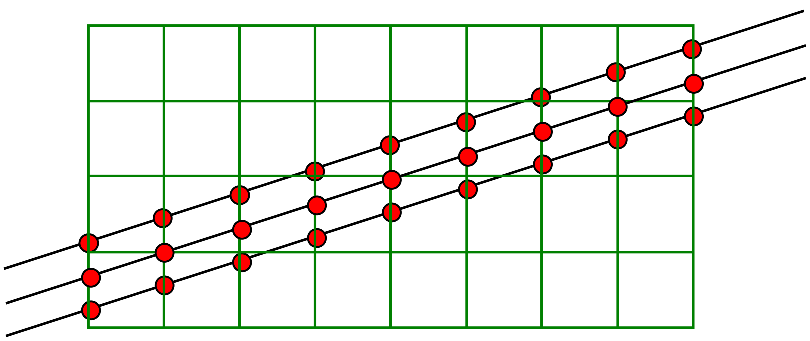
Analytic: True coincidences

normalization factors ε



$$t_{\mathrm{E}} = \frac{AF}{\varepsilon} \sum_{\mathrm{obj}\,i} \int_{\mathrm{LOR}} e_i(\mathbf{x})\,\mathrm{d}\,l$$

$$AF = \mathrm{e}^{-\sum_{\mathrm{obj}\,i} \int_{\mathrm{LOR}} \mu_i(\mathbf{x})\,\mathrm{d}\,l}$$



