Formation Flying Blimps Project

AA449 Milestone Report 2

Beth Boardman

Linh Bui

Kyle Odland

Matthew Walker

Maggie Wintermute

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Introduction

The fundamental goal of the Formation Flying Blimps project is to design and implement two autonomously controlled blimp vehicles into the University of Washington Distributed Space System Laboratory. The focus of this report is to identify the system model parameters to adequately characterize the system. First the fundamental system inputs and outputs are defined in terms of the desired control modes. Next, the state vector is identified and the dynamics of the blimp are derived. The relationship between the input state and the dynamics are then linearized about a characteristic operating point, and then represented in a classical state space model. With this model, the controllability and observability of the blimp system are then examined.

Following the discussion of the system model, a modified budget list reflecting changes since MS1 and a project status update regarding current work and difficulties from each subsystem are provided.

Nomenclature

α: thrust, voltage slope constant [N/V]

β: torque, voltage slope constant [Nm/V]

A_c: cross sectional area [m^2]

C_D: coefficient of drag

C_m: controllability matrix

D: drag parameter, defined for convenience as

$$D = 0.5\frac{C_D \rho A_c}{M} \text{ [m}^{-1}\text{]}$$

J: moment of inertia [kgm^2]

M: mass [kg]

r: blimp radius [r]

r_f: distance from blimp center of mass to center of motor/propeller [m]

r_B: distance from blimp center of mass to blimp center of buoyancy [m]

F_B: force of buoyancy [N]

F: motor force [N]

O_m: observability matrix

n: rank of matrix

τ: motor torque [Nm]

V: motor voltage [V]

ρ: density [kg/m^3]

x: position along x-axis [m]

y: position along y-axis [m]

z: position along z-axis [m]

θ_x: rotation about x-axis [°]

θ_y: rotation about y-axis [°]

θ_z: rotation about z-axis [°]

$\dot{x}$: velocity along x-axis [m/s]

$\dot{y}$: velocity along y-axis [m/s]

$\dot{z}$: velocity along z-axis [m/s]

$\dot{\theta}_x$: rotation rate about x-axis [°/s]

$\dot{\theta}_y$: rotation rate about y-axis [°/s]

$\dot{\theta}_z$: rotation rate about z-axis [°/s]

$\ddot{x}$: acceleration along x-axis [m/s^2]

$\ddot{y}$: acceleration along y-axis [m/s^2]

$\ddot{z}$: acceleration along z-axis [m/s^2]

$\ddot{\theta}_x$: rotation rate about x-axis [$°/s^2$]

$\ddot{\theta}_y$: rotation rate about y-axis [$°/s^2$]

$\ddot{\theta}_z$: rotation rate about z-axis [$°/s^2$]

Subscripts:

b: body reference frame

e: inertial reference frame

Determination of Model Parameters

Figure 1 shows the system diagram as the blimp undergoes translational motion in one direction.

