EE 547: Linear Systems

Control of a Fan-Powered Vehicle

Homework 9

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Assigned: Dec. 2, 2009
Due: in class, Dec. 11, 2009

1 Instructions

1. Your project writeup should be in the form of a *term paper*. Please write in complete sentences and explain what you are doing. Choose your figures and captions carefully and show the relevant details of any derivations. Part of your grade will be based on presentation.

2. Your write-up should clearly address each of the problems listed below.

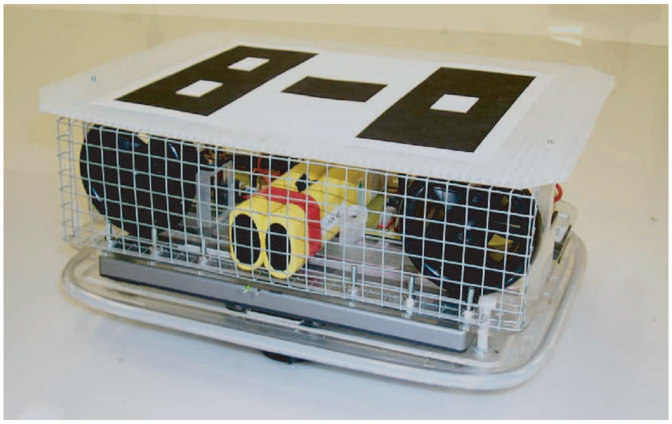
3. The write-up is limited to 6 SINGLE SIDED PAGES! Therefore, you may wish, for example, to scale your figures so that several can fit onto one page.

4. The work you turn in must be yours and your alone. You are encouraged to discuss the project with other students, but you must work through all the problems yourself and do all the MATLAB and Simulink problems yourself. Please cite the names of anyone who provided substantial assistance or input.

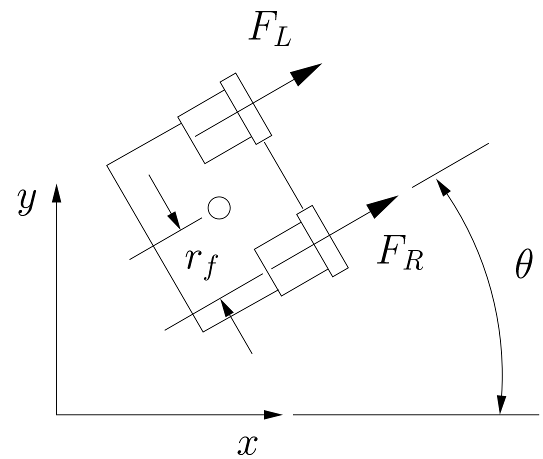
2 Introduction

The goal of this project will be to control the heading and speed of the fan-powered vehicle, named *Kelly*, shown in Figure 1. The vehicle moves on a flat surface on low-friction omni-directional casters. Two high-performance fans are mounted on the top of the vehicle and provide differential thrust to propel the vehicle forward and to turn it. The fans are controlled locally by an onboard computer that is able to communicate with a control station and with other vehicles. A positioning system consisting of an overhead camera broadcasts the locations and orientations of the vehicles in the lab.

Several copies of the vehicle were developed at Caltech by the instructor and his collegues to create a *multi-vehicle wireless testbed*. The goal was to test formation control with vehicles whose dynamics are non-trivial (the Kelly dynamics are similar to an aircraft's) but still inexpensive and easy to use in a laboratory setting.



(a)



(b)

Figure 1: Kelly: a fan-powered vehicle. (a) A photo of the Kelly vehicle. (b) The geometry and variable denoations used to describe the vehicle.

In this project you will control the heading and speed of the vehicle to follow an input command. You will develop various separate controllers using state space techniques, integral feedback and eventually an observer.

> **PROBLEM 1:** Make a Simulink model of the full nonlinear system and turn it into a subblock. The subblock should take as input the two forces and as ouput the state (or just the velocity and heading, depending on what problem you are working on). You will use this block in several of the problems below. Each problem builds on the previous, so you will make a series of simulink models that build on the simpler ones. You should also set up some "scopes" to visulaize various quantities. Show the step response of the vehicle with different magnitude step inputs for each fan.

3 A Model of the Vehicle

The vehicle is modeled by the following nonlinear, state space equation:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ \omega \\ \frac{1}{m}\left(-\eta v_x + F\cos(\theta)\right) \\ \frac{1}{m}\left(-\eta v_y + F\sin(\theta)\right) \\ \frac{1}{J}\left(-\psi\omega + Tr_f\right) \end{pmatrix}, \tag{1}$$

where (x, y) is the position of the center of mass of the vehicle, θ is the orientation of the vehicle, (v_x, v_y) is the velocity vector of the center of mass of the vehicle and ω is the angular velocity of the vehicle (see Figure 1(b)).

parameter	value	meaning
m	5.1 kg	mass
J	0.050 kg m^2 / s	moment of inertia
η	4.0 kg / s	coefficient of linear friction
ψ	0.055 kg m^2 / s	coefficient of rotational friction
r_f	0.123 m	distance from COM to fan

Table 1: The parameter values and definitions for the vehicle model.

