LONG-TERM STORAGE

An Experimental Study

by

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The authors have an unrivalled knowledge of the regime of the Nile, having in the course of their work collectively explored the whole Basin. They have worked on the hydrology of every major Nile project since 1920, at first as senior staff of the Nile Control (formerly Physical) Department of the Egyptian Ministry of Public Works, and later as Consultants to the Ministry. They are also, with the late Dr. P. Phillips, the authors of *The Nile Basin*, of which nine volumes and eighteen supplements have been published by the Ministry. These books contain a general description of the Basin, and a detailed account of its hydrology, with a collection of the principal data. Dr. Hurst is also the author of *The Nile* published by Constable.

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Preface

The research described in this book began about 1936 and arose out of the need to make use of the Nile waters to the fullest possible extent, involving the question of the volume of storage required to do this. As far as was known this problem had never been investigated. The method employed by us was to make experiments with sets of measurements of the discharge of the Nile and other rivers, and later with rainfall and other natural phenomena, without making any assumptions as to their properties. From this it was found that some time-honoured beliefs were not in accord with the facts, and during the course of the investigations a new statistical distribution, which covers a wide range of phenomena, was discovered. Although in this book the principles are mostly applied to storage of water, they also apply to storage generally and to some of those cognate problems dealt with on the assumption that the events are independent and randomly distributed. The work was done under the auspices of the Ministry of Public Works, Cairo, in which, we have served for the greater part of our lives. We thank the successive Ministers for their interest and for the help which we have always received, also we would thank our colleagues, past and present who have worked on the study of the Nile over the last forty years, in Nile Control (formerly Physical Dept.) and in the Irrigation Service. In particular we would mention Hussein Khalil Fahmy, Assistant Under-Secretary for Nile Control and Naguib Boulos, Director of Observations, who have spent most of their lives on this work. The engineers who have measured discharges all over the Nile Basin, often under difficult conditions, and the computers, who have done the important work of checking and reducing the field observations to the form in which they are most useful, also deserve the recognition and thanks of all who work or depend on the Nile. We also thank draftsman Hanna Ghobrial who drew all the figures for the book.

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Cairo 1964

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1. Sir M. MacDonald, *Nile Control.* Ministry of Public Works, 2nd edition, Cairo, 1921.
2. Sir W. Garstin, *Report upon the Basin of the Upper Nile.* Cairo, 1904, and *Blue Book, Egypt*, No. 2, 1904.
3. A. D. Butcher, *Lake Albert Reservoir Project.* Ministry of Public Works, Cairo (unpublished), 1938.
4. Hurst and Phillips, *The Nile Basin,* Vol. V. Physical Department, Ministry of Public Works, Cairo, 1938, p. 81.
5. Hurst, Black and Simaika, *The Nile Basin*, Vol. VII. Physical Department, Ministry of Public Works, Cairo, 1946.
6. Allen Hazen, Storage to be provided in Impounding Reservoirs for Municipal Water Supply. *Trans. Amer. Soc. Civ. Engrs.*, **77,** 1914, p. 1,539.
7. Hurst, Long-term Storage Capacity of Reservoirs. *Trans. Amer. Soc. Civ. Engrs.*, **116,** 1951, p. 770.
8. W. Feller, The Asymptotic Distribution of the Range of Sums of Independent Random Variables. *Ann. of Math. Stat.*, **22,** 1951.
9. A. A. Anis and E. H. Lloyd, On the Range of Partial Sums of a Finite Number of Independent Normal Variates. *Biometrica*, **40,** Parts 1 and 2, June 1953.
10. P. A. P. Moran, *The Theory of Storage.* London, 1959.
11. H. E. Hurst, Methods of using Long-term Storage in Reservoirs. *Instn. Civ. Engrs.*, Part **I, 5,** September 1956, pp. 519–90.
12. Hurst, A Suggested Statistical Model of some Time Series which occur in Nature. *Nature*, **180,** September 7, 1957, p. 494.
13. C. E. Sudler, Storage required for the Regulation of Stream Flow. *Trans. Amer. Soc. Civ. Engrs.*, **91,** 1927, p. 622.
14. F. B. Barnes, Storage required for a City Water Supply. *Jour. Inst. Engrs.*, Australia, 1954.
15. M. R. Brittan, *Probability Analysis applied to the Development of Synthetic Hydrology for the Colorado River.* University of Colorado, Boulder, 1961.
16. Maas, Hufschmidt, Dorfman, Thomas, Marglin, Fair, *Design of Water-resource Systems.* Harvard University, London, 1962.
17. Wm. Popper, The Cairo Nilometer. University of California, *Publications in Semitic Philology*, 1951.
18. Prince Omar Toussoun, Mémoire sur l'histoire du Nil. *Institut d'Egypte*, Cairo, 1925.
19. H. H. Turner, On a Long Period (240 years) in Chinese Earthquake Records. *Monthly Notices*, R. Astron. Soc., **79,** May 1919.
20. C. E. P. Brooks, Periodicities in the Nile Floods. *Memoirs R. Meteorological Soc.*, **II,** No. 12, May 1928.
21. C. S. Jarvis, Discussion on Flood Stage Records of the River Nile. Paper 1944, *Proc. Amer. Soc. Civ. Engrs.*, August 1935.

22. C. E. P. Brooks, The Difference Periodogram. *Proc. Roy. Soc. A.*, London, **105**, 1924, p. 346.
23. V. M. Yevdjevich, *Investigation Methods of Water-power Resources.* Institute of Hydraulic Engineering, Belgrade, 1955.
24. V. M. Yevdjevich, *Past and Present Variations in Stream Flow of the Upper Colorado River.* Bureau of Economic Research, University of Colorado, 1961.
25. Luna B. Leopold, Probability Analysis applied to a Water-supply Problem. *U.S. Geological Survey*, Circular 410, Washington, 25, D.C., 1959.
26. J. I. Craig, England—Abyssinia—the South Atlantic.—A Meteorological Triangle. *J. R. Meteorological Soc.*, **36,** 1910.
27. C. E. P. Brooks and S. T. A. Mirrlees, Atmospheric Circulation over Central Africa. *Geophysical Memoirs*, No. 55, British Meteorological Office, 1932.
28. E. W. Bliss, Nile Flood and World Weather. *Memoirs R. Meteorological Soc.*, **I,** No. 5.
29. Sir Gilbert Walker, Indian Monsoon and Nile Flood—Correlations with World Weather. *Indian Meteorological Memoirs*, 1910.
30. H. A. Morrice and W. N. Allan, Planning for the Ultimate Hydraulic Development of the Nile Valley. *Proc. Instn. Civ. Engrs.*, **14,** p. 101, October 1959. Discussion by A. N. M. Robertson.
31. Hudson and Roberts, The 1952–55 Illinois Drought. *Bulletin 43, State Water Survey Division*, State of Illinois.
32. H. E. Hurst and R. P. Black, Report on a Hydrological Investigation on how the Maximum Volume of the Nile Water may be made available for Development in Egypt and the Sudan. *Sadd el Aali Authority*, Cairo, 1955.
33. A. Fathy and A. S. Shukry, The Problem of Reservoir Capacity for Long-term Storage. Journal of the Hydraulics Division. *Proc. Amer. Soc. Civ. Engrs.*, Paper 1082, **82,** October 1956.
34. I. E. Houk, *Irrigation Engineering*, Vol. I. New York, 1951.
35. Israelson, *Irrigation Principles and Practices.* New York, 1950.
36. M. E. Solari and A. A. Anis, The Mean and Variance of the Maximum of the Adjusted Partial Sums of a Finite Number of Independent Normal Variates. *Ann. of Math. Stat.*, **28,** No. 3, September 1957.
37. V. M. Yevdjevich, Fluctuations of Wet and Dry Years. *Hydrology Papers 1*, Colorado State University.
38. V. M. Yevdjevich, Climatic Fluctuations studied by using Annual Flows and Effective Annual Precipitations. *United States National Science Foundation.*
39. V. M. Yevdjevich, Fluctuations of Effective Annual Precipitation. *Proceedings Western Snow Conference*, Cheyenne, Wyoming, April 1962.
40. Hurst and Phillips, *Nile Basin*, Vol. I, Cairo, Government Press, 1931.
41. Yule and Kendall, *Introduction to the Theory of Statistics.* London.
42. W. A. Whitworth, Choice and Chance. Cambridge 1901.
43. W. B. Schostakowitch, Leningrad, *Memoirs of the Hydrological Institute.* Summary in British Meteorological Magazine, Vol. 70. 1935. p. 134.

1

Introduction

Interest in long-term storage problems began in Egypt when the Physical Department of the Egyptian Government was working on the hydrological data for the scheme of projects proposed by Sir Murdoch MacDonald, Adviser to the Ministry of Public Works, for the extension of irrigation from the Nile. This scheme was published in Nile Control(1) in 1920. Up to that time the only reservoir on the Nile was that formed by the Aswan Dam, which stored water from the excess of the annual flood for use in the following low period (annual storage). Nile Control contained proposals for annual storage reservoirs on the Blue and White Niles above their junction at Khartoum. These were built a few years later, and enabled expansion of cultivation to take place. There is a limit, however, to expansion on annual storage, and this was foreseen by Sir Wm. Garstin(2), who suggested the possibility of reservoirs in the Great Lakes of Central Africa. This limit is set by the amounts of water, natural and stored, available after low floods for cultivation during the following low stage of the Nile.

In Nile Control tentative proposals were made for reservoirs in Lakes Tana and Albert, with the possibility of further storage in Lake Victoria. The data did not then exist for definite projects, though estimates were made of the capacities required, based on the discharges of 1913–14 to 1915–16, which contained the two lowest years of which there was definite information at the time. Later a project was drawn up by A. D. Butcher, based on the discharges of 1904–38(3), who wrote: "There is no reliable mathematical process by which the best value for the quota, or the necessary size of the reservoir can be calculated, since these figures depend not only on the mean discharge but on the exact sequence of high and low years which may occur in the future."

Much time was spent on this problem and some preliminary results were published in the Nile Basin Vol. V(4) and in Vol. VII(5). The long series of heights of floods recorded on the Roda gauge in Cairo showed that their frequency of distribution was the normal Gaussian one, if their order of occurrence was ignored. The discharges of the Nile at Aswan and out of Lake Albert, however, showed that when order of occurrence was considered their distributions could not be the normal one, but something more complicated. In order, therefore, to study the relation between reservoir content and quota it was necessary to compare results from a number of rivers. Work of this nature was done by Allen Hazen(6) using the records of some rivers in the Eastern United States. Unfortunately only periods of about 35 years were available. The assumption was that their frequency distributions followed the normal error law, but the existence of what were called cycles of high and low years was recognised. In plotting results probability paper was used. This paper seems to have been invented by Hazen and is very useful.

In our investigations the method used was to take the series of annual discharges of a river and calculate the storage which would have been required to give a discharge each year equal to the mean for the period. The method for doing this is well known and consists in taking the departures of the yearly discharges from the mean in order and forming their continued sums. The difference (R) between the maximum and minimum of these sums is the storage which would have been required to maintain the mean discharge throughout the period. This is an ideal unattainable in practice since it involves no spilling and no reduction of draft, for which the reservoir must be full at the time of maximum excess and empty at the time of maximum deficit. The practical question of regulation will be discussed later. As long records of river discharges were scarce the work was extended to records of rainfall and other meteorological phenomena. The results derived from 20 series, which included annual discharges, rainfalls, temperatures and pressures, were given in Vol. V. Similar results were found from all of these, and there was nothing to distinguish the various phenomena. What appeared was that R/σ increased with the length of the series, where σ is the standard deviation (the square root of the mean of the squares of the deviations). If a draft less than the mean was sufficient this could have been guaranteed with less storage than R, but the reservoir would have filled and water would have had to be spilled and consequently wasted. The relations between this reduced storage S, the storage R and the reduced draft were found for ten of the phenomena, and the results plotted on a curve.

The scatter of the points in these two diagrams (see later p. 11) showed that it was necessary to compute results from more phenomena. The results for 60 phenomena were shown in Vol. VII of the Nile Basin(5) and were there used in drawing up a scheme of projects for the utilisation of the Nile waters. By analogy with the accumulated errors of a line of levelling R/σ was assumed to increase proportionally to $\sqrt{N}$, but actually N was not large enough compared with the scatter to distinguish between $\sqrt{N}$ and N. At this point it seemed useful to go into the mathematics and find the relation between R/σ and N for a series whose members followed the normal Gaussian distribution and were independent of each other. The result of this was that for large values of N*

$$R/\sigma = \sqrt{\tfrac{1}{2} N\pi} = 1{\cdot}25\sqrt{N} \quad \ldots 1$$

The value for R/σ which had been found by the trial of 60 phenomena mentioned above was $1{\cdot}65\sqrt{N}$, which is very definitely larger. This unexpected result left a doubt as to whether in the case of natural phenomena the form of the result was correct, a doubt which could not be settled except by analysis of phenomena where series with large values of N existed. One such series was the Nile Gauge records at Roda, and others were provided by the annual rings of the big American trees and the annual mud layers in ancient lake beds (varves). From the analysis of many cases of these there was no doubt that R/σ increased more rapidly than $\sqrt{N}$, and was expressed by the form (see fig. 4)

$$R/\sigma = (N/2)^K \quad \ldots 2$$

where K was variable, but its values were normally distributed about a mean of 0·73, with a standard deviation of about 0·09, and appeared to be randomly distributed, since there was no correlation between values of K for successive periods of the same phenomenon.

Equations 1 and 2 were published in a paper to the American Society of Civil Engineers(7). Equation 1 was given later by W. Feller(8) and by A. A. Anis and E. H. Lloyd(9) using more abstruse methods. Equation 2 was based on the analysis of 75 phenomena and 690 portions of these, for which R, σ, and K were calculated. This work has since been continued when time permitted. It will be noticed that the method is that of physical science and not of mathematics, and has led to the discovery that the law of distribution of many time series occurring in nature is not the normal law for random events.

During and since the last war much mathematical work has been done on time series where the events are independent of each other. Examples of this are stockpiling of materials and queues, which present problems analogous to storage of water. In the case of stockpiling, the withdrawals are random and it is required to find the input to meet them. In the case of queues the queue corresponds to the reservoir but must be kept as low

* See appendix 2.

as possible. On the assumption that the terms of the time series are randomly distributed mathematical solutions to many of the problems have been discovered. An account of these will be found in The Theory of Storage by Professor P. A. P. Moran(10), who studied the question in connection with the Snowy Mountains Project in South-east Australia and produced a number of papers on Dams and Storage Systems. The mathematics is difficult and it is interesting to see that nearly all the references in Moran's bibliography date from 1950 onwards.

Returning to equation 2, the variable quantity K has a range from 0·46 to 0·96 with a mean of 0·726, from which the means of various groups of phenomena do not differ very much. In matters concerning Nile projects the value of K adopted was 0·72, and the equation is

$$R/\sigma = (N/2)^{0\cdot72} = 0\cdot61\ N^{0\cdot72} \qquad \ldots 3$$

0·72 is the mean value of K from statistics relating to rivers and rainfall, amounting to 72 phenomena and 329 cases. Equation 3 gives the most likely value of R for use in designing a storage project, but it does not contain any factor of safety. Values of R based on this equation were tried on 51 phenomena, three-quarters of which were rivers or rainfall, and for most of which the record was longer than 100 years. R, σ and the mean were obtained from the first 30 years and the regulation was then applied to the remainder, starting with the reservoir half-full and giving the preceding mean discharge. The result was that in about 40 per cent of the cases the reservoir filled and water was spilt, while in another 40 per cent the reservoir emptied, leaving 20 per cent when it neither filled nor emptied. This obviously was no use and further regulations were made with the object of finding a system which could be applied automatically to give as large an annual draft as possible without emptying the reservoir over a long term of years. The prevention of floods was also investigated. The results were given in a paper to the Institution of Civil Engineers, London(11), and are discussed later in this book.

We do not know whether the problems which have been solved in the case of random events, to which equation 1 applies, can be solved for those from which equation 3 was derived, but in view of the great development of mathematical statistics there is good reason to suppose that they can. A model has been made which produces series with properties similar to those of the natural time series already analysed, at any rate for the first 6,000 terms(12). This is described later and might be of assistance in mathematical studies.

In concluding this introduction it may be said that this book deals with

quantitative relations concerning long-term storage and its use, but the actual volume of storage to be created depends also on other considerations. Of these, important ones are: the nature of the dam and reservoir sites, topography and geology; relation of the height and cost of the dam and reservoir to the volume which will be impounded; water losses, and relation of these to impounded volume; potential area which can be cultivated; political and financial circumstances.

2

Method of Computation of R and Some Examples

1. Method

The definition of R and the method of computing it have already been briefly given. It is necessary, however, to amplify the descriptions. Given a series of annual total discharges covering N years they are tabulated along with their departures from the mean, or more usually from a convenient base near the mean. In our Nile statistics discharges are usually given to three significant figures but have not this degree of accuracy. For computations of storage, owing to the variability of the results, it is not necessary in general to use more than two figures, and departures are taken from the nearest two-figure number to the mean, which is called the base, (B). This makes no significant difference to the result and saves labour and possible mistakes in computation.

An example of the process is given in table 1. The mean (M) for the whole period is 23·72, so 24 is taken as the base of departures (B). The effect of the difference between the base and the mean on the standard deviation and the accumulated departures is corrected afterwards. Two digits in the departures lead to three or four in the squares, which can be reduced to two or three significant ones with a negligible effect on the standard deviation. The table is arranged in two halves and totals of each half are taken. This helps in the detection of mistakes and facilitates the determination of R and σ for portions of the series. Means and accumulated totals are self-checking, and if squares of numbers up to 25 are memorised the checking can be done very quickly. When the base differs from the mean a correction is sometimes necessary. If d is a departure from the base and d_1 the corresponding departure from the mean

$$\sigma^2 = \Sigma d_1{}^2/N = \Sigma d^2/N - (M - B)^2$$

These corrections are made at the bottom of table 1.

Making use of table 1 the successive accumulated departures from the base, 24 mlrds., are plotted in fig. 1 as ordinates to the axis OO' with

TABLE 1

Accumulated Departures of the Annual Discharges from Lake Albert
(Milliards of cubic metres)

Year	Disch. Q	Depart. d Base 24 +	−	d^2	Acc. sums of d	Year	Disch. Q	Depart. d Base 24 +	−	d^2	Acc. sums of d
1904	35	11		120	11	1931	26	2		0	31
5	31	7		50	18	32	28	4		20	35
6	34	10		100	28	33	29	5		20	40
7	33	9		80	37	34	23		1	0	39
8	26	2		0	39	35	20		4	20	35
9	29	5		20	44	36	20		4	20	31
1910	26	2		0	46	37	24	0		0	31
11	22		2	0	44	38	26	2		0	33
12	19		5	20	39	39	24	0		0	33
13	20		4	20	35	1940	20		4	20	29
14	21		3	10	32	41	19		5	20	24
15	24	0		0	32	42	29	5		20	29
16	27	3		10	35	43	26	2		0	31
17	47	23		530	58	44	18		6	40	25
18	48	24		580	82	45	15		9	80	16
19	29	5		20	87	46	16		8	60	8
1920	23		1	0	86	47	25	1		0	9
21	17		7	50	79	48	28	4		20	13
22	13		11	120	68	49	24	0		0	13
23	14		10	100	58	1950	18		6	40	7
24	18		6	40	52	51	17		7	50	0
25	16		8	60	44	52	25	1		0	1
26	19		5	20	39	53	21		3	10	−2
27	25	1		0	40	54	19		5	20	−7
28	21		3	10	37	55	20		4	20	−11
29	19		5	20	32	56	21		3	10	−14
30	21		3	10	29	57	23		1	0	−15
Sums	677	102	73	1,990			604	26	70	490	
$N = 27$		29				$N = 27$			44		
Means	25·07	1·07		73·7			22·37		1·63	18·1	
	$(M-B)^2 = 1{\cdot}14$			$\sigma^2 = 72{\cdot}6$			$(M-B)^2 = 2{\cdot}66$			$\sigma^2 = 15{\cdot}4$	
				$\sigma = 8{\cdot}52$						$\sigma = 3{\cdot}92$	

$N = 54$ $M = 23{\cdot}72$ $M - B = -0{\cdot}28$ $(M-B)^2 = 0{\cdot}1$ $\Sigma d^2 = 2{,}480$

$\Sigma d^2/N = 45{\cdot}9$ $\sigma^2 = 45{\cdot}8$ $\sigma = 6{\cdot}77$

dates as abscissae. The curve produced is OPX, which shows the result of a steady outflow equal to the base of departures, and the reservoir would have finished 15 mlrds. below its starting content. If the basic discharge had been the mean (23·72 mlrds.) the reservoir would have finished all square. With the mean as draft each year the accumulated departures increase by 0·28 mlrds. and the variation of reservoir content is shown by the ordinates of OPX referred to OX as axis. The content of the reservoir increases from the beginning of 1904 to the end of 1919, when the gain is PM, and decreases back to its starting content by the end of 1957. The storage R required to carry out the regulation is $PM = 91$ mlrds. If we want to know the value of R for the first 30 years we join the point A on the curve at the end of 1933 to the starting point O. OA is then the axis for accumulated departures from the mean of this period, and $R = PM_1 + M_2P_1 = 73$ mlrds. For the second period the axis is AX and the maximum excess is P_2M_3 while there is a small deficit M_4P_3 so that $R = P_2M_3 + M_4P_3 = 16$ mlrds. R may be defined as (*a*) the maximum

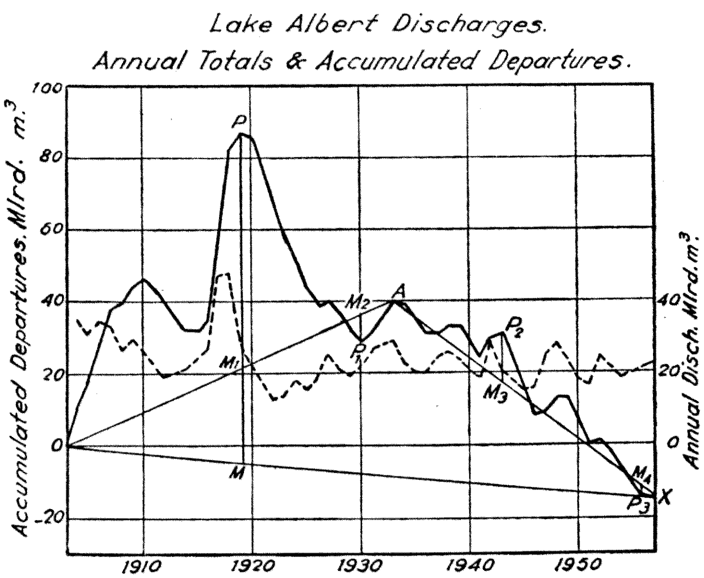


FIG. 1

accumulated storage when there is never a deficit, or (*b*) the maximum accumulated deficit when there is never any storage, or (*c*) their sum when there is both storage and deficit.

There is a noticeable difference between R for the two halves of the period and the dotted time–discharge curve of the first half is much more variable than that of the second; in fact the standard deviations are 8·5 and 3·9 mlrds. respectively.

The above explanation shows that when a curve of accumulated departures from any base has been drawn, by a change of axis the

accumulated departures from any other base, including the mean, can be found. Moreover the process can be applied to any continuous series of years which forms a part of the original series. More examples of the use of this principle will appear later. An approximate value of R can be computed without plotting a curve.

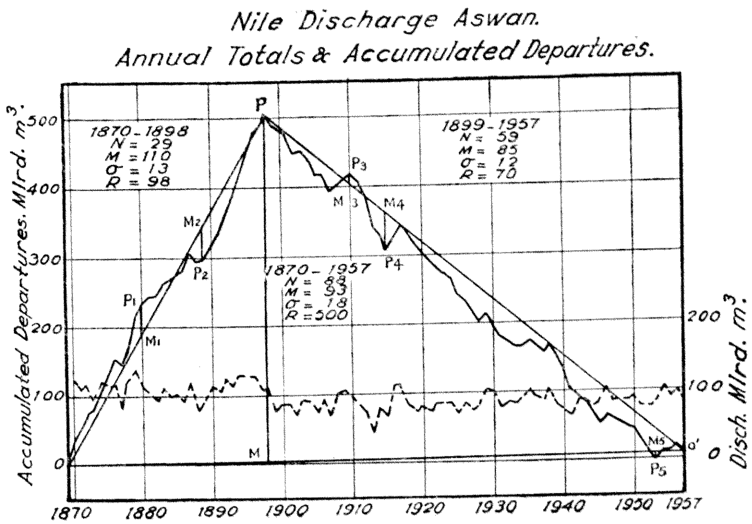


FIG. 2

2. OTHER EXAMPLES

Two more examples of curves of accumulated departures and the discharges from which they are computed are shown in figs. 2 and 3. The natural river discharge at Aswan is the actual discharge as estimated up to 1902, when the reservoir was first used, and after that the discharge corrected for the effect of the reservoir, and later for abstractions in the Sudan and losses in the Gebel Aulia reservoir. The curve is similar to that for the outflow from Lake Albert. If a large reservoir had come into use at the beginning of 1870 and had passed a steady discharge equal to the mean of the next 88 years, it would have filled steadily up to the end of 1898. From then on with slight interruptions the stored water would have been used, until by the end of 1957 the condition at the beginning of 1870 would have been reached. The storage required for this would have been $PM + M_5P_5$, which is 500 mlrds. The period has been divided into two parts at P. The axis OP and the curve show the conditions for 1870–98 giving $R = P_1M_1 + M_2P_2 = 98$ and $\sigma = 13$. PO' represents the conditions of the second half, for which $R = P_3M_3 + M_4P_4 = 70$ and $\sigma = 12$. For the whole period $\sigma = 18$. The curve for the actual discharges shows a remarkable difference between the discharges up to 1898 and those afterwards, the mean for 1870–98 being 110, and for the remainder 85 mlrds., a difference of 30 per cent. The discharges previous to the completion of the Dam in 1902 depend on a gauge–discharge curve made from discharges

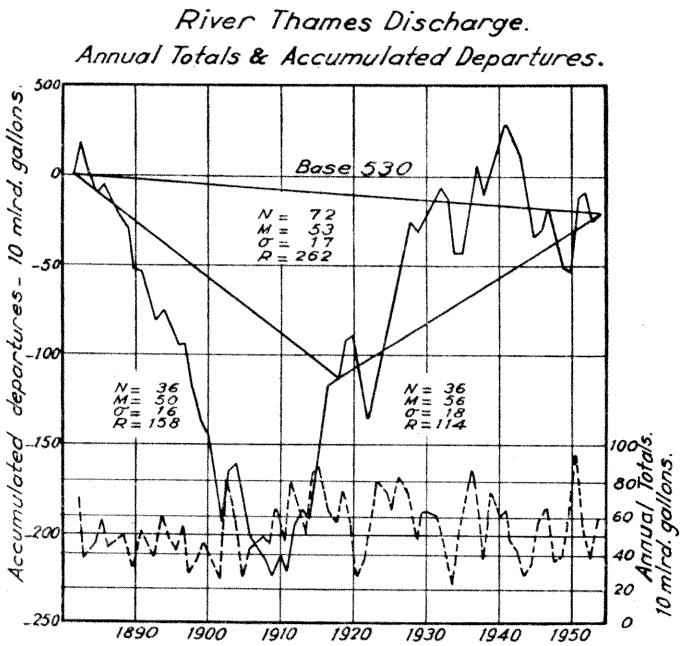


FIG. 3

measured later. From 1902 onwards the discharges depend on measurements by means of the sluices and have a high degree of accuracy. The earlier ones are not so accurate, though the accuracy is good enough for the present purpose. There is no doubt whatever that the earlier period was one of high floods from the evidence both of gauges in Egypt and of the records of disaster owing to breaches of the Nile banks.

Another example of a curve of accumulated departures is that of the annual discharges of the Thames over Teddington Weir taken from the

Statistics of the Thames compiled by the Thames Conservancy (1960). This is shown in fig. 3 and is more or less the opposite of the curve for the Nile, since the first 29 years are on the whole low, and the next 41 high, the average daily discharges being 1,240 and 1,440 million gallons per day, a difference of 16 per cent.

The three examples just given show such variations in means, standard deviations and values of R that it is clear, as already written in chapter 1, that much more information is necessary before we can infer anything about the form of R. The work outlined in chapter 1 was therefore carried out and its results will be described in the following chapters.

3

Values of *R* from Some Natural Time Series

1. Rivers and Rainfall

Equations 1 and 2 in chapter 1 employ the well-known device of using the standard deviation as a unit and R/σ as a function of N. River discharges and levels and rainfall seem to be the natural phenomena of greatest practical interest in connection with storage, so they are analysed first. From figs. 1, 2 and 3 values of R and σ for $N = 10$ and 20 years have been found in order to extend the range downwards, and all the values for larger N's from 53 sets of discharges taken from 37 rivers have been divided into approximately equal groups in order of N's. The data relating to these are given in the appendices and the results have been plotted in the form log R/σ against log N in fig. 4. Owing to the scatter of individual points only the means of groups have been plotted. It is clear that the means are well fitted by the line $\log (R/\sigma) = 0{\cdot}72 \log (N/2)$ which is shown and passes through the centre of gravity of the groups and the point $R/\sigma = 1$ $N = 2$. At this point R/σ has only the one value. Its choice has been criticised and it has been suggested that the line to represent such a set of statistics should be found from the individual values by the method of least squares, using two parameters, as giving a more accurate representation. This was discussed in correspondence on ref. 7 where it was shown that in the case of 168 values of log R/σ, derived from records of rainfall from different stations, owing to the scatter of the points, the difference produced in the mean line by the introduction of another parameter was not significant. This is shown in fig. 5, where the line $\log (R/\sigma) = 0{\cdot}70 \log (N/2)$ derived from means of groups of points, and the line $\log (R/\sigma) = \log 0{\cdot}43 + 0{\cdot}79 \log N$ from least squares, are both given. An examination of probable errors shows that the difference between the lines is not significant. There is therefore a strong preference for the simpler formula with only the parameter K. In this case, as in many others, the use of least squares, instead of the simpler method of dividing the

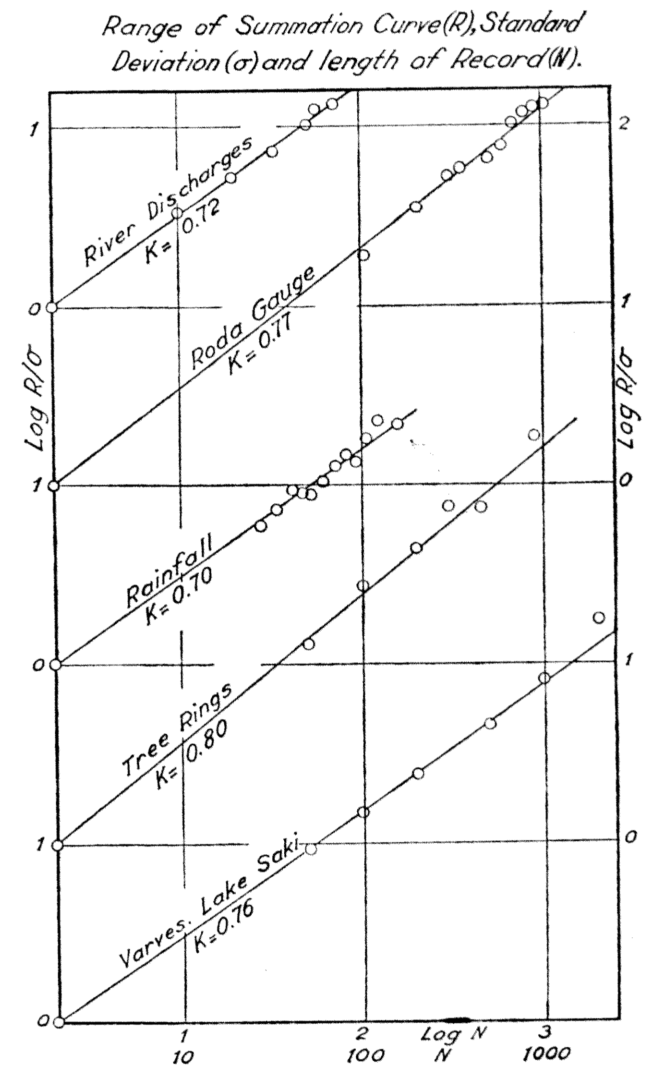


FIG. 4

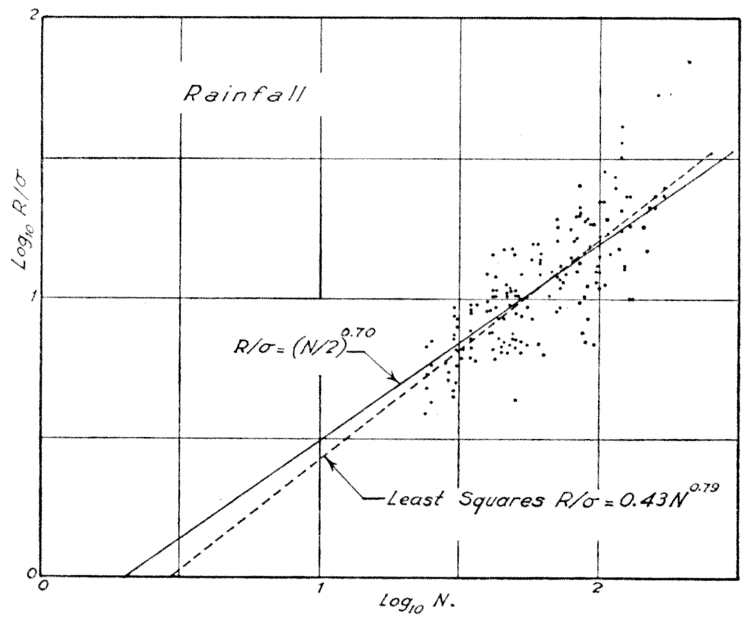


FIG. 5

observations into groups, whose centres of gravity are determined, is not worth the extra labour which is involved.