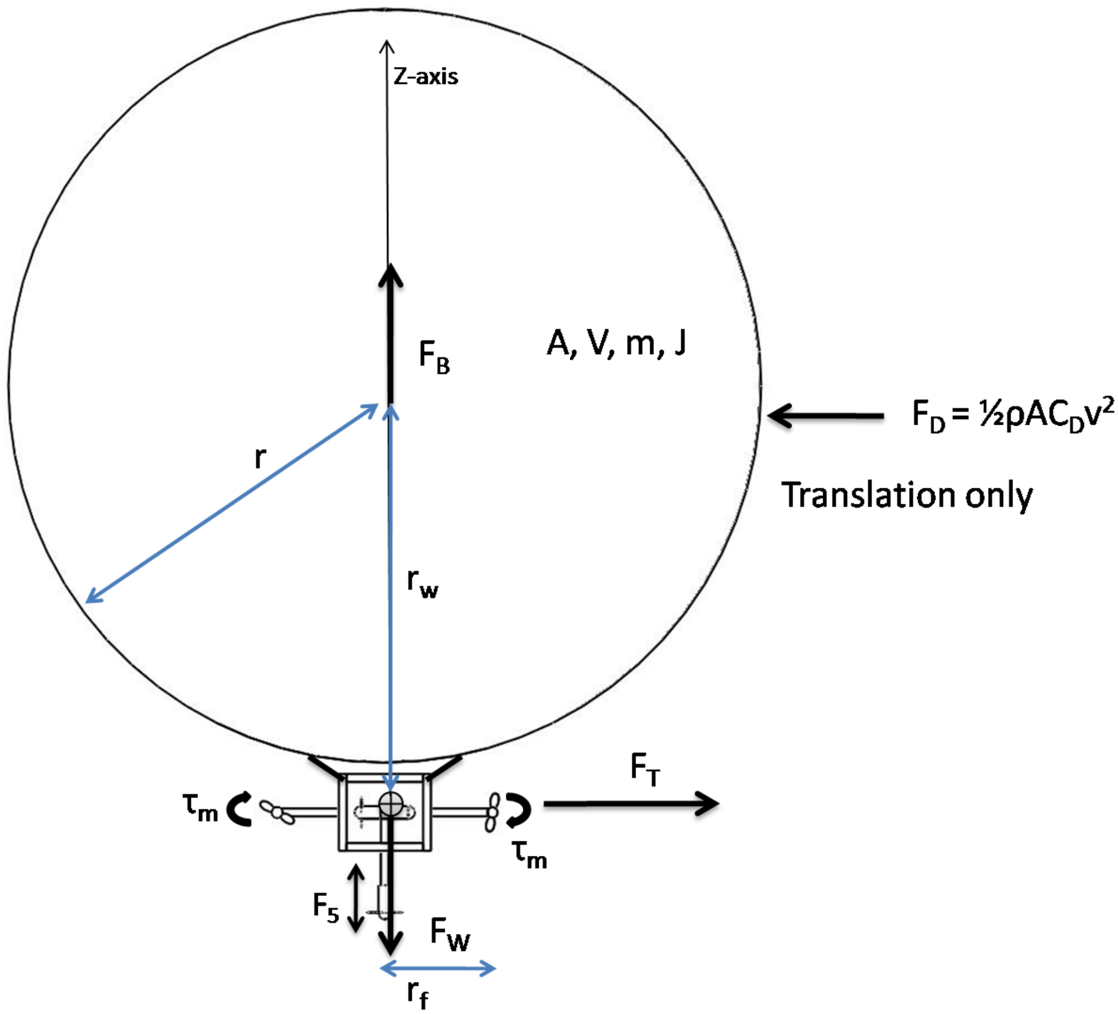

Figure 1. Identification of system parameters.

The parameters of the system will be estimated and calculated in order to produce an accurate model of the system. The following parameters have been identified.

- Mass (.5 kg) : estimated by summing part masses from component list
- Volume of envelope (1 m^3)
  - directly related to mass, since blimp is neutrally buoyant, left must equal weight

- Blimp envelope will use He with known density, so specifying lift specifies volume
- Blimp is spherical, so radius and cross sectional area known

- Drag
  - C_D of sphere = 0.5, area and density known, so drag is a function of velocity only
- Moment of Inertia (.03 kgm^2)
  - Current estimate calculated using solid sphere approximation with added gondola volume
  - Solidworks model can also provide moment of inertia estimate
  - Testing of physical plant will validate estimated values
- Motor Properties
  - Tests were conducted to determine thrust and moment as a function of input voltage

System Modeling

System Inputs and Outputs

As detailed in MS1, each blimp will only operate in the Distributed Space System Laboratory Vicon test area. The six Vicon cameras can track the translational and angular position over a period of time for any object in this region of space over. Direct line-of-sight to only a minimum of three cameras is necessary to determine these positions. This means that even if both potential blimps are flying in close proximity, accurate tracking results can be readily achieved. By numerically differentiating the position tracking results, the translational and angular rates of each blimp can be determined by the Vicon system.

The project goals outlined in MS1 are reiterated in this report here to identify the necessary system outputs.

1. Construct one working blimp vehicle that can complete simple waypoint tracking within the confines of the Distributed Space System Laboratory testbed facility, robust enough to be used for the future plans of the lab.
2. Construct a second working blimp vehicle that can complete simple waypoint tracking within the confines of the Distributed Space Systems Laboratory testbed facility, robust enough to be used for the future plans of the lab.