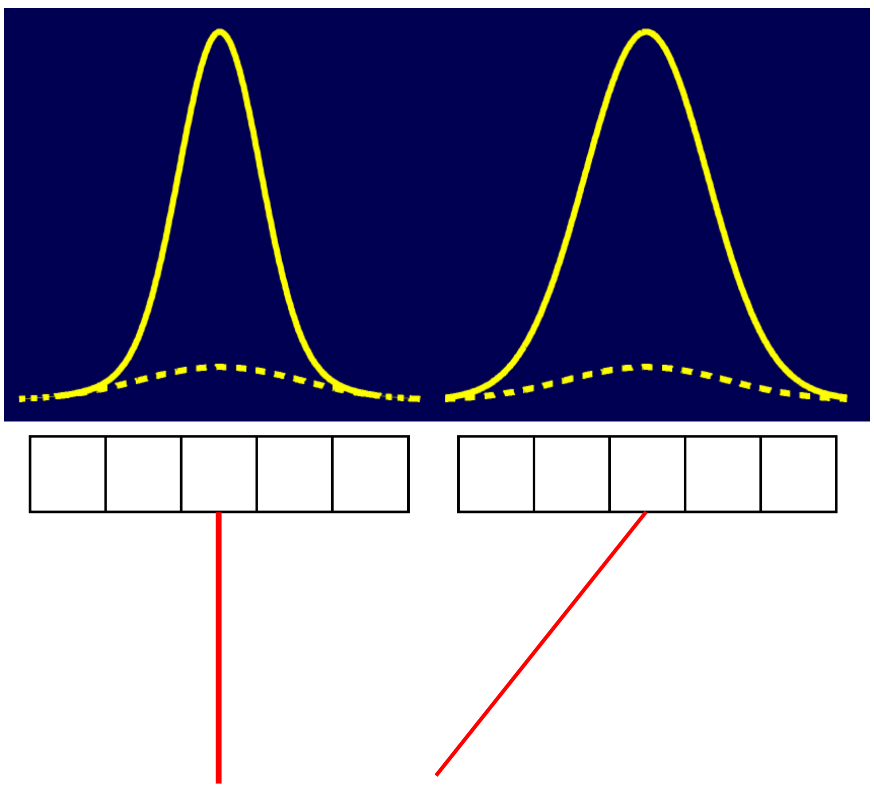
Ray driven 3D forward-projection

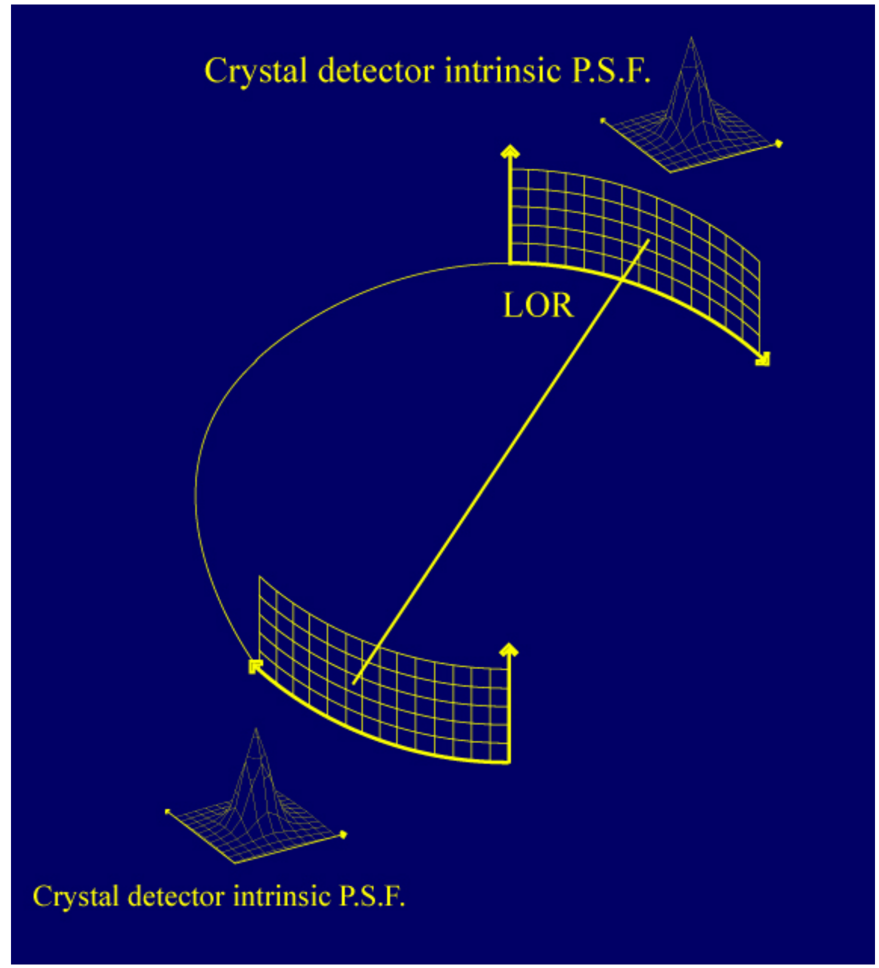
$$t_{\mathrm{E}} = \frac{AF}{\varepsilon} \mathrm{FwdProj}_{\mathrm{LOR}} \{e_j\}$$

$$AF = \mathrm{e}^{-\mathrm{FwdProj}_{\mathrm{LOR}} \{\mu_j\}}$$

Intrinsic Scanner Resolution

Two 2-D convolutions by the crystal detector intrinsic P.S.F.





Analytic: Random & scatters

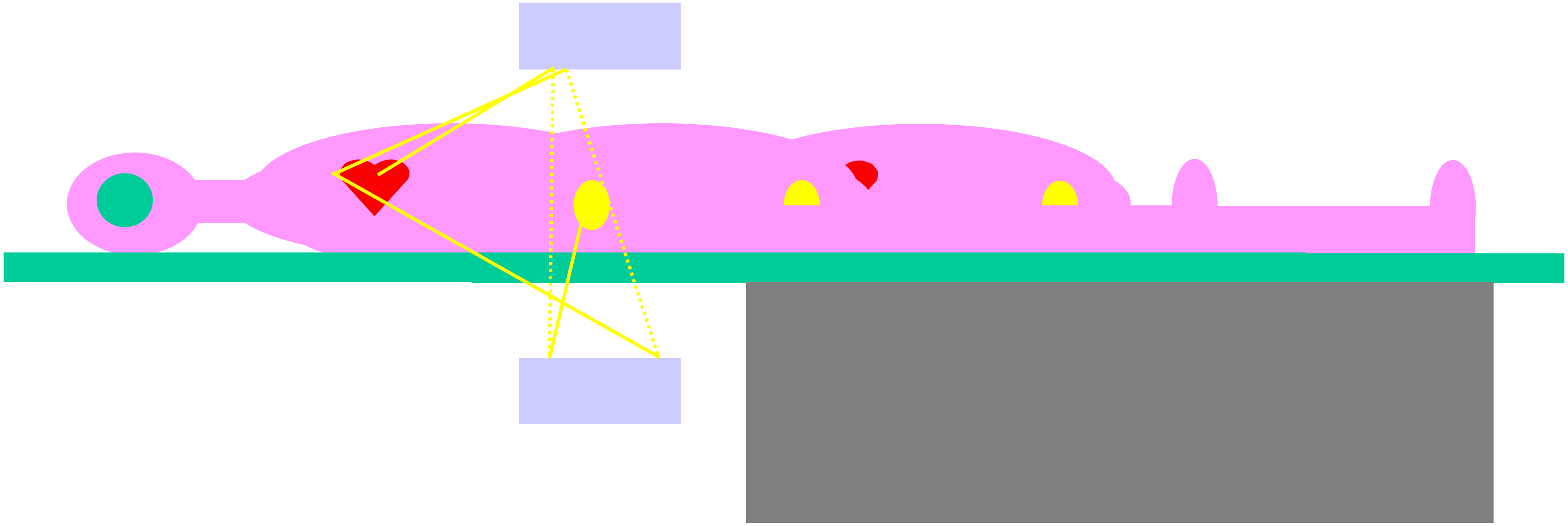
- Not accurate
- Effect on the noise of emission data
- Same analytical distribution to simulate the contamination and the correction term