In fig. 5 the index of $N/2$ is 0·70. In table 2 some additional results have been added and the value of K has been computed for all the cases. Its mean value is 0·70, agreeing with that of the figure.

No river discharges cover more than 100 years and only a few rainfall records extend beyond 150 years. However, the flood records of Roda (Cairo) river gauge cover 800 years with only a few years missing, and 1,000 if large gaps are ignored. These offer a wide range as a test of whether equation 2 applies. The results from 66 series of observations, divided into ten groups in order of N, are plotted in fig. 4 and the individual cases are given in appendix 4. The line through the point log $(R/\sigma) = 0$, log $N = 0{\cdot}3$, and through the centre of gravity of the groups is a good fit and its equation is log $(R/\sigma) = 0{\cdot}77$ log $(N/2)$.

The Roda Gauge is situated on the Nile just upstream of Cairo. It consists of a marble column carrying a scale and standing in a well connected with the river. Records began about A.D. 620 and have continued

with gaps to the present time, though from the end of the nineteenth century readings have been taken on a metric gauge nearby. They were first read on a gauge to the south of the present gauge which was built about A.D. 711. The maximum and minimum levels of the year were recorded, and those used here are the maxima, as being less liable to accidental changes. The actual readings have been corrected for the progressive rise of the river bed due to deposition of silt (see chapter 8). Although they contain many sources of error, and naturally have not the precision of modern scientific observations, nevertheless they are probably as reliable as many of the statistics collected today about such less well-defined phenomena as health or social conditions.

2. OTHER NATURAL PHENOMENA

The fact that river discharges, rainfall and Nile flood levels all satisfy an equation of the same form with not very different values of its parameter K leads one to think that the equation may be of more general application. The following classes of natural phenomena have been examined to test this and to see what is the nature of K. The results of the computations are given in tables in the appendices and the means of groups are plotted in fig. 4. The previous results are also included in this.

Meteorological information which includes temperatures and pressures is taken from World Weather Records and its supplements, published by the Carnegie Institution of Washington. Additional information was supplied by the British Meteorological Office, including the long series of records of annual deposits (varves) in Lake Saki in the Crimea[43]. The records of annual growth of trees are from the work of Dr. A. E. Douglass, Climatic Cycles and Tree Growth, published by the Carnegie Institution of Washington. Dr. K. S. Sandford of the Geological Department, Oxford University, kindly supplied references to geological literature on lake deposits.

With regard to the annual growth rings of American trees, many factors affect their rate of growth—temperature, the sun's radiation and precipitation, both the amount and its distribution in time. The relative importance of each of these factors varies with the locality. The thicknesses of the rings of different trees in the same locality are found to correspond closely, and this cross-identification is applied in choosing the trees to form representative groups. The thickness of rings as measured is affected by (a) rapid growth when the tree is young, which decreases with age, (b) more rapid growth of some trees due to favourable environment, and (*c*) spread of the tree at its base. The actual measurements were corrected by Dr. Douglass so as to eliminate as far as possible non-climatic factors such as the above. There are possibilities of error due to missing rings or doubling of rings, but these are not serious and only occasional trees are discarded for these reasons. The figures given are always the means of groups of trees.

The trees whose measurements have been used are the *Sequoia Washingtonia*, or Giant Californian Redwood, and various kinds of pine. The sequoia is an enormous tree and is the longest-lived plant in existence. Rings have been identified in different trees corresponding to times before 1000 B.C. The Flagstaff, Arizona, trees are a group of pines covering the period A.D. 1400–1900. The group from Pike's Peak are pines and Douglas firs covering A.D. 1570–1920 and the group from Meadow Valley, California, are California pines with records from A.D. 1620–1920.

Annual Layers of Mud (Varves). The figures for Lake Saki were originally taken from W. B. Schostakowitch, Leningrad, Memoirs of the Hydrological Institute, and appear also in "Boden ablagerungen der Seen und periodischen Schwankungen der Naturer Scheinungen". A summary is given in the British Meteorological Magazine 1935, Vol. 70, p. 134. They cover the period 2294 B.C. to A.D. 1894 and vary in thickness from 2 to 56 tenths of a millimetre, with an average thickness of 1·3 millimetres. The first two centuries were omitted as the average thickness was very much greater than any other century but one. There seemed to be no systematic trend and the measurements were used as they stood.

Stratified lake deposits are due to seasonal changes, sometimes the change from a definite rainy to a dry season, and sometimes to the melting of winter snows. In both cases there is a change in the suspended matter brought into the lake by its tributaries. The thickness and nature of a band will depend on the distribution of a run-off as well as its amount. A band therefore represents some complex climatic condition. There are various effects tending to confuse the record from varves. For example, very thin layers may be missed entirely, and the river whose flow into a lake produced a series of bands may change the position of its mouth and so affect the thickness of the deposits. The loss of layers is not important and changes of the topography must be taken as similar to the long period changes of rainfall or temperature. They will cause the mean, standard deviation and R to vary but will have less effect on K.

Other data from varves have been taken from Geochronologia Suecica, Principles, by Baron Gerard de Geer, which was obtained through the kindness of Dr. Borgquist and Mr. Munding, Swedish engineers. They

relate to the period 7000 to 5000 B.C. approximately. There is a great deal of material in this work available for statistical analysis, and the same may be said in regard to tree-rings. Fig. 4, which contains examples from five classes of phenomena, shows clearly that for every class the mean of many cases is represented by a line expressed by $\log (R/\sigma) = K \log (N/2)$. It should be mentioned, and this appears in the tables, that the number of cases for small values of N is greatest, and that it decreases as N increases, until for values of N of 500 or more there are very few cases and the group values which are plotted tend to be more variable. In these examples K varies from 0·70 to 0·80. The cases could be multiplied. However, instead of plotting all the cases, K has been computed for each one, and the results are given in tables in the appendices. Altogether 872 cases from

TABLE 2

Properties of K from Natural Phenomena

Phenomenon	Range of N Years	Number Pheno-mena	Number Sets	K Mean	K Std. devn.	K Range	Coeff. of auto-correl-ation
River discharges	10–100	39	94	0·72	0·091	0·50–0·94	
Roda Gauge	80–1,080	1	66	0·77	0·055	0·58–0·86	0·025±0·26
River and lake levels	44–176	4	13	0·71	0·082	0·59–0·85	n=15
Rainfall	24–211	39	173	0·70	0·088	0·46–0·91	0·07±0·08* n=65
Varves							
Lake Saki	50–2,000	1	114	0·69	0·064	0·56–0·87	−0·07±0·11 n=39
Moen and Tamiskaming	50–1,200	2	90	0·77	0·094	0·50–0·95	
Corintos and Haileybury	50–650	2	54	0·77	0·098	0·51–0·91	
Temperatures	29–60	18	120	0·68	0·087	0·46–0·92	
Pressures	29–96	8	28	0·63	0·070	0·51–0·76	
Sunspot numbers	38–190	1	15	0·75	0·056	0·65–0·85	
Tree-rings and spruce index	50–900	5	105	0·79	0·076	0·56–0·94	
Totals and means of sections							
Water statistics		83	346	0·72	0·08	0·46–0·94	
Varves		5	258	0·74	0·09	0·50–0·95	
Meteorology and trees		32	268	0·72	0·08	0·46–0·94	
Grand totals and means	10–2,000	120	872	0·726	0·082	0·46–0·95	

* Includes also river discharges.

120 natural phenomena have been computed. This is a large mass of statistics and represents a formidable amount of computation.

The characteristics of K can now be examined.

3. Characteristics of K

Table 2 gives a summary of the results from the preceding tables, together with the statistical properties of K.

The table is divided into: (*a*) a group of water statistics, which is what specially concerns the object with which this work was started; (*b*) a group of varves, which contains the largest amount of information based on large values of N; and (*c*) a miscellaneous group mainly connected with meteorology. These groups are of the same order of size.

The most striking feature of the table is that the means and standard deviations of K in the three groups are practically the same. In the cases of three phenomena which were tried the coefficients of correlation between K for a set and K for its successor have been computed, the largest being $0{\cdot}07 \pm 0{\cdot}08$, which of course is entirely negligible. Thus values of K from different sets of a phenomenon are independent of each other. An examination of the tables of long-term records in the appendix will confirm this.

As a further test of the validity of equation 2, and the distribution of K, the phenomena which cover long periods have each been divided into two approximately equal numbers of sets, by length of period, and the mean values of K have been computed for each section. The results were:

	Number of sets	Mean N		Mean K
	168	81 years		0·749
	187	324 years		0·766
Total	355		Mean	0·76

The phenomena were Roda Gauge readings, three varves and three tree-ring thicknesses; and the figures show that there is very little difference between the mean values of K for the short and the long periods.

It is of interest to see what the frequency distribution of K is like, as we have already seen from a sample of 119 pairs of successive variates that there is no correlation between them. This distribution is shown in fig. 6 where the stepped line shows the actual frequency per thousand in classes whose interval is 0·03, and the continuous line is the normal frequency curve based on the standard deviation of K. It is evident that the normal curve is a good fit for the actual frequencies. The slight appear-

ance of skewness is probably due to the sample not being large enough. When separate curves are drawn for water statistics (346 cases) and the remaining phenomena (526 cases) one curve is slightly skew to the right and the other to the left. We may say therefore that K is normally distributed and the evidence of the correlations shows that it is a random quantity. In Nile projects we have used the value 0·72 which gives a little more weight to quantities such as water and meteorological statistics, where measurements are likely to be a little more accurate than those for varves and tree-rings.

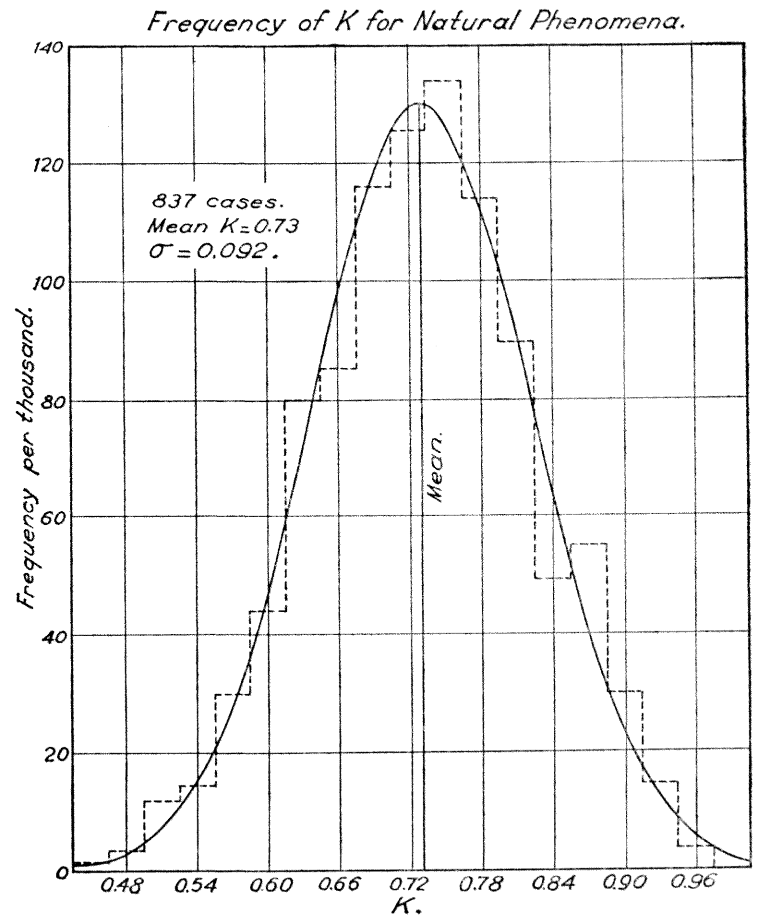


FIG. 6

In the cases where correlations have been computed a value of K is independent of the value of its predecessor. This has been examined in more detail by considering the means for the groups given in table 2. Taking these as individuals the standard deviation of a group mean from the mean of all groups is 0·04. If, however, we deal with a group consisting on the average of 74 values of K on the basis of the standard deviation of individual values of K, whose average is 0·082, the probable deviation of the mean of the group is $\pm 0{\cdot}006$. Thus the variation of the groups among themselves is 4·5 times what would be expected from the variation derived from individual values of K. The largest variations of group means occur with Roda Gauge, some of the varves and the tree-rings. So although the evidence is that there is no correlation between successive values of K, it may be that in some cases there are small systematic effects on K between one phenomenon and another. This might be due to a greater tendency for high or low values of the phenomenon to occur together, which might not show on the correlation of successive years.

As a matter of practice, these differences are not very important. In the case of river discharges and rainfall the means of K differ very little from the mean of all values.

It has been suggested that, in choosing a value of K in the case of a projected reservoir on a river, the value chosen should be that obtained from the series of past records of that river only, disregarding evidence obtainable from other rivers or other phenomena. The preceding discussion shows that the best value to adopt for K is 0·72 or 0·73, and the further one departs from this the less likely is the value. Naturally this does not mean that the records of the river itself are ignored, since the mean of the discharges determines the draft, and their standard deviation is used in calculating the capacity of the reservoir. The only quantity depending on outside information is K. If we compare equation 1 for random events with equation 2 for natural events, in 1 R is a function of N and σ, while in 2 it is a function of N, σ and K where K depends on the order in which the variates occur. The difference between these two classes of events is that with natural phenomena high or low values tend to be more grouped together than is the case with random events. With regard to this grouping it is not shown by a year-to-year correlation, but rather by a persistence over an indefinite period of something which causes values on the whole to be, for example, high and then changes in the course of a few years to the opposite. An extreme example is the case of the discharge of the Nile at Aswan (fig. 2) where there is a change of the mean discharge after 1898, the means and standard deviations being:

	N	Mean	Standard deviation	Auto-correlation coefficient
1870–1898	29	110 mlrds.	13·4 mlrds.	$0{\cdot}11 \pm 0{\cdot}12$
1899–1957	59	85 mlrds.	12·2 mlrds.	$0{\cdot}12 \pm 0{\cdot}09$
1870–1957	88	93 mlrds.	17·5 mlrds.	$0{\cdot}49 \pm 0{\cdot}05$

Here there is no significant correlation between successive years either before or after the change of mean, but when the whole period is considered the change of mean produces a spurious correlation.

In the discussion on paper 11, Professor G. A. Barnard (Professor of Mathematical Statistics, Imperial College) said that "It was possible to prove that no set of simple correlations could account for those figures (i.e. the values of K), so that in fact the author had shown that not only was the series of rainfall figures not random, but that no simple type of non-randomness could account for the figures. That was a fascinating problem for climatologists."

Several contributors have suggested that for very large values of N the index K may perhaps tend towards $\frac{1}{2}$. The following are the only values of K for large N

N	1,080	900	1,000	1,000	1,000	1,000	2,000	2,000	1,000	1,200
K	0·78	0·86	0·86	0·75	0·77	0·72	0·78	0·87	0·72	0·88

Mean $K = 0{\cdot}80$

Ten is a small sample, but it certainly affords no evidence of a trend towards $K = 0{\cdot}5$.

While on this subject a paper by A. Fathy and A. S. Shukry(33) may be mentioned. In the discussion on the paper Hurst wrote, summing up his contribution, "The preceding relates to the statistical description of natural phenomena and it may be emphasised that there is only one method of investigation, and that is the analysis of a large number of phenomena with records covering long periods. Instead of this the authors make R a function of four quantities (equation 10) and then proceed to make hypotheses about all of them and base their conclusions on these." These assumptions led to results not in accordance with the known characteristics of natural phenomena and the paper therefore is useless as a solution of practical problems. Dr. Gamal Mustafa(33) showed that the application by the authors of their method to the High Aswan Reservoir produced a storage capacity which was much too low. Messrs. Y. M. Simaika and N. Boulos(33) showed, by dividing a number of records of phenomena into two halves, that the equation $R/\sigma = (N/2)^K$ produced a more accurate result for the second half when K was given the mean value 0·72 than when the value computed from the first half was used.

It is worth while to give some details about the sequence of Nile discharges from 1899 to 1957. The value of K for this period is 0·50 and this is the lowest out of 61 cases from 39 different rivers (see appendix 4). The next lowest value is 0·57. From the frequency curve for K (fig. 6), and the tables, the probability of occurrence of a value as low or lower than this is about 1/200, and therefore its chance of recurrence during the life of present structures is very small.

We can consider the matter further by comparing this value of K with the values found for periods of 100 years from the records of flood levels of Roda Gauge. The mean value of K for 11 sets is 0·74 and the extreme values are 0·65 and 0·86. It is clear therefore that in the long run the Nile behaves in a similar manner to the rivers and other natural phenomena contained in the tables in the appendix, and summarised in table 2.

The plain fact of the case is that to make the maximum use of the Nile discharge, with reasonable safety against low years, far more long-term storage is required than is possible at Aswan. Consequently we have to do the best with what we can produce at Aswan, and there can be no question of reducing the capacity.

4. Frequency Distribution of Annual Values of Some Natural Phenomena

It seems relevant to give some data on this subject, which has an indirect bearing on storage problems, but a more direct one on the expectation of extreme values. While it was thought that natural phenomena behaved as independent random events their distribution followed the normal probability curve and hence could be derived theoretically, resulting in equation 1 as will be shown later. Feller[8], however, has shown that in the limit when N is large R is independent of the form of the distribution. This probably also applies to the form of R found experimentally, and given in equation 2.

The following table and curves figs. 7 and 8, which were given in the paper to the Institution of Civil Engineers, show a frequency distribution based on 5,916 annual values from 51 phenomena all of which have been used in deriving the values of R and K already tabulated. In order to have all the quantities on the same basis the departures from the mean are expressed in terms of their standard deviations. The curve is given in two forms, the usual one by plotting frequencies against departures from the mean and the less known one where probability paper is used. This paper,

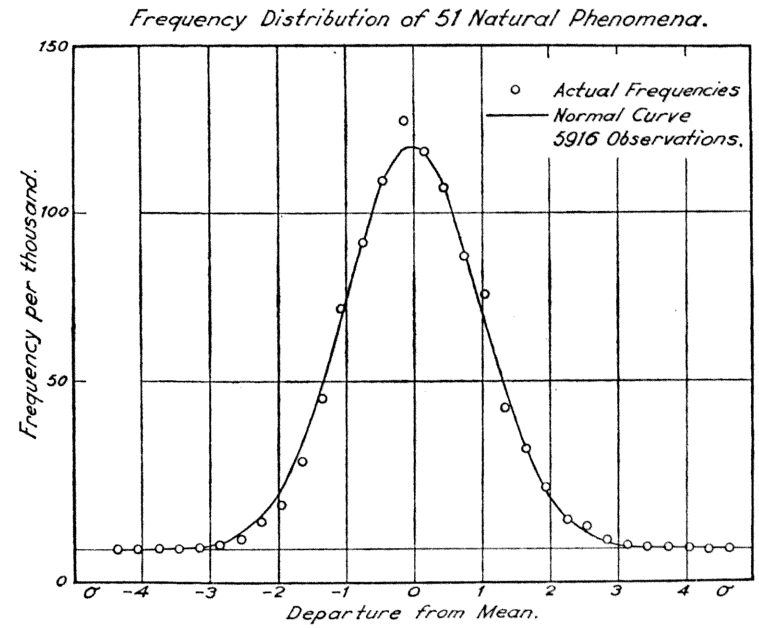


Fig. 7

which we first saw mentioned by Hazen[6], has its scale representing frequencies or probabilities so expanded from the middle outwards that a normal distribution plots as a straight line. This is a very useful device, even where distributions are skew, since for many of these the line will be only slightly curved. In all these cases it is easier to draw the line or curve, and different draughtsmen will produce more concordant results, than in drawing the usual humped frequency curves. The phenomena used are chosen largely on the length of record available, and consist of rainfall, temperature and some river statistics. It may be mentioned that no cases of small rainfall are used, as these, bounded at one end by zero, tend to be skew. The table shows that there is no distribution which does not approximate to the normal or is more than slightly skew. Between $+3\sigma$ and -3σ the curve on probability paper is a straight line passing through the point 0 and 50 per cent, showing that the ordinary form of the curve will be symmetrical. Both ends of the probability paper curve bend away from the straight line in the directions of making extreme values appear to be more probable than they would be in a normal distribution.

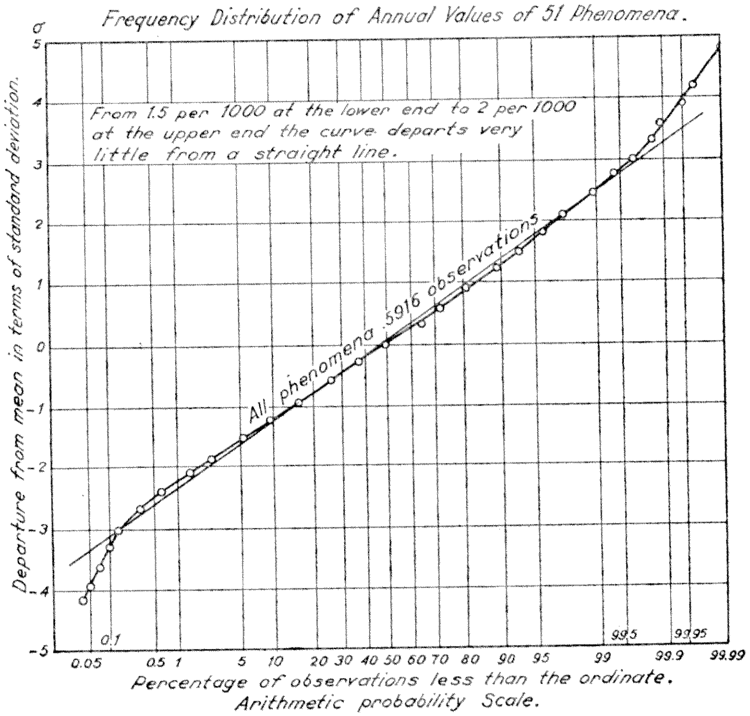


FIG. 8

The part which is normal includes all the observations except about two per thousand at both ends. The extremes of the curve depend on a few observations, and of the two extreme groups of departures of more than $4{\cdot}2\sigma$ at each end three observations are from the old records of Roda Gauge dating from the Middle Ages. In the case of any records extending a long way into the past the possibility of mistakes cannot be excluded, and the effect of these, which is negligible in the classes which contain many observations, becomes considerable, if they occur at the ends of the distribution where the observations are few.

The curve of fig. 7 has been computed from the mean and standard deviation and its equation is

$$y = \frac{N}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} \qquad \ldots 4$$

It will be seen that it is a good fit for the actual observations and any skewness is negligible. There is no doubt that the curve on probability paper is a more useful device than the normal curve, since it shows the nature of the curve at its extremities, an important matter for some purposes.

With regard to skewness, the frequencies of the individual phenomena in the table approximate to the normal curve, order of occurrence being ignored. A rough analysis showed that in 24 cases Pearson's measure of skewness—(mean-mode)/standard deviation—was positive; in 22 cases negative; in four cases zero; and in one case uncertain. The mean values of skewness for the groups were:

Rainfall	0·09
River discharges and levels	—0·23
Temperature	—0·04

K was also compared with skewness. For the cases of positive skewness the mean value of K was 0·74 and for the negative cases 0·73. If numerical values of skewness ignoring sign were taken the mean for $K \leqslant 0{\cdot}73$ was 0·26, and for $K \geqslant 0{\cdot}74$ it was 0·32. It is clear from these facts that the effect on K of the small amount of skewness in the data was negligible.

5. The Variability of Means and Standard Deviations

Until recent times the belief was current among engineers that the means of series of rainfall and river discharges, and presumably of other meteorological phenomena, tended to approach constant values as the lengths of the series increased. For example, it was thought that a record of 35 years of rainfall gave a good enough approximation to the supposed mean. This was based on the assumption that natural phenomena were independent and random events, for which means and standard deviations approach limiting values as N increases. Figures 2 and 3 showing the discharges of the Nile and the Thames prove that this opinion is incorrect, since the mean discharges of the two parts of the records are: Nile, 101 and 86 milliard cubic metres, and Thames, 500 and 560 milliard gallons. However, as J. K. Hunter remarked(*) "such expressions as average annual run-off had no precise meaning except in relation to the particular records from which they had been derived". Other examples will be found in appendices 4, 5 & 6. Rainfall is one of the primary storage interests and the following table shows the variation of means in the case of some of the longest records.

* Discussion on paper 11.

TABLE 3

Variation of the Mean in the Case of Rainfall

Station	Duration of records Years	Length of period Years	No. of periods	Max. value	Mean Min. value	Mean value
Philadelphia	126	42	3	42·7	41·4	42
Boston	128	43	3	44·2	38·2	42
Madras	133	44	3	51·3	48·4	50
Rome	151	50	3	886	777	830
Stockholm	161	40	4	57·8	38·3	49
Padua	171	43	4	940	793	850
Milan	171	49	6	104	95	100
Zwanenbourg	211	53	4	77	69	74

The average range of variation of the means of rainfall in table 3 is about 14 per cent, which is more than would be expected for purely random events. Unfortunately 200 years is about the longest record, and few stations exceed 150 years. They are divided into periods of 40 or 50 years to illustrate recorded variations. This division is chosen because quite frequently water supply or hydro-electric projects have to be based on even shorter records, because nothing longer is available. Further evidence of the variability of means is given by the long-term records of tree-rings and varves. In the case of rings of the sequoia, 200-year means range from 0·84 to 1·0, while in the case of varves at Lake Saki 500-year means range from 11 to 16 and even 2,000-year means differ considerably. W. B. Langbein in the discussion on paper 11 approached the question of variation of means from this point of view, making use of the annual discharges of ten American and two European rivers. He computed the ratios of the standard deviations of the means for periods of from 2 to 20 years to the standard deviations for single years, and plotted the results against the lengths of the periods. He also plotted the similar ratios from data given in the paper, and included lines to show the same ratios for independent random quantities. The graph shows that the variance of the means of river discharges is usually greater than the variance of means of independent random quantities, and that the ratio of these variances increases with the length of the period. For example, while the standard deviation of the mean of 100 years of a random event is on the average one-tenth of the standard deviation of a single year, that for river discharges is a quarter. Langbein's graph confirms the previous results.

In the following table the standard deviations of 50- and 100-year means from some of the long records of thickness of tree-rings and varves are compared with the standard deviations to be expected if the data

TABLE 4

Variations of Means of Random and Natural Phenomena

Phenomenon	No. of years	Standard deviations of 50-year means Random	Standard deviations of 50-year means Natural	Standard deviations of 100-year means Random	Standard deviations of 100-year means Natural
Tree-rings					
Sequoia	900	0·04	0·12	0·02	0·10
Pines, Flagstaff	500	0·04	0·09	0·03	0·07
Varves					
Lake Saki	1,000	0·9	2·0	0·6	1·8
	1,000	0·9	1·5	0·6	1·0
	1,000	1·3	2·2	0·9	1·6
	950	1·2	2·2	0·9	2·0
Moen	1,000	1·9	4·1	1·4	3·5
Tamiskaming	1,200	0·8	4·2	0·6	3·5
Roda Gauge (free of trend)	1,050	0·10	0·24	0·07	0·21
Mean ratio: natural/random			2·5		3·2

were random. In such a case the standard deviation of a 50-year mean is the standard deviation of a single observation divided by $\sqrt{50}$.

It appears from the table that the actual means are two and a half to three times as variable as would be expected if the phenomena considered were independent random events.

The variation of the standard deviation is relatively greater than the variation of the mean, and examples from the same phenomena are given in the following tables.

The mean range is about 25 per cent of the mean standard deviation. The following table compares the variation of the standard deviations of natural and random events.

TABLE 5

Variation of the Standard Deviation in the Case of Rainfall

Station	Standard deviation Maximum	Standard deviation Minimum	Standard deviation Mean
Philadelphia	7·0	5·6	6·3
Boston	7·4	5·0	6·5
Madras	15·6	14·3	15·0
Rome	17·1	15·7	16·3
Stockholm	13·5	7·8	9·8
Padua	179	147	166
Milan	21·0	15·1	18·6
Zwanenbourg	13·1	10·8	12·0

TABLE 6

Variations of Standard Deviations of Random and Natural Phenomena

Phenomenon	No. of years	Standard deviation of the standard deviation for a 50-year period: Random	Natural	Standard deviation of the standard deviation for a 100-year period: Random	Natural
Tree-rings					
Sequoia	900	0·014	0·034	0·011	0·032
Pines, Flagstaff	500	0·029	0·040	—	—
Varves					
Lake Saki	3,950	0·6	2·6	0·5	3·1
Moen	1,000	1·2	5·6	0·9	4·5
Tamiskaming	1,200	0·36	1·11	0·3	1·6
Roda Gauge (free of trend)	1,050	—	—	0·04	0·14
Mean ratio: natural/random			3·2		4·6

In the case of independent random events the standard deviation of the standard deviation σ is $\sigma/\sqrt{2N}$, and this is what is given in the table, which shows that the value for natural events is three to four and a half times as great.

Another difference between the standard deviations of random and natural events is that while the standard deviations of random events approach a limiting value as N increases, the standard deviations of many natural events show an increase as N increases. This is shown in table 7.

In addition the mean of ratios from 30 rainfall and river observations gave

N/N_0	2	3	4	(5, 6, 7)
σ/σ_0	1·04	1·06	1·06	1·05

The increase is most marked in the long-term records of tree-rings and varves, though it is not very pronounced in the record of Roda Gauge Flood Levels. It is also shown in rainfall and river statistics but these records are fewer and much shorter, and only half of them cover 100 years or more, so that they correspond practically to the first two columns of the table, with which they agree fairly well.

Although in these phenomena of great practical importance the progressive increase of standard deviation is not definitely established, it is a sound precaution to increase the observed standard deviation in designing storage projects.

TABLE 7

Increase of Standard Deviation with Length of Record

Phenomenon	Length of record in years (N): 50	100	200	300	400	500	1,000	2,000
	Standard deviations and their ratios with the standard deviations for 50 years							
Tree-rings								
Sequoia	0·143	0·151	0·170	0·180	—	0·170		
	1·0	1·06	1·19	1·26	—	1·19		
Flagstaff, Pines	0·29	0·31	0·31					
	1·0	1·07	1·07					
Pike's Peak, Pines	0·269	0·285	0·317					
and Douglas Firs	1·0	1·06	1·18					
Meadow Valley,	0·485	0·54	0·575					
Pines	1·0	1·11	1·19					
Varves								
Lake Saki	6·22	6·79	7·06	—	—	7·42	7·60	7·50
	1·0	1·09	1·14	—	—	1·19	1·22	1·21
Moen	11·90	12·52	13·14			14·0		
	1·0	1·05	1·11			1·18		
Tamiskaming	3·63	4·15	4·73	4·72	5·07	5·25	600 years	
	1·0	1·14	1·30	1·30	1·40	1·70		
Mean ratios	1·0	1·08	1·17	1·28		1·19		
Roda Gauge	0·59	0·60	0·59	0·60	0·59	0·65		
	1·0	1·02	1·0	1·02	1·0	1·10		

6. THE POSSIBLE MAXIMUM AND MINIMUM VALUES OF K

The distribution of observations of natural phenomena is normal if their order of occurrence is ignored. In the following a normal distribution is assumed as an average case.