3. Derive and implement the control algorithms for a series of coordinated tasks for both operational blimp vehicles to conduct. This will start as a simple Lead/Follow scheme, and evolve to steadily more difficult tasks should project time allow.

To fulfill the first two goals, the only necessary system outputs will be the position and planar orientation of the blimp in relation to a desired point in the DSSL test area. To fulfill the final goal, the rates of position and orientation will also be necessary outputs. At this stage in the project, a linearized model with inertial position and planar orientation as outputs has been determined, fulfilling the first two project objectives. A model that satisfies the final project goal is currently under investigation.

A neutrally buoyant blimp platform for a controllable aerial vehicle was chosen due to its inherent stability and simplicity. The translational and rotational dynamics of each blimp are dominated by the thrust and torque generated from the motor/propeller actuators. As will be demonstrated in the derivation of the blimp dynamics, aerodynamic effects cause nonlinear terms in the plant model. The inputs to the blimp vehicle system will be the voltages across each motor.

Reference Frames

In order to achieve waypoint tracking control, it was necessary to determine the dynamics of the blimp in an inertial reference frame. This is due to the fact that commands to the system will necessarily be given in an earth centered reference frame, since positions commands are meaningless in a reference frame with a moving origin. The equations of motion for the blimp system are more simply formulated in a frame of reference fixed to the body. The two reference frames are related rotationally by 3 Euler angles (typically called roll, pitch, and yaw). These angles represent the deviation of the body fixed axes from some fixed inertial axes. Figure 2 shows the difference between the inertial reference frame and the blimp reference. A diagram demonstrating the forces and moments acting in the x-y plane in the blimp reference frame is shown in Figure 3.