$$\underbrace{\text{Poisson}\left[t_E + r_E + s_E\right]}_{\text{Prompt coincidences}} - \underbrace{\text{Poisson}\left[r_E\right]}_{\text{Delayed coincidences}} - s_E$$

$$\underbrace{\text{Poisson}\left[t_E\right]}_{\text{True coincidences}}$$

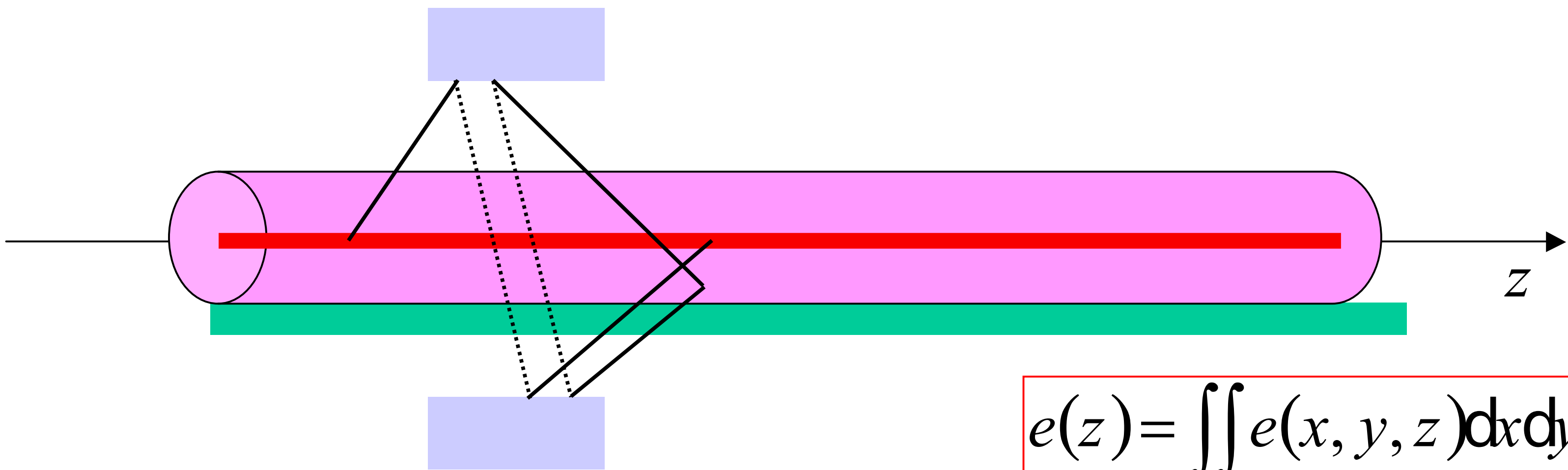
	Mean	Variance
$\text{Poisson}[t_E + r_E + s_E] - \text{Poisson}[r_E] - s_E$	t_E	$t_E + 2\cdot r_E + s_E$
$\text{Poisson}[t_E]$	t_E	t_E