The greatest possible value of R from a given set of observations will occur when all the positive values of departures from the mean occur together in one group, and all the negative values in another. In this case R is the sum of either all the positive or all the negative values of the departures, and it can be shown that when N is large,

$$K = \frac{\log N - \frac{1}{2}\log 2\pi}{\log N - \log 2}$$

Therefore K very slowly approaches 1 as $\log N$ increases. Its value for $N = 100{,}000$ is 0·98.

With regard to the minimum value, for any particular series the least value of R is not less than the maximum departure from the mean. From tables this limit can be calculated for different values of N. For $N = 143$ K is not less than 0·25, for $N = 19{,}000$ not less than 0·15 and for $N = 100{,}000$ not less than 0·14, on the average.

Changes in the maximum and minimum are inversely proportional to changes in $\log N$, which are small compared with changes in N. In the case of the minimum the change of the probability at the extremities of the normal curve is also involved and this further reduces the change of K with N.

This computation is for the normal distribution, but variants from this could alter the approach and also the lower limit.

7. Combination of Storage from Different Sources

The previous theory has dealt with storage of one variable supply, as for example the water flowing out of Lake Albert. For this we have

$$R = \sigma(N/2)^K$$

Where K has a mean value of 0·72 and a standard deviation of 0·08. Suppose there are two streams which join together, and on these there are reservoirs with capacities R_1 and R_2, capable of giving steady discharges equal to their averages over a period of N years. Let R be the capacity on the combined stream which would produce the same effect. Using subscripts for the tributary streams we have:

$$R_1 = \sigma_1(N/2)^{K_1} \qquad R_2 = \sigma_2(N/2)^{K_2} \qquad R = \sigma(N/2)^K$$

and

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2r\,\sigma_1\,\sigma_2$$

Where r is the coefficient of correlation between the discharges of 1 and 2. Then

$$\frac{R^2}{(N/2)^{2K}} = \frac{R_1^2}{(N/2)^{2K_1}} + \frac{R_2^2}{(N/2)^{2K_2}} + 2r\frac{R_1\,R_2}{(N/2)^{(K_1+K_2)}}$$

In general K, K_1 and K_2 will be different, and not much use can be made of this equation. In this case to compare R with the sum of R_1 and R_2 it will be necessary to compute each separately. If, however, the K's are all the same

$$R^2 = R_1^2 + R_2^2 + 2r\,R_1\,R_2$$

If the tributaries vary together so that $r = 1$, then

$$R = R_1 + R_2$$

If there is no relation between them, then $r = 0$ and

$$R^2 = R_1^2 + R_2^2$$

So that R is less than $R_1 + R_2$. If r is negative, so that the tributaries vary oppositely, their variations tend to neutralise each other and R is still smaller.

The maximum value of R is therefore $R_1 + R_2$, so that in general the storage which is required to equalise the discharge of the main stream is less with one dam on the main stream than with dams on each of the tributaries, except in the one case above.

8. Use of the Absolute Mean Deviation

It has been suggested by Professor L. M. Laushey* that computation would be saved if, instead of σ, the absolute mean deviation (μ), i.e. the mean of the deviations ignoring their sign, was used. If this were done there would be no need to write down the squares of the deviations. In the case of the normal curve

$$\mu/\sigma = \sqrt{2/\pi} = 0{\cdot}798$$

This ratio is said to be stable for normal and moderately skew distributions.

In practice, however, there are difficulties, since departures are usually taken from a base near the mean, as this simplifies computation and lessens liability to error. Also departures from a suitable base near the mean are computed for a long record, and afterwards the record may be divided into several sets, in some of which the base may differ considerably from the mean. It was therefore necessary to find how the ratio μ/σ varied with departures from a base other than the mean, and the following results were obtained from varves and some rivers and rainfalls.

$$\mu/\sigma$$

49 cases. Mean 0·777. Extremes 0·29 and 1·49. 40 cases between 0·60 and 1·0 inclusive. Standard deviation, omitting two extremes, 0·093. The use of the mean ratio 0·8 instead of 0·6 would make a difference in K of 0·07 with $N = 100$, or 10 per cent. Considering this evidence it seems inadvisable to use the absolute mean deviation, as it is inconvenient and could cause considerable variations in the computed values of K.

* Discussion on paper 7.

4

Calculations of R and K for Some Phenomena involving Human Factors

The time series studied so far depend directly on climatic processes. It is of theoretical interest also to examine some time series where human factors play a part, to see whether equation 2 also applies to them. Among such series are records of sport and of historical events. An example of the first is the annual Oxford and Cambridge Rugby football match, and of the second the lengths of the reigns of the Kings of England. In the match the annual terms of the series are points scored by Oxford minus points scored by Cambridge. The earlier scores have been corrected to accord with present scoring. Similarly with the University Boat Race, where the result is usually given as won by x lengths, x is taken positive if Oxford won. When the result is given in seconds this has been converted approximately into lengths. Table 8 gives the results of these investigations.

The figures relating to trout are from the records of Overscaig Hotel on Loch Shin which have been carefully kept and summarised. Prices of wheat were compiled by Sir William Beveridge and are taken from Advanced Theory of Statistics, Vol. II, by Kendall. The effect of trend has been eliminated. Those for cost of living and reigns of the Popes are from Chambers's Encyclopaedia. The lengths of reigns of Ancient Egyptian Kings are taken from Egyptian Pharaohs by Sir Alan Gardiner.

The number of phenomena and cases used in table 8 are not as great as those of the natural phenomena investigated in the preceding sections, as data are not so easy to obtain. The following is a comparison of the results obtained for the two classes.

	Number of phenomena	Number of cases	Mean	K Standard deviation	Range
Natural phenomena	107	837	0·72	0·08	0·46 to 0·95
Phenomena with a human element	11	47	0·71	0·11	0·52 to 0·97

TABLE 8

Accumulated Departures. Phenomena involving a Human Element

Phenomenon	Period	N	M	σ	R	K
Lengths of reigns						
Kings of England	924–1952	51	20	15·6	136	0·67
Kings of Ancient Egypt		41	29	16·1	132	0·70
		41	23	20·2	138	0·65
		40	19	16·6	153	0·74
		40	14	12·9	88	0·64
		82	26	18·6	238	0·69
		81	21	18·7	150	0·56
		80	16	15·3	154	0·63
		162	21	17·6	464	0·74
Popes	217–1939	55	7·3	8·5	52	0·55
		55	4·6	5·3	61	0·74
		55	4·6	4·9	53	0·72
		55	6·6	6·2	63	0·70
		55	9·4	7·2	80	0·73
		110	5·9	7·2	89	0·63
		110	4·6	5·1	73	0·66
		110	5·6	5·7	84	0·67
		110	8·0	6·8	114	0·70
		275	6·5	6·8	265	0·74
Rugby Football Matches						
England *v.* Scotland	1870–1959	75	1·7	7·7	105	0·72
Wales *v.* Ireland	1881–1960	63	0·2	8·7	107	0·73
Oxford *v.* Cambridge	1871–1959	83	−0·5	9·8	118	0·67
University Boat Race						
Oxford *v.* Cambridge	1852–1957	47	2·0	6·5	49	0·52
		47	−2·8	4·5	42	0·58
		94	−0·4	6·1	134	0·81
Trout caught at Overscaig, Loch Shin						
Annual total, unit 100	1887–1922	36	53·6	23·0	215	0·77
	1923–1959	37	24·1	9·6	137	0·91
	1887–1959	73	38·6	22·8	614	0·92
Average weight per fish	1887–1922	36	0·41	0·053	0·56	0·81
	1923–1959	37	0·50	0·081	0·92	0·84
	1887–1959	73	0·46	0·114	1·90	0·81
Average weight per fish omitting last three years, when effect of dam appeared	1923–1956	34	0·48	0·044	0·47	0·85
	1887–1956	70	0·45	0·084	1·56	0·82

TABLE 8

Accumulated Departures. Phenomena involving a Human Element (continued)

Phenomenon	Period	N	M	σ	R	K
Prices of wheat	1500–1589	90	98	22	210	0·60
	1590–1679	90	100	19	240	0·67
	1680–1769	90	99	21	270	0·67
	1770–1869	100	100	20	290	0·69
	1500–1679	180	99	20	290	0·59
	1590–1769	180	100	20	290	0·60
	1680–1869	190	99	20	370	0·64
	1500–1769	270	99	21	280	0·54
	1590–1869	280	100	20	310	0·55
	1500–1869	370	99	20	380	0·56
Cost of living	1818–1859	42	96	12·6	160	0·84
	1860–1901	42	84	15·5	295	0·97
	1902–1945	44	114	41·6	538	0·83
	1818–1881	64	97	11·1	150	0·75
	1882–1945	64	100	40·2	910	0·90
	1818–1945	128	98	29·5	958	0·84
Number of pennies minted annually	1860–1909	50	14·3	11·7	212	0·91
	1910–1960	51	37·8	40·4	505	0·78
	1860–1960	101	26·1	32·1	918	0·86
Number of pennies per head of population	1860–1909	50		0·42	7·0	0·87
	1910–1960	51		0·89	11·1	0·78
	1860–1960	101		0·76	22·7	0·86

A comparison can also be made with the detailed tables for natural phenomena already given. The figures above show that the means, standard deviations and ranges of K are very similar in the two kinds of phenomena, and that equation 2 applies to a number of what may be called social phenomena. In this class also it is reasonable to suppose that the mean of K for a large number of cases will be near 0·72, and so equation 2 may be of very wide application.

In the case of sporting events there is obviously a similarity to natural events in the existence of a tendency to persistence; success may be due to good coaching and also itself attracts good players. So in reigns of kings or popes conditions favourable to long or short reigns may persist for long periods.

5

Storage required to guarantee a Draft less than the Mean Supply

1. METHOD OF CALCULATION

The preceding chapters all refer to the properties of R, the range of accumulated sums of departures from the mean. A different problem arises when a discharge less than the mean would meet requirements, or

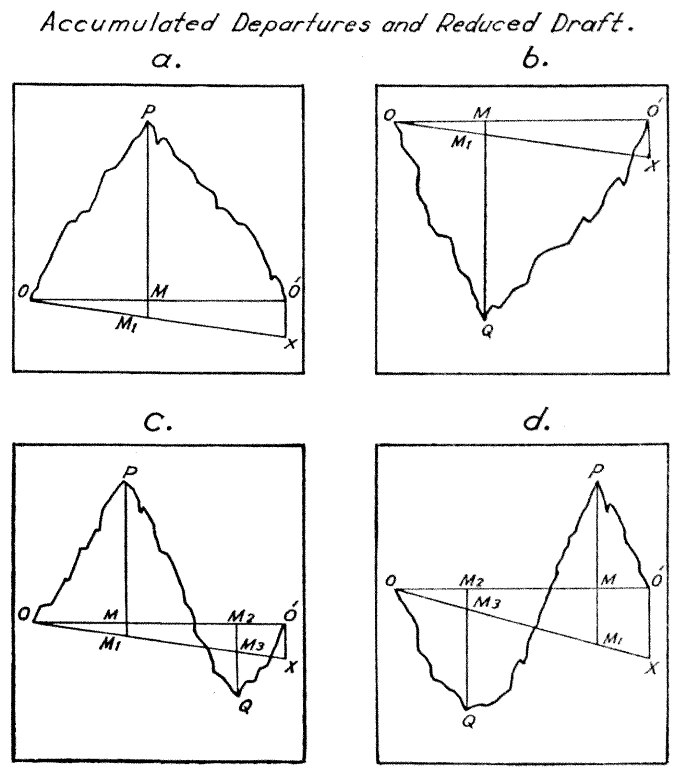


FIG. 9

it is not possible to produce the storage necessary on the average to guarantee the mean. In order to solve this it is necessary to find the greatest accumulated deficit S with a draft B less than the mean M. This can easily be done if accumulated departures from the mean or some other base have been plotted. Examples of four cases are shown diagrammatically in fig. 9 where, to simplify the question, accumulated departures from the mean are plotted. If the discharge is not to be allowed to fall below a draft B less than the mean, this corresponds to taking departures from B, which can be done by using the curve of accumulated departures from the mean. In the figure OO' is the axis for departures from the mean, the final sum of departures being zero. OX represents the axis for departures from B, where $O'X = N(M-B)$, N being the number of years in the record. The curve of fig. 9a is of the same type as the outflow from Lake Albert and the discharge of the Nile at Aswan (figs. 1 and 2), while that of b is similar to fig. 3, which is for the Thames. In c the discharge is mainly above the mean in the first part and mainly below the mean in the second part, while in d the reverse is the case. In curve c the maximum precedes the minimum, and in d this is reversed. In a the maximum deficit S takes place in the period MO' and has the value $PM_1 - O'X$. In b, $S = M_1Q$; in c the maximum deficit takes place from M to M_2, and is equal to $PM_1 + M_3Q$; while in d, $S = M_3Q$.

It is obvious that cases similar to d could arise where the maximum deficit would be $PM_1 - O'X$ as in a.

2. The Relation between Reduced Draft and Storage for Natural Phenomena

To find this relation the procedure is similar to that used to find the mean relation between R, σ and N given in equation 2 (p. 3).

That is to calculate values of S/R for many phenomena with several different values of $(M - B)$ for each. Table 9 gives the mean results of the calculations, and the details are given in the appendix.

In addition to these, observations on twelve American rivers were supplied by Mr. W. B. Langbein, of which those with the lowest values of S/R have been divided into three groups of four observations each and are shown in fig. 11. Langbein's other observations are fairly close to the mean line of the figure and have not been plotted.

In the original paper two types of curve fitted the observations over their range ($(M - B)/\sigma = 0{\cdot}05$ to $0{\cdot}7$) very closely.

For the case of 38 phenomena taken from river discharges, rainfall and

Table 9

Relation between Draft (B) and Storage (S). Mean Values

Phenomenon		Mean values					
Rivers, rainfall and temperature $N = 96$ years (38 phenomena)	$(M-B)/\sigma$	0·06	0·09	0·20	0·23	0·26	0·29
	S/R	0·76	0·77	0·56	0·49	0·47	0·46
	$\log(S/R)$	−0·12	−0·11	−0·26	−0·32	−0·34	−0·37
	$\sqrt{(M-B)/\sigma}$	0·24	0·30	0·45	0·48	0·51	0·54
Rivers, rainfall and temperature $N = 96$ years (38 phenomena)	$(M-B)/\sigma$	0·30	0·37	0·42	0·43	0·52	0·53
	S/R	0·42	0·32	0·34	0·32	0·24	0·28
	$\log(S/R)$	−0·40	−0·51	−0·47	−0·52	−0·64	−0·58
	$\sqrt{(M-B)/\sigma}$	0·55	0·61	0·65	0·66	0·72	0·73
Rivers, rainfall and temperature $N = 96$ years (38 phenomena)	$(M-B)/\sigma$	0·55	0·63	0·66	0·68	0·88	
	S/R	0·24	0·23	0·20	0·20	0·09	
	$\log(S/R)$	−0·62	−0·67	−0·73	−0·76	−0·96	
	$\sqrt{(M-B)/\sigma}$	0·74	0·79	0·81	0·82	0·94	

Number of observations: 118.

Roda Gauge, tree-rings and varves $N = 430$ years (7 phenomena)	$(M-B)/\sigma$	0	0·1	0·2	0·3
	S/R	0·89	0·64	0·48	0·35
	$\log(S/R)$	−0·05	−0·19	−0·32	−0·46
	$\sqrt{(M-B)/\sigma}$	0	0·32	0·45	0·55
Roda Gauge, tree-rings and varves $N = 430$ years (7 phenomena)	$(M-B)/\sigma$	0·4	0·5	0·6	0·7
	S/R	0·27	0·21	0·15	0·12
	$\log(S/R)$	−0·57	−0·68	−0·82	−0·92
	$\sqrt{(M-B)/\sigma}$	0·63	0·71	0·78	0·84

Number of observations: 96.

temperature, comprising the 118 observations of the preceding table, the curves were

$$\log_{10}(S/R) = -0{\cdot}08 - 1{\cdot}00\,(M - B)/\sigma \quad \ldots 5$$

$$S/R = 0{\cdot}97 - 0{\cdot}95\sqrt{(M - B)/\sigma} \quad \ldots 6$$

The first of these two curves is shown in fig. 10.

Now when $M - B = 0$ $S/R = 1$, so that neither equation fits exactly at this point.

Similarly at the other end of the curve the smallest value of a series requires no storage to guarantee it over the whole series, so $S/R = 0$. For this lowest value, however, $(M - B)/\sigma$ will depend on the number of observations in the series; the larger the number of observations the greater will be the departure of the smallest from the mean. If there is a

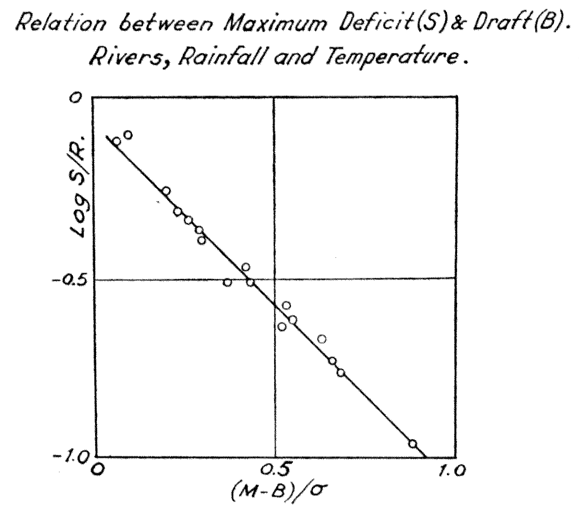


FIG. 10

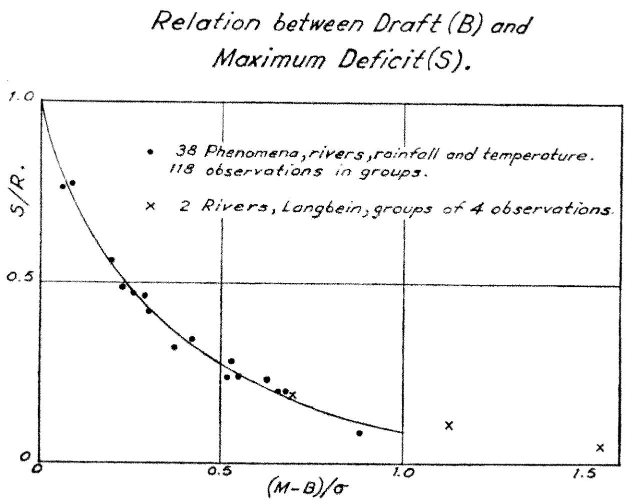


FIG. 11

number of phenomena all with the same number of observations there will be a mean value of $(M - B)/\sigma$ for $S = 0$.

In general, however, the position of the zero is indefinite, and consequently the curve of fig. 11 has not been carried beyond $(M - B)/\sigma = 1$.

An important feature of the curve is the large reduction in the storage required when the draft is a little below the mean of the series, a reduction of $0{\cdot}1\sigma$ producing a 30 per cent reduction in S, and $0{\cdot}2\sigma$ 45 per cent.

This is a valuable property, since in many cases it will be impossible to provide a storage capacity as large as R. In these cases it will not be possible to make full use of the average supply. Methods of using storage to the best advantage are discussed in chapter 9.

6

Experiments with a Statistical Model of Some Time Series which occur in Nature

1. Introduction

In the study of time series of independent events normally distributed, experimental series were made by cutting cards from what we called a probability pack. In one of these packs the cards received the numbers +1, −1, +3, −3, etc., to +9, −9, which represented departures from a mean. The numbers of each kind of card were made proportional to the corresponding ordinates of a normal frequency curve. There were thirteen of each of +1 and −1; ten of ±3; six of ±5; three of ±7; and one of ±9. The approximation of these numbers is fairly close to the normal curve. Experiments with this pack were made by drawing a card, noting its number, replacing the card and shuffling the pack before drawing the next number. This procedure produces a series of independent numbers normally distributed and the results are described in appendix 2, where equation 1 of chapter 1 is obtained theoretically and agrees with the experiments.

Such methods where problems are solved by experiments on samples are known as Monte Carlo methods. The name is of recent origin, but the method has been known for more than a century, and examples are given by Rouse Ball in Mathematical Essays and Recreations of two experimental methods of finding π based on probability theory applied to random events. Moran in the Theory of Storage[10] has a chapter on Monte Carlo and other Statistical Methods and gives references to papers on the subject. It is a means of solving practical problems which are very difficult by mathematical analysis. The method has been used to produce time series to imitate annual values of river flow by Sudler[13], Barnes[14], Brittan[15] and Maas and others[16]. All these series consist of independent random numbers, for which on the average $R/\sigma = 1{\cdot}25\, N^{\frac{1}{2}}$. It has been shown in this book and preceding papers that time series from many natural phenomena, including rainfall and river discharges, are not of this type. In the following section a method is described which produces results closely resembling natural series.

2. Description of the Model

A preliminary account of some of the work described in this chapter was given in Nature[12]. In this the probability cards were used to imitate series occurring with natural phenomena, whose characteristics have already been described. The method of doing so will now be given.

The pack is shuffled and a card is cut, and after its number has been noted it is replaced in the pack. Two hands are then dealt and if for example the card cut was +3, the three highest positive cards in one hand are transferred to the other, and from this the three highest negative cards are removed. This hand then has a definite bias. A joker is now placed in it and it is shuffled and a card is cut from it. The number on this card is the first of the series. It is replaced and the hand then reshuffled, and another card is cut, recorded and replaced. This cutting and shuffling goes on until the joker is cut. Then the joker is replaced and all the cards are

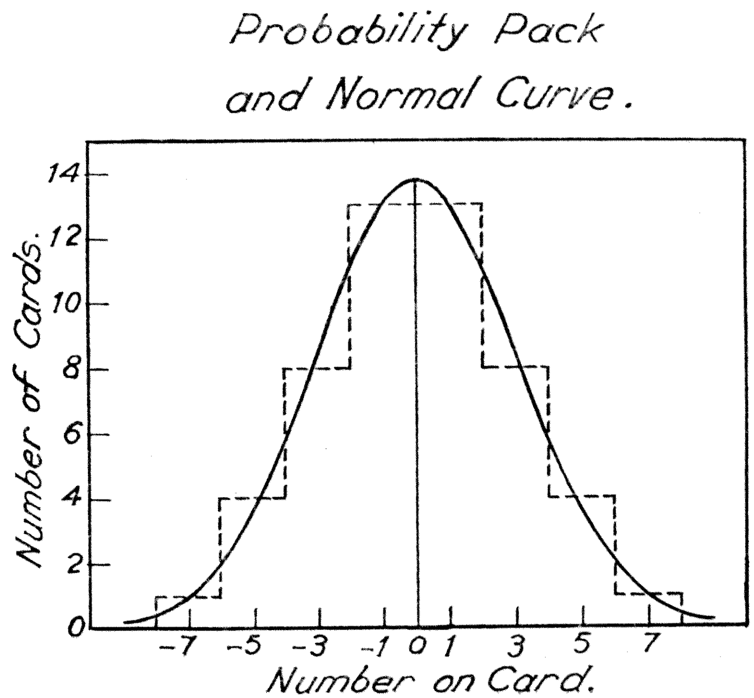


FIG. 12

put together and the pack is reshuffled, after which the process is repeated. The amount of bias, the number of cuts for which the bias acts, and the actual cards which are cut all depend on random processes.

There were six experiments, each consisting of 1,000 cuts, which were afterwards combined in order of occurrence to form one long series. The first four experiments were made with a pack which consisted of 52 cards numbered ±1 (thirteen of each), ±3 (eight), ±5 (four), and ±7 (one of each). This distribution compared with the normal is shown in fig. 12 and it will be seen that the correspondence is close. For the last two experiments a pack was used consisting of 62 cards made up of twelve ±1, nine ±3, six ±5, three ±7 and one ±9. These also are a close approximation to the normal distribution. The pack of 62 was dealt into three hands, one of which containing 21 cards was biased in experiments

TABLE 10

Mean Values of R/σ and K from Experimental Time Series

N / Expt.	50 (log R/σ)	100	200	500	1,000	50 (K)	100	200	500	1,000
1	0·98	1·23	1·45	1·76	1·95	0·70	0·72	0·72	0·73	0·72
2	0·95	1·18	1·37	1·62	1·81	0·68	0·70	0·68	0·68	0·67
3	0·98	1·31	1·64	1·94	2·06	0·70	0·77	0·82	0·81	0·76
4	0·92	1·20	1·40	1·83	1·88	0·66	0·70	0·70	0·76	0·70
5	1·01	1·35	1·57	2·03	2·27	0·72	0·79	0·79	0·85	0·84
6	0·90	1·11	1·33	1·82	2·18	0·66	0·66	0·66	0·76	0·81
Means	0·96	1·23	1·46	1·83	2·02	0·69	0·72	0·73	0·76	0·75
log N/2	1·40	1·70	2·00	2·40	2·70					

N / Expt.	2,000 (log R/σ)	3,000	4,000	6,000	2,000 (K)	3,000	4,000	6,000
1–2	1·72				0·64			
1–3		2·05				0·65		
1–4			2·09				0·64	
3–4	2·14				0·79			
1–6				2·67				0·77
3–6			2·58				0·78	
4–6		2·56				0·80		
5–6	2·49				0·92			
Means	2·12	2·30	2·34	2·67	0·78	0·72	0·71	0·77

Mean of 276 values of K: 0·714
Mean standard deviation of K: 0·091

Natural Phenomena
Mean of 872 values of K: 0·726
Mean standard deviation of K: 0·082

by the previous cutting of a card from the pack and the exchange of cards as before from the other two hands, which were combined.

The joker is placed in the biased hand, which is shuffled and cut as before. In experiment 5 the cards numbered 1, 3, 5, 7, 9 were given the bias of their numbers, but in experiment 6 they had bias 1 to 5 respectively.

3. Results of Experiments with the Model

These are given in table 10.

The results are also shown in fig. 13.

The most remarkable fact about this method of experimenting with probability cards is that it produces a distribution identical with that already found in the case of natural phenomena, with practically the same mean value and standard deviation of the parameter K.

In the table each thousand cuts is considered as a single experiment, as this was the way in which the work was done, and the mean for each

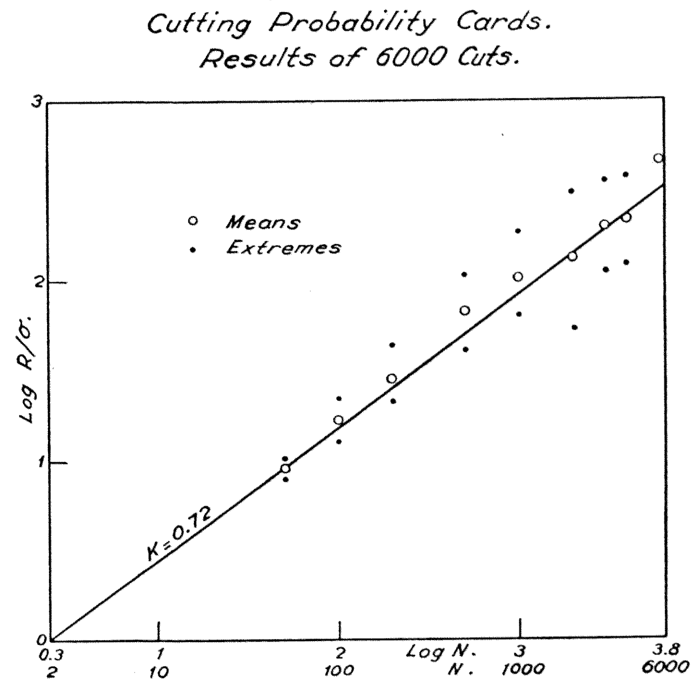


FIG. 13

experiment is given. In fig. 13 the means from all the experiments are plotted for each value of N, and with these the extreme means from the individual experiments. The weights of the points decrease as N increases, since for $N = 50$ there are 20 values of R/σ and K, while for $N = 1{,}000$ there is only one value for each experiment. The points indicating extremes therefore show an increasing dispersion as N increases. All the means, however, lie fairly close to the line $K = 0{\cdot}72$, and there is no significant departure for $N = 6{,}000$. It was written in chapter 3, section 5, "The evidence shows that neither means nor standard deviations have definite long-term values" and the tables in the appendices give ample proof of this. It is of interest therefore to see how the card model compares in this respect with some phenomena for which we have long records—varves and tree-rings. For this comparison we use 100-year means of the natural phenomena, and the means of 100 cuttings of the cards. The standard deviations of these are computed both from the observed means and from the standard deviations of individual observations, on the assumption that they are independent random quantities, i.e. by dividing by $\sqrt{100}$. The following are the results:

TABLE 11

	Long-term means	Standard deviations of 100-year means From means	From single observations	Ratio
Varves				
Saki	13	2·3	0·75	3·1
Moen	17	3·5	1·4	2·5
Tamiskaming	18	3·3	0·6	5·5
Tree-rings				
Sequoia	0·93	0·10	0·018	5·6
Cutting cards	–0·33	0·8	0·31	2·6

In all these examples the 100-year means are much more variable than would be the case if the individual observations were independent. The cards therefore give results which show the indefinite nature of the mean and in this respect also resemble natural phenomena.

The method may therefore be safely used in the investigation of storage problems where ordinary Monte Carlo methods based on the assumption of independent random variables are inapplicable.

An example is the case of a long-term storage reservoir on the Main Nile, where K for the period 1899–1957 is 0·50. A reference to the curve of fig. 6 shows that the probability of a value as low as this is less than 1 per cent. It is therefore very unlikely that the next 60 years will have such

a low value of K, so that a scheme of regulation based only on the past 60 years might be a very unreliable guide to the future.

One course which can be followed is to rearrange the order of the series so as to give a value of K nearer to the mean. Another course is the use of probability cards, and the experiments already made would give many series suitable for testing schemes of regulation. By comparing means and standard deviations of the cards and river discharges a correspondence can be established whereby card numbers can be converted into river discharges. By this means samples can be prepared for the trial of methods of regulation. This has been done for the High Aswan Reservoir and an account of the experiment is given in chapter 9.

7

The Effects of Losses and Trends

1. The Effect of Losses

So far all our investigations have ignored any losses which would be caused by storage. In the case of storage of water, evaporation and seepage are the main losses, though sometimes the growth of aquatic plants may add to these by transpiration. Other substances suffer losses from different causes; for example grain may be eaten by rats or weevils, or made useless by fungi. In drawing up storage projects estimates must be made of probable losses and their effects on the required storage capacity and on the output. Attention was drawn to this by Mr. W. N. Allan*.

The following deals with losses in the long-term storage of water, but some of the results may have applications to other kinds of storage. We shall deal first with losses by evaporation. These are greatest in the arid regions of the world where rainfall is scanty and long-term storage is likely to be most important. In other regions rainfall may equal or exceed evaporation so that there is no need to consider evaporation losses for long-term storage. If, however, there are definite rainy and dry seasons evaporation loss will be important for annual storage.

Two types of instrument are used for measuring evaporation: those like Piche or Wild evaporimeters placed in meteorological screens, and pans fixed in the ground or floating in water. These all require reduction factors in order to give the corresponding amounts for large sheets of open water in the neighbourhood. The most satisfactory of these instruments is the pan floating in the reservoir itself, on account of the fact that the temperature of the water in the pan is very nearly the same as that of the surrounding water. Experiments made at Denver with land pans of different sizes showed that the depth of evaporation tended to decrease asymptotically as the size of the pan increased and the same was found with pans floating in a reservoir near Cairo[40]. The asymptotic value appeared to be reached for a tank 12 feet square. From these experiments it was inferred that the evaporation from a floating tank of 1 metre cube

* Discussion on paper 11.

must be multiplied by 0·80 to give an approximation as good as could be obtained to the evaporation from a large sheet of water. As a confirmation of this we may quote I. E. Houk, Irrigation Engineering[34], who gives reduction factors from several sources for floating pans 3 feet square and circular pans 4 feet in diameter. The mean values for these are respectively 0·82 and 0·83. The chapter on evaporation in Houk's book contains much useful information.

It is not practicable to transport tanks larger than a metre cube. Consequently this size has been used in the river at Cairo, Aswan, Wadi Halfa and other stations on the Nile, to obtain comparisons with Piche evaporimeters in screens. These are the standard instruments used at second order meteorological stations in Egypt and the Sudan. From these comparisons the factor of 0·5 was adopted as the best value for reducing Piche observations on the Main Nile to evaporation from open water.