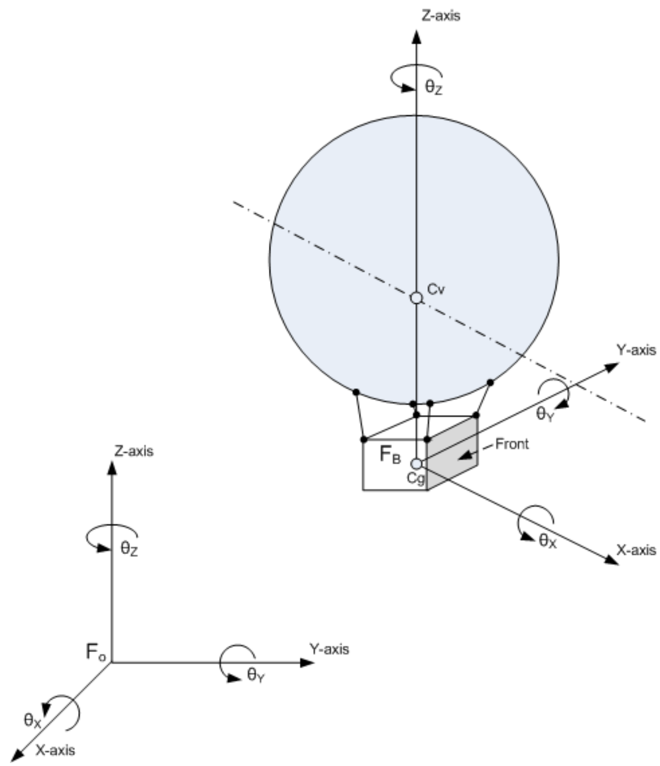

Figure 2. Blimp system reference frames

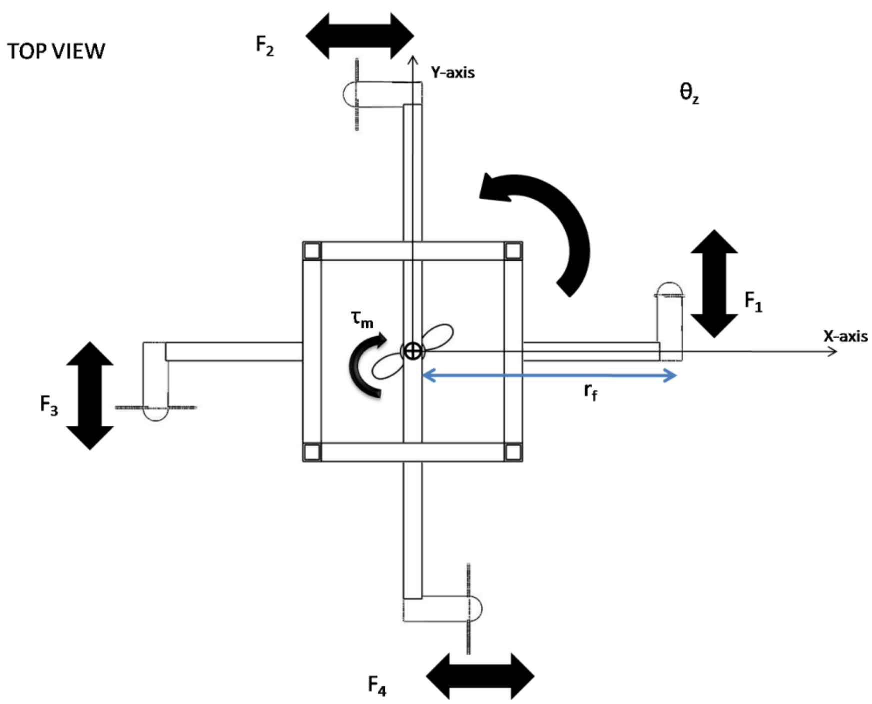

Figure 3. Top view of physical system

Control of the blimp must be formulated in the inertial reference frame in order to facilitate way point tracking. This means that the output states of the plant model must include the inertial frame positions and the Euler angles (specifically yaw about the z axis). In order to relate the body-fixed equations of motion to the inertial frame, two transformations are applied. The transformations are represented as matrices whose elements are functions of the Euler angles. The transformation matrices are multiplied by the velocities and angular velocities in the body frame to output these same quantities in the inertial frame[1,2]. In order to represent the dynamics of the system (both its equations of motions and the coordinate transformations) the state vector of our system will include variables from both the body-fixed and inertial frames, as well as the Euler angles which relate the two. Applying the two transformations creates a system of non-linear equations relating the two frames, which (from the definition of system states) is part of the larger equation $\dot{x} = f(x, u)$. The states of the system are shown below.

$$\vec{x} = \begin{bmatrix} x_b \\ y_b \\ z_b \\ \dot{x}_b \\ \dot{y}_b \\ \dot{z}_b \\ \dot{\theta}_{xb} \\ \dot{\theta}_{yb} \\ \dot{\theta}_{zb} \\ \theta_x \\ \theta_y \\ \theta_z \\ x_e \\ y_e \\ z_e \end{bmatrix} \quad (1)$$

The full description of the system state relationships is given in equations 1-15. Equations 5-10 describe the dynamics of the system in the body frame, including thrust and drag forces, moments due to the torques of the motors, and moments due to potential displacement between the centers of buoyancy and gravity (where the always-vertical lift and weight forces are applied).

$$\dot{x}_b = \dot{x}_b \quad (2)$$

$$\dot{y}_b = \dot{y}_b \quad (3)$$

$$\dot{z}_b = \dot{z}_b \quad (4)$$

$$\ddot{x}_b = F_2 - F_4 - D\dot{x}_b{}^2 \quad (5)$$

$$\ddot{y}_b = F_1 - F_3 - D\dot{y}_b{}^2 \quad (6)$$

$$\ddot{z}_b = F_5 - D\dot{z}_b{}^2 \quad (7)$$

$$\ddot{\theta}_{xb} = \tau_2 - \tau_4 + F_B r_B \cos\theta_y \, sin\theta_x \quad (8)$$