The force F is equal to the sum of the forces on the left and right fans:

$$F = F_R + F_L;$$

and the torque T is equal to their difference:

$$T = F_R - F_L.$$

The paramters m, J, η, ψ and r_f were measured in the lab and are described in Table 1.

For this project, we will suppose that the speed and heading of the vehicle are available via sensors. Note that *the heading is not the same as the orientation.* The vehicle can be facing one direction and moving in a different direction. The nonlinear output map for the system is thus

$$\begin{pmatrix} v \\ \alpha \end{pmatrix} = \begin{pmatrix} \sqrt{v_x^2 + v_y^2} \\ \text{atan}(v_y, v_x). \end{pmatrix}. \tag{2}$$

Note that we are using the two-place arc-tangent function (look up `atan2` in the MATLAB documentation) to give the heading.

4 Linearization and Controllability

To design a controller, you must obtain a linear model of the system that is appropriate to the task at hand. The main issue is that the system may not be controllable if careful attention is not given to this process.

> **PROBLEM 2:** Show that the vehicle model in Equation 1 is not controllable around any equilibrium. That is, suppose $\mathbf{x}^*$ is an equilibrium and determine the A, B and C matrices for the error dynamics using Equation 1. Show that the resulting system is uncontrollable.

One reason that the system is not controllable is that the position cannot be determined from velocity and heading alone. Thus, define a new model as follows:

$$\begin{pmatrix} \dot{\theta} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} \omega \\ \frac{1}{m}\left(-\eta v_x + F\cos(\theta)\right) \\ \frac{1}{m}\left(-\eta v_y + F\sin(\theta)\right) \\ \frac{1}{J}\left(-\psi\omega + Tr_f\right) \end{pmatrix}, \tag{3}$$

In this model, we have simply taken out the position variables, which were just integrating the velocity and which have no effect on the heading and velocity.

PROBLEM 3: Determine the A, B and C matrices for the error dynamics using Equation 3. Show that resulting system is still not controllable around any equilibrium.

The intuitive reason that this new model is not controllable is that the vehicle cannot move sideways when it is not moving. Thus, we define a third model:

$$\begin{pmatrix} \dot{\theta} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} \omega \\ \frac{1}{m}\left(-\eta v_x + (F_{nom} + \Delta F)\cos(\theta)\right) \\ \frac{1}{m}\left(-\eta v_y + (F_{nom} + \Delta F)\sin(\theta)\right) \\ \frac{1}{J}\left(-\psi\omega + T r_f\right) \end{pmatrix}, \tag{4}$$

where we change the control inputs to $(\Delta F, T)$ instead of (F, T). The constant F_{nom} is the *nominal force* required to keep a nominal velocity v_{nom}.

In this project, we will control the system around the following equilibrium

$$\begin{pmatrix} \theta^* \\ v_x^* \\ v_y^* \\ \omega^* \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

That is, the nominal situation is where the vehicle is facing along the x-axis, is moving with unit velocity along the x-axis and is not rotating. To maintain the velocity $v_{nom} = 1$, a nominal force of $F_{nom} = \eta v_{nom} = \eta$ is required.

PROBLEM 4: Determine the A, B and C matrices for the error dynamics using Equation 4. Show that the resulting system is controllable around the equilibrium defined above. The input to the system should be ΔF and T and the output should be $v_e = v - v_{nom}$ and α.

5 Full-State Feedback

PROBLEM 5: Design a full-state feedback controller for the system you found in Problem 4. Choose the poles to make the system stable and so that its step response does not saturate the fans, which we suppose cannot deliver a force more than 5 N. Plot the impulse response and/or step response to the linear and nonlinear models. For the nonlinear Simulink model, use the state $\vec{x}$ from the vehicle model block and ignore v and α.

PROBLEM 6: Determine the effect of a disturbance applying a -1 N input on the force input F and a 0.5 $N \cdot m$ torque on the torque input T . That is, determine the steady state error formally with the linear model and show it in simulation.

PROBLEM 7: Add integral action to your controller and show in simulation that it rejects the above disturbance.

PROBLEM 8: Show that the system you found in Problem 4 is observable using v and α and build an observer for your system. Demonstrate the observer dynamics with your controller with plots using the nonlinear model.

Note: Once again, be sure to adjust the input and output of the Simulink model before interfacing it with your controller. The output velocity of the Simulink model is v_x and v_y, but you will need $v_{x,e}$ and $v_{y,e}$, the velocity errors.