In the case of the Aswan, Gebel Aulia and Sennar Reservoirs, discharges of the river above and below them have been measured regularly for many years. The losses are taken as the differences between these discharges, which, however, also contain the errors of the measurements. Sufficient numbers of accurate measurements exist to make accidental errors insignificant on their means, and it is found that on these Nile reservoirs evaporation can account for the differences between inflow and outflow, so seepage losses can be ignored.

Seepage losses or gains depend on the permeability of the rocks forming the reservoir basin, which can be determined by observations of water levels in wells. They also depend on the area wetted and on the slope of the water table in the subsoil. The mathematical theory concerning seepage losses is given in books on hydraulics, under the heading flow of subsoil water. Both evaporation and seepage losses increase with the surface area of the reservoir. They may consequently be taken as some function of the reservoir content, though they are not due to the content as such. Some investigations on the effect of losses have been made, and these will now be described.

The relation of inflow (Q), outflow (B), losses (L) and capacity of the reservoir (C) is given by

$$dC = (Q - B - L)\,dt \qquad \ldots 7$$

In this equation L is a function of C and in some cases can be represented by lC where l is a constant. Over a limited range such as occurs in many reservoirs this may be accurate enough. In some cases approximating to geometrical figures, $L = lC^{2/3}$ would be a reasonable approximation.

In the computations by which mean values of R and S (equations 2, 5 and 6, pp. 3 and 37) have been found, B is a constant, equal to or less than the mean value of Q. Q has the properties already described. If values of Q were independent of each other and normally distributed it might be possible to find a solution of equation 7 by an extension of the method given in appendix 2, or by the methods given by Moran[10] in his chapters on dams. If Q is periodic the equation can be solved exactly.

An attempt was made to get a relation by combining the above equation and the equation $R = \sigma\ (N/2)^K$ but nothing of much value resulted. Consequently it was necessary to do as before and make experiments, using actual phenomena. The results of some of these are given in table 15 which follows. In the computations for this, losses are assumed to be a fixed percentage of the content at the beginning of the year. This is an approximation made to lessen the labour of computation, while giving a general idea of the effect of losses on the range of content. The computation uses the preceding equation to find the increase of content year by year. The greater part of the table makes use of the annual discharges of the Nile at Aswan, which are arranged in different orders.

For negative contents losses are taken as 0. If the contents at the beginning and end of the storage are C_o and C_n, n being the number of years, and equation 7 is summed, we get

TABLE 12

Effects of Losses

(Milliards of cubic metres)

Phenomenon	Draft	Range	Mean loss
Aswan discharge	93	500	0
1870–1957	81	53*	0
loss 10 per cent	79	290	14·3
loss 5 per cent	81	413	12·4
Aswan discharges			
1899–1957	85	84	0
loss 10 per cent	79	78	5·9
loss 5 per cent	82	74	2·9
Transposition 1	85	129	0
loss 10 per cent	79	129	5·8
Transposition 2	85	129	0
loss 10 per cent	79	151	6·1
Thames discharge	53	262	0
1878–1960 loss 5 per cent	47	228	5·9
Sava (tributary of Danube)	52	132	0
1878–1950 loss 10 per cent	45	110	7·4
loss 5 per cent	48	125	4·2

* This is the maximum accumulated deficit.

$$C_n - C_o = \Sigma Q - nB - \Sigma L \quad \text{or} \quad 1/n\,(C_n - C_o) = \bar{Q} - B - \bar{L}$$

where $\bar{Q}$ and $\bar{L}$ are mean annual inflow and mean annual loss respectively. If initial and end contents are equal $B = \bar{Q} - \bar{L}$.

In the table initial contents were guessed from the values of R already given in previous tables, and various drafts, were tried. In the result final contents were never quite the same as the initial contents because losses cannot be estimated exactly, and consequently neither can the suitable draft.

Several trials were made with Aswan discharge 1870–1957 with losses of 10 per cent and 5 per cent and various drafts, and the same procedure was used with the order of the years reversed.

Aswan discharge 1899–1957 has been tried with losses of 10 per cent and 5 per cent; also the years have been divided into three groups, which have been transposed and ranges computed with 10 per cent loss for two of the five possible new arrangements of these groups.

Table 12 gives the results of six computations with no losses and 14 with losses. The general result is that there are 12 cases where the range of accumulated departures is less with losses than without, one case where it is greater, and one case where the ranges are the same.

These two anomalous cases occur with the transpositions of the Aswan

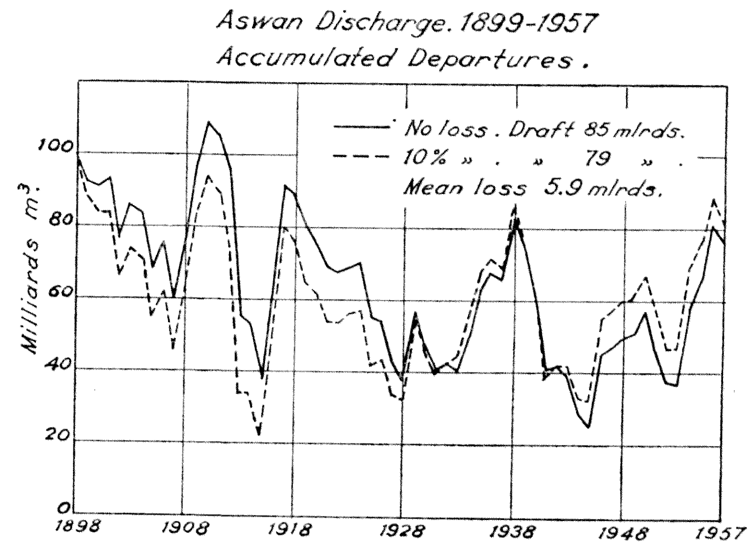
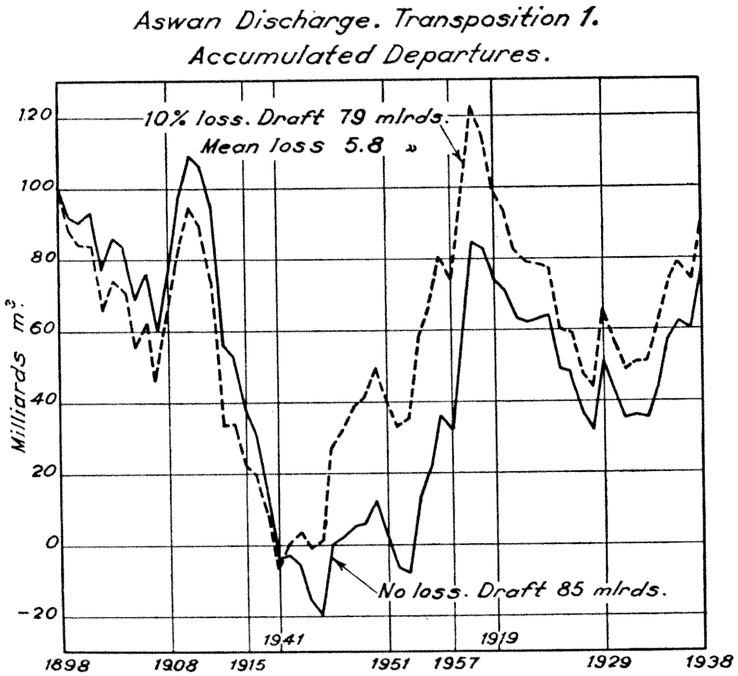


FIG. 14



Transposition 1. 1898–1917; 1939–1957; 1919–1938.

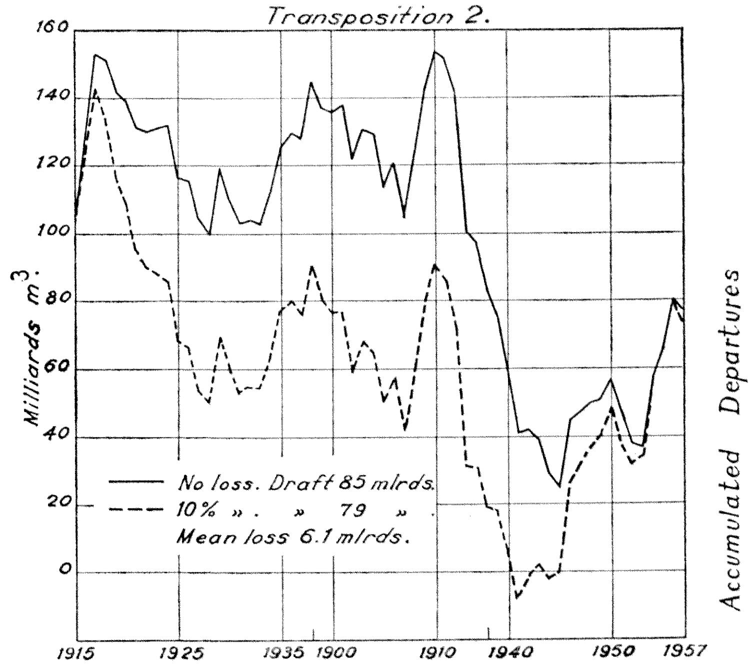
FIG. 15

discharges 1899–1957, and the contents in the original and the transpositions are shown in figs. 14, 15 and 16.

Further experiments were made with five 5's and five 10's, arranged in order to show extreme cases, of which the results are given in table 13.

Where a negative content would occur an initial content just large enough to prevent this has been chosen. In these extreme arrangements of Q in each case the range is a little greater with than without losses. An arrangement similar to the first in the above table is the one shown in fig. 16 where the content is high for about the first 40 years and low for the last 20, and this is the case given in table 12, in which the range is greater with losses than without.

The general conclusion from the previous few examples is that the addition of losses can both decrease and increase the amount of storage required to give the maximum draft obtainable with the losses. The



Transposition 2. 1915–1938; 1898–1914; 1939–1957.

FIG. 16

TABLE 13

Experiments on the Effect of Losses

Order of Q's	Loss per cent	Draft	Contents Initial	Final	Max.	Min.	Range
15 . . . 15, 5 . . . 5	0	10	0	0	25	0	25
	10	8·7	0	0	25·8	0	25·8
	5	9·3	0	0·5	25·7	0	25·7
5 . . . 5, 15 . . . 15	0	10	25	25	25	0	25
	10	8·7	26	26	26	0·2	25·8
15, 5, 15, 5, . . .	0	10	0	0	5	0	5
	10	9·7	0	0·2	5·5	0	5·5
	10	9·8	0	−0·5	5·2	−0·5	5·7
5, 15, 5, 15, . . .	0	10	0	0	0	−5	5
	10	9·7	6	5·7	6	0·5	5·5

circumstances which regulate this effect of losses, however, require further study. As far as can be seen at present there is no way of doing this other than by a number of experiments.

2. The Effect of Trends

A trend is a persistent and fairly regular annual change due to some physical cause. In the data discussed in this book there are a few cases where trends occur: the levels of the Nile on Roda (Cairo) Nilometer, wheat prices in Europe, the lengths of the reigns of Pharaohs and the thickness of some varves. As an example the following figures are quoted from the Cairo Nilometer[17], table 22, p. 166. Means of decades are also given in table 37, p. 221.

Table 14

Cairo Nilometer (Roda Gauge)

Means of maximum levels measured from the foot of the column

Period A.D.	Mean date	Mean level inches
641–850	746	338
851–1040	946	340
1041–1080	1060	346
1081–1250	1166	352
1251–1330	1290	353
1331–1380	1356	362
1381–1522	1451	385
1587–1625	1605	430
1658–1720	1689	435
1721–1840	1780	440
1841–1890	1866	453
1870–1890	1880	450

There are many gaps in the records, and some uncertainties introduced by repairs to the gauge and vagaries of the gauge readers. However, taking it all in all it is a remarkable record. It has been analysed for periodicities by several people and some have been found, but all of small amplitude of the order 10 centimetres, which is insignificant compared with the irregular variations.
The levels of the table are plotted in fig. 17.

The levels of the table are plotted in fig. 17.

From this it is obvious that the trend before A.D. 1300 is much less than the trend since that date. The trend is generally supposed to be due to a general rise of the river due to the deposition of silt. In flood time the bed is in continual motion, so such a small effect as a millimetre or two a year can only be separated from the much larger variations of level due to the variation of discharge, by observations extending over centuries. When the

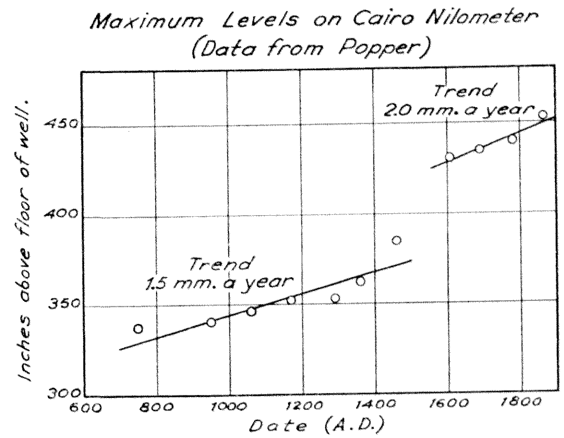


Fig. 17

above discontinuity occurs there is some uncertainty about the readings and the discontinuity may be due to this.

In the computations of R, σ and K for the observations of the Roda Gauge trend has been removed. The observations which were used were taken from Prince Omar Toussoun's book[18] and a single overall trend was used. An examination of the tables of data in the appendix shows that the variations of the means of periods of 50 or 100 years usually appear to be irregular and what can be recognised as definite trends are rare. The following examples illustrate this.

Table 15

Variation of Means

Phenomenon	Date	No. of years	Mean
Milan rainfall	1764–1813	48	95
	1814–1861	48	100
	1862–1911	50	100
	1912–1936	25	96
Sequoia, or Californian Redwood. Thickness of annual rings	1000–1099	100	1·12
	1100–1199	100	0·93
	1200–1299	100	0·86
	1300–1399	100	1·07
	1400–1499	100	0·84
	1500–1599	100	0·85
	1600–1699	100	0·83
	1700–1799	100	0·89
	1800–1899	100	0·91

The other statistics for rainfall, tree-rings and other phenomena give similar results. In the case of varves, however, there is occasional evidence of a steady progression of the mean. For example in the case of Lake Saki the first 700 years give average annual thicknesses per 100 years of 16, 14, 13, 12, 12, 12, 12, 10 tenths of a millimetre. The 1,000-year means are 12, 12, 13, 16.

There is no doubt that the variations of these means are due to climatic or topographic changes, and series such as the first could be said to have a trend, though this trend did not persist throughout the record.

The thickness of varves at Haileybury, Canada, also shows a trend, as appears from the following 50-year means covering 650 years: 23·3, 23·0, 22·0, 22·0, 21·7, 23·2, 20·6, 19·3, 17·5, 16·0, 16·4, 14·9, 15·0. In general, however, it may be said of the phenomena which appear in storage problems that definite trends are not very common, and if they exist are often masked by effects of an irregular nature. It is interesting, however, to examine what are the effects of removing trends, real or hypothetical, from the time series with which we are concerned. The results of some trials are given in table 16.

TABLE 16

Effect of Trends on R and K

	Actual observations					Trend removed			
	N	R	σ	K	Trend	t/σ	R	σ	K
Varves									
Haileybury −350 to A.D. 300	650	99	3·9	0·92	0·14 (per 10 years)	0·036	37	2·57	0·78
10-year means									
Lake Saki 2990–1941 B.C.	1050	133	6·8	0·69	−0·027	0·004	118	6·7	0·66
Outflow from Lake Albert 1904–57	54	92	6·8	0·79	−0·2	0·03	67	63	0·72
Trier Rainfall 1806–1940	117	214	11·5	0·72	0·11	0·01	165	11·3	0·66

As already shown the varves appear to contain steady trends. The trends in Lake Albert and Trier rainfall are not so pronounced and are probably accidental and due to fluctuations of the lake of the same character as those shown by all the phenomena included in our data. However, they serve to illustrate the effect of removing a trend. The case of Lake Albert is shown in fig. 18, where the irregularities are so great as to make it practically certain that the trend is accidental, and it can certainly not be relied upon to repeat itself in the next period. The same is true of tree-rings as may be seen by looking at the 100-year means in table 15.

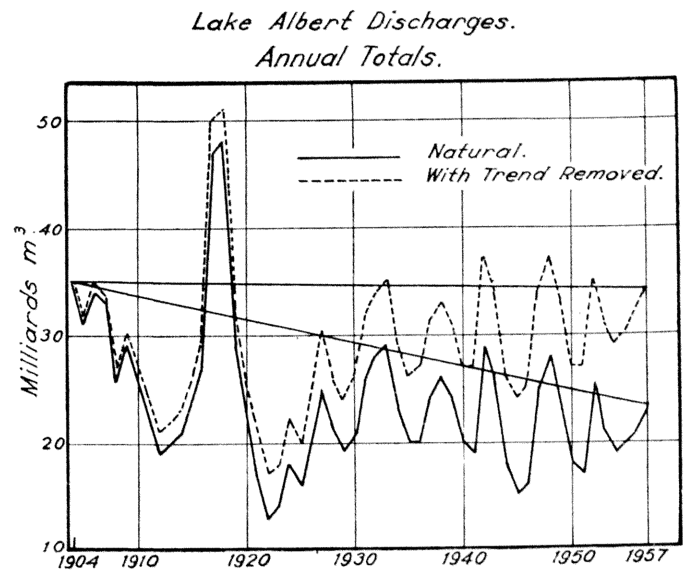


FIG. 18

The table shows that whether trends are due to any permanent physical cause or not their removal reduces R, σ and K.

The general conclusion, however, is that in the case of storage of water no case of a definitely established trend of practical importance has been established. The trend of Roda Gauge, due to a definite physical cause, is of historical interest.

8

The Regulation of Long-term Storage, mainly in relation to the High Aswan Reservoir

1. Introduction

It is necessary to try to devise a scheme of regulation which will reduce the danger, both from flood and from drought, to a minimum. This is a difficult matter, to which a large amount of research has already been devoted, as will have been realised from Nile Basin, Vol. VII, and previous chapters in this book. To produce such a scheme we must depend on past records, not only of the Nile, but of other natural phenomena, to see how far the present and future can be predicted from the past. Failing definite means of prediction possible schemes of regulation must be tried out on existing records and the possibilities arising from probability analysis must be studied.

With regard to prediction a great deal of work has been done on two lines: (*a*) the search for periodicities in Nile phenomena, and (*b*) the attempt to correlate these with world weather. A review of work done on these lines is given in the following sections.

2. The Search for Periodicities

Sir Henry Lyons mentions this in the Physiography of the Nile, 1906. A five-year fluctuation of the rainfall in India and Mauritius had been suggested by Sir Norman Lockyer and his son, who also thought this applied to the Nile floods. After examining the record of floods from 1825 to 1903, Lyons wrote: "The most marked feature is the way in which the flood varies; passing from a value above the normal to one below it in almost successive years. It is this irregularity, this rapid oscillation of the curve, which makes of small practical value any argument based on periodicity shown by the five-year curve." As an example of this oscillation we may quote the floods of 1877, the third lowest flood in 93 years, and 1878, the highest. The difference in height between these was $2\frac{3}{4}$ metres, and the peak discharge of one was nearly twice that of the other.

Since then the long series of flood heights recorded on Roda Gauge (Cairo) has been analysed for periodicities by J. I. Craig, H. H. Turner[19], C. E. P. Brooks[20] and others[21]. The series used by Brooks covered about 800 years, and many periodicities were found ranging from lengths of about three to 77 years, beyond which he did not carry his analysis. The following is a short summary of Brooks' paper. The method used for finding periodicities was Brooks' difference–periodogram[22] and the series used was A.D. 641 to 1451, which is practically complete. The series was divided into sections of lengths about 100 years for the shorter periodicities and about 150 for the longer. These sections were analysed separately for periodicities of the form $y = a \sin (2\pi t/p + \phi)$ where p is the length of the period and a the amplitude. Eighteen periodicities were found varying from two to 78 years. The characteristics of these were: small values of a, of the order 10 centimetres, varying very much from section to section, with values of ϕ also varying. There is a suggestion in some of the periodicities of a cyclic variation of the length of the period which shows itself in the computations as a variation of the phase ϕ, with a period of about 500 years. Another feature is that about two-thirds of the periods discovered are multiples or sub-multiples of 22·1 years, which is twice the sunspot period. These facts are interesting and may perhaps have some physical basis. The periodicity with the largest mean amplitude, about 17 centimetres, and a fairly steady phase is one of 76·8 years. The above amplitudes may be compared with the standard deviation of the gauge readings of the 800 years, which is about 0·6 metre, so that the periodicities, whether real or not, contribute only a small part of the variations of the flood levels.

In order to estimate the value of periodicities for practical use in forecasting, Brooks' periodicities of $5\frac{1}{2}$, 11, 22 and 40 years were chosen for application to the series A.D. 1824 to 1955, 500 years later than the former series. The procedure for any periodicity, for example 11 years, is to divide the series into sections of 11 years and write these in horizontal lines one below the other, so that the columns contain all the first, all the second, etc., years of the sections. The means of columns are found and can then be used for harmonic analysis. The departures of these means from the mean of the whole series are plotted in fig. 19. Harmonic analysis has been applied to the means of columns for 11 years and 22 years, and the results of this are also shown in fig. 19.

It is clear from fig. 19 that the means of columns are very irregular, and cannot be closely represented by single sine curves. This becomes more obvious as the length of period increases. For example, the curve for 40

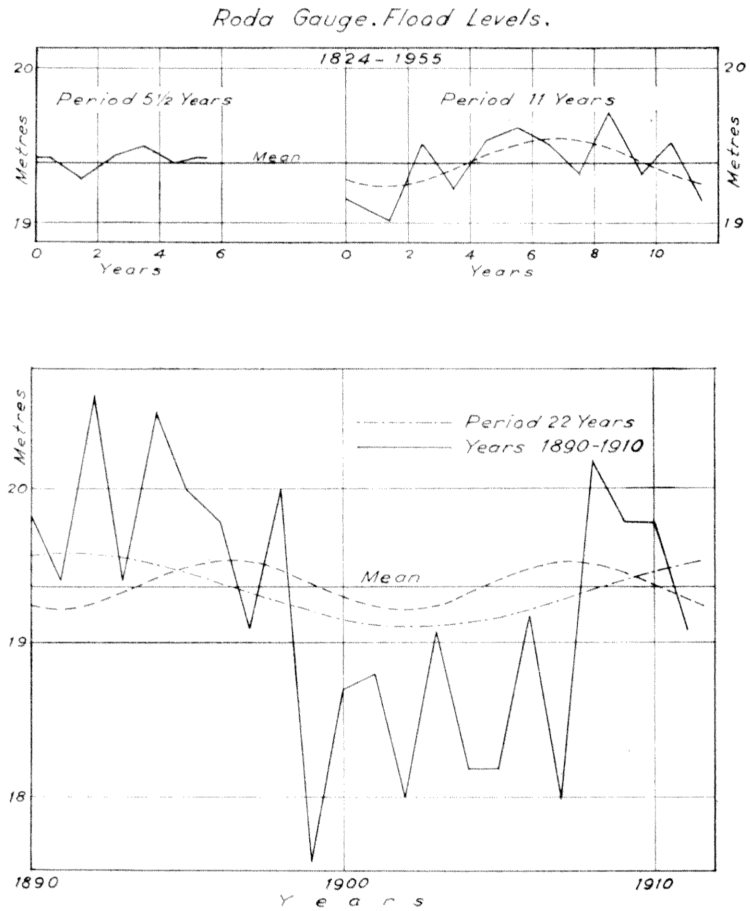


FIG. 19

years could not be distinguished from one which represented a series of independent random events.

The 11-year and 22-year periods derived from 1824 to 1955 are plotted along with the flood levels for 1890 to 1911, which are representative of the variations which occurred in the longer period. This shows that what one may call accidental variations are very much greater than any variations due to periodicity. Apart from the small average amplitudes of all

the periodicities, in most of them the amplitudes found by Brooks for different sections of the record varied from less than half to more than twice their mean values.

This question of periodicity has been examined at some length, because from the time of Joseph up to the present day there has been belief that the behaviour of the Nile goes in cycles and can therefore be predicted. Unfortunately the above analysis shows that there is no periodic effect large enough or regular enough to be of any practical use for forecasting. Studies of some other rivers have led to the same conclusion.

Yevdjevich[(23)] writes: "The complex nature of meteorological phenomena creates difficulties in the determination of fluctuations of hydrological magnitudes over a period of many years. Contemporary technical literature abounds in 'successful cases'—cases in which it was possible to determine certain cyclic fluctuations (usually a large number of cycles)—although there are probably disproportionally more cases of treatments abandoned by their authors because no traces of the regularity of fluctuations could be found. It is nearly always found that the new observations do not confirm the cyclic fluctuations of long intervals of time determined by previous observations.

"The study of the data supplied by the rainfall gauging and water stage observation stations in Yugoslavia has led the author to the initial statement that there are no determined regular cyclic oscillations with a periodicity covering a number of years."

Yevdjevich in collaboration with others also studied the discharge of the Colorado River[(24)]. He concludes that "There is no statistical evidence that the fluctuations of annual flows may be composed of hidden periodicities or of some regular patterns in the fluctuations, which can be extrapolated in the future with a reasonable expectancy that they would occur and would be verified by future flow records."

Luna B. Leopold who also studied the records of the Colorado River[(25)] wrote: "It should be emphasised that a statement of probability is not a forecast. Extensive studies of the variation of hydrologic phenomena clearly indicate that values of any hydrologic factor tend to vary with time, but these variations are not sufficiently regular to be deemed cyclic. For forecasting future hydrologic events there must be repetitive cyclical phenomena; . . . repetitive cycles are, for all practical purposes, absent in hydrologic data. Therefore, the past record should be used as an indication only of the probability that certain events will occur in the future, not as a forecast."

W. B. Langbein in an unpublished paper on variations of annual streamflow based on records of twelve American and two European rivers remarks: "Attempts to discern cycles lead chiefly to subjective results".

3. The Search for Correlations between the Nile Flood and Atmospheric Changes

By the end of the nineteenth century it began to be recognised that the conditions regulating such major phenomena as the Indian Monsoon and the Nile Flood must be looked for in the general circulation of the atmosphere. The manifestations of this are barometric pressures, winds, temperatures and rainfall. Mahmoud Pasha el Falaki in 1882 was the first person to suggest that the Nile Flood might be predicted by a study of temperatures and pressures in Cairo. Later Ventre Pasha thought that the force and direction of the winds at Aden and Zanzibar might be a base for a forecast. Sir Henry Lyons discussing the information existing in 1905 (Physiography of the Nile) thought that the Indian Ocean was the source of the Ethiopian rains which cause the flood. In 1910 J. I. Craig(26) put forward the theory that the flood was caused by a current of air from the South Atlantic crossing Africa and depositing its moisture on the Ethiopian Plateau. Balloon observations by H. G. Bambridge at Lake Tana during the rainy season of 1923 showed upper winds from the south-west. More recently C. E. P. Brooks and S. T. A. Mirrlees have studied the atmospheric circulation over Central Africa(27) with much more data at their disposal. Major Solot of the American Army during the late war also studied the question. The general conclusion from these papers is that the main rain-bearing current to Ethiopia during flood-time is from the South Atlantic. There is also a current from the Indian Ocean which would affect the eastern Lake Plateau and Southern Ethiopia, and perturbations of this might occasionally cause Indian Ocean rain to fall in the Nile Basin.

There is another line of research started by Sir Gilbert Walker and used in Egypt, and this is to look for correlations between the Flood and other phenomena. If significant correlations are found an equation can be made connecting the Flood and the other phenomena. The best result up to the recent war was given by E. W. Bliss(28). The phenomena which are connected are:

1. Nile Flood. Total discharge, Aswan Natural River, August to October.
2. Temperature, Dutch Harbour, Alaska, March to May, preceding.

3. Temperature, Samoa, Pacific Ocean, December to February, preceding.
4. Pressure, Port Darwin, Australia, March to May, preceding.

The coefficients of correlation between 1 and 2, 3 and 4, which were found were

$$\text{Bliss.}\quad r_{12} = -0{\cdot}56;\ r_{13} = -0{\cdot}56;\ r_{14} = -0{\cdot}54;$$

which give a joint coefficient of 0·72.

We have recently recomputed these coefficients with data up to 1950, about 60 years, with the following results:

$$r_{12} = -0{\cdot}29;\ r_{13} = -0{\cdot}48;\ r_{14} = -0{\cdot}49;$$

which give a coefficient of 0·46 between the observed and predicted values, and the probable error of a prediction as 16 per cent. This means that supposing the differences between predicted and actual were due to one single cause, the coefficient of correlation between this cause and the Flood would need to be 0·89.

The prediction may be looked at in another way. Suppose we take the difference actual–predicted and compute its standard deviation, which comes out to be 7·5. The standard deviation of the Nile Flood itself is 10·7, so the prediction has reduced the variation only by one-fifth. Moreover it predicts the very low flood of 1913 as 53 mlrds. whereas the actual was 26, and in five out of eight low floods the prediction was a failure being 15 mlrds. or more too high. Similarly in the three highest floods the prediction averaged 11 mlrds. too low. Recently A. N. M. Robertson(30) has referred to the work of Colonel Rawson and J. I. Craig and thinks that the Nile Flood has a cyclical variation of period of about 19 years, corresponding to the movements of the anti-cyclonic belt of the Southern Hemisphere. The remarks on periodicity apply to this. Craig found a correlation between pressures at St. Helena and the Nile Flood. Pressure at St. Helena is presumably influenced by the movements of the high pressure in the South Atlantic. This correlation has recently been computed for the years 1892 to 1958; the value of the coefficient is $-0{\cdot}19 \pm 0{\cdot}08$, and is not significant.

The general experience of relations between meteorological phenomena all over the world is that usually coefficients of correlation are not very high, and that a relation which has persisted for a number of years may die out altogether, or even be reversed. To sum up the matter, in the cases here discussed there is nothing which is of any use for prediction.

4. Use of Probability Methods

In the cases of many rivers, under certain circumstances, forecasting flow is possible for a short interval in advance. For example, in the case of the Nile, after the rains are over and the fall of the river is well established the discharge for several months in advance can be forecast from the discharge at the time, owing to the regular nature of the fall. Analysis of rainfall data over a catchment may lead to the possibility of short-term forecasts. However, such methods are not able to provide long-term forecasts, and as far as is known there are at present no methods of making such forecasts. The only possibility, therefore, is the analysis of masses of data and the application of methods based on the theory of probability to the results of the analysis.

Allen Hazen(6) seems to have been one of the first to adopt this method, making use of records from 14 American rivers which he combined to make a series covering 300 years. Langbein, whose work has already been mentioned, in an unpublished paper "Variations of Annual Streamflow and their Effects on Water Supply and Storage", which he very kindly lent, discussed annual totals of precipitation for 16 groups of stations covering the U.S.A. and also the discharges of 12 rivers. L. B. Leopold(25) has discussed the discharge of the Colorado River and obtained "confidence limits" giving the probability that the future mean will be between them.

A comprehensive study of the flow of the Colorado River has been carried out by the Bureau of Economic Research, University of Colorado(24). It includes hydrometeorological studies, construction of synthetic records, and studies of precipitation. The 47-year record of discharge deviates from randomness ($K = 0{\cdot}81$). The investigation is confined to the Colorado River but does not discuss the utilisation of its water. Another American study(31) is the 1952–55 Illinois Drought with special reference to Impounding Reservoir Design, which has a bibliography of references and a list of bulletins. It has some information on evaporation losses, and loss of capacity due to deposit of sediment.

The application of probability theory to storage problems has usually been based on the assumption that the quantities dealt with are independent of each other and have a Gaussian distribution. The work so far described in this book depends on the examination of records of many natural phenomena and leads to a new distribution. The same method has been continued in the attempt to find means of using storage suitable for practical use. Experimental regulations have been made on a large number

of phenomena and the results of these are described in the following pages. An account of some of this work was given in Methods of using Long-term Storage in Reservoirs(11) and use is made of this in the following section.

5. First Trial Regulations

In general, water requirements will vary from month to month and consequently also the draft from the reservoir; the same will also happen with storage of other substances. The following investigation considers only over-year storage, and seasonal storage must be considered separately. For example, in practice a maximum capacity may be fixed for over-year storage, and a sufficient additional capacity be provided for seasonal working. In the case of the High Aswan Reservoir the storage above a certain level will be used, when necessary, for containing the excess water of flood time for use in the dry season, and also for retaining water from the peak of a dangerous flood, to be escaped later when the discharge has fallen to a safe amount.

There are several conditions which affect regulations and their results in practice and of these the following are important.