$$\ddot{\theta}_{yb} = -\tau_1 + \tau_3 - F_B r_B sin\theta_y \quad (9)$$

$$\ddot{\theta}_{zb} = -\tau_5 + r_f(F_1 + F_2 + F_3 + F_4) \quad (10)$$

$$\begin{bmatrix} \dot{\theta}_{xe} \\ \dot{\theta}_{ye} \\ \dot{\theta}_{ze} \\ \dot{x}_e \\ \dot{y}_e \\ \dot{z}_e \end{bmatrix} = \begin{bmatrix} 1 & t\theta_y s\theta_x & t\theta_y c\theta_x & 0 & 0 & 0 \\ 0 & c\theta_x & -s\theta_x & 0 & 0 & 0 \\ 0 & \frac{s\theta_x}{c\theta_y} & \frac{c\theta_x}{c\theta_y} & 0 & 0 & 0 \\ 0 & 0 & 0 & c\theta_y c\theta_z & c\theta_z s\theta_x s\theta_y - c\theta_x s\theta_z & c\theta_x c\theta_z s\theta_y + s\theta_x s\theta_z \\ 0 & 0 & 0 & c\theta_y s\theta_z & c\theta_x c\theta_z + s\theta_x s\theta_y s\theta_z & -c\theta_z s\theta_x + c\theta_x s\theta_y s\theta_z \\ 0 & 0 & 0 & -s\theta_y & c\theta_y s\theta_x & c\theta_x c\theta_y \end{bmatrix} \begin{bmatrix} \dot{\theta}_{xb} \\ \dot{\theta}_{yb} \\ \dot{\theta}_{zb} \\ \dot{x}_b \\ \dot{y}_b \\ \dot{z}_b \end{bmatrix} \quad (11)$$

The above relationship for the transformation of rates applies the two Euler angle rotation matrices. The resulting system is highly non-linear, and must be linearized before a state space model can be formulated. Representing the system in state space form will aid in the development of an effective controller at a later state in the design process. Though any arbitrary point of linearization is technically feasible, the linearization points for the following state space model were chosen for simplicity and to reflect the equillibirium position of the blimp. The equations for $\dot{x} = f(x, u)$ are linearized about 0 for all states except θ_z and $\dot{x}_b$, which are left as symbolic constants. Future work on the model will include linearizations about non-zero velocities in all translational directions. Due to the nature of the original non-linear equations, choosing non-zero operating points means that some constant terms due to drag and rotations will remain in the linearized system. These terms are treated as noise to the system, and are introduced to the state space model in the vector *w*. The form of the state space model and the linearized A and B matrices are shown below[3].

$$\dot{x} = Ax + Bu + w \quad (12)$$

$$y = Cx + Du \quad (13)$$

The inputs to the system, u, will be the voltages applied to the five motors. The forces and torques shown in equations 5 through 10 are currently modeled as being directly proportional to the input voltages. The outputs of the system will be only the inertial positions and the Euler angles, so the C matrix from equation 13 will contain mostly zeros, and ones only in the diagonal entries for the yaw and inertial positions.

$$A = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2D\tilde{\dot{x}}_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r_b F_b & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r_b F_b & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathrm{c}\tilde{\theta}_z & -\mathrm{s}\tilde{\theta}_z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\dot{x}}_b\left(-\mathrm{s}\tilde{\theta}_z\right) & 0 & 0 & 0 \\
0 & \mathrm{c}\tilde{\theta}_z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\dot{x}}_b & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{\dot{x}}_b & 0 & 0 & 0 & 0
\end{bmatrix} \quad (14)$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\alpha & 0 & 0 & \alpha & 0 \\ \alpha & 0 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha \\ 0 & -\beta & 0 & \beta & 0 \\ -\beta & 0 & \beta & 0 & 0 \\ \alpha r_f & \alpha r_f & \alpha r_f & \alpha r_f & \beta \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

Controllability and Observability

The state space representation detailed in equations 12-15 models the blimp as a linear, time-invariant system about the operating point. In the process of designing a control system, once a linear model has been achieved, classical linear control design techniques become applicable in this regime. However, additional verifications to the state space representation are necessary before control design can commence.