Activity outside FOV



	Brain	Heart	Liver
Scatters/Trues	0.3	1.0	0.8
Randoms/Trues	0.2	1.3	1.2

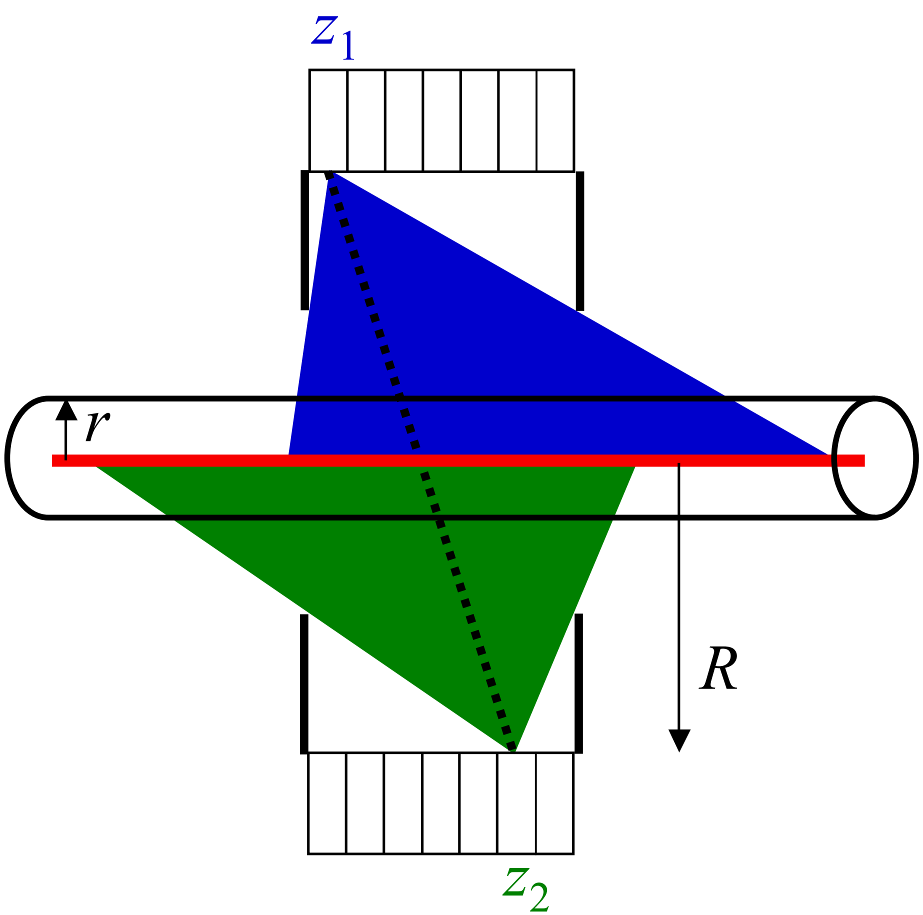
1D model



$$e(z) = \iint_{\perp} e(x, y, z)\,\mathrm{d}x\,\mathrm{d}y$$

$$r_{\mathrm{E}}(s, \phi, z_1 + z_2, z_1 - z_2) = r_{\mathrm{E}}^{\perp}(s)\, r_{\mathrm{E}}^{//}(z_1 + z_2, z_1 - z_2)$$
$$s_{\mathrm{E}}(s, \phi, z_1 + z_2, z_1 - z_2) = s_{\mathrm{E}}^{\perp}(s)\, s_{\mathrm{E}}^{//}(z_1 + z_2, z_1 - z_2)$$

Analytic Axial Random profile



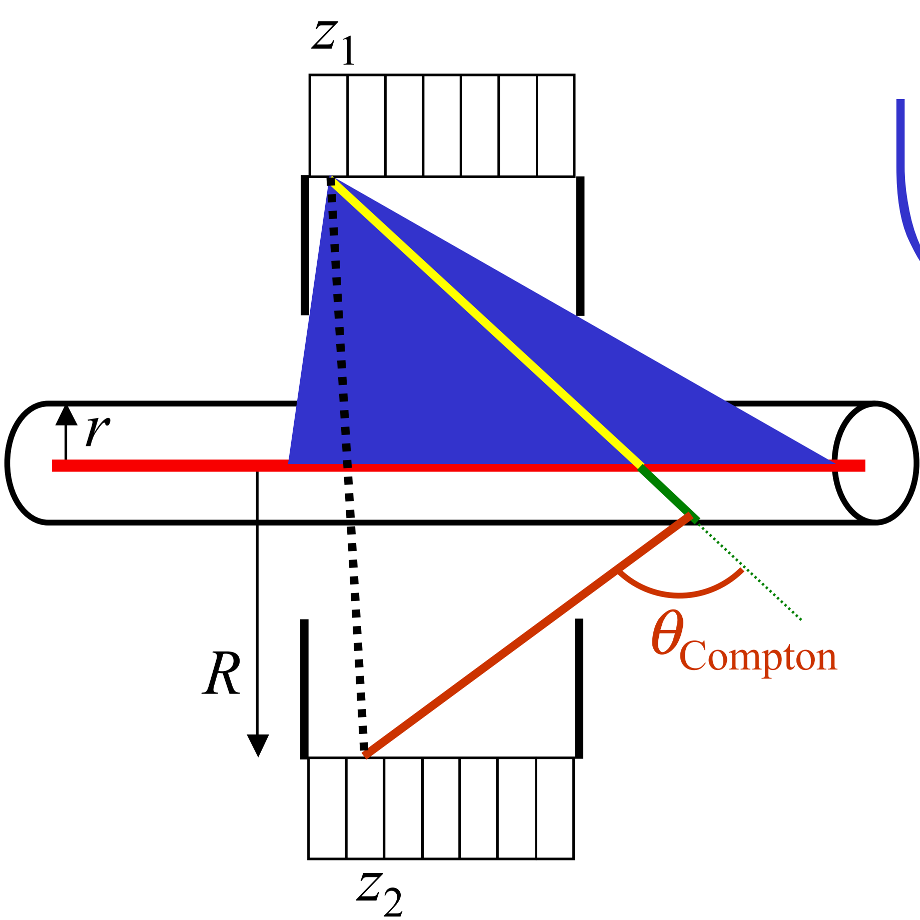
$$r_E^{//}(z_1+z_2, z_1-z_2) =$$

$$\int_{z_{\min(z_1)}}^{z_{\max(z_1)}} \frac{e(z')\cdot e^{-r\sqrt{1+\left(\frac{z'-z_1}{R}\right)^2}\mu}}{1+\left(\frac{z'-z_1}{R}\right)^2} dz'$$