1. The capacity is large enough to enable a steady draft equal to the average discharge to be given. There are two cases: (*a*) when losses are negligible and (*b*) when losses which will reduce the draft must be allowed for.
2. The available capacity (S) is less than that required to maintain a draft equal to the average, which involves spilling of water when the reservoir is full. There are again two cases: (*a*) losses negligible and (*b*) losses important.

A reservoir in Lake Victoria is a case of 1a,* on account of its large capacity and the fact that evaporation and rainfall on the lake surface are approximately equal. It seems likely that such cases are few, and in general with reservoirs on large rivers it may not be possible to find enough capacity to control the whole discharge.

Cases of 1a were examined in paper 11 presented to the Institution of Civil Engineers and were directed towards understanding the problem, rather than to providing methods of regulation of general use.

The first trials were made on a series of observations of 32 phenomena from those given in the appendices using the equation $R/\sigma = (N/2)^{0{\cdot}72}$ and

* This may no longer be true, as the recent very high levels have increased the recorded range of the lake by about 60%.

taking N as 100 which gives $R_{100} = 16{\cdot}7\sigma$. The values of the mean M and the standard deviation σ are those for the whole recorded period. This means that with an unlimited storage capacity a steady draft equal to the mean M would on the average make use of a capacity $16{\cdot}7\sigma$ over 100 years. All the following trials were made with this capacity.

The results were:

With a reservoir of R_{100} capacity starting half-full the reservoir filled in 66 per cent of the cases, and emptied in 59 per cent.
Starting full the reservoir emptied in 41 per cent of cases.

Such results were to be expected, since K is variable and the value used is a mean. In the above experiments regulation takes place after the events, and M and σ are known. The case is different when past values of M and σ have to be applied to the future. The following are possible modes of regulation:

(*a*) The draft is kept constant.
(*b*) The draft varies with the inflow.
(*c*) The draft varies with the amount in the reservoir (content).
(*d*) The draft varies with inflow and reservoir content.

Some results of trying regulations of types (*a*) and (*b*) are given in table 20, which is based on 52 trials on a series of observations taken from 42 phenomena. The procedure was to suppose that data from the first 30 years of a series were known, from which a mean and standard deviation were computed, which were used in regulating the remainder of the series. In accordance with section 5 of chapter 3 the standard deviation used in the regulation was 1·1 times the standard deviation of the first 30 years. The capacity of the reservoir, R_{100}, was taken as $16{\cdot}7\sigma$ corresponding to $K = 0{\cdot}72$ and $N = 100$ years.

TABLE 17

Results of Regulations based on Initial Data from 30 Years

Regulation	Starting content	Draft	Percentage of cases Reservoir fills	empties
1. Type a	$0{\cdot}5\ R_{100}$	M_{30}	44	38
2. Type b	$0{\cdot}5\ R_{100}$	M_{10} changing every five years	23	19
3. Type b	$0{\cdot}75\ R_{100}$	M_{10} changing every five years	56	5
4. Type b	R_{100}	M_{10} changing every five years	—	2
5. Type b	$0{\cdot}5\ R_{100}$	M_{10} changing every year	12	15

Subscripts show number of years concerned.

The table shows that in the first case, starting half-full, it is not economical to try to give a draft equal to the 30-year mean. At this point a promising line seemed to be a draft depending on the supply already received of which the last four cases are the results. The best of these is No. 4 where in only one case out of 52 did the reservoir empty. A disadvantage of this procedure, however, appeared later in the practical problem of trying to devise a suitable regulation for the High Aswan Reservoir. This was the large variation of draft which can take place, of which examples will be given later. The table shows the advantage of starting with the reservoir full when a large content like R_{100} is available.* If, however, a reservoir is to be used for flood protection as well as water supply, a start from half-full may be desirable.

From the regulations summarised in the preceding table the most difficult were chosen for further experiment in which reductions of the draft were tried. Table 18 gives the reductions which are required to prevent the reservoirs from emptying.

A regulation (6) of type (*d*) draft varying both with inflow and with reservoir content was tried in the six worst cases of the preceding table. The regulation started with the reservoir half-full, and the draft was M_{10}

TABLE 18

Difficult Cases. Reduction of Draft required to prevent Emptying

Phenomenon	Reduction of draft to prevent emptying: Regulation 2	Regulation 4	Regulation 5
Albany rainfall	$0{\cdot}06\ \sigma'$	NE	$0{\cdot}03\ \sigma'$
Aswan discharge	$0{\cdot}43\ \sigma'$	NE	$0{\cdot}19\ \sigma'$
Berlin temperature	$0{\cdot}18\ \sigma'$	NE	$0{\cdot}03\ \sigma'$
Dalalven lake levels	$0{\cdot}05\ \sigma'$	NE	NE
Lake Huron outflow	$1{\cdot}4\ \sigma'$	$0{\cdot}62\ \sigma'$	$0{\cdot}9\ \sigma'$
Roda Gauge A.D. 641–740	$0{\cdot}31\ \sigma'$	NE	$0{\cdot}14\ \sigma'$
Roda Gauge A.D. 1141–1245	$0{\cdot}03\ \sigma'$	NE	Very small
Rome rainfall	$0{\cdot}04\ \sigma'$	NE	NE
Stockholm rainfall	$0{\cdot}07\ \sigma'$	NE	NE
Zwanenberg rainfall	$0{\cdot}02\ \sigma'$	NE	$0{\cdot}007\ \sigma'$
Means	$0{\cdot}25\ \sigma'$		$0{\cdot}13\ \sigma'$

NE means that the reservoir did not empty.

Regulation 5 is the better of the two which start with the reservoir half-full and a reduction of draft of $0{\cdot}1\ \sigma'$ would have prevented the reservoir emptying in all but 6 per cent of the cases.

* The advantage is less when the available capacity is less than R, so that the standard draft is less than the mean. Then, starting with the reservoir only partly full, it will at some time fill.

changing every year as in regulation 5, and further altered by a sliding scale. The scale was made by dividing the content of the reservoir into nine parts. If the content is in the middle ninth the discharge is M_{10}, changing every year as in regulation 5, if in the ninth below $M_{10} - a$, and so on down to $M_{10} - 4a$, where a is the step of the sliding scale. In table 19 the values of the steps of the scales which would prevent the reservoir from emptying with regulation 5 have been calculated for six of the cases in table 18. In the other cases a scale with a very small step would have prevented emptying.

TABLE 19

A Sliding Scale applied to Regulation 5

Phenomenon	Step	Phenomenon	Step
Albany rainfall	$0{\cdot}001\ \sigma'$	Lake Huron outflow	$0{\cdot}5\ \sigma'$
Aswan discharge	$0{\cdot}06\ \sigma'$	Roda Gauge A.D. 641–740	$0{\cdot}4\ \sigma'$
Berlin temperature	$0{\cdot}01\ \sigma'$	Roda Gauge A.D. 841–1245	$0{\cdot}04\ \sigma'$

As a measure of safety against floods the scale could be applied to contents in the upper four-ninths of the reservoir to increase the draft. The causes of difficulty in regulation are important and table 20 gives the result of an analysis of the previous cases.

It will be seen that in all the seven cases of the table there is a decrease of the mean after the regulation starts. In two cases there is an increase of the standard deviation, and in five cases an increase of K. In all these cases the supply is diminished, and the changes of standard deviation and K mean that a larger capacity is needed to smooth out the variations of the supply.

6. Practical Conditions governing the Size of Reservoirs

In the examples given of regulations the capacity of the reservoir is taken as $16{\cdot}7 \times 1{\cdot}1 \times \sigma$. This, however, is a theoretical case taking no account of the practical conditions which will determine what size of reservoir can be constructed. Such conditions may be:

1. Topography. Shape of the basin and volume available for storage; existence of a suitable site for a dam.
2. In arid climates where evaporation exceeds rainfall, in some cases a limit will be reached beyond which it is not economical to increase the depth of water in the reservoir, owing to the rapid increase of area of the water surface and consequent increase of evaporation loss. This occurs in the case of the High Aswan Reservoir.

TABLE 20

Data relating to Difficult Cases of Regulation

	Date	Mean M	Standard deviation σ	K	Causes of difficulty
Lake Huron outflow.	1860–1889	211	8·7	0·83	Decrease of M
10^3 cubic feet per second	1890–1948	177	12·6	0·87	Increase of σ
	1860–1948	188	20·0	0·94	Large K
Nile. Aswan annual	1870–1899	109	14·5	0·65	Decrease of M
discharge.	1900–1952	83	12·6	0·53	Increase of σ
10^9 cubic metres	1870–1952	93	18·4	0·90	Increase of K
Nile. Roda Gauge.	641–670	9·1	0·46	0·60	Decrease of M
Metres	671–696	8·6	0·76	0·77	Increase of σ
	641–740	9·0	0·72	0·68	and K
Berlin temperature.	1769–1798	9·1	0·87	0·61	Decrease of M
0°C.	1799–1817	8·2	0·90	0·71	
	1769–1939	9·0	0·88	0·74	
Stockholm rainfall.	1785–1814	55·4	12·4	0·73	Decrease of M
Centimetres	1815–1874	39·1	8·2	0·61	
	1785–1946	49	12·3	0·91	Increase of K
Albany rainfall.	1826–1855	40·7	5·2	0·52	Decrease of M
Inches	1891–1945	33·5	4·4	0·67	and increase
	1826–1945	37	6·0	0·85	of K
Rome rainfall.	1783–1810	82·5	14·7	0·49	Decrease of M
Centimetres	1826–1852	69·9	15·2	0·59	Increase of K
	1783–1945	83	16·8	0·72	

3. Existing interests in the proposed reservoir area, e.g. towns, buildings or valuable agricultural land. This affects the height of the Lake Victoria Reservoir.
4. In some cases the existence of established rights to water may put limits to the size of reservoirs which can be constructed. An example of this is the case of Nebraska *v.* Wyoming and the State of Colorado, which was decided in the Supreme Court of the United States of America. Nebraska complained that Wyoming and Colorado, in which are the head-waters of the North Platte River, were taking water which by priority of use belonged to Nebraska. The Court by decree established limits on the acreage which could be irrigated from the North Platte River and the water which could be stored in the various reservoirs on the river.
5. Financial considerations may determine the size of the works which can be built, and consequently the storage which can be provided.

Having regard to these circumstances it seems likely that except in the case of small streams, it will not usually be possible to construct enough storage to make full use of the discharge. In fact, ignoring these limiting factors, the preceding examples of regulations based on capacities of R_{100} show that even with this capacity it is often not possible to maintain a steady discharge equal to the mean over a long period. In other words, full use of the available water is often impossible because we do not know the future values of the parameters M and σ nor the order in which events will occur. Another point which affects a programme for regulation is the existence in many cases of losses. The effect of these was discussed in chapter 8. Following this a section on trends has been inserted, as these are somewhat similar in form to losses, though they have no bearing on methods and results of regulation of storage.

Case 2b of section 5, where the capacity is smaller than R_{100} and losses are considerable, is probably of the greatest practical importance, and for this reason the example of the High Aswan Reservoir is examined at some length in the next chapter.

9

The Application of a Monte Carlo Method to the Regulation of the High Aswan Reservoir

1. The Use of the Card Model

The possibility of this has already been mentioned in chapter 7, and the method was indicated. It was shown that the system of cutting probability cards therein described produced series indistinguishable statistically from natural time series, since they are both fitted by the same equation.

$$R/\sigma = (N/2)^K \quad \ldots 8$$

and K has the same mean value and standard deviation for the large collections of natural phenomena as for the many experiments with cards.

By comparing means and standard deviations of the cards and of the river discharges a correspondence can be established, so that card numbers can be converted into river discharges.

Let $c_1, c_2, \ldots, c_n$ be a series of numbers obtained from n cuttings of cards whose mean is $\bar{c}$ and $q_1, q_2, \ldots, q_n$ the corresponding series of river discharges.

From equation 8 $R_q/\sigma_q = R_c/\sigma_c$ and R_q and R_c are sums of sets of corresponding departures from their means. Therefore for corresponding departures

$$\frac{(q - \bar{q})}{\sigma_q} = \frac{(c - \bar{c})}{\sigma_c} \quad \ldots 9$$

and the values found in an experiment with cards can be converted into corresponding Aswan values by multiplying each by σ_q/σ_c. Each side of equation 9 is dimensionless, and it applies to any two phenomena whose distribution is approximately Gaussian or is represented by equation 8.

Consequently any series of annual values of a natural phenomenon may be transformed to produce a possible series of values of another phenomenon. We might have done this for Aswan, but each phenomenon would have needed its own transformation table, which would have increased the computation considerably.

The Aswan discharge has been measured directly from 1902 onwards. Previous to this from 1869 the discharges may be inferred from gauge readings at Aswan. These are less accurate than the measured discharges, but are of sufficient accuracy for our purpose. The discharges in the high years 1874 and 1878 are confirmed by measurements made during the high floods of 1946 and 1954, which reached approximately the same levels on Aswan gauge. Moreover the high floods previous to 1900 cannot be disputed, as there are records on Roda, Delta Barrage and other gauges, which correlate well with Aswan. Many other records of the damage which was done owing to breaches in the river bank confirm the height of these floods.

If we examine the flood levels on Roda Gauge (Cairo) given in chapter 8, section 2, and apply the value of the trend, we find that the average flood levels from 1587 to 1890 taken in periods of roughly 90 years are only a few inches lower than the mean level for 1870 to 1890. Data concerning Roda Gauge, taken from the work of Prince Omar Toussoun[18], are given in appendix 5. These were corrected for a general trend used before the investigations of chapter 8 were made. This correction makes only insignificant differences over short periods like 100 years, and the following are mean and extreme values for 100-year periods, including one of 80 years.

		Minimum	Maximum	Mean
Standard deviations	metres	0·38	0·86	0·58
R	metres	5	18	11
K		0·65	0·86	0·74

1867 to 1946

Standard deviation 0·68 metre R 14 metres K 0·75

These tables show that K is close to the average, and the value of R has occurred three times in eleven sets of 100 years. The probable departure of R is 2·4 metres and its departure from the mean is 3 metres, so that its value is normal.

The standard deviation, however, is the largest in the set and this is due partly to the occurrence of an unusual number of high floods, including what appear to be the two highest recorded even when trend is removed, and also the lowest recorded.

On the whole therefore as regards storage problems the long series of Nile flood levels shows that the period 1870 to 1957 is a fair sample. Consequently it is clear that we must take all the discharges from 1870 onwards to the present time as our basis on which to transform the card results to Aswan discharges.

From these we have, using hydrological years, August to July:

1870–71 to 1957–58. Mean 93·0 mlrds., standard deviation 17·5 mlrds.

The discussion in chapter 3, section 5, shows the probable variability of the standard deviation with length of record. To counteract this and to give a factor of safety in the following calculations the standard deviation of the Aswan discharge has been increased by 10 per cent in finding the factor of transformation. Having regard to the information from the long record of flood levels, the factor of safety seems to be ample.

For the card experiments we use 2,000 cuttings from a pack of 62 cards (experiments 5 and 6), for which the factor of transformation was 4·6, and a table for transformation was made. The 2,000 cuttings were taken in sets which gave a sample of 20 sets of 100 annual Aswan discharges. These sets are not series which will happen, but they are series which might happen. That is to say they are not predictions, but possibilities on which trial regulations can be made. These trials will show what is the most suitable regulation, having regard to the conditions which will control the use of the reservoir.

Also, from regulating 20 different sets of possible Aswan discharges, a good idea will be obtained of the possible range of conditions and the probable value of the scheme of regulation.

2. The Aswan Discharges obtained by Transformation

The following table (table 21) gives the principal characteristics of the 20 sets of 100 annual Aswan discharges obtained by the transformation just described.

In the table R_{100} is the storage which would have been required to give a steady discharge equal to the mean of the set. The main features shown in the table are:

Range of mean of a set	80 to 112 mlrds.
Range of standard deviation	13·0 to 23·5
Range of R_{100}	120 to 877
Range of K	0·49 to 0·93

Commenting on these, we may say that the range of K is a little less than the range in the large number of natural phenomena which have been examined, which is from 0·46 to 0·96 with a mean of 0·726. This again confirms that the results of the card experiments are the same statistically as those from natural phenomena. Some remarks on card experiments 1 to 4 are made in appendix 1.

TABLE 21

Characteristics of derived Aswan Discharges
(Milliards of cubic metres a year)

Serial No. of set	Mean discharge M	Standard deviation σ	K	R_{100}
13	80	15·2	0·76	291
15	82	19·8	0·58	194
17	82	21·2	0·62	240
20	83	18·5	0·67	149
24	85	20·0	0·80	462
10	87	19·6	0·86	577
28	89	20·0	0·85	554
29	90	23·5	0·93	877
16	90	21·9	0·60	226
14	91	20·0	0·49	134
21	91	16·8	0·72	182
26	93	19·8	0·83	512
12	94	20·4	0·74	370
19	95	18·9	0·66	231
27	97	22·8	0·84	614
18	98	19·9	0·63	231
25	101	20·5	0·80	475
22	102	20·2	0·78	425
23	105	14·2	0·79	314
11	112	13·0	0·57	120
Aswan actual discharge				
30	93	19·3	0·86	512
Means of transformations				
	92·4	19·3	0·72	359

Another point to be noted is the relatively small amount of long-term storage available in the High Aswan Reservoir, about 90 mlrds., compared with the average amount of 360 mlrds. in the table required to allow a draft equal to the mean discharge. This average agrees with the figure of 330 mlrds. calculated from equation 8, and given by Hurst and Black in their report on the Hydrology of the Sadd el Aali[32].

The conclusion is that the Aswan discharges transformed from the card experiments are possible cases which might occur. Also that with their wide ranges of mean, standard deviation and K, they can therefore be used with confidence as material for trial regulations of the High Aswan Dam. In chapter 6, section 2, data from experiments 1 to 4 are also given. Further information about these and a comparison with experiments 5 and 6 which were used in the trials are given in appendix 1.

The results of these trials are given in the following section.

3. Trial Regulations

The following are important data relating to the High Aswan Reservoir.

Maximum water level (Egyptian Survey Department datum; mean sea level at Alexandria) 182 metres. Total capacity 157 mlrd. cubic metres. Of this the lowest 30 mlrds. are reserved to contain the silt which will be deposited, and this capacity is considered to be large enough to contain the deposit of several hundred years. The turbines have also been designed

with this as a normal minimum working content. The top 37 mlrds. will be reserved to hold back water so as to reduce the peaks of dangerously high floods, and also for annual working, i.e. storing flood water for use in low stage, when these may be necessary.

Losses have been taken as 10 per cent of the content of the reservoir at the end of the year and this amount is deducted in finding the content at the end of the following year. This is approximate and is based on available information rounded off to simplify computation.

The draft is the amount available, as at Aswan, for division between Egypt and the Sudan. The share of the latter would be taken in the Sudan and would not enter the reservoir, while Egypt's share would pass through the dam. The draft does not take account of extra water or storage provided by the Southern projects discussed in the Nile Basin, Vol. VII, or the demands of other countries in the Basin.

The first lot of regulations was made using the mean of each set as a basis for the draft to be used on that set. This type was carried out to get information about the sets, so as to devise some practicable scheme.

It has already been said that the long-term storage capacity (S) of the High Aswan Reservoir is much less than R, as found from the 88-year record. Using this and the information in chapter 5 we find that the available storage corresponds to a minimum draft less than the mean discharge by about 13 mlrds. This is an average quantity and has been subtracted from the means of the sets to give the trial drafts which have been used in the first lot of regulations.

On considering this first lot of regulations it was possible to decide on a trial draft to be used throughout a second lot of regulations to include all the sets. The draft chosen was 72 mlrds., which seemed a reasonable beginning as there were four drafts less than 72 and 16 greater, and more difficulties had arisen with the low drafts. The draft was kept at 72 unless the content at the end of the previous year was less than 41 mlrds., in which case the draft was reduced by 20 per cent. In some cases it would be clear by the end of September that the flood was a high one and consequently that the reduction need not be continued. Similarly if the flood was very low a reduction of draft of 20 per cent would also be made, unless the reservoir was nearly full. If a low flood and low content occurred together, which was very rare, the reduction of draft would be 30 per cent. Similar regulations were made for a draft of 74 mlrds. The main information required from the second lot of regulations was the number of times reductions of draft were necessary, and also the number of times the computed content fell below 30 mlrds., the lower limit of the usable content.

As the mean draft plus loss is less than the mean discharge in each of the 20 sets there were always years in which the reservoir was full and water had to be spilt and passed to sea.

Table 22 shows the results of this second lot of regulations.

The table is arranged in order of the mean discharges of the sets, and the upper part contains those sets with discharges less than the mean. It will be seen that this part contains nearly all the cases where it was necessary to reduce the draft. The reductions included are those of 20 per cent or more, since reductions of 5 or 10 per cent of the water would have very little effect on the crops, and might even be beneficial. One-quarter of the reductions occur in set 29, although the natural supply is not much below the mean. The reasons for this can be deduced from the

TABLE 22

Derived Aswan Discharges. Second Lot of Trial Regulations
(Milliards of cubic metres)

No. of set	Mean discharge Q	K	Mean drafts 72	Mean drafts 74	Mean loss 72	No. of reductions 72	No. of reductions 74	No. of years. Contents less than 30: 72	No. of years. Contents less than 30: 74	Spill 72
			Basic drafts							
13	80	0·76	70·8	72·4	7·6	7	11	4	4	90
15	82	0·58	71·6	73·3	8·8	3	5	0	0	140
17	82	0·62	71·3	72·6	8·7	5	9	0	2	200
20	83	0·67	71·6	73·1	9·0	3	6	0	0	190
24	85	0·80	70·8	72·3	9·6	7	11	0	2	450
10	87	0·86	71·3	73·1	9·5	4	6	1	2	630
28	89	0·85	71·6	73·1	8·9	3	6	0	2	670
29	90	0·93	70·5	71·1	9·2	11	19	0	2	1,030
16	90	0·60	72·0	74·0	10·8	0	0	0	0	690
14	91	0·49	71·7	73·8	12·0	1	1	0	0	730
21	91	0·72	71·9	73·8	11·6	1	1	0	0	670
Means or totals		0·72	71·4	73·0	9·6	45	75	5	14	500
26	93	0·83				0	2	0	0	
12	94	0·74				0	0	0	0	
19	95	0·66				0	0	0	0	
27	97	0·84				1	3	0	1	
18	98	0·63				0	0	0	0	
25	101	0·80				0	0	0	0	
22	102	0·78				0	0	0	0	
23	105	0·79				0	0	0	0	
11	112	0·57				0	0	0	0	
Means or totals		0·74	72·0	73·9		1	5	0	1	
Trial means or totals omitting set 29	93	0·73	71·7	73·4		35	61	5	13	

preceding table 21 where it appears that this set has the greatest values of standard deviation, K and R of all the sets. Among 60 sets of card experiments no value of K was as high, and there were only five in 800 cases among natural phenomena. It may therefore be considered a very rare phenomenon. If we omit this the remaining 19 sets are the equivalent of 1,900 years of discharges.

If set 29 is excluded for the reasons already given, practically all the reductions of discharge and deficient contents occur with mean discharges below 90 mlrds., and losses tend to increase as the mean discharge increases. These features can be inferred from the equation of continuity relating supply, draft, content and losses.

On the average a reduction of 20 per cent or more would occur once in 54 years with a basic draft of 72 mlrds. and once in 31 years with one of 74 mlrds. It may be noted that these correspond to Allen Hazen's 98 per cent and 97 per cent dry years. Contents below 30 mlrds. occur once in 380 years and once in 146 years respectively. The significance of these will be discussed later; at present we are only making experiments. It will be noticed that the average value of K is about the same in both parts of the table, and K has very little relation to the number of reductions of draft or deficiencies of content. The effect of using a sliding scale relating draft and reservoir content has been tried in the worst cases which occur in table 22, and these are shown in table 23, in view of the preceding results, for basic drafts of 74 mlrds. and 76 mlrds. It was obviously not necessary to apply the scale to a draft of 72 mlrds., though its application to a draft of 76 might be useful.

The difference between the systems of regulation whose results are summarised in the preceding two tables can be seen by a comparison, which follows, of the figures for the worst cases.

The mean intervals are the averages (approximate) corresponding to 1,900 years.

The following are some general remarks on tables 22 to 24. The first lot of regulations were computed completely but afterwards it was not necessary to do this for every regulation. For example, in cases where the mean discharge of a set was much greater than (draft plus losses) the reservoir was often at its maximum level and spilling took place. In these cases it was only necessary to compute the regulations for the successions of low years. The mean drafts apply only to the six sets of 100 years which table 26 summarises, but as nearly all the important reductions take place in these years, approximate mean drafts for the 1,900 years can be calculated.

TABLE 23

Comparison of Regulations with Sliding Scale. Worst Cases. Derived Aswan Discharges
(Milliards of cubic metres)

Set number	13	17	24	10	28	27	Means
Mean discharge	79·8	82·2	85·4	86·8	88·9	96·8	86·6
		Basic draft 74					
Mean draft	70·8	70·9	71·6	72·0	72·6	73·0	71·8
Mean loss	7·9	9·3	9·5	9·3	10·0	10·7	9·4
Spill total	5·1	112	373	571	579	1,271	
No. of years	7	14	20	21	24	49	
No. of reductions of draft							Totals
Reduction 20 per cent	7	3	3	4	2	2	21
Reduction 25 per cent	7	12	8	5	5	2	39
Reduction total	14	15	11	9	7	4	60
No. of contents less than 30	2	0	1	1	0	0	4
		Basic draft 76					
							Means
Mean draft	71·8	72·2	73·0	73·2	74·1	74·5	73·1
Mean loss	7·4	8·7	9·0	8·6	9·2	10·5	8·9
Spill total	34	110	292	517	503	1,139	
No. of years	3	9	16	20	24	46	
No. of reductions of draft							Totals
Reduction 20 per cent	12	7	8	8	5	3	43
Reduction 25 per cent	7	12	8	5	5	2	39
Reduction total	19	19	16	13	10	5	82
No. of contents less than 30	2	3	2	1	0	0	8

Sliding Scale

Content	Per cent reduction	Draft 74	Draft 76
	0	74	76
60–56	5	70	72
55–51	10	67	68
Below 51	20	59	61
Very low year Q less than 59	25	56	57

With regard to the sliding scale it may appear strange that a reduction of 25 per cent in draft reduces 74 mlrds. to 56, while what is described as the class of low years, to which the reduction applies, begins at 58 mlrds. It must be remembered, however, that a natural river, discharge of 58 mlrds., is reduced by losses varying from 13 to 3 with a mean of about 10 mlrds.

TABLE 24

Summary of Results of Regulations. Approximations for 19 Sets

No.				
	Basic draft	74 mlrds.	74 mlrds.	76 mlrds.
	Regulations	Table 22	Table 23	Table 23
			Sliding scale	
	Mean draft	73·6	73·4	75·0
	No. of reductions	46	60	82
	Mean interval between reductions, years	41	32	23
	No. of contents less than 30 mlrds.	13	4	8
	Mean interval, years	146	380	230

4. DISCUSSION OF THE RESULTS OF THE TRIAL REGULATIONS

To illustrate the discussion fig. 20 has been drawn. It shows the two regulations of set 28 with a draft of 74 mlrds. summarised in the preceding tables. Its mean discharge is 89 mlrds. and this is about one-third of the way up the range of discharges.

The regulation of table 22 is shown in continuous line and that with the sliding scale in table 23 is shown in broken line.

In the first 25 years of this set six reductions of draft were necessary, owing to the occurrence of as many as five low years, which would be

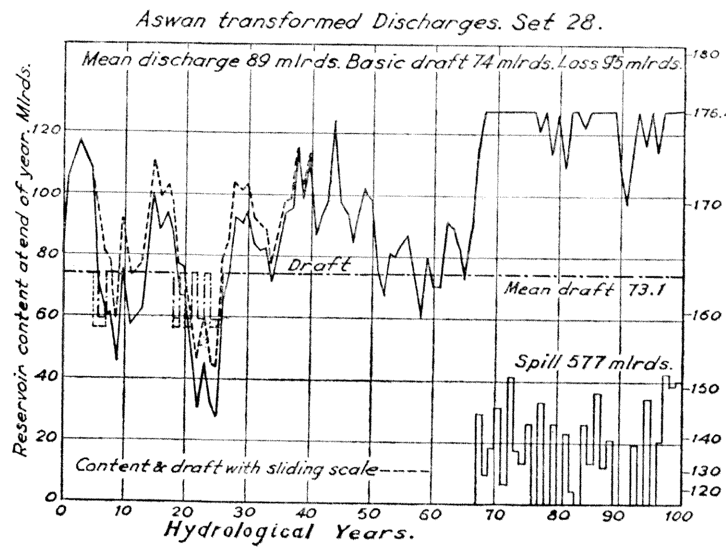


FIG. 20

unusual. The content would have fallen twice to slightly below 30 mlrds. In the last 35 years the reservoir would have been full for most of the time and it would have been necessary to escape water to sea. During the middle period the level would have been in the upper part of the reservoir, and there would have been no reduction of draft and no spilling. The effect of the sliding scale on draft and content is also shown. By this, reduction of content below 30 mlrds., which occurred twice in the regulation of table 22, is avoided.

A few words may be written about "spill", though this has not been investigated in detail in order to lessen the amount of computation. Some information on this is given in table 22.

The water included under spill is the amount which is in excess of normal requirements, but this is not necessarily all escaped to sea. Experience has shown that crops have a range of tolerance over which variations of 10 per cent or so in the water supply produce only insignificant changes in the yields. It is also believed in Egypt that if crops are grown on a minimum supply of water for several successive years the land and future crops will benefit from a copious watering. Consequently it will always be the case that some of the excess water not retained in the reservoir will be used on the land.

The present practice in Egypt is that during the time when the natural river supply is below requirements and stored water is used, some of the land lies fallow. The amount of this varies according to the supply, since there is a variable area which is planted with rice. Permits for this are given depending on successive forecasts of the low stage supply. The area which it is possible to cultivate under rice depends on the water and labour available and the time left for planting. At a maximum it may be 1,250,000 acres. When the High Aswan Reservoir is working this amount will be cultivated in every year except those where the draft has been reduced.

The question arises—Can anything be done to use the surplus which in many years will go to sea? As far as Egypt is concerned some thought has been devoted to this, but no solution has been found. The difficulty will be apparent from fig. 20. In this case, with a basic draft of 74 mlrds., there were 67 years with no available excess, followed by 33 years in 24 of which spilling would have taken place, and nine of these years were consecutive.

The mean discharge of the first period was 81 mlrds., and of the second 104 mlrds. a year. The variability of means has already been discussed, and is now a well-known property of natural time series. This, coupled with the fact that in general there is no known method of predicting the mean of a single year, or of a series of years, leads to the deduction that a mean is never known until it has become past history. Statements about what will be done if the mean changes are therefore illusory, and this should be emphasised, since ancient beliefs die slowly.

The procedure already investigated of making the draft depend on the previous 10-year mean supply, although it produces a more complete use of the water, has the disadvantage that the draft varies considerably from period to period.

The High Aswan Dam has four functions: (*a*) it will store a large quantity of flood water for use in the following low stage, (*b*) it will give protection against dangerous floods, (*c*) it will store some of the excess water in high years to increase the supply of low ones, and (*d*) it will produce a large amount of power for use in Egypt.

Functions (*a*) and (*b*) have been practised to some extent with the existing dam and will be completely carried out by the High Dam. Functions (*c*) and (*d*) are interrelated and in their details are sometimes opposed to each other. In these last two functions the High Dam acts as a partial insurance against the effects of low years.

The present position is that Egypt and the Sudan make use of 52 mlrds. out of an average discharge of 93 mlrds. Referring to table 26 it is evident that more than 20 mlrds. of additional water can be used with the probability that losses from low years will not happen very often. Considering a basic draft of 76 mlrds. modified by the given sliding scale (third column), a reduction of 22 per cent to 25 per cent would occur on the average of a very long period once in 23 years. An occurrence at this rate of four reductions in 100 years would be supportable, but in deciding on a regulation weight must be given to the three sets of 100 years with an average of 18 reductions per set (table 26). The probability of occurrence of one of those sets is about once in 600 years.

Of the difference between the mean discharge and the mean draft, about 10 mlrds. is due to evaporation loss in the High Reservoir, and the remainder is excess for which no storage is at present foreseen. The evaporation loss cannot be avoided since storage of flood water must be either on the Main Nile or on the lower courses of the Blue Nile and Atbara, in a region where evaporation is heavy. There is, however, the possibility that the use of mono-molecular films may make substantial reductions in evaporation.