In general, a given state space representation of a linear, time-invariant system is not unique. In theory, any combination of states, inputs and outputs that characterizes the behavior of the system can modeled in the traditional state space form. Before control can be implemented on a system that is represented by a given state space model, this representation of the system must be completely state controllable and completely observable.[5]

A system is defined as completely state controllable if the unconstrained control inputs can transfer the system between any initial and final state.[6] Essentially, if a state variable exists in a system that cannot be effected by the inputs, the system is not controllable. The controllability of a system can be rapidly determined from the A and B matrices of a state space model by comparing the rank of the controllability matrix, which is shown in equation 16, to the number of state variables.[3] Rank is defined or the number of linear independent equations represented by a certain matrix. The system is controllable if the rank of the control matrix is equal to the order of the system, or the number of state variables.[6]

$$C_M = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B] \; (16)$$

Where *n* is the order of the system

A system is defined as completely observable if every state can be found from the system output. Fundamentally, a system is completely observable if all relations on the system state effect the system output. As with controllability, observability matrix of a state space representation can be rapidly determined, in this case with the A and C matrices, which is shown in equation 17. A system is observable if the rank of the observability matrix is equal to the order of the system.[6]

$$O_M = [C \quad CA \quad CA^2 \quad \ldots \quad CA^{n-1}]^T \; (17)$$

Where n is the order of the system

The state space representation used in this determination of the plant model incorporated dynamics and inputs with respect to the body reference frame, and the transition between these states. As there are 15 separate state variables, 5 control inputs and 4 system outputs for waypoint tracking, it was expected that the initial state space representation would be neither fully controllable nor observable. Using the control functions in Matlab, the rank of the C_M for this representation is 9, while the rank of the O_M is 13.

This result does not mean that the system is will not be controllable in waypoint tracking. It only means that the representation must be modified in such a way that the controllability and observability conditions are satisified. The process of modifiying the realization of the system to meet these criteria is known as minimum realization, which can be accomplished with a sequence of matrix row operations on the total system matrix.[5]

While there may be many representations for a system that are controllable and observable, the transformation from a specific realization to a minimized state is unique. Thus for each linearized operating point for the blimp system, a minimum realization on the state will be performed.

Performing these minimum realizations and identifying state variable reductions that result are under further investigation under direct guidance of the graduate students employed by the Distributed Space Systems Lab.

Simulation Results

There are currently two working simulators for the blimp system. The fully non-linear model utilizes the Euler angles block to automatically transform from body-fixed inputs to inertial frame outputs. The block diagram for this system is shown in Figure 4.

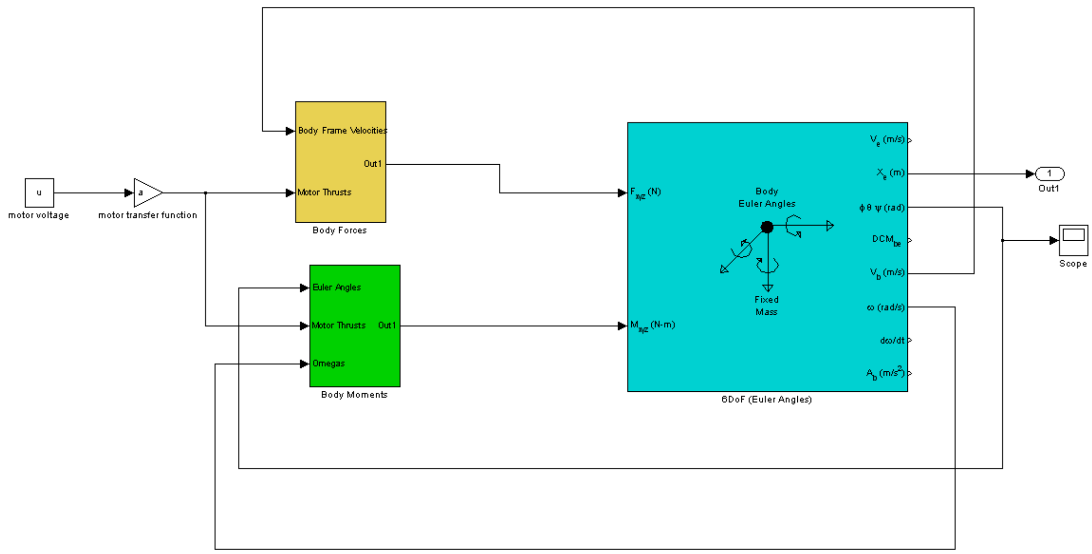

Figure 4. Non-linear system simulation

The results of this simulation with three motors on (motors 2, 4, and 5) are shown in figure 5.