$$\times$$

$$\int_{z_{\min(z_2)}}^{z_{\max(z_2)}} \frac{e(z'')\cdot e^{-r\sqrt{1+\left(\frac{z''-z_2}{R}\right)^2}\mu}}{1+\left(\frac{z''-z_2}{R}\right)^2} dz''$$

Analytic Axial Scatter profile



$$s_E^{//}(z_1 + z_2, z_1 - z_2) =$$

Annihilation location

Compton scatter location

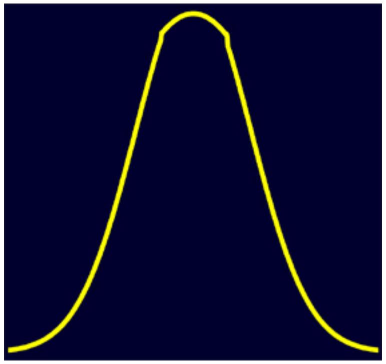
Compton scatter angle

$\Rightarrow$ Monte Carlo integration technique

Analytic scatter & random radial profiles

$r_E^{\perp}(s), s_E^{\perp}(s)$

- Not calculated
- Input to the simulation
  - 20-cm diameter cylinder scan
    - random: uniform profile
    - scatter: bell-shaped profile



Noise model

Analytical sinograms (true, random, and scattered coincidences)

- Arbitrary unit, does NOT predict the number of coincidences

Noise level

- Total numbers of true unscattered $(N_{t\mathrm{E}})$, random $(N_{r\mathrm{E}})$, and scattered $(N_{s\mathrm{E}})$ coincidences for some range of bed positions $[b_1,b_2]$ or frames $[f_1,f_2]$
- Half-life of isotope $T_{½}$ ($\lambda = \ln 2/ T_{½}$)
- Scan duration Δt and start time t_b

$$DF_b = \frac{1-\mathrm{e}^{-\lambda \Delta t}}{\mathrm{e}^{\lambda t_b} \cdot \lambda}$$

$\Rightarrow$ calibration factors $\alpha_{t\mathrm{E}}$, $\alpha_{r\mathrm{E}}$, and $\alpha_{s\mathrm{E}}$

$$\alpha_{t\mathrm{E}} \cdot \sum_{b=b_1}^{b_2} DF_b \sum_{s,\varphi,z_1,z_2} t_E(s,\varphi,z_1,z_2) = N_{t\mathrm{E}}$$

Noise model

Alternatively

Noise level

- Calibration factors α_{tE}, α_{rE}, and *SF*
- Half-life of isotope $T_{½}$ ($\lambda = \ln 2 / T_{½}$)
- Scan duration Δt and start time t_b

$$\alpha_{t\mathrm{E}} \cdot \sum_{b=b_1}^{b_2} DF_b \sum_{s,\varphi,z_1,z_2} t_E(s,\varphi,z_1,z_2) = N_{t\mathrm{E}}$$

Full noise simulation

1) Acquired data

$$\tilde{p}_{\mathrm{E}} = \mathrm{Poisson}\left[\alpha_{t\mathrm{E}} \cdot DF \cdot t_E + \alpha_{r\mathrm{E}} \cdot DF' \cdot r_E + \alpha_{s\mathrm{E}} \cdot DF \cdot s_E\right]$$

$$\tilde{d}_{\mathrm{E}} = \mathrm{Poisson}\left[\alpha_{r\mathrm{E}} \cdot DF' \cdot r_E\right]$$

2) Corrections

$$\tilde{c}_{\mathrm{E}} = \frac{\varepsilon}{\alpha_{t\mathrm{E}} \cdot DF \cdot AF}\left(\tilde{p}_{\mathrm{E}} - \tilde{d}_{\mathrm{E}} - \alpha_{s\mathrm{E}} \cdot s_{\mathrm{E}}\right)$$