One point may be mentioned with regard to evaporation losses. It has been thought that the reservoir could be regulated so as to minimise these, by confining working as much as possible to the lower levels and contents. It is obvious from the tables that losses tend to increase as the mean supply increases and decrease as the draft increases. The draft is limited by the frequency of low years which can be tolerated, and consequently when a scheme has been chosen on probability grounds, on the lines indicated in this chapter, there is nothing the controlling body can do to reduce levels.

5. General Effects of Very Low Years

The studies described in this book arose from the need to make the maximum use of Nile water. All the projects except those of recent years were designed to store water for irrigation. Latterly it has become necessary also to consider the needs of power production. Supplies for domestic use are also involved, but the requirements of these are negligible compared with those of the other two. Power and irrigation needs are not entirely compatible, since in some cases, as at Aswan, a minimum reservoir level is required for power, whereas in years of low natural supply occurring with a small stored supply, irrigation might be best served by completely emptying the reservoirs. The Nile in Egypt carries much merchandise and for the greater part of the year irrigation requirements are enough for navigation. In January, however, there is no need for water for irrigation, but except in very low years it is possible to spare water to maintain navigation. As development proceeds this will become more and more difficult. In the case of the water supply of a large town, where the main use is for domestic purposes, it might be disastrous if the reservoirs were completely emptied. The various cases, however, merge into each other. In the case of the Nile, the natural supply of the main river varies very greatly from flood to low stage and considerably also from one flood to another. However, it never runs completely dry, though some of its tributaries do. So even with empty reservoirs and a very poor flood the greater part of a minimum supply would be available.

There would, however, be reduced crops, and reduced industrial output in both Egypt and the Sudan, and Ethiopia would suffer from the effects of the low rainfall, but the rainfall of the Lake Plateau is not closely correlated with that of Ethiopia. Storage in Lakes Victoria, Albert and Tana[5] would be of great benefit to both Egypt and the Sudan. Reduction of power and industrial output could be met by standby thermal power. The desirable amount of this is a probability question. The same applies to flood storage as a means of meeting shortage on crops.

Similar problems to those just mentioned will arise whatever is the substance being stored, and the wide range of phenomena covered by the fundamental equations given in this book leads one to think that these, and the methods of using them, may have still wider applications.

6. An Extension of the Experiment with the Card Model

The method of using the Card Model to simulate the discharge of the river at Aswan can be extended to cover the case where there are tributaries, for which simulated discharges are needed. For example with two tributaries (like the Blue and the White Niles), if their discharges are not correlated two sets of cuts entirely independent of each other can be made, and be transformed to simulate discharges of the tributaries. These can be added to give the discharge of the main river. All three sets can then be regulated as a combination. Suitable material to represent each of the tributaries can be taken from the results of the experiments which have already been described.

If the discharges of the tributaries are correlated we can proceed as follows. Let a_1 a_2 etc. and b_1 b_2 etc. be the above sets of the simulated discharges of the tributaries. The b's are then corrected by the addition of terms μa_1, μa_2 etc. where μ is a function of the coefficient of correlation r, which is to be determined. The resulting values $b_1 + \mu a_1$ etc. are then transformed so as to have the same standard deviation as the original set b_1 b_2 etc. The value of μ increases with r, and the relation between the two can be determined experimentally.

7. Conclusion

This chapter describes a method which can be used to find a suitable programme for regulating a long-term storage reservoir. Not all the trials which were made have been given, and still others will be necessary with variations of draft and sliding scales, to fill in gaps and give a complete and well-finished picture of the Aswan problem. The final choice of a detailed scheme of regulation will be modified by conditions in the Nile Basin which are unknown at present; for example, the possibilities of projects on the Upper Nile, and the developments of agriculture and industry in the Basin.

It is hoped that the work described in this book, besides being an investigation into the properties of natural time series, will be of assistance in the solution of some of the other storage problems of practical importance which arise.

The possibilities for future work which are suggested by this account are considerable. The hypothesis that the terms of natural time series are independent random events has been the basis of an enormous amount of mathematical work based on the theory of probability, from the time of Laplace onwards. The large mass of data concerning natural phenomena, and including some results relating to human activities, which has been discussed in this book has produced a new and more complicated relation. The application of this will provide mathematicians with a new field. The solutions given here to some practical problems are founded on the solid ground of experiments with masses of data. These can well be extended by the use of modern computing machinery to give further valuable results for practical use.

Appendices

1. Card experiments.
2. Random events.
    1. Theoretical investigation of the range of accumulated departures of independent events having the binomial distribution.
    2. Correction to the range for the case of tossing a set of coins.
    3. Experiments with random events.
    4. Further discussion of R for independent random events.
3. Some recent publications on natural water supplies and storage.
4. Accumulated departures. River discharges.
5. Accumulated departures. Lake and river levels.
6. Accumulated departures. Rainfall.
7. Accumulated departures. Temperature.
8. Accumulated departures. Atmospheric pressure.
9. Accumulated departures. Tree-rings.
10. Accumulated departures. Varves.
11. Accumulated departures. Sunspot numbers.
12. Relation between draft and storage.

Appendix 1

Card Experiments

Card experiments 1 to 4 have not been used for the trial regulations described in the previous chapter, but have been transformed to Aswan values, as further evidence for comparison with experiments 5 and 6 as to the value of the data and the trial regulations. The results are shown in table 25.

The sets in the table and their constituent observations are in the order in which they were made. The first four experiments were made on a pack of 52 cards and the last two with a pack of 62. The range of cards in the first pack was +7 to −7, and in the second +9 to −9.

The mean values of mean, standard deviation and K for the ten sets in each experiment did not vary very much, though there were rather greater ranges of set means in experiments 5 and 6 than in experiments 1 to 4. It is an advantage that the former were chosen as the subject of trial regulations, since the variability of the results was not minimised. It is possible that some part of these differences of variability is due to the slight difference in the constitution of the two packs and in the bias arising in their use.

The conclusion is that experiments 1 to 4 provide additional evidence of the value of the regulations based on experiments 5 and 6.

TABLE 25

Data from Card Experiments 1 to 6 transformed to Corresponding Values of Aswan Annual Total Discharge in milliards of cubic metres. Each Set contains 100 Observations. Total 6,000

No. of expt.	Mean	Std. devn.	K	No. of expt.	Mean	Std. devn.	K	No. of expt.	Mean	Std. devn.	K
1	*86*	20	0·63	3	89	19	0·83	5	*112*	*13*	0·57
	96	19	0·79		89	20	0·88		105	14	0·79
	98	17	0·64		97	19	0·76		94	20	0·74
	99	*15*	0·59		*84*	16	0·69		101	20	0·80
	91	18	0·81		86	19	0·79		93	20	0·83
	96	20	*0·82*		*104*	17	0·64		87	20	0·86
	89	18	0·66		90	*22*	0·81		97	23	0·84
	94	21	0·81		101	18	0·83		90	*24*	*0·93*
	93	20	0·74		86	19	0·72		*80*	15	0·76
	92	19	0·76		92	21	0·76		85	20	0·80
Means	93·3	18·9	0·72		91·8	18·9	0·77		94·3	18·9	0·79
2	94	20	0·65	4	98	20	0·63	6	90	22	0·60
	88	16	0·68		99	18	*0·60*		83	18	0·67
	92	20	0·65		87	18	0·74		82	20	0·58
	95	*24*	0·71		94	*22*	*0·89*		82	21	0·62
	94	23	0·77		90	*22*	0·70		89	20	0·85
	98	20	0·80		91	20	0·61		91	20	*0·49*
	96	20	*0·58*		103	*15*	0·76		102	20	0·78
	93	19	0·61		96	18	0·74		98	20	0·63
	90	19	0·68		89	*22*	0·74		95	19	0·66
	92	20	0·75		88	17	0·65		91	17	0·72
Means	93·2	20·0	0·69		93·0	19·2	0·71		90·3	19·7	0·66
Means	93·2	19·4	0·70		92·4	19·0	0·74		92·3	19·3	0·72
			Highest and lowest values								
Highest	99	24	0·82		104	22	0·89		112	24	0·93
Lowest	86	15	0·58		84	15	0·60		80	13	0·49

Appendix 2

The Theory of Long-term Storage for Independent Random Events

1. THEORETICAL INVESTIGATION OF THE RANGE OF ACCUMULATED DEPARTURES OF INDEPENDENT EVENTS HAVING THE BINOMIAL DISTRIBUTION

Gauss' normal frequency curve (which is the curve for the distribution of accidental errors of observation) gives the frequency of departures of a quantity from its arithmetic mean in relation to their magnitudes. If y is the number of departures lying between $x + \frac{1}{2}$ and $x - \frac{1}{2}$ when the total number of values of the quantity is N, the equation of the curve is

$$y = (N/\sigma\sqrt{2\pi})\, e^{-x^2/2\sigma^2} \quad \ldots 1$$

It is well known that an approximation to this curve is given by the terms of a binomial expansion. When a set of m coins is tossed the probability of the occurrence of r heads and $m - r$ tails at any throw is ${}^mC_r\, (\frac{1}{2})^r\, (\frac{1}{2})^{m-r}$ in which ${}^mC_r$ is the number of combinations of m different things taken r at a time, $= \left(\dfrac{m!}{r!\,(m-r)!}\right)$

If the set is tossed N times (the number N being fairly large) the average frequency of occurrence of zero heads and m tails, one head and $(m - 1)$ tails, etc., is given by the terms of the binomial expansion

$$N(\tfrac{1}{2} + \tfrac{1}{2})^m = N(1 + {}^mC_1 + {}^mC_2 + \ldots + {}^mC_{m-1} + 1)(\tfrac{1}{2})^m \quad \ldots 2$$

As m increases equation 2 approximates to the normal curve, and if $m = 10$ the approximation is fairly good; in fact for many purposes it is good enough. Table 26 shows how the number of heads would be expected to be distributed on the average when a set of ten coins is tossed 1,024 times—that is the binomial distribution. The distribution derived from the normal curve and one actually obtained by trial are also given. In the normal distribution the numbers represent the nearest integer.

The relation between the binomial and the normal distribution has been treated by G. Udny Yule[41] who defines the normal distribution as the

TABLE 26

Random Events—Comparison of Frequencies

Distribution	Number of heads, r: 0	1	2	3	4	5	6	7	8	9	10	Total
Binomial	1	10	45	120	210	252	210	120	45	10	1	1,024
Normal Gaussian	2	10	43	116	212	258	212	116	43	10	2	
Actual experiment	0	16	53	122	209	240	194	138	46	5	1	

limit of the binomial distribution when m is indefinitely increased. For theoretical purposes computers can use whichever form is convenient.

Assume a number of consecutive values of an element whose variation from its mean is distributed approximately normally, as in the binomial expansion already discussed, but where there is no correlation between successive values or groups of values, as in tossing coins. For example, consider the definite case of tossing a set of $2m$ coins N times, calling a head a gain and a tail a loss. For present purposes each toss of the $2m$ coins is recorded as the number of heads minus the number of tails, and the departures of this quantity from its mean are added to find the range R between the maximum and the minimum of this summation curve. The range thus found, from N tosses of a set of $2m$ coins, may differ slightly from the range found by the summation of heads minus tails from $2Nm$ tosses of a single coin. For the present this difference is ignored, but it will be considered later.

In the case selected for analysis $2m$ coins are tossed N times, and heads and tails or gains and losses are equal at the end of the trial, and n is written for Nm. The number of orders or arrangements in which n gains and n losses can occur is the number of different ways in which we can choose n out of $2n$ places, and for this we use the symbol ${}^{2n}C_n$, which is the number of combinations or selections of n out of $2n$ different things.

$${}^{2n}C_n = \frac{(2n)!}{(n!)\,(n!)}$$

If we take a particular arrangement and form the continued sums of gains (positive) and losses (negative) in serial order, if these sums are plotted as ordinates we get three types of curve.

(*a*) Losses never exceed gains and the curve does not go below the axis of abscissae.

(*b*) Gains never exceed losses and the curve does not go above the axis.

(*c*) Losses exceed gains and gains exceed losses, so that the curve is partly above and partly below the axis.

There are ${}^{2n}C_n/(n + 1)$ examples of each of types (*a*) and (*b*) and $(n - 1)\,{}^{2n}C_n/(n + 1)$ of type (*c*).

The range R in (*a*) is the maximum of (gains minus losses), in (*b*) it is the maximum of (losses minus gains), and in (*c*) the sum of the maxima of (gains minus losses) and (losses minus gains). The mean range can be got by adding all the maxima of (gains minus losses) to all the maxima of (losses minus gains) and dividing the sum by the total number of arrangements, ${}^{2n}C_n$. The method of doing this is as follows:

Among all the arrangements there are ${}^{2n}C_{n+h}$ in which at some point in the tossing gains exceed losses by h or more, and ${}^{2n}C_{n+h}$ where at some point losses exceed gains by h or more. (See Whitworth, Choice and Chance, chapter on Priority[42].) Thus there are ${}^{2n}C_{n+1}$ where gains exceed losses by one or more, ${}^{2n}C_{n+2}$ where the excess is two or more, and therefore the number where the excess is one is ${}^{2n}C_{n+1} - {}^{2n}C_{n+2}$, etc., as in table 27.

TABLE 27

Random Events—Computations to determine the Mean Range (R)

Gains minus losses	No. of arrangements	Products
1	${}^{2n}C_{n+1} - {}^{2n}C_{n+2}$	$1\,({}^{2n}C_{n+1} - {}^{2n}C_{n+2})$
2	${}^{2n}C_{n+2} - {}^{2n}C_{n+3}$	$2\,({}^{2n}C_{n+2} - {}^{2n}C_{n+3})$
. . .	. . .	. . .
$n - 1$	${}^{2n}C_{2n-1} - {}^{2n}C_{2n}$	$(n - 1)\,({}^{2n}C_{2n-1} - {}^{2n}C_{2n})$
n	${}^{2n}C_{2n}$	$n\,({}^{2n}C_{2n})$
Sums	${}^{2n}C_{n+1}$	${}^{2n}C_{n+1} + {}^{2n}C_{n+2} + \ldots + {}^{2n}C_{2n}$

The table includes all except those where gains never exceed losses, whose maxima of gains minus losses are zero.

The sum of gains minus losses is

$${}^{2n}C_{n+1} + {}^{2n}C_{n+2} + \ldots + {}^{2n}C_{2n-1} \pm {}^{2n}C_{2n} \qquad \ldots 3$$

and this is also the sum of losses minus gains, which can be written

$${}^{2n}C_{n-1} + {}^{2n}C_{n-2} + \ldots + {}^{2n}C_1 + 1$$

Now

$$1 + {}^{2n}C_1 + \ldots + {}^{2n}C_{n-1} + {}^{2n}C_n + {}^{2n}C_{n+1} + \ldots + {}^{2n}C_{2n} = (1 + 1)^{2n} \qquad \ldots 4$$

The left-hand side of equation 4 is the binomial distribution. The sum of gains minus losses together with losses minus gains is

$$2^{2n} - {}^{2n}C_n$$

and the mean range of the ${}^{2n}C_n$ arrangements is

$$R = 2^{2n}/{}^{2n}C_n - 1 \qquad \ldots 5$$

Since n is large we can simplify this by Stirling's approximation for $n!$

$$n! = \sqrt{2n\pi}\,(n/e)^n \left(1 + \frac{1}{12n} + \frac{1}{288n^2} + \ldots\right) \qquad \ldots 6$$

If n is large $n! = \sqrt{2n\pi}\,(n/e)^n$ is a close approximation. Hence

$${}^{2n}C_n = \frac{(2n)!}{(n!)^2} = \frac{\sqrt{4n\pi}\,(2n/e)^{2n}}{[\sqrt{2n\pi}\,(n/e)^n]^2} = \frac{2^{2n}}{\sqrt{n\pi}}$$

and

$$R = \sqrt{n\pi} - 1 = \sqrt{Nm\pi} - 1 \qquad \ldots 7$$

Thus the average range of the accumulated sums (number of heads minus number of tails), when $2m$ coins are tossed N times (Nm being large), increases as the square root of the number of tosses, exactly like the accumulated error on a line of levelling.

The standard deviation of the binomial distribution produced by tossing $2m$ coins is

$$\sqrt{2m \times \tfrac{1}{2} \times \tfrac{1}{2}} \qquad \ldots 8$$

This is the standard deviation of the number of heads (r) or tails ($2m - r$). The standard deviation σ of the number of heads minus the number of tails is twice this, or $\sqrt{2m}$. Substituting for m in equation 7,

$$R = \sigma\sqrt{\tfrac{1}{2}N\pi} - 1 = 1{\cdot}25\,\sigma\sqrt{N} - 1 \qquad \ldots 9$$

When N is large the asymptotic value of R is $\sigma\sqrt{\tfrac{1}{2}N\pi}$.

The above investigation and the value of R when N is large was published in the Proceedings of the American Society of Civil Engineers of April 1950[7]. The same result was obtained by Professor W. Feller by more advanced methods[18]. Doctors Anis and Lloyd have also investigated the problem for finite values of N and arrive at the same asymptotic value[9].

2. Correction to the Range for the Case of Tossing a Set of Coins

In equation 7, R is the range of the continued sums of $2mN$ tosses of a single coin, and is a little larger than the range obtained from tosses of a set of $2m$ coins. The average difference between the two computations for range can be found as follows:

Toss a coin a large number of times and plot the continued sums of heads minus tails, marking a given number of them off in sets of $2m$. The set of $2m$ that contains the maximum sum is selected for scrutiny. If there are r heads and t tails the end-point of the set is $r - t$ higher than the end-point of the previous set. The number of orders in which r heads and t tails can occur is ${}^{2m}C_r$.

The number of orders in which at some stage of the tossing the number of heads shall exceed the number of tails by h or more is ${}^{2m}C_{m+h}$. At the end of the set, heads exceed tails by $r - t = 2r - 2m$. The number of orders in which at some stage heads exceed tails by more than $2r-2m$ is ${}^{2m}C_{2r-2m} + 1$. In all these cases the maximum of the summation curve of individual tosses will exceed the curve derived from sets of $2m$ tosses. The mean value of the excess d can be obtained by a procedure similar to that already employed in finding the mean range of the summation curve. The result of this is:

$$d = \frac{1}{2^{2m}}\left[\frac{(2m)!}{(m-1)!(m+1)!} + 3\frac{(2m)!}{(m-2)!(m+2)!} + 5\frac{(2m)!}{(m-3)!(m+3)!} + \ldots + (2m-3)\,2m + (2m-1)1\right]$$

There is a similar effect on the minimum of the summation curve so that the average range of the summation curve of individual tosses is greater than the range of the curve of sets of tosses by $2d$. The correction $2d$ to the range is independent of N, and so has a less proportionate effect as N increases. The following are some values of $2d$.

m	4	5	6	10
$2d$	1·46	1·71	1·93	2·71
Percentage correction to R	4·1	4·3	4·5	4·8

3. Experiments with Random Events

Table 28 gives the results of an actual experiment in which sixpences were shaken in a box and thrown on to a table. They have been analysed in some detail to give a practical example of the foregoing theory. Their frequency distribution has been given in table 29, in which there were 1,024 throws of ten coins ($m = 5$) which resulted in a distribution close to the theoretical.

The observations were recorded as they occurred and the continued sums were computed from zero to 1,000 observations. In this case (because, at the finish, heads and tails are nearly equal) the actual values (heads minus tails) are taken as departures, and not the departures from the mean (column 1, table 28). At the finish there are 74 more heads than tails; so the mean value of $h - t$ per toss is 0·074. This is corrected graphically by the aforementioned method. Figure 21, discussed subsequently, shows the result of one of the experiments.

In each case R (column 3, table 28) is corrected to what it would have been if the departure had been reckoned from the mean. The theoretical value of σ is 3·16 for this binomial distribution.

Table 28 shows that $R/(\sigma\sqrt{N})$ tends to be constant and the values from 1,000 trials (column 4) agree closely with the theoretical values (column 5). The trials illustrate a point not examined in the theoretical work, and that is the variability of σ, R and $R/\sigma(\sqrt{N})$ from set to set of 100 throws.

The maximum variations, as percentage departures from the mean, are as follows: σ, 11 per cent; R, 50 per cent; and $R/(\sigma\sqrt{N})$, 39 per cent. The standard deviation (or standard error) or σ for a set of 100 throws is about 7 per cent.

Another device for obtaining a normal frequency distribution is what may be called a probability pack of cards (table 31). In this pack the cards are numbered +1, −1, +3, −3, +5, −5, +7, −7, +9 and −9, and the numbers of each kind are proportional to the corresponding ordinates of a normal frequency curve. There are: thirteen 1's; ten 3's; and six, three and one, respectively, of the others. The approximation of these numbers is fairly close. The cards are first well shuffled and then cut, and the number on the exposed card is recorded. The cards are reshuffled slightly and cut again, and so on. The numbers recorded may be taken as corresponding to observations of a quantity whose frequency distribution conforms to the normal Gaussian curve.

This is a quicker process than tossing and counting coins. To toss ten coins 100 times required about 35 minutes, whereas shuffling and cutting for 100 cards required 20 minutes. Table 28 records the results of one trial of cutting 1,000 times. As in column 3, table 28, in each case R is corrected to what it would have been if the departures had been reckoned from the mean; and, again, $R/(\sigma\sqrt{N})$ in column 4 approximates to the value obtained theoretically for the case when N is large (column 5).

In each case R (column 3) has been corrected to what it would have been if the departures had been reckoned from the mean. The theoretical value of $R/(\sigma\sqrt{N})$ (column 5) has been corrected for the effect of tossing in sets in (*a*).

Although the mean value of $R/(\sigma\sqrt{N})$ found from the two sets of random events in table 28 is very close to the value found by mathematical

TABLE 28

Experiments with Random Events

Definitions of column headings

Columns 1. Primary observations:
Table (*a*), excess of heads minus tails = $h - t$.
Table (*b*), sum of recorded numbers = Σ.

Columns 2. Standard deviation of corresponding primary observations (columns 1) = σ.

Columns 3. Range of accumulated deviations = R.

Columns 4. Computation of $R/(\sigma\sqrt{N})$ from observed data.

Columns 5. Theoretical values of data in columns 4 corrected in (*a*) for the effect of tossing in sets.

No. of trials	(*a*) Ten sixpences tossed 1,000 times					(*b*) Probability cards cut 1,000 times				
	$h-t$ (1)	σ (2)	R (3)	$R/(\sigma\sqrt{N})$ (4)	(5)	Σ (1)	σ (2)	R (3)	$R/(\sigma\sqrt{N})$ (4)	(5)
1–100	26	3·08	39	1·28	1·16	−26	3·16	40	1·27	1·23
101–200	24	3·10	25	0·81		+38	3·00	30	1·00	
201–300	−26	3·36	44	1·31		− 2	3·99	35	0·88	
301–400	−30	2·88	29	1·01		−44	4·06	36	0·89	
401–500	+24	3·59	40	1·11		+28	3·87	65	1·68	
501–600	−32	3·06	31	1·01		− 2	3·58	48	1·34	
601–700	+12	3·56	44·5	1·25		+10	3·57	33	0·92	
701–800	+42	3·02	28·5	0·94		−22	3·11	44	1·42	
801–900	+38	3·03	25·5	0·84		+ 4	3·82	78	2·04	
901–1,000	− 4	3·50	54	1·54		−14	4·25	48	1·13	
Mean		3·22	36·1	1·11	1·16		3·64	45·7	1·26	1·23
1–500	+18	3·20	87	1·21	1·21	− 6	3·62	81	1·00	1·24
501–1,000	+56	3·23	94	1·30	1·21	−24	3·67	111	1·35	1·24
1–1,000	+74	3·22	121	1·19	1·22	−30	3·64	116	1·01	1·24
Mean				1·19	1·20				1·15	1·24

analysis (columns 5 compared to columns 4) yet individual values for sets of 100 observations vary considerably.

4. Further Discussion of R for Independent Random Events

The preceding discussion is based on the simplest frequency distribution of random events, the binomial, and in the case where N, the number of events, is very large leads to the asymptotic result

$$R = \sigma\sqrt{\tfrac{1}{2}N\pi} \qquad \ldots 12$$

as was shown in the first part of this chapter. Anis and Lloyd[9] also gave expressions for R when N is finite, using the normal distribution.

This work was extended by Anis, and Anis and Solari(36), for higher moments of R (standard deviation, etc.). The use of the normal distribution leads to more difficult mathematics, so that it is interesting to find how the results of the simpler method compare with those of the more abstruse for smaller values of N. From sections 1 and 2 we deduce:

(*a*) Tossing a single coin $2n$ ($= N$) times.
Head $= 1$, tail $= -1$. Binomial distribution.

$$R/\sigma = \frac{2^{2n}}{{}^{2n}C_n} - 1 \quad \ldots 13$$

which, with Stirling's approximation for large n, becomes

$$R/\sigma = \sqrt{\tfrac{1}{2} N\pi} - 1$$

If $N = 10$ these two formulae give $R/\sigma = 3{\cdot}06$ and $2{\cdot}96$ respectively.

(*b*) Tossing a set of $2m$ coins N times. After applying the correction $2d$ for tossing in sets, and using Stirling's approximation and $\sigma = \sqrt{2m}$, we find

$$R/\sigma = \sqrt{\tfrac{1}{2} N\pi} - \frac{1 + 2d}{\sqrt{2m}} \quad \ldots 14$$

In this case if $N = 10$ and $m = 5$ the exact value of R/σ is 3·125, and 3·10 with Stirling's approximation, a difference of less than 1 per cent. So whether the event is simple, like tossing a single penny, or more complex, like tossing a set of pennies, the values of R/σ are nearly the same. The term independent of N only becomes negligible when N becomes fairly large.

(*c*) The normal distribution. In the paper by Solari and Anis they give a formula for R/σ developed from the normal distribution, from which the following is obtained

$$R/\sigma = \sqrt{\frac{N}{2\pi}} \sum_{s=1}^{s=N-1} s^{-\frac{1}{2}} (N-s)^{-\frac{1}{2}} \quad \ldots 15$$

The formula is said to be valid for $N \geqslant 2$. They have computed exact and asymptotic approximation values from $N = 10$ to $N = 150$, the asymptotic values corresponding to equation 12. The following table compares some values computed from our investigation with some taken from Solari and Anis.

We do not know the reason for the constant difference of 0·31.

TABLE 29

Values of R/σ computed from the Normal and from the Binomial Distributions

No. of variates	Values of R/σ Solari and Anis Normal distribution: Exact (1)	Asymptotic (2)	Tossing set of ten coins using Stirling's approximation (3)	Differences (3) − (1)
10	2·79	3·96	3·10	0·31
50	7·70	4·63	8·01	0·31
100	11·37	12·53	11·67	0·31
150	14·18	15·35	14·49	0·31

The following table compares results computed from Anis and Solari's formula and from equation 13 (with corrections for tossing in sets) for small values of N.

N	Anis and Solari	Equation 13 (corrected)	Difference
2	0·56	0·95	0·39
3	0·98	1·33	0·35
4	1·33	1·67	0·34
5	1·62	1·96	0·34

When there are only two events, say a and b, it is clear that $R/\sigma = 1$, so that for this case the value from equation 13, as corrected for tossing in sets, is approximately correct. The difference of about 0·31 found in the previous table persists for these low values of N.

It is important to compare R/σ for natural and random events, and this is done in fig. 21, where log R/σ is plotted against log N for both.

The whole mass of data from natural phenomena, which is given in this book, is represented by the mean line $K = 0{\cdot}73$. The line for random events is plotted from the results of tossing a set of ten coins, which have just been discussed. The two lines coincide from $N = 2$ to $N = 6$, above which R/σ for natural events is greater than for random events by an amount increasing as N increases.

Table 2, p. 17, shows that the standard deviation of K is 0·081, so that the probable departure of a single value is $\pm 0{\cdot}054$. The line $K = 0{\cdot}62$ represents a value of R/σ lower than the mean by twice the probable departure, and is therefore such that on the average the probability of a value below this is $\frac{1}{11}$. The two lines intersect at $N = 126$. If there are only 50 years recorded, $K = 0{\cdot}65$ gives the same as the mean for random

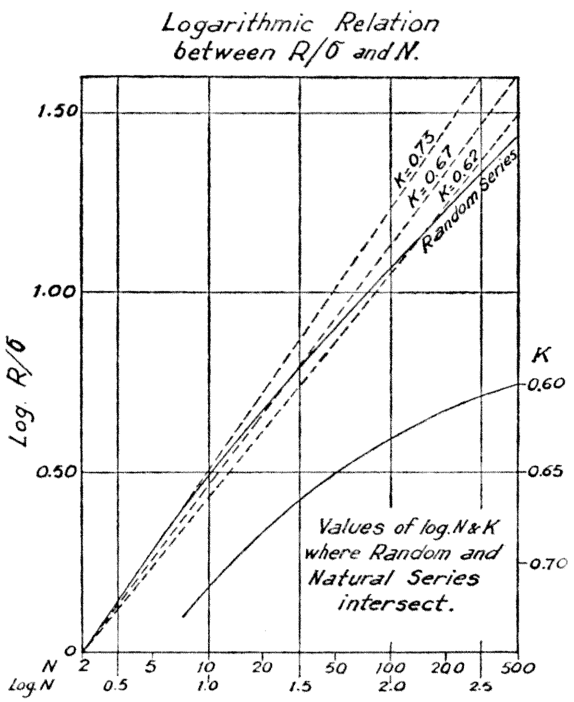


FIG. 21

events. Short records with low values of K are therefore not distinguishable from random records. The probability of K being less than 0·65 is about $\frac{1}{8}$. In fig. 21 a separate curve has been drawn by plotting the values of log N and K at which the equations for natural and random series give identical results. At points below and to the right of this a random series is less than a natural one. The figure shows that when K is greater than 0·73 the natural series is greater than the random for all values of N.

Appendix 3

Some Recent Publications on Natural Water Supplies and Storage

(*a*) By V. M. Yevdjevich.
Fluctuations of Wet and Dry Years. Part I—Hydrology Papers. Colorado State University. 1. 1963.[37]
Climatic Fluctuations studied by using Annual Flows and Effective Annual Precipitations. Sponsored by U.S.A. National Science Foundation.[38]
Proceedings 30th Annual Meeting, Western Snow Conference, Cheyenne, Wyoming, April 1962.[39]

A very valuable part of the first paper is the tables of data from 140 rivers, the majority of which are in Europe and America. It includes annual discharges with means, standard deviations and the first serial correlation coefficients for each of the rivers. The length of record varies from 37 to 150 years, with a mean of 55 years. It provides the means of increasing considerably the number of values of R and K given in appendix 3.

In order to obtain annual values in a set in a dimensionless form they are divided by the mean of all the years in the set. From the point of view of storage it would have been preferable to divide annual discharges by their standard deviation, as is done in this book.

The data also contain annual values of discharge corrected for carry-over of water stored in lakes, marshes, or the river trough, likewise divided by period means to produce dimensionless data. This carry-over, where it is significant, would tend to produce a correlation (r_1) between successive discharges, and these have been computed in the paper. Their mean values are:

Actual discharges. Mean $r_1 = 0{\cdot}177$
Discharges without carry-over. Mean $r_1 = 0{\cdot}136$.

The values of r_1 for single rivers are usually not large enough to be significant, though the mean from all the rivers is significant. There are

28 rivers out of the 140 with values of r_1 greater than 0·3. In 22 of these the mean r_1 for the original data is 0·45, and for the data with carry-over removed 0·41, so in these carry-over has little effect. In the remaining six the means are respectively 0·46 and 0·09. It is curious that the effects of carry-over due to the lakes have quite different effects on the St. Lawrence and on the Nile; on the first the serial correlation is reduced from 0·70 to 0·09, and on the second, Lake Victoria 0·65 to 0·42 and Lake Albert 0·64 to 0·49. The Göta, which is the outflow from Lake Vener, also shows a large effect, from 0·46 to 0·01.

In the second and third papers the author analyses the data given in the first, and in particular has investigated serial correlations by means of the correlogram, a device in which the coefficients of correlation between a year and the next (r_1), a year and the next but one (r_2), r_3, etc., are plotted against the intervals 1, 2, 3 years and so on.

Some of his conclusions are:

1. "Distribution of first serial correlation coefficient, correlogram analysis, and distribution of maximum range have shown that the sequence of effective annual precipitations is very close to random sequence.
2. Most of dependence between the successive values of annual flows can be explained (*a*) by changing of water carry-over from one water year to another in the form of different water storage in a river basin; (*b*) by non-homogeneity in data; (*c*) by some systematic errors in computation of annual flows; and (*d*) by error from regional sampling. After these factors are taken care of the room left for the causes of this linkage, which come from either the atmosphere or solar activities, remains small.
4. There is no statistical evidence that the fluctuations of annual flows or effective annual precipitations may be composed of hidden periodicities, or of some regular patterns in the fluctuations, which can be extrapolated in the future with a reasonable expectancy that they would occur and would be verified by future flow records."