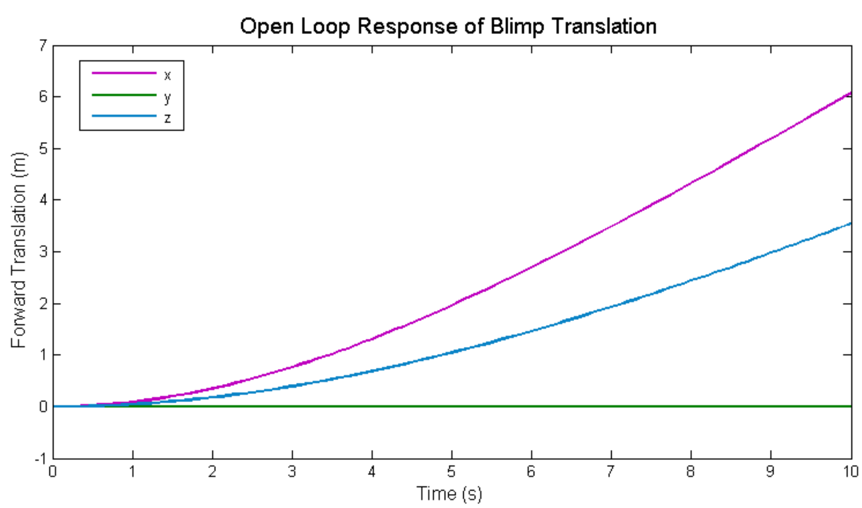

Figure 5. Open loop non-linear blimp simulation

The linearized state space model was also implanted in Simulink, and is shown in Figure 6.

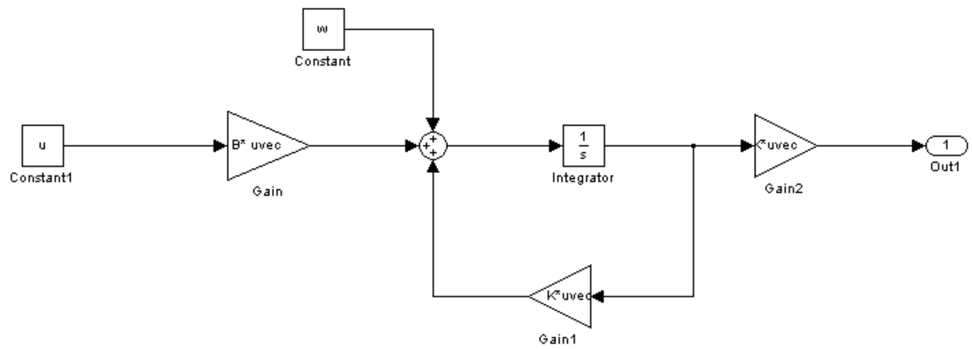

Figure 6. State space Simulink model

Revised Budget for Hardware and Electrical Components

The components necessary for the construction of the overall system specified by the goals of this project have been selected and ordered. The customer has specifically requested purchasing these items in a staggered manner, acquiring the parts necessary to construct one blimp first. Once in it is clear that the primary and secondary goals can be satisfactorily reached, the remaining necessary components will be purchased. The revised budget for the two blimp system is $902.47.

Subsystem Status Update

Hardware Subsystem Lead: Beth Boardman

After evaluating the motors it was determined that the ducted propellers would not be ideal for the blimp. Bi-directional propellers are the best for our situation. This would allow the blimp to rotate clockwise and counter-clockwise, hence rotating to the desired heading quickly. The ducted propellers did not have bi-directional propellers. The ducted propellers were removed from the motor and replaced by a bi-directional propeller. Upon doing a thrust test, the motor became over heated and was drawing too much current, consequently this burned out the motor. For this reason, we decided to go with a motor that are better equipped to run the bi-directional propellers.

The new motors-propeller system chosen is the GWS EDP-50XC electric motor which runs at 6V drawing 1.35A and produces 0.5N of thrust.

The thrust test is performed by attaching the motor to a rigid, vertical stand which is then taped to a scale. The motor is run as various voltages and the thrust is measured. The voltage, current, and thrust are all recorded.

For the first thrust test, it was found that the voltage vs. thrust curve was mostly linear, figure 7. This curve will give a reasonable estimation for the thrust produced as voltages that were not tested explicitly.