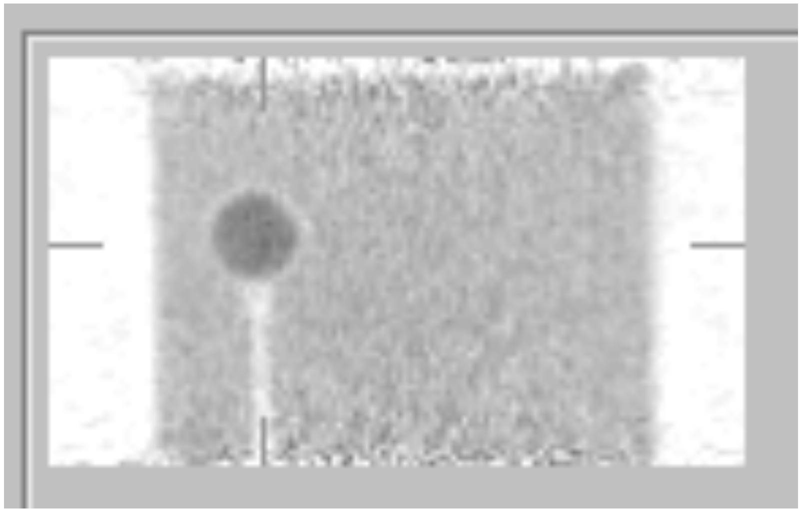
Use

Uses

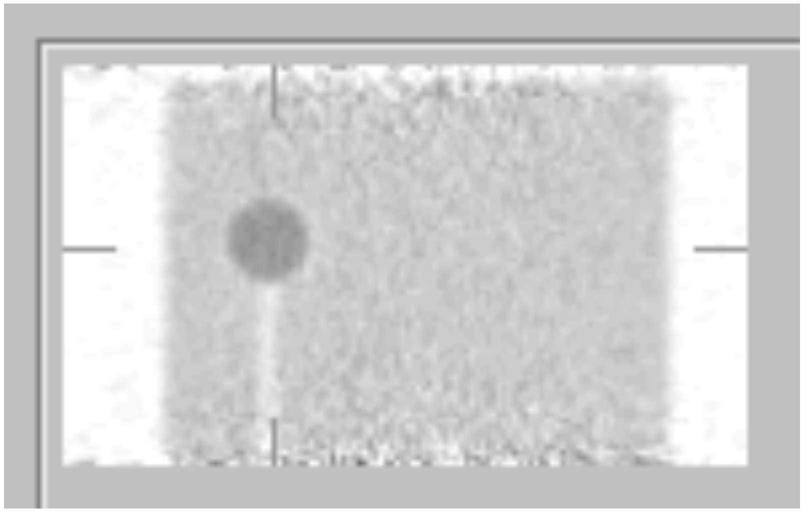
- Multiple realizations of a same acquisition
  - noise and SNR
  - human observer studies

Don’t use:

- To evaluate Scatter correction techniques
- To predict Detector response

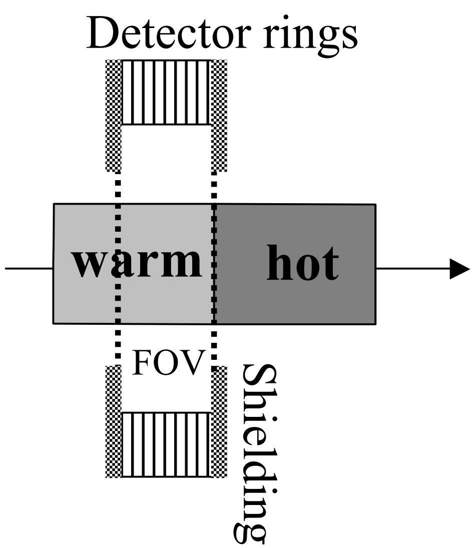


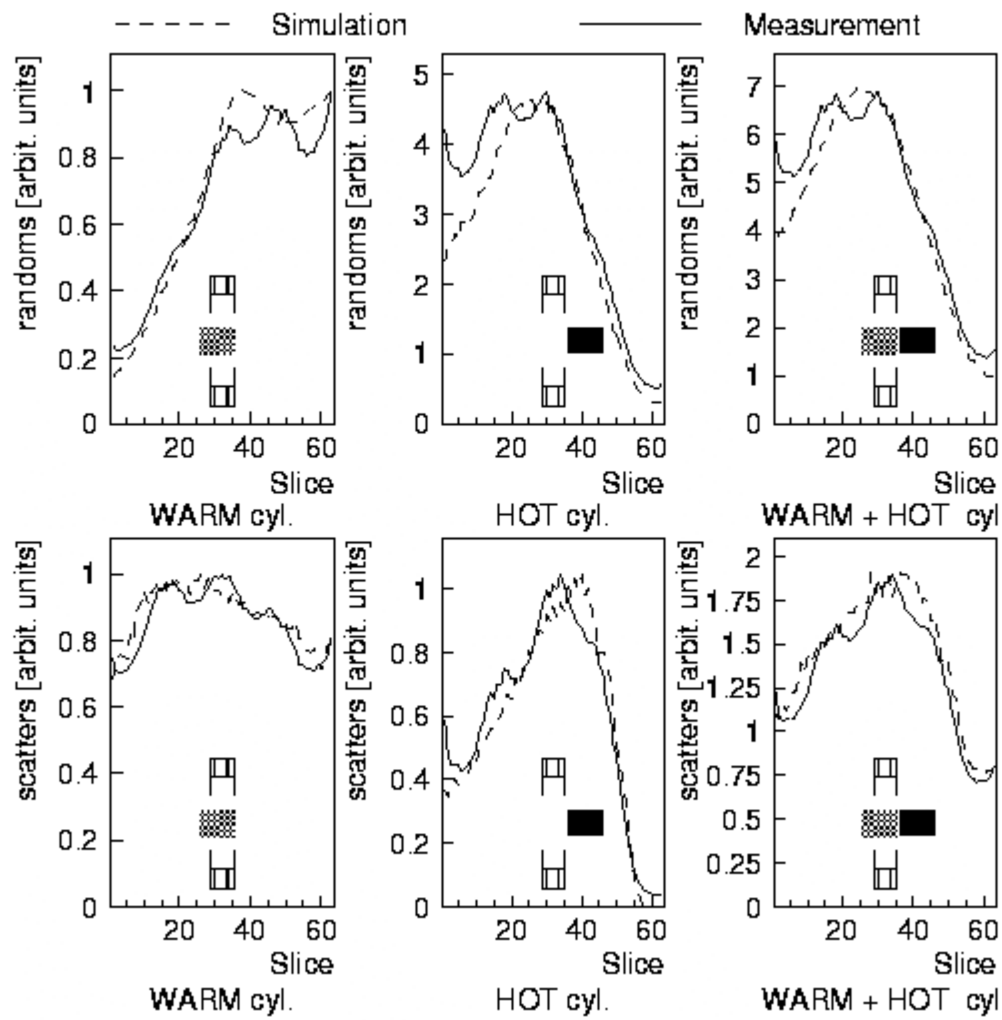
measured



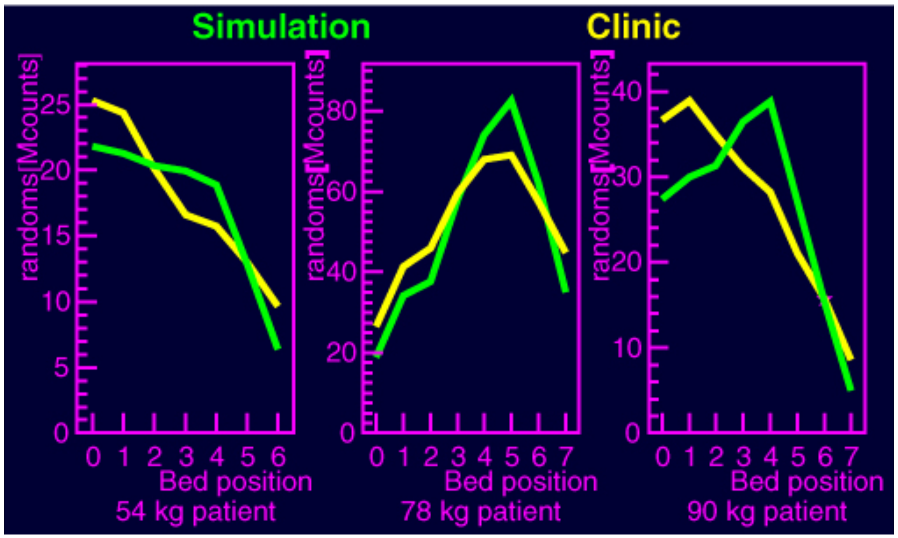
simulated

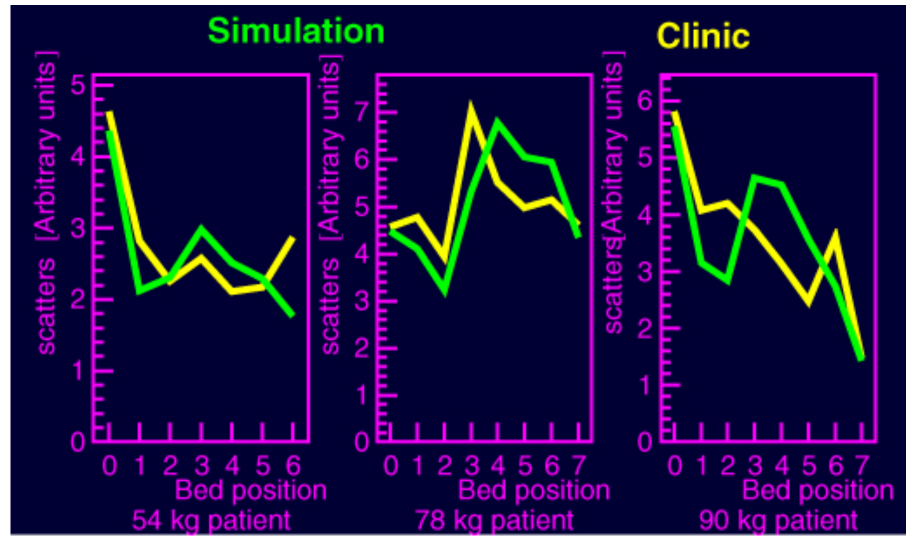
Scatter & random: comparison with cylinders