The following are some comments on these conclusions:

4. The evidence quoted in chapter 8, section 2, of this book confirms this unpredictability.

1 and 2. The analysis of data given in this book is definitely against these two conclusions, and the following is some further evidence in the same direction.

The persistence which exists in many natural time series, as well as in others of social origin, is not well described by serial correlations. It cannot, however, be explained away by the causes mentioned in 2. As already shown in the case of a variety of natural time series, periods occur when on the whole values are above the long-term average, and others when they are below it, to a greater extent than is to be expected from a series of independent random variates. These periods are irregular in length and in their departure from the mean, and they cannot be predicted (see chapter 2). It is this grouping which is mainly responsible for the value of the index K, and also affects the serial correlations.

An example which illustrates this is the Niger for which data are given in Yevdjevich's first paper.

Serial correlations

$$r_1 = 0{\cdot}554 \pm 0{\cdot}10; r_2 = 0{\cdot}48 \pm 0{\cdot}11; r_3 = 0{\cdot}43 \pm 0{\cdot}12; r_4 = 0{\cdot}23 \pm 0{\cdot}14.$$

The coefficient of serial correlation when carry-over has been removed is $r_1 = 0{\cdot}548$, so that the effect of carry-over is negligible. The first three coefficients for the uncorrected data are significant, and this is due to grouping, which occurs as follows:

Departures from mean: 1906–21, 13 negative, 3 positive; 1922–32, 11 positive; 1933–49, 3 positive, 14 negative; 1950–56, 6 positive, 1 negative. N is 51 years and K is 0·75. Reference to fig. 21 shows that R/σ for these is 35 per cent greater than its random value.

In the case of the St. Lawrence K has the value 0·90 for the discharge below the lakes. When carry-over is removed K is 0·684 and R/σ is 23 per cent above the corresponding mean random value.

Another important example is the Nile at Aswan, with which parts of this book are concerned, and which Professor Yevdjevich gives as an example of inconsistency in the data, produced by the building of the Aswan Dam, which caused degradation downstream and changed the rating curve. Professor Yevdjevich is mistaken about this, as will be seen from the evidence, given in chapter 10, of the reality of the high floods of last century, and that they occurred also at Wadi Halfa, 250 miles above.

A little more detail may be added to this. The Aswan river gauge is just below the foot of the cataract and about 6 kilometres from the dam. The dam is at the head of the cataract, which is formed by a barrier of igneous rock crossing the valley, through which the river has cut channels. In flood, however, the barrier is submerged. The slope over the cataract averages about 1 metre to the kilometre. Until recently the reservoir was empty by the end of July and all sluices were opened and kept fully open until the end of November, when they were gradually closed in order to refill the reservoir. This allowed the flood to pass without obstruction, so that during this time conditions below the cataract were the same as before the building of the dam. About 70 per cent of the annual discharge passed during the time of fully open sluices. At the top of the flood the mean depth of the river below the cataract is 9 or 10 metres, while at its lowest it is 3 or 4 metres. During the low stage the gauge–discharge curve is affected in an irregular manner by the movement of sandbanks. During the flood season there are changes of the curve caused by the changes of slope during the passage of flushes; these changes appear as peaks on the curve.

It is clear, therefore, that the dam can have had no effect on the gauge–discharge curve below the cataract due to degradation, and that any changes are the transient ones which occur at most sites in the Nile system. As shown in chapter 10 collateral historical evidence shows that 1870 to 1898 was a period of high floods, and consequently a high average discharge, while 1899 up to the present time has had few high floods. This fact of grouping affects the serial correlations, and also K, as the following figures show.

Period	Mean	Std. devn.	K	r_1	r_2	r_3	r_4
1870–1898	110	13	0·74	0·11 ±0·12	−0·002	—	—
1899–1957	83	12	0·50	0·12 ±0·09	−0·06 ±0·09	—	—
1870–1957	92	18	0·88	0·49 ±0·05	0·44 ±0·06	0·34 ±0·11	0·18 ±0·10

In this case the serial correlations for the whole period are entirely due to grouping, and the changes of mean which this produces. In the two sections the serial correlations are not significant. Examination of cases where r_1 and K have been computed, of which we have 19, gives a coefficient of correlation between r_1 and K of $0{\cdot}35 \pm 0{\cdot}14$, which is probably not significant.

Further information on Yevdjevich's conclusions 1 and 2 can be obtained from water supply data given in appendices 4, 5 and 6. In appendix 4 there are 15 values of K where N is 60 years or more, with an average of 75. The mean value of K is 0·694 and the corresponding value of R/σ is 23 per cent greater than the random value. More information is available from rainfall, where there are 86 cases of $N = 60$ or more. These have mean values of $N = 99$ and $K = 0{\cdot}71$ with a corresponding value of R/σ 38 per cent greater than the random value.

The long series of flood levels on Roda Gauge (Cairo), where 800 years

have few missing readings, divided into periods of 100 years, have a mean value of $K = 0{\cdot}74$, which is 55 per cent above the random line of fig. 21.

It seems clear that the serial correlation–random value scheme is of very limited application in connection with long-term storage. On the other hand the equation $R/\sigma = (N/2)^K$, which holds from $N = 10$ to 2,000 and probably further, covers all the data, climatological and sociological, which is collected in the previous appendices, in one simple formula.

(*b*) By members of Harvard University.
Design of Water-Resource Systems. New Techniques for relating Economic Objectives, Engineering Analysis and Government Planning. Macmillan & Co. London. 1962.

Part I of this book describes the objects of development and the economic and technical considerations involved. Part II gives an account of methods and techniques. A large amount of this section is concerned with the preparation of the material in a form suitable for the computer. Part III, which is short, is entitled Governmental Factors and deals with political matters.

As an example for experiment and the derivation of techniques for the design of projects to give optimum results, the Clearwater river system in Idaho has been selected. Twelve rivers were considered and the Clearwater is thought to be typical in its hydrology of a major region of the United States. The basin is still largely in its natural condition, and in it are placed hypothetical reservoirs, hydro-electric power schemes, areas to be irrigated, and means of flood protection. Costs of works and values of benefits are hypothetical but are based on actual experience. On this basis a simplified model was constructed of such a form that it could be analysed by high-speed digital computers.

In the chapter on Objectives and Concepts it is written: "Although we do not deal with over-year storage here, we must mention that in dry regions the storage of water in years of ample rainfall for use in periods of drought is often far more important than the storage of wet-season water for use in the dry season of the same year". The latter is what is principally discussed in the book. There are, however, a few remarks on over-year storage which may be quoted.

(Page 259.) "In general over-year storage can be accounted for in one of three ways: (1) by using the historic or a synthetic stream-flow record . . .; (2) by analysing the "critical period" (or periods) of an historical or synthetic record; and (3) by introducing the probability distribution of the inflow record to determine probabilities of outputs of reservoirs of different sizes."

The historic record used in the book covers the 38 years of streamflows of the Clearwater, and it is stated (page 256): "But no matter how poorly a brief record may identify the time frequency of years or seasons of unusually low and high floods, unless it is very short indeed, it will permit fairly reliable estimates of mean annual and seasonal flows and their variances".

There is a fallacy here as may be seen by a reference to the appendices of data in this book which show the variability of means and standard deviations of a very large number of natural phenomena. It will also show that analysis of critical periods, particularly on a short record, may give very misleading results in regard to over-year storage, though applicable to annual storage reservoirs.

A synthetic record of 510 years (called the stochastic sequential model) was produced by the use of a table of random numbers, though there is mention of a probability card method, which appears to be identical with that first described in reference (7) of the table at the beginning of this book. As was shown in our first chapter, in the matter of storage, natural time series do not in general behave like random numbers, unless their order of occurrence is ignored. The use of sequences of random numbers of 50 years from which to compute the most probable storage required will give results for R/σ, on the average, 20 per cent below those derived from the examination of natural time series, as described in the present book. The Harvard book is mainly occupied with the use of high-speed digital computers, and has little to say on the hydrological data, which after all are the very foundation of the matter. It appears that problems have to be severely simplified by assumptions so as to be within the capacity of the computing machine.

(*c*) By E. H. Lloyd, Imperial College (now University of Lancaster).
A Probability Theory of Reservoirs with Serially Correlated Inputs. Journal of Hydrology, I (1963). Amsterdam.
Reservoirs with Serially Correlated Inflows. Technometrics. Vol. 5, No. 1 (1963).

These papers extend the mathematical work of Moran[10] on long-term storage of flows which are independent random quantities, to flows which are serially correlated; that is to say where a flow is related to its predecessors or there is "persistence". The kind of persistence considered is that the inflow of a year is correlated with that of its predecessor only, i.e. that the sequence of flows forms a Markov Chain.

A solution is given for a simple case by the use of matrices, and indications are given of how this might be extended to more complicated cases, including allowance for losses.

Notes on the Data

Annual values, usually means or totals, are used throughout. The meteorological data are taken mainly from World Weather Records published by the Smithsonian Institution. Additional information has been supplied by the British Meteorological Office. The long record of temperatures in Central England is from a paper by Professor Gordon Manley in Archiv für Meteorologie, Geophysik und Bioklimatologie. Series, B, B, and I. Vienna, 1959.

Hydrological information is from various sources, the principal of which are: the Nile Basin Volumes, Ministry of Public Works, Cairo; Water Supply Papers of the U.S. Geological Survey, and additional information from W. B. Langbein, Hydraulic Engineer; Papers by Professor V. M. Yevdjevich, University of Colorado.

In chapter 3, section 2, some information has been given about the data for tree-rings and varves.

Appendix 4

Accumulated Departures. River Discharges

River	Period	*N*	*M*	σ	*R*	*K*
Campaspe, Australia	1886–1945	60	1·8	1·4	13	0·65
Cedar, U.S.A.	1896–1950	55	69	15	190	0·78
Danube at Orsava	1840–1876	37	165	31	284	0·76
	1877–1913	37	169	26	221	0·74
	1914–1950	37	173	39	232	0·61
	1840–1950	111	169	33	420	0·64
Derwent, England		50				0·62
Drina, Sava—	1890–1950	61	11·2	3·1	36	0·72
tributaries of Danube	1878–1914	37	51	9·5	75	0·71
	1915–1950	36	52	14·3	89	0·63
	1878–1950	73	52	12·1	143	0·69
Godavari, India	1881–1936	55	36	12	150	0·76
Goulburn, Australia	1882–1945	64	2·2	1·0	10	0·66
Lake Huron	1860–1948	89	188	20	720	0·94
Kistna, India	1881–1936	56	234	100	1,500	0·80
Merrimack, U.S.A.	1880–1945	66	70	16	210	0·74
Mississippi, U.S.A.	1874–1936	63	48	13	190	0·77
Niger, Africa	1906–1956	51	1,540	370	4,220	0·75
Nile, Aswan	1870–1898	29	110	13·4	98	0·74
	1899–1957	59	85	12·2	70	0·50
	1870–1913	44	101	19·3	292	0·88
	1914–1957	44	86	11·0	82	0·65
	1870–1957	88	93	17·5	500	0·88
Nile tributaries						
Outflow,						
Lake Victoria	1899–1957	59	21	4·4	35	0·61
Net inflow,						
Lake Victoria	1904–1958	54	20	15·9	87	0·52

Accumulated Departures. River Discharges (continued)

River	Period	*N*	*M*	σ	*R*	*K*
Outflow, Lake Albert	1904–1930	27	25	8·6	70	0·81
Outflow, Lake Albert	1931–1957	27	22	4·0	24	0·68
Outflow, Lake Albert	1904–1957	54	24	6·8	91	0·79
Net inflow,						
Lake Albert	1904–1958	54	23	7·1	119	0·86
Sobat	1905–1952	48	14	2·7	27	0·73
Atbara	1905–1952	48	12	3·5	21	0·60
Pripet, U.S.S.R.	1882–1917	36	122	34	210	0·64
Pripet and Dnieper,						
U.S.S.R.	1878–1924	47	132	28	200	0·61
Rhine, Germany	1807–1856	50	1,030	144	960	0·59
	1857–1906	50	990	147	1,070	0·61
	1907–1956	50	1,050	189	1,280	0·60
	1881–1930	50	1,030	162	1,380	0·66
	1807–1957	150	1,030	164	2,280	0·61
Spokane, U.S.A.	1892–1950	59	6·7	2·0	25	0·75
Sudbury, U.S.A.	1876–1945	70	20	6·1	78	0·72
Tennessee, U.S.A.	1875–1935	61	38	9	67	0·60
Thames, England	1883–1921	39	50	16·1	167	0·79
	1922–1960	39	56	17·1	122	0·66
	1883–1960	78	53	16·9	248	0·73
Truckee, U.S.A.	1839–1871	33	107	98	580	0·64
	1872–1904	33	107	59	320	0·60
	1905–1938	34	87	52	500	0·80
	1839–1904	66	107	81	580	0·57
	1872–1938	67	97	57	760	0·74
	1839–1938	100	100	73	1,110	0·69
Discharges, percentage of median						
Brazos, U.S.A.	1899–1952	54	124	81	720	0·66
Chattahoochee, U.S.A.	1896–1952	57	104	31	260	0·63
Colorado, U.S.A.	1896–1952	57	101	28	410	0·80
Columbia, U.S.A.	1896–1952	57	99	19	290	0·82
Kings, U.S.A.	1896–1952	57	104	45	470	0·69
Mississippi (Keokuk),						
U.S.A.	1896–1952	57	104	29	370	0·77

Accumulated Departures. River Discharges (continued)

River	Period	*N*	*M*	σ	*R*	*K*
Missouri, U.S.A.	1896–1952	57	100	28	460	0·84
Niagara, U.S.A.	1896–1952	57	101	7	110	0·81
Pamigewasset, U.S.A.	1896–1952	57	100	16	180	0·72
Red, U.S.A.	1896–1952	57	126	83	1,530	0·87
Rio Grande, U.S.A.	1896–1952	57	96	30	440	0·80
Sacramento, U.S.A.	1896–1952	57	106	42	610	0·79
Susquehanna, U.S.A.	1896–1952	57	102	20	190	0·68
39 rivers. 63 cases. Mean 0·71.						
Results for short periods						
Nile, Lake Albert	1904–1913	10	27	5·5	24	0·90
	1914–1923	10	26	11·7	46	0·96
	1924–1933	10	22	4·3	17	0·86
	1934–1943	10	23	3·2	9	0·65
	1944–1953	10	21	4·3	16	0·80
	1904–1923	20	27	9·2	44	0·68
	1924–1943	20	23	3·8	20	0·72
	1934–1953	20	22	4·0	17	0·63
Nile, Aswan	1870–1879	10	114	14·9	50	0·75
	1880–1889	10	101	10·5	25	0·54
	1890–1899	10	112	13·7	45	0·74
	1900–1909	10	86	12·5	40	0·72
	1910–1919	10	83	18·7	65	0·77
	1920–1929	10	83	8·7	20	0·52
	1930–1939	10	87	8·4	35	0·88
	1940–1949	10	82	10·5	40	0·83
	1870–1889	20	108	14·6	75	0·71
	1890–1909	20	99	18·8	165	0·94
	1910–1929	20	83	14·6	65	0·65
	1930–1949	20	85	9·8	56	0·76
Thames	1886–1895	10	46	8·5	24	0·65
	1896–1905	10	42	16·7	53	0·72
	1906–1915	10	56	14·7	49	0·75
	1916–1925	10	61	18·2	60	0·74
	1926–1935	10	57	15·5	50	0·73
	1936–1945	10	54	18·5	66	0·79

Accumulated Departures. River Discharges (continued)

River	Period	*N*	*M*	σ	*R*	*K*
Thames (continued)	1946–1954	10	52	17·7	48	0·62
	1886–1905	20	44	13·4	70	0·72
	1906–1925	20	58	16·7	86	0·71
	1926–1945	20	56	17·1	74	0·63
	1935–1954	20	54	17·6	92	0·72

31 cases. Mean 0·735.
$N = 10$. Mean 0·746.
$N = 20$. Mean 0·715.

Appendix 5

Accumulated Departures. Lake and River Levels

Station	Period	N	M	σ	R	K
Lake on Dalalven River, Sweden	1765–1808	44	209	33	430	0·84
	1809–1852	44	192	28	210	0·65
	1853–1896	44	182	26	160	0·59
	1897–1940	44	186	26	240	0·71
	1765–1852	88	200	32	600	0·78
	1809–1896	88	187	28	310	0·64
	1853–1940	88	184	26	360	0·69
	1765–1896	132	194	31	740	0·76
	1809–1940	132	187	27	330	0·59
	1765–1940	176	192	30	840	0·75
Rhine, Germany	1881–1930	50	247	32	310	0·70
Lake Runn, Sweden	1852–1938	87	17	2·1	27	0·67
Lake Vattern, Sweden	1858–1939	82	49	14	160	0·66
	13 cases. Mean 0·71.					
Roda Gauge, Egypt	1867–1946	80		0·86	14	0·75
	641–740	100		0·72	10	0·68
	741–840	100		0·61	8	0·65
	841–940	100		0·51	9	0·74
	941–1041	100		0·38	5	0·65
	1042–1142	100		0·44	11	0·82
	1143–1242	100		0·48	14	0·86
	1243–1344	100		0·44	7	0·72
	1345–1445	100		0·66	14	0·78
	1446–1741	100		0·73	11	0·69
	1742–1866	100		0·69	18	0·84

Accumulated Departures. Lake and River Levels (continued)

Station	Period	N	M	σ	R	K
Roda Gauge, Egypt (continued)	1742–1946	180		0·77	22	0·74
	641–840	200		0·68	17	0·70
	741–940	200		0·57	8	0·58
	841–1041	200		0·58	16	0·76
	941–1142	200		0·45	16	0·78
	1042–1242	200		0·47	20	0·82
	1143–1344	200		0·46	15	0·76
	1243–1445	200		0·65	34	0·86
	1345–1741	200		0·74	40	0·86
	1446–1886	200		0·72	21	0·74
	1446–1946	280		0·76	32	0·75
	741–1041	300		0·53	20	0·72
	641–940	300		0·63	21	0·70
	941–1142	300		0·47	24	0·78
	941–1242	300		0·45	29	0·83
	1042–1344	300		0·47	24	0·78
	1143–1445	300		0·61	46	0·86
	1243–1741	300		0·68	42	0·82
	1345–1866	300		0·72	43	0·82
	1345–1946	380		0·76	40	0·76
	641–1041	400		0·60	31	0·74
	741–1142	400		0·52	26	0·74
	841–1242	400		0·51	24	0·72
	941–1334	400		0·45	30	0·80
	1042–1445	400		0·58	47	0·83
	1143–1741	400		0·65	46	0·81
	1242–1866	400		0·69	40	0·77
	1243–1946	480		0·72	41	0·74
	641–1142	500		0·58	30	0·72
	741–1242	500		0·51	22	0·68
	841–1344	500		0·47	26	0·72
	941–1445	500		0·56	50	0·81
	1042–1741	500		0·61	46	0·78
	1143–1866	500		0·66	49	0·78
	1143–1946	580		0·69	58	0·79
	641–1242	600		0·56	34	0·72

Accumulated Departures. Lake and River Levels (continued)

Station	Period	N	M	σ	R	K
Roda Gauge, Egypt (continued)	741–1344	600		0·50	24	0·68
	841–1445	600		0·55	57	0·81
	941–1741	600		0·59	59	0·81
	1042–1866	600		0·63	52	0·78
	1042–1946	680		0·66	62	0·78
	641–1344	700		0·54	40	0·74
	741–1445	700		0·56	56	0·79
	841–1741	700		0·58	64	0·80
	941–1866	700		0·61	68	0·81
	941–1946	780		0·64	80	0·78
	641–1445	800		0·59	74	0·81
	741–1741	800		0·58	64	0·78
	841–1866	800		0·60	74	0·80
	841–1946	880		0·63	80	0·80
	641–1741	900		0·61	78	0·80
	741–1866	900		0·60	76	0·79
	741–1946	980		0·63	83	0·79
	641–1866	1,000		0·69	86	0·78
	641–1946	1,080		0·64	88	0·78

66 cases. Mean 0·77.

Appendix 6

Accumulated Departures. Rainfall

Station	Period	N	M	σ	R	K
Europe						
Greenwich, U.K.	1841–1870	30	610	120	530	0·55
	1871–1900	30	600	83	780	0·82
	1901–1930	30	630	110	500	0·57
	1841–1900	60	610	100	720	0·59
	1871–1930	60	620	98	1,070	0·70
	1841–1930	90	620	100	1,060	0·61
Portsmouth, U.K.	1830–1881	49	40	7·2	55	0·63
	1882–1930	49	42	6·4	47	0·62
	1830–1930	98	41	6·9	86	0·65
Carlsbad, Czechoslovakia	1860–1944	83	60	13	160	0·68
Copenhagen, Denmark	1821–1875	55	57	11	76	0·59
	1876–1930	55	58	7·7	72	0·67
	1821–1930	110	58	9·4	140	0·67
	1821–1938	118	58	9·3	140	0·67
Helsingfors or Helsinki, Finland	1845–1873	29	550	110	600	0·64
	1874–1902	29	640	110	560	0·60
	1903–1930	28	680	88	520	0·67
	1845–1902	58	600	120	1,630	0·78
	1874–1930	57	660	100	880	0·64
	1845–1930	86	630	120	2,500	0·82
Frankfurt, Germany	1837–1883	47	64	14	89	0·59
	1884–1930	47	60	10	110	0·74
	1837–1930	94	63	12	190	0·71
Trier, Germany	1806–1859	36	660	120	1,120	0·78
	1860–1895	36	680	100	640	0·62

Accumulated Departures. Rainfall (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
Trier, Germany	1896–1930	35	730	120	1,050	0·77
(continued)	1806–1895	72	670	110	1,220	0·67
	1860–1930	71	700	110	1,380	0·69
	1806–1930	107	690	120	1,990	0·71
Vanersburg, Sweden	1860–1944	85	72	13	170	0·69
Stockholm, Sweden	1785–1824	40	52	14	160	0·84
	1825–1864	40	38	7·8	70	0·73
	1865–1904	40	47	9·2	90	0·76
	1905–1946	41	58	8·8	80	0·73
	1785–1864	80	45	19	340	0·78
	1825–1904	80	43	15	210	0·73
	1865–1946	81	53	15	240	0·75
	1785–1904	120	46	12	420	0·88
	1825–1946	121	48	12	490	0·91
	1785–1946	161	49	12	660	0·91
Rome, Italy	1782–1831	50	830	160	1,400	0·68
	1832–1881	50	780	160	1,600	0·72
	1882–1932	51	890	170	1,800	0·72
	1782–1881	100	800	160	2,800	0·72
	1832–1932	101	830	180	3,900	0·79
	1782–1932	151	830	170	4,000	0·71
Padua, Italy	1764–1806	43	940	180	2,170	0·81
	1807–1849	43	790	160	1,750	0·78
	1850–1892	43	840	150	900	0·59
	1893–1934	42	820	170	1,850	0·77
	1764–1849	86	870	190	3,900	0·80
	1850–1934	85	830	160	1,550	0·60
	1764–1892	129	860	180	4,200	0·76
	1807–1934	128	820	170	3,000	0·70
	1764–1934	171	850	180	4,500	0·73
Milan, Italy	1764–1789	24	91	14	75	0·68
	1790–1813	24	99	15	58	0·55
	1814–1837	24	100	19	90	0·63
	1838–1861	24	100	21	140	0·77
	1862–1886	25	100	21	120	0·69
	1887–1911	25	100	18	75	0·57

Accumulated Departures. Rainfall (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
Milan, Italy (continued)	1912–1936	25	96	20	140	0·77
	1764–1813	48	95	15	140	0·71
	1790–1837	48	100	17	120	0·61
	1814–1861	48	100	20	220	0·75
	1838–1886	49	100	21	220	0·75
	1862–1911	50	100	20	140	0·62
	1887–1936	50	100	19	200	0·74
	1764–1837	72	97	19	180	0·62
	1890–1861	72	102	19	250	0·72
	1814–1886	73	103	20	240	0·69
	1838–1911	74	104	20	230	0·68
	1862–1936	75	100	20	240	0·69
	1764–1861	96	100	18	340	0·76
	1790–1886	97	102	19	260	0·66
	1814–1911	98	102	20	230	0·63
	1838–1936	99	102	20	260	0·65
	1764–1886	121	100	19	340	0·70
	1790–1911	122	102	19	240	0·62
	1814–1936	123	102	20	270	0·63
	1764–1911	146	101	19	340	0·68
	1790–1936	147	101	19	280	0·63
	1764–1936	171	100	19	440	0·71
Zwanenberg, Holland	1735–1787	53	76	13	130	0·70
	1788–1840	53	69	11	130	0·77
	1841–1893	53	77	13	120	0·67
	1894–1945	52	74	11	72	0·57
	1735–1840	106	73	12	240	0·74
	1841–1945	105	76	12	140	0·60
	1735–1893	159	74	13	270	0·70
	1788–1945	158	74	12	280	0·72
	1735–1945	211	74	12	280	0·66
Africa						
Accra, Ghana	1888–1920	32	27	8	53	0·68
Cape Town,	1838–1868	30	24	4·4	32	0·73
South Africa	1869–1899	31	27	5·9	34	0·64
	1900–1930	31	24	4·3	34	0·76

Accumulated Departures. Rainfall (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
Cape Town, South Africa (continued)	1838–1899	61	25	5·5	74	0·76
	1869–1930	62	25	5·5	79	0·78
	1838–1930	92	25	5·2	100	0·78
Freetown	1875–1920	46	160	29	440	0·87
Lagos	1892–1940	49	71	17	98	0·54
Salisbury, Rhodesia	1896–1940	45	32	6·8	46	0·61
Zanzibar	1892–1930	39	60	13·4	123	0·75
Asia						
Bangalore, India	1835–1885	50	36	8·2	36	0·46
	1886–1930	45	34	7·1	48	0·58
	1835–1930	95	35	7·8	54	0·50
Batavia, East Indies	1864–1940	77	181	33	242	0·55
Calcutta, India	1829–1879	50	66	11	120	0·74
	1880–1930	51	63	11	80	0·60
	1829–1930	101	64	12	160	0·68
Cherrapunji, India	1872–1940	69	431	52	390	0·47
Colombo, Ceylon	1870–1930	61	84	21	306	0·79
Madras, India	1813–1856	44	49	16	110	0·64
	1857–1900	44	48	15	130	0·69
	1901–1945	45	51	14	93	0·60
	1813–1900	88	49	15	120	0·54
	1857–1945	89	50	15	110	0·52
	1813–1945	133	50	15	150	0·55
Australia						
Adelaide	1839–1884	46	21	4·1	35	0·68
	1885–1930	46	21	4·6	32	0·62
	1839–1930	92	21	4·4	38	0·57
Darwin	1870–1930	61	60	11·6	91	0·60
Sydney	1840–1899	50	49	14	91	0·58
	1890–1930	41	46	10	77	0·66
	1840–1930	91	48	12	140	0·63
United States of America						
Albany	1826–1860	35	40	5·3	38	0·69
	1861–1895	35	39	6·0	4·6	0·71

Accumulated Departures. Rainfall (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
Albany (continued)	1896–1930	35	32	4·2	26	0·64
	1826–1895	70	40	5·7	52	0·62
	1861–1930	70	36	6·1	120	0·84
	1826–1930	105	37	6·2	180	0·72
	1826–1945	120	37	6·0	200	0·85
Boston	1818–1860	43	43	7·4	68	0·73
	1861–1903	43	44	7·0	72	0·76
	1904–1945	42	38	5·0	52	0·77
	1818–1903	86	44	7·1	140	0·80
	1861–1945	85	41	6·9	180	0·87
	1818–1945	128	42	7·0	220	0·83
Charleston	1832–1881	50	50	12	170	0·83
	1882–1930	49	44	9·3	140	0·85
	1832–1930	99	47	11	290	0·84
	1832–1945	114	47	11	290	0·82
New York	1826–1860	35	40	7·0	64	0·77
	1861–1895	35	45	6·1	41	0·67
	1896–1930	35	41	4·3	41	0·79
	1826–1895	70	42	7·1	130	0·82
	1861–1930	70	43	5·7	92	0·79
	1826–1930	105	42	6·3	140	0·79
	1826–1945	120	42	6·3	130	0·74
Philadelphia	1820–1861	42	43	6·2	47	0·67
	1862–1903	42	43	7·0	100	0·88
	1904–1945	42	41	5·6	37	0·62
	1820–1903	84	43	6·6	130	0·80
	1862–1945	84	42	6·3	100	0·75
	1820–1945	126	42	6·3	150	0·76
St. Louis	1837–1867	31	43	9·4	80	0·78
	1868–1898	31	38	6·8	34	0·59
	1899–1930	32	37	6·6	41	0·67
	1837–1898	62	41	8·6	120	0·76
	1868–1930	63	38	6·7	42	0·53
	1837–1930	94	40	8·2	150	0·76
St. Paul	1837–1883	47	28	6·2	65	0·74
	1884–1930	47	27	5·4	61	0·77

Accumulated Departures. Rainfall (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
St. Paul (continued)	1837–1930	94	27	5·9	92	0·71
Washington	1824–1869	30	38	8·7	64	0·73
	1870–1899	30	43	8·2	71	0·80
	1900–1930	31	40	5·9	39	0·69
	1824–1899	60	40	8·8	140	0·81
	1870–1930	61	42	7·2	89	0·74
	1824–1930	91	40	8·0	150	0·77
South America						
Fortaleza, Brazil	1849–1920	72	14·3	5·5	60	0·67
La Serena, Chile	1869–1930	62	13·7	8·8	103	0·72
General rainfall, England	1837–1946	110	102	13·6	149	0·60

39 stations. 173 cases. Mean 0·70.