Once the new motors are in, a new thrust test will be run for both directions of the propeller. It is preferred that the thrust produced be the same whether the propeller is spinning clockwise or counter-clockwise. The resulting curves from the thrust test will show what voltages the motor needs to be run at to achieve this goal. The thrust test with the previous motors indicates that the relationship between voltage and thrust is linear, as shown in Figure 7.

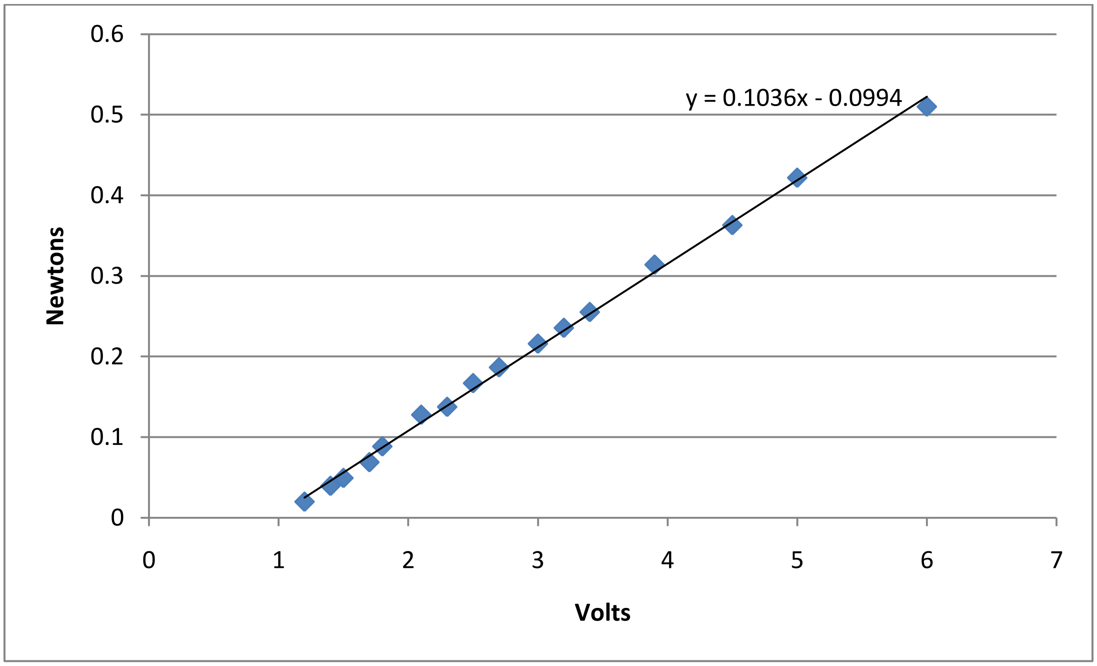

Figure 7: Motor thrust as a function of applied voltage

Software Subsystem Lead: Kyle Odland

The software team has begun to develop a client to communicate with the Vicon camera system. Using Matlab, marker positions and their changes in time can now be observed. Eventually a relationship between the reflective markers and the desired states of the system will be developed. The next step is to establish communication between the Vicon system and the Atmel microprocessor. The other task that has been completed was to verify successful communication between the xBee wireless module, the Arduino I/O board, and an actuator.

Controls Subsystem Lead: Maggie Wintermute

Controls system development is presented in the previous sections of the paper.

Power Subsystem Lead: Linh Bui

From the finalized motor configuration and performance requirements, the power subsystem has determined an initial battery configuration and circuit design. A previous model of the blimp vehicle used a total of four 7.4 V lithium batteries were used to support the whole system, with two in series and two in parallel. One of the primary differences between the current and previous projects is the onboard controller hardware. The motors' maximum voltages are 7.2 V while the Arduino's minimum voltage was somewhere between 6 and 7 V, as opposed to the Gumstix, which could operate at a lower voltage. This made an 11.1 V battery choice seem more reasonable.

The motors will be controlled with pulse with modulated power to ensure that the RMS voltage across the motors will always stay within the operating limits. The system will use optocouplers to reduce the noise in the logic signals, and H-bridges to regulate the voltage across the motors. Figure 8 shows a diagram of the preliminary circuit design.