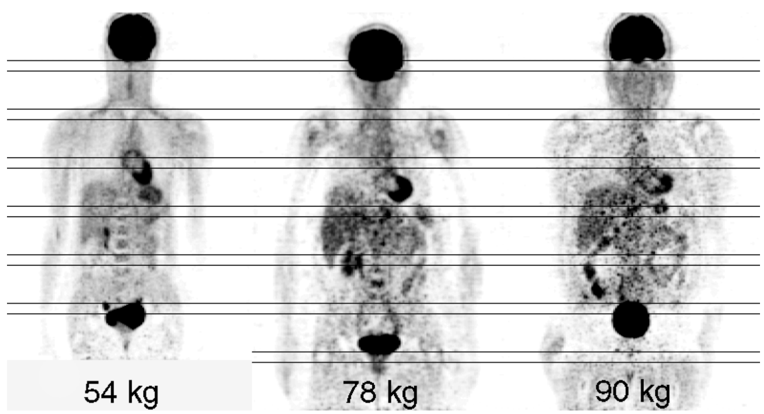




Scatter & random: comparison with patients

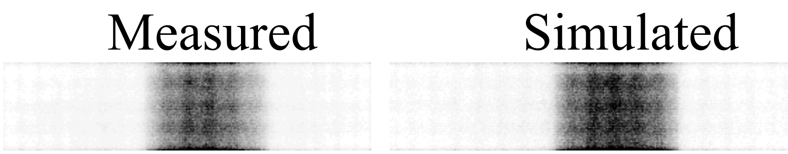




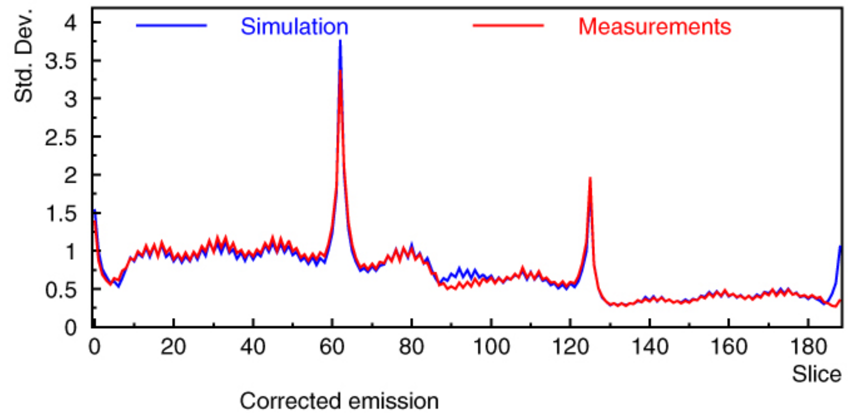


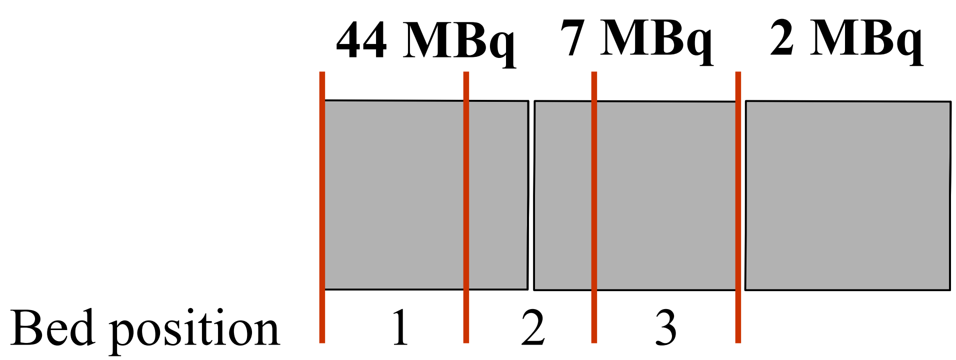
Validation

- Statistical distribution of the simulated events (c_E)



Std. Dev.





Gated acquisitions

Programs

1) simul
   - analytic emission scan (t_E, s_E, r_E)
   - voxelized image (to check geometric phantom)
   - attenuation correction factors (ACF)
   - run also under MPI

2) noise
   - prompt and delayed emission data

3) normalize
   - normalization factors
   - scatter and random correction
   - attenuation correction