Appendix 7

Accumulated Departures. Temperature

Station	Period	*N*	*M*	σ	*R*	*K*
Adelaide, Australia	1857–1924	68	63	0·89	11	0·72
Greenwich, U.K.	1841–1885	45	50	1·1	8·2	0·64
	1886–1930	45	50	1·0	9·6	0·74
	1841–1930	90	50	1·0	13	0·67
St. Louis, U.S.A.	1900–1930	31	56	1·4	7·6	0·62
	1836–1867	32	55	1·2	6·8	0·62
	1868–1899	32	56	1·3	6·4	0·58
	1868–1930	63	56	1·3	15	0·69
	1836–1899	64	55	1·3	12	0·65
	1836–1930	95	56	1·4	24	0·64
Washington, U.S.A.	1820–1866	32	54	1·6	14	0·80
	1867–1898	32	55	1·1	6·6	0·64
	1899–1930	32	55	1·3	9·2	0·72
	1820–1898	64	54	1·4	18	0·73
	1867–1930	64	55	1·2	12	0·67
	1820–1930	96	55	1·4	25	0·74
Helsingfors, Finland	1829–1862	34	4·0	0·86	3·9	0·54
	1863–1896	34	4·2	1·1	7·1	0·65
	1897–1930	34	4·7	0·87	3·6	0·50
	1829–1896	68	4·1	1·0	7·4	0·57
	1863–1930	68	4·4	1·0	11	0·68
	1829–1930	102	4·3	1·0	18	0·74
	1829–1940	102	4·4	1·1	26	0·78
Vienna, Austria	1825–1938	114	9·3	0·53	14	0·80
Rome, Italy	1811–1850	39	16	0·70	5·1	0·67
	1851–1890	40	15	0·42	4·1	0·77
	1891–1930	40	15	0·32	5·0	0·92

Accumulated Departures. Temperature (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
Rome, Italy	1811–1890	79	15	0·59	9·2	0·74
(continued)	1851–1930	80	15	0·38	6·1	0·76
	1811–1930	119	15	0·54	14	0·79
Albany, U.S.A.	1813–1857	40	48	1·4	12	0·74
	1860–1904	41	48	1·5	12	0·70
	1905–1945	41	48	1·4	14	0·76
	1813–1904	81	48	1·5	22	0·73
	1860–1945	82	48	1·4	24	0·75
	1813–1945	122	48	1·4	21	0·66
Charleston, U.S.A.	1823–1863	41	66	1·3	13	0·76
	1864–1904	41	66	1·0	8	0·69
	1905–1945	41	66	1·0	12	0·82
	1823–1904	82	66	1·2	22	0·78
	1864–1945	82	66	1·0	12	0·67
	1823–1945	123	66	1·1	28	0·77
Wilno, U.S.S.R.	1885–1937	45	6·4	0·81	5·6	0·62
	1781–1832	52	6·3	1·1	8·8	0·64
	1833–1884	52	6·5	1·0	4·8	0·49
	1833–1937	97	6·5	0·92	9·2	0·57
	1781–1884	104	6·4	1·1	14	0·65
	1781–1937	149	6·3	0·97	16	0·65
Copenhagen, Denmark	1768–1810	29	7·9	0·87	3·4	0·51
	1811–1853	43	7·5	0·98	10	0·77
	1854–1896	43	7·4	0·73	3·0	0·47
	1897–1940	44	8·1	0·74	6·4	0·70
	1768–1853	72	7·6	0·95	12	0·71
	1854–1940	87	7·8	0·80	19	0·84
	1768–1896	115	7·6	0·88	18	0·74
	1811–1940	130	7·7	0·87	24	0·80
	1768–1940	159	7·7	0·73	26	0·81
Newhaven	1781–1821	40	49	1·3	18	0·87
	1822–1862	41	49	1·3	11	0·71
	1863–1903	41	49	1·4	14	0·78
	1904–1945	42	50	1·3	13	0·84
	1781–1862	81	49	1·4	18	0·70
	1863–1945	83	50	1·5	30	0·81

Accumulated Departures. Temperature (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
Newhaven	1781–1903	122	49	1·4	25	0·71
(continued)	1822–1945	124	50	1·5	44	0·83
	1781–1945	164	49	1·5	46	0·78
Paris, France	1764–1804	41	11	0·92	13	0·88
	1805–1845	41	11	0·83	6·0	0·66
	1846–1886	41	11	0·57	4·5	0·69
	1887–1930	44	10	0·61	5·8	0·73
	1764–1845	82	11	0·89	15	0·76
	1846–1930	85	10	0·64	12	0·79
	1764–1886	123	11	0·80	15	0·71
	1805–1930	126	10	0·70	14	0·73
	1764–1930	167	11	0·78	18	0·72
Berlin, Germany	1769–1810	42	8·9	0·96	9·9	0·77
	1811–1852	42	8·7	0·88	6·4	0·65
	1853–1895	43	9·1	0·81	5·2	0·61
	1896–1938	43	9·4	0·61	3·4	0·56
	1769–1852	84	8·8	0·92	12	0·69
	1853–1938	86	9·2	0·71	7·0	0·61
	1769–1895	127	8·9	0·90	18	0·76
	1811–1938	128	9·1	0·82	18	0·74
	1769–1938	170	9·0	0·87	25	0·76
Stockholm, Sweden	1899–1942	43	6·1	0·93	9·0	0·74
	1764–1808	45	5·7	0·99	9·0	0·71
	1809–1853	45	5·7	0·99	11	0·77
	1854–1898	45	5·6	0·95	8·0	0·68
	1854–1942	88	5·8	0·98	17	0·76
	1764–1853	90	5·7	0·99	10	0·62
	1764–1898	135	5·7	0·98	14	0·64
	1809–1942	122	5·8	0·98	23	0·75
	1764–1942	178	5·7	0·97	20	0·68
Zwanenburg, Holland	1735–1787	53	9·0	0·75	7·0	0·68
	1788–1840	53	8·8	0·81	8·0	0·70
	1841–1893	53	9·0	0·74	4·5	0·55
	1894–1945	52	9·1	0·53	3·7	0·59
	1841–1945	105	9·1	0·65	4·6	0·49
	1735–1840	106	8·9	0·78	12	0·70

Accumulated Departures. Temperature (continued)

Station	Period	N	M	σ	R	K
Zwanenburg, Holland	1788–1945	158	9·0	0·71	12	0·65
(continued)	1735–1893	159	9·0	0·77	14	0·66
	1735–1945	211	9·0	0·72	16	0·66
Central England,	1698–1762	65	9·2	0·60	8·6	0·76
Annual	1763–1827	65	9·0	0·60	4·7	0·60
	1828–1892	65	9·1	0·65	6·2	0·65
	1893–1957	65	9·4	0·46	5·4	0·70
	1698–1827	130	9·1	0·61	9·6	0·66
	1763–1892	130	9·0	0·63	10·2	0·67
	1828–1957	130	9·2	0·59	12·8	0·74
	1698–1957	260	9·2	0·60	18·3	0·70
Central England,	1698–1762	65	3·7	1·21	11·0	0·64
Winter	1763–1827	65	3·3	1·23	6·1	0·46
	1828–1892	65	3·8	1·37	11·4	0·61
	1893–1957	65	4·1	1·21	11·4	0·64
	1698–1827	130	3·5	1·24	17·0	0·63
	1763–1892	130	3·5	1·33	23·1	0·68
	1828–1957	130	3·4	1·31	19·4	0·64
	1698–1957	260	3·7	1·29	38·6	0·66

120 cases. Mean 0·68.

Appendix 8

Accumulated Departures. Atmospheric Pressure

Station	Period	N	M	σ	R	K
Adelaide, Australia	1857–1924	68	30	0·030	0·38	0·72
Buenos Aires,	1858–1899	42	759·5	0·48	4·85	0·76
South America	1900–1940	41	759·3	0·48	4·15	0·71
	1858–1940	83	759·4	0·49	6·50	0·70
Cape Town,	1842–1870	29	30	0·014	0·082	0·66
South Africa	1871–1900	30	30	0·014	0·090	0·75
	1901–1930	30	30	0·012	0·067	0·63
	1842–1900	59	30	0·013	0·110	0·63
	1871–1930	60	30	0·012	0·088	0·58
	1842–1930	89	30	0·013	0·120	0·58
Copenhagen,	1842–1891	50		9·7	73	0·53
Denmark	1892–1937	46		9·7	55	0·63
	1842–1937	96		9·8	77	0·55
de Bilt, Netherlands	1849–1894	46		9·2	44	0·50
	1895–1940	46		10·3	75	0·63
	1849–1940	92		9·8	106	0·62
Madras, India	1842–1871	30	820	14	94	0·70
	1872–1901	30	820	14	68	0·60
	1902–1930	29	810	11	66	0·68
	1842–1901	60	820	14	110	0·60
	1872–1930	59	820	14	160	0·75
	1842–1930	89	820	13	200	0·69
Stikkisholm, Iceland	1846–1895	50		16·6	104	0·57
	1896–1940	45		14·5	88	0·58
	1846–1940	95		16·0	188	0·64
Vienna, Austria	1851–1894	44	744	0·72	4·9	0·62
	1895–1937	43	744	0·85	4·1	0·51
	1851–1937	87	744	0·78	7·2	0·59

28 cases. Mean 0·63.

Appendix 9

Accumulated Departures. Annual Growth of Trees, Thickness of Rings

Station and Phenomenon	Period	*N*	*M*	*σ*	*R*	*K*
Meadow Valley,	1620–1669	50	2·9	0·71	13	0·90
California, U.S.A.	1670–1719	50	3·0	0·38	5·0	0·80
California pines	1720–1769	50	2·7	0·27	3·0	0·74
	1770–1819	50	3·1	0·67	12	0·91
	1820–1869	50	2·6	0·40	4·0	0·71
	1870–1919	50	2·2	0·48	7·5	0·85
	1620–1719	100	3·0	0·58	15	0·84
	1720–1819	100	2·9	0·55	18	0·89
	1820–1919	100	2·4	0·49	16	0·89
	1620–1819	200	2·9	0·56	19	0·77
	1720–1919	200	2·6	0·59	31	0·86
	1620–1919	300	2·8	0·61	52	0·89
	12 cases. Mean 0·84.					
Pike's Peak, U.S.A.	1570–1619	50	1·0	0·30	3·4	0·75
Pines and Douglas firs	1620–1669	50	0·90	0·38	6·5	0·88
	1670–1719	50	1·0	0·37	7·0	0·91
	1720–1769	50	0·83	0·27	1·1	0·64
	1770–1819	50	1·3	0·23	1·7	0·62
	1820–1869	50	0·99	0·19	2·4	0·79
	1870–1919	50	0·80	0·14	1·3	0·69
	1570–1669	100	0·95	0·35	7·2	0·77
	1670–1769	100	0·93	0·34	12	0·90
	1770–1869	100	1·2	0·26	9·3	0·91
	1820–1919	100	0·90	0·19	5·4	0·85
	1570–1769	200	0·94	0·34	11	0·76
	1670–1869	200	1·0	0·32	15	0·84

Accumulated Departures.
Annual Growth of Trees, Thickness of Rings (continued)

Station and Phenomenon	Period	*N*	*M*	*σ*	*R*	*K*
Pike's Peak, U.S.A., Pines	1720–1919	200	0·98	0·29	19	0·91
and Douglas firs (cont.)	1570–1919	350	0·99	0·32	21	0·81
	15 cases. Mean 0·80.					
Flagstaff, Arizona, U.S.A.	1400–1449	50	1·0	0·35	5·9	0·87
Pines	1450–1499	50	0·87	0·27	2·5	0·69
	1500–1549	50	0·85	0·26	3·0	0·76
	1550–1599	50	1·2	0·34	4·4	0·79
	1600–1649	50	1·0	0·30	4·8	0·86
	1650–1699	50	0·95	0·25	3·5	0·82
	1700–1749	50	1·1	0·30	4·9	0·86
	1750–1799	50	0·92	0·25	3·6	0·83
	1800–1849	50	0·87	0·25	2·6	0·73
	1850–1899	50	1·0	0·35	6·7	0·92
	1400–1499	100	0·95	0·33	7·6	0·80
	1500–1599	100	1·0	0·34	8·6	0·82
	1600–1699	100	0·98	0·28	7·6	0·85
	1700–1799	100	1·9	0·29	7·2	0·82
	1800–1899	100	0·96	0·31	8·7	0·85
	1400–1599	200	0·98	0·33	17	0·85
	1500–1699	200	0·99	0·31	13	0·82
	1600–1799	200	0·99	0·29	9·6	0·76
	1700–1899	200	0·98	0·30	11	0·78
	1400–1899	500	0·98	0·31	18·2	0·74
	20 cases. Mean 0·81.					
California, U.S.A.	1000–1049	50	1·25	0·20	3·8	0·91
Sequoia, or Californian	1050–1099	50	0·99	0·19	3·4	0·90
Redwood	1100–1149	50	0·98	0·13	1·7	0·79
	1150–1199	50	0·88	0·21	1·9	0·68
	1200–1249	50	0·84	0·11	1·5	0·82
	1250–1299	50	0·85	0·15	1·2	0·64
	1300–1349	50	1·13	0·13	0·8	0·56
	1350–1399	50	1·02	0·13	1·8	0·82
	1400–1449	50	0·88	0·15	2·5	0·88

Accumulated Departures.
Annual Growth of Trees, Thickness of Rings (*continued*)

Station and Phenomenon	Period	*N*	*M*	σ	*R*	*K*
California, U.S.A.	1450–1499	50	0·80	0·09	1·8	0·93
Sequoia, or Californian	1500–1549	50	0·92	0·14	1·0	0·61
Redwood (continued)	1550–1599	50	0·78	0·15	1·9	0·79
	1600–1649	50	0·88	0·11	1·2	0·74
	1650–1699	50	0·77	0·08	1·0	0·77
	1700–1749	50	0·83	0·14	2·3	0·86
	1750–1799	50	0·96	0·18	2·2	0·77
	1800–1849	50	0·88	0·13	1·9	0·84
	1850–1899	50	0·94	0·15	3·2	0·94
	1000–1099	100	1·12	0·23	7·0	0·87
	1100–1199	100	0·93	0·16	3·6	0·79
	1200–1299	100	0·86	0·14	2·2	0·71
	1300–1399	100	1·07	0·13	3·2	0·82
	1400–1499	100	0·84	0·13	3·7	0·85
	1500–1599	100	0·85	0·13	3·9	0·87
	1600–1699	100	0·83	0·12	2·9	0·81
	1700–1799	100	0·89	0·17	4·3	0·82
	1800–1899	100	0·91	0·15	4·5	0·87
	1000–1199	200	1·0	0·22	12	0·86
	1200–1399	200	0·97	0·18	12	0·91
	1400–1599	200	0·84	0·14	4·9	0·77
	1600–1799	200	0·86	0·15	7·6	0·86
	1700–1899	200	0·90	0·16	4·7	0·73
	1000–1299	300	0·97	0·21	17	0·87
	1300–1599	300	0·92	0·18	17	0·90
	1600–1899	300	0·88	0·15	9·3	0·82
	1000–1499	500	0·96	0·20	18	0·81
	1400–1899	500	0·86	0·14	13	0·82
	1000–1899	900	0·93	0·18	34	0·86
	38 cases. Mean 0·81.					
	85 cases of tree-rings. Mean 0·81.					
Southern California	1385–1434	50	10·4	4·4	43	0·71
Big Cone spruce index	1435–1484	50	10·3	3·4	28	0·65
	1485–1534	50	9·2	2·7	22	0·65

Accumulated Departures.
Annual Growth of Trees, Thickness of Rings (*continued*)

Station and Phenomenon	Period	*N*	*M*	σ	*R*	*K*
Southern California	1535–1584	50	10·1	3·4	39	0·76
Big Cone spruce index	1585–1634	50	9·6	3·7	52	0·82
(continued)	1635–1684	50	9·7	3·5	42	0·77
	1685–1734	50	10·1	2·2	28	0·80
	1735–1784	50	9·5	3·3	34	0·73
	1785–1834	50	10·8	3·6	39	0·74
	1835–1884	50	9·3	4·4	26	0·56
	1885–1934	50	10·6	3·2	45	0·78
	1385–1484	100	10·4	3·9	67	0·72
	1485–1584	100	9·6	3·1	56	0·74
	1585–1684	100	9·6	3·6	52	0·69
	1685–1784	100	9·8	2·8	32	0·62
	1785–1884	100	10·1	4·1	53	0·65
	1835–1944	110		3·9	64	0·70
	1385–1684	300		3·6	80	0·62
	1685–1944	260		3·5	68	0·61
	1385–1944	560		3·5	102	0·60
	20 cases. Mean 0·70.					
	105 cases. Mean 0·79.					

Appendix 10

Accumulated Departures. Thickness of Annual Layers of Mud, Varves

Station	Period	*N*	*M*	*σ*	*R*	*K*
Lake Saki, Crimea, U.S.S.R.	2090 B.C.–A.D. 1889	50	16	7·9	65	0·65
		50	13	5·5	70	0·78
		50	14	6·9	100	0·83
		50	10	2·8	30	0·74
		50	14	7·1	60	0·66
		50	11	6·3	55	0·67
		50	11	4·6	45	0·71
		50	10	3·2	25	0·64
		50	10	3·2	30	0·69
		50	10	3·0	25	0·66
		50	14	7·0	85	0·77
		50	10	4·3	40	0·69
		50	14	15	105	0·61
		50	13	5·6	35	0·56
		50	14	6·6	60	0·68
		50	10	3·9	25	0·58
		50	12	4·4	35	0·64
		50	14	5·3	55	0·73
		50	11	4·8	45	0·69
		50	11	4·4	45	0·72
		50	13	7·1	65	0·68
		50	11	3·7	45	0·78
		50	16	12	85	0·60
		50	12	4·8	45	0·69
		50	11	4·0	45	0·75
		50	13	7·3	55	0·63
		50	12	4·9	50	0·72

Accumulated Departures.
Thickness of Annual Layers of Mud, Varves (continued)

Station	Period	*N*	*M*	*σ*	*R*	*K*
Lake Saki, Crimea, U.S.S.R. (continued)	2090 B.C.–A.D. 1889 (continued)	50	10	3·9	35	0·68
		50	12	4·1	35	0·66
		50	16	6·6	65	0·71
		50	14	6·3	80	0·78
		50	16	7·6	65	0·66
		50	20	12	120	0·70
		50	18	7·1	70	0·71
		50	13	5·6	45	0·64
		50	15	11	110	0·71
		50	15	7·8	105	0.81
		50	16	6·4	60	0·69
		50	14	6·5	100	0·86
		50	17	8·1	90	0·75
		100	16	7·2	65	0·56
		100	14	6·3	95	0·69
		100	13	7·1	120	0·69
		100	12	9·4	190	0·77
		100	12	6·3	140	0·78
		100	12	5·6	100	0·75
		100	10	4·7	55	0·63
		100	11	3·4	65	0·76
		100	11	5·2	80	0·70
		100	10	3·7	65	0·74
		100	12	6·7	120	0·73
		100	11	5·4	80	0·69
		100	14	12	200	0·73
		100	12	5·8	65	0·62
		100	13	5·5	110	0·76
		100	11	3·7	45	0·64
		100	12	4·0	45	0·62
		100	13	6·1	180	0·86
		100	12	5·8	100	0·72
		100	11	4·9	80	0·71
		100	14	6·2	70	0·62
		100	11	4·8	75	0·70

Accumulated Departures.
Thickness of Annual Layers of Mud, Varves (*continued*)

Station	Period	*N*	*M*	*σ*	*R*	*K*
Lake Saki, Crimea, U.S.S.R. (continued)	2090 B.C.– A.D. 1889 (continued)	100	14	9·7	140	0·69
		100	12	5·3	83	0·70
		100	11	4·8	63	0·65
		100	13	6·2	55	0·56
		100	12	4·8	62	0·65
		100	11	3·9	45	0·62
		100	16	21	280	0·65
		100	14	6·1	110	0·74
		100	14	5·3	100	0·75
		100	15	6·8	75	0·61
		100	21	11	170	0·78
		100	18	8·3	100	0·64
		100	14	5·8	90	0·70
		100	16	10	140	0·66
		100	15	7·4	110	0·69
		100	15	5·7	80	0·68
		100	16	12	170	0·66
		100	16	7·8	140	0·75
		200	15	6·8	160	0·68
		200	13	8·3	270	0·76
		200	12	6·0	150	0·70
		200	11	4·1	110	0·72
		200	11	4·6	110	0·69
		200	12	6·2	180	0·74
		200	12	9·4	180	0·64
		200	12	4·8	150	0·75
		200	12	5·2	200	0·80
		200	12	5·4	120	0·66
		200	12	5·6	160	0·73
		200	13	7·9	200	0·70
		200	12	5·6	120	0·66
		200	12	4·4	120	0·72
		200	15	16	330	0·66
		200	15	6·2	120	0·64
		200	20	9·8	270	0·72

Accumulated Departures.
Thickness of Annual Layers of Mud, Varves (*continued*)

Station	Period	*N*	*M*	*σ*	*R*	*K*
Lake Saki, Crimea, U.S.S.R. (continued)	2090 B.C.– A.D. 1889 (continued)	200	15	8·3	210	0·70
		200	15	6·6	120	0·62
		200	16	10	170	0·61
		500	13	7·5	400	0·72
		500	11	4·6	190	0·67
		500	12	7·6	270	0·65
		500	12	5·0	220	0·69
		500	12	6·5	270	0·67
		500	13	11	520	0·70
		500	16	8·2	710	0·81
		500	16	9·0	300	0·63
		1,000	12	6·4	720	0·86
		1,000	12	6·4	400	0·75
		1,000	13	9·0	620	0·77
		1,000	16	8·6	780	0·72
		2,000	12	6·4	820	0·78
		2,000	14	8·6	2,000	0·87
	114 cases. Mean 0·69.					
Moen, Sogn district, Norway	100–51	50	18	7·8	90	0·76
	50–1	50	15	9·4	130	0·81
	0–51	50	27	14	160	0·76
	52–101	50	23	14	170	0·78
	102–151	50	16	6·4	80	0·79
	152–201	50	16	10	90	0·68
	202–251	50	16	12	95	0·63
	252–301	50	18	17	210	0·78
	302–351	50	24	28	200	0·61
	352–401	50	15	8·4	75	0·68
	402–451	50	19	20	200	0·72
	452–501	50	19	16	140	0·66
	502–551	50	13	6·5	42	0·58
	552–601	50	8·8	5·0	52	0·72
	602–651	50	12	4·8	62	0·79
	652–701	50	17	16	180	0·75

Accumulated Departures.
Thickness of Annual Layers of Mud, Varves (*continued*)

Station	Period	*N*	*M*	σ	*R*	*K*
Moen, Sogn district, Norway (continued)	702–752	50	19	10	110	0·74
	753–802	50	17	6·6	65	0·71
	803–852	50	13	11	150	0·80
	853–902	50	17	15	180	0·78
	100–1	100	16	8·7	150	0·73
	0–101	100	25	14	280	0·76
	102–201	100	16	8·5	140	0·72
	202–301	100	17	15	230	0·70
	302–401	100	19	21	300	0·68
	402–501	100	19	18	240	0·66
	502–601	100	11	6·2	160	0·82
	602–701	100	14	12	230	0·76
	702–802	100	18	8·8	140	0·71
	803–902	100	15	13	340	0·82
	100–101	200	21	13	600	0·84
	102–301	200	16	12	260	0·66
	302–501	200	19	20	310	0·60
	502–701	200	13	9·7	320	0·76
	702–902	200	16	11	400	0·78
	100–401	500	19	15	790	0·72
	402–902	500	16	13	800	0·75
	100–902	1,000	17	14	1,210	0·72
	38 cases. Mean 0·73.					
Tamiskaming, Canada	0–49	50	22	4·8	50	0·73
	50–99	50	24	3·3	25	0·63
	100–149	50	23	4·2	55	0·80
	150–199	50	25	3·1	21	0·60
	200–249	50	20	3·6	35	0·72
	250–299	50	20	3·8	44	0·76
	300–349	50	18	3·0	28	0·70
	350–399	50	18	4·4	45	0·72
	400–449	50	15	2·4	28	0·76
	450–499	50	19	3·1	32	0·72
	500–549	50	19	2·7	41	0·84

Accumulated Departures.
Thickness of Annual Layers of Mud, Varves (*continued*)

Station	Period	*N*	*M*	σ	*R*	*K*
Tamiskaming, Canada (continued)	550–599	50	18	5·0	74	0·84
	600–649	50	19	2·3	30	0·80
	650–699	50	18	3·1	31	0·72
	700–749	50	14	3·0	15	0·50
	750–799	50	13	3·2	38	0·77
	800–849	50	13	2·7	32	0·77
	850–899	50	11	2·4	21	0·67
	900–949	50	12	7·0	140	0·95
	950–999	50	24	5·9	70	0·77
	1000–1049	50	22	3·8	22	0·54
	1050–1099	50	21	3·3	33	0·72
	1100–1149	50	19	3·9	51	0·80
	1150–1199	50	10	3·3	52	0·86
	0–99	100	23	4·5	97	0·79
	100–199	100	24	3·8	85	0·80
	200–299	100	20	3·7	52	0·68
	300–399	100	18	3·8	51	0·66
	400–499	100	17	3·4	100	0·87
	500–599	100	19	4·0	100	0·83
	600–699	100	18	2·8	50	0·74
	700–799	100	14	3·2	62	0·76
	800–899	100	12	2·8	79	0·85
	900–999	100	18	8·8	360	0·95
	1000–1099	100	22	3·6	50	0·68
	1100–1199	100	15	5·4	200	0·92
	0–199	200	23	4·1	120	0·73
	200–399	200	19	3·8	93	0·69
	400–599	200	18	3·8	200	0·86
	600–799	200	16	3·7	220	0·89
	800–999	200	15	7·2	560	0·94
	1000–1199	200	18	5·8	420	0·93
	0–299	300	22	4·3	320	0·86
	300–599	300	18	3·8	200	0·79
	600–899	300	15	3·9	350	0·89
	900–1199	300	18	6·9	780	0·94

Accumulated Departures.
Thickness of Annual Layers of Mud, Varves (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
Tamiskaming, Canada (continued)	0–399	400	21	4·6	500	0·89
	400–799	400	17	3·9	420	0·88
	800–1199	400	17	6·7	1,070	0·96
	0–599	600	20	4·6	700	0·88
	600–1199	600	16	5·9	1,130	0·92
	0–1199	1,200	18	5·6	1,530	0·88
	52 cases. Mean 0·79.					
Lake Corintos, Argentina	B.C.					
	1200–1151	50	19	6·8	69	0·72
	1150–1101	50	15	6·0	39	0·58
	1100–1051	50	20	9·0	131	0·83
	1046–1001	46	22	5·4	48	0·70
	1000–951	49	27	8·4	151	0·90
	950–901	50	20	5·4	58	0·74
	900–851	49	19	6·1	86	0·83
	850–801	49	22	6·8	125	0·91
	1200–1101	100	17	6·8	135	0·76
	1100–1001	100	21	5·7	116	0·77
	1000–901	100	23	7·9	285	0·92
	900–801	100	21	6·6	132	0·76
	1200–1001	200	20	6·6	286	0·82
	1000–801	200	22	7·4	435	0·88
	1200–801	400	20	7·2	528	0·81
	15 cases. Mean 0·80.					
Haileybury, Canada	−349–300	50	23	7·9	134	0·88
	−299–250	50	23	3·7	32	0·68
	−249–200	50	22	3·8	24	0·58
	−199–150	50	22	4·5	23	0·51
	−149–100	50	22	4·4	56	0·79
	−99–50	50	23	3·6	50	0·82
	−49–0	50	21	3·6	26	0·61
	1–50	50	19	3·4	31	0·68
	51–100	50	18	3·7	37	0·72

Accumulated Departures.
Thickness of Annual Layers of Mud, Varves (continued)

Station	Period	*N*	*M*	σ	*R*	*K*
Haileybury, Canada (continued)	101–150	50	16	4·6	70	0·84
	151–200	50	16	3·3	43	0·80
	201–250	50	15	2·8	29	0·72
	251–300	50	15	2·8	37	0·80
	−349–250	100	23	6·2	162	0·84
	−249–150	100	22	4·2	35	0·54
	−149–50	100	22	4·1	78	0·75
	−49–50	100	20	3·6	62	0·73
	51–150	100	17	4·3	71	0·72
	151–250	100	16	3·1	68	0·79
		200	23	5·3	191	0·78
		200	22	4·2	92	0·67
		200	22	4·2	96	0·68
		200	21	4·1	146	0·78
		200	20	4·1	204	0·85
		200	17	4·0	136	0·76
		200	16	4·8	80	0·61
		300	23	4·9		0·73
			22	4·1		0·66
			21	4·1		0·72
			21	4·3		0·83
			22	3·9		0·89
			19	4·5		0·86
			17	4·1		0·82
			17	4·0		0·81
		500				0·86
						0·87
						0·89
						0·89
		650				0·91

39 cases. Mean 0·76.
Total varves: 258 cases. Mean 0·74.

Appendix 11

Accumulated Departures. Sunspot Numbers

Station	Period	N	M	σ	R	K
	1751–1788	38	56	39	300	0·70
	1789–1826	38	28	26	260	0·79
	1827–1864	38	57	34	230	0·65
	1865–1902	38	40	34	280	0·72
	1903–1940	38	45	31	260	0·72
	1751–1826	76	42	36	820	0·85
	1789–1864	76	42	34	730	0·85
	1827–1902	76	49	35	510	0·73
	1865–1940	76	43	32	450	0·72
	1751–1864	114	47	36	910	0·80
	1789–1902	114	42	34	800	0·78
	1827–1940	114	48	34	750	0·77
	1751–1902	152	45	36	840	0·73
	1789–1940	152	43	33	760	0·72
	1751–1940	190	45	35	840	0·70

15 cases. Mean 0·75.

Appendix 12

Relation between Draft (B) and Storage (S) for Rivers, Rainfall and Temperature

Phenomenon	No. of years N	$\frac{M-B}{\sigma}$	S/R
River discharges			
Nile at Aswan	75	0·1	0·82
		0·2	0·63
		0·3	0·45
		0·4	0·28
		0·5	0·14
		0·6	0·12
Sobat and Atbara	66	0·15	0·66
		0·28	0·42
		0·42	0·29
		0·55	0·23
		0·68	0·17
Lake Albert outflow	41	0·1	0·77
		0·2	0·61
		0·3	0·51
		0·4	0·40
		0·5	0·32
		0·6	0·27
Run-off of Pripet and Dnieper	47	0·16	0·43
		0·27	0·40
		1·11	0·21
Water levels at Strasbourg	50	0·22	0·46
		0·84	0·26
Discharge of Kistna	56	0·34	0·20

Relation between Draft and Storage (continued)

Phenomenon	No. of years N	$\frac{M - B}{\sigma}$	S/R
Discharge of Kistna (continued)		1·01	0·08
Discharge of Godavari	56	0·43	0·37
		0·60	0·29
Discharge of Tennessee	61	0·51	0·27
		0·97	0·15
Discharge of Mississippi	63	0·07	0·87
		0·14	0·74
		0·22	0·66
		0·29	0·55
		0·44	0·38
		0·67	0·28
		0·89	0·19
Rainfall			
Greenwich	90	0·14	0·72
		0·62	0·22
		1·09	0·17
Frankfurt	94	0·30	0·35
		0·54	0·29
		0·78	0·24
Copenhagen	100	0·04	0·88
		0·26	0·39
		0·46	0·27
		0·79	0·20
Rome	88	0·06	0·86
		0·12	0·75
		0·18	0·69
		0·30	0·55
		0·42	0·42
		0·60	0·27
		0·78	0·16
Cape Town	87	0·04	0·89
		0·23	0·43
		0·43	0·17

Relation between Draft and Storage (continued)

Phenomenon	No. of years N	$\frac{M - B}{\sigma}$	S/R
Cape Town (continued)		0·62	0·11
		0·82	0·07
Bangalore	95	0·25	0·50
		0·63	0·39
		1·02	0·03
Sydney	91	0·31	0·32
		0·55	0·21
Adelaide	86	0·05	0·66
		0·28	0·34
		0·52	0·24
		0·75	0·15
New York	105	0·29	0·49
		0·60	0·23
		0·92	0·13
Washington	91	0·32	0·47
		0·57	0·26
Albany	105	0·03	1·00
		0·36	0·50
		0·68	0·19
Boston	113	0·04	0·55
		0·33	0·29
		0·61	0·16
		0·89	0·06
St. Paul	94	0·37	0·20
		0·71	0·16
Portsmouth	98	0·04	0·57
		0·34	0·28
		0·63	0·20
		0·92	0·14
Charleston	99	0·27	0·47
		0·55	0·18
		0·82	0·09

Relation between Draft and Storage (continued)

Phenomenon	No. of years N	$\frac{M-B}{\sigma}$	S/R
Temperature			
Greenwich	90	0·25	0·78
		0·45	0·45
Vienna	156	0·24	0·48
		0·45	0·15
Copenhagen	149	0·30	0·42
		0·54	0·35
Helsingfors	102	0·26	0·28
		0·56	0·16
Frankfurt	96	0·33	0·55
		0·60	0·28
Berlin	162	0·24	0·51
		0·58	0·28
Rome	119	0·24	0·48
		0·42	0·38
Wilno	142	0·18	0·42
		0·39	0·34
Zwanenberg	116	0·23	0·65
		0·46	0·49
		0·60	0·09
Charleston	108	0·31	0·27
		0·57	0·15
St. Louis	95	0·23	0·47
		0·44	0·23
Albany	107	0·44	0·36
		0·80	0·19
Washington	96	0·27	0·45
		0·59	0·24
Newhaven	143	0·10	0·74
		0·24	0·41
		0·48	0·26
		0·82	0·18

Number of phenomena: 38. Number of values of S/R: 118.

Relation between Draft and Storage for Roda Gauge, Varves and Tree-rings

	$(M-B)/\sigma$	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7
Phenomenon	$\sqrt{(M-B)/\sigma}$	0	0·32	0·45	0·55	0·63	0·71	0·77	0·84
	N	Values of S/R							
Roda Gauge	300	1·0	0·72	0·51	0·28	0·26	0·23	0·19	0·15
	500	1·0	0·55	0·37	0·26	0·21	0·16	0·10	0·07
Tamiskaming	400	0·92	0·74	0·56	0·42	0·29	0·19	0·12	0·10
varves	400	0·91	0·68	0·60	0·50	0·42	0·35	0·25	0·21
	399	0·70	0·61	0·53	0·45	0·35	0·29	0·23	0·17
Moen varves	500	0·88	0·51	0·23	0·14	0·08	0·06	0·04	0·02
	500	1·0	0·71	0·45	0·25	0·16	0·09	0·04	0·02
Flagstaff tree-rings	500	0·94	0·73	0·55	0·40	0·29	0·20	0·14	0·13
Pike's Peak tree-rings	350	0·55	0·43	0·34	0·24	0·14	0·14	0·03	0·03
Meadow Valley tree-rings	300	0·78	0·68	0·55	0·48	0·43	0·39	0·34	0·29
Sequoia tree-rings	500	1·0	0·64	0·55	0·47	0·38	0·28	0·20	0·16
	500	1·0	0·69	0·51	0·36	0·22	0·18	0·16	0·13

Number of phenomena: 7. Number of cases of S/R: 96.

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