2.2 (1) The ideal lowpass filter is not causal.
For an input signal $x(t)$, the correspondent output $y(t) = x(t) * g(t) = \int_{-\infty}^{+\infty} x(\tau) g(t-\tau) d\tau$

From the form of $g(t)$, we can see the convolution result at t_0 also depends on $x(t), t>t_0$

(2) It's NOT possible to build a noncausal filter with infinite tail in the reel world

2.4 For $t>\alpha$,

$$P_\alpha y = P_\alpha H u = P_\alpha H P_\alpha u = 0 \quad ①$$

Now consider $t<\alpha$,

$\because$ $P_\alpha u = u$, $\forall t<\alpha$

$\because$ H is causal

$\therefore$ $HP_\alpha u = Hu$, $\forall t<\alpha$ (according to causality definition)

$\therefore$ $P_\alpha H P_\alpha u = P_\alpha H u$, $\forall t<\alpha$ ②

Hence, combine ① and ②

when H is causal

$$P_\alpha y = P_\alpha H u = P_\alpha H P_\alpha u$$

2.6 $\quad y(t) = f\{u(t)\} = \begin{cases} u^2(t)/u(t-1), & \text{if } u(t-1) \neq 0 \\ 0, & \text{if } u(t-1) = 0 \end{cases}$

① Homogeneity

For $u(t-1) = 0$.

$\alpha u(t-1) = 0$

$\therefore \; f\{\alpha u(t)\} = 0 = \alpha f\{u(t)\}$

For $u(t-1) \neq 0$

$$f\{\alpha u(t)\} = \frac{\alpha^2 u(t)}{\alpha u(t-1)} = \alpha \frac{u(t)}{u(t-1)} = \alpha f\{u(t)\}$$

$\therefore$ The system is homogeneous.

② Additivity

Consider $u_1(t) = 1$, $u_2(t) = t$

$$f\{u_1(t) + u_2(t)\} = f\{1+t\} = \frac{1+2t+t^2}{t}$$

$$f\{u_1(t)\} + f\{u_2(t)\} = f\{1\} + f\{t\} = 1 + \frac{t^2}{t-1}$$

At $t=2$. $f\{u_1(t) + u_2(t)\} = 9/2$

$f\{u_1(t)\} + f\{u_2(t)\} = 5$

$\therefore \; f\{u_1(t) + u_2(t)\} \neq f\{u_1(t)\} + f\{u_2(t)\}$.

2.7 For $\forall\ m \in \mathbb{Z}^{+}$,

$$f(mt) = f(\underbrace{t+\cdots+t}_{m}) = \underbrace{f(t)+\cdots+f(t)}_{m} = mf(t) \quad ①$$

For $m=0$

$$f(0) = f(0+0) = f(0) + f(0)$$

$$\therefore\ f(0) = 0 \quad ②$$

For $m=-1$

$$f(-t) = f(-t) + f(t) - f(t)$$

$$= [f(-t) + f(t)] - f(t)$$

$$= [f(t-t)] - f(t) \quad \text{[Additivity]}$$

$$= f(0) - f(t)$$

$$= -f(t) \quad \text{[from ②]}$$

$$\therefore\ f(t) = f(-t) \quad ③$$

$\therefore$ Combine ①②③, we get

$$\text{for } \forall\ m \in \mathbb{Z} \quad f(mt) = mf(t) \quad ④$$

For $\forall n \in \mathbb{Z}$

$$f(t) = f(n\tfrac{t}{n}) = f(\underbrace{\tfrac{t}{n}+\cdots+\tfrac{t}{n}}_{n}) = nf(\tfrac{t}{n})$$

$$\therefore\ \frac{1}{n}f(t) = f(\tfrac{t}{n}) \quad ⑤$$

Hence combine ④ and ⑤, for any rational number α.

$$f(\alpha t) = \alpha f(t)$$

2.8 Define $\begin{cases} x = t+\tau \\ y = t-\tau \end{cases}$

Then we can get

$$\begin{cases} t = \frac{x}{2} + \frac{y}{2} \\ \tau = \frac{x}{2} - \frac{y}{2} \end{cases}$$

$$\therefore \quad g(t,\tau) = g\left(\frac{x}{2}+\frac{y}{2}, \frac{x}{2}-\frac{y}{2}\right)$$

$$= g\left(\frac{y}{2}, -\frac{y}{2}\right)$$

$\because$ $g\left(\frac{y}{2}, -\frac{y}{2}\right)$ is not a function of x

$$\therefore \quad \frac{\partial g(t,\tau)}{\partial x} = \frac{\partial g(y/2, -y/2)}{\partial x} = 0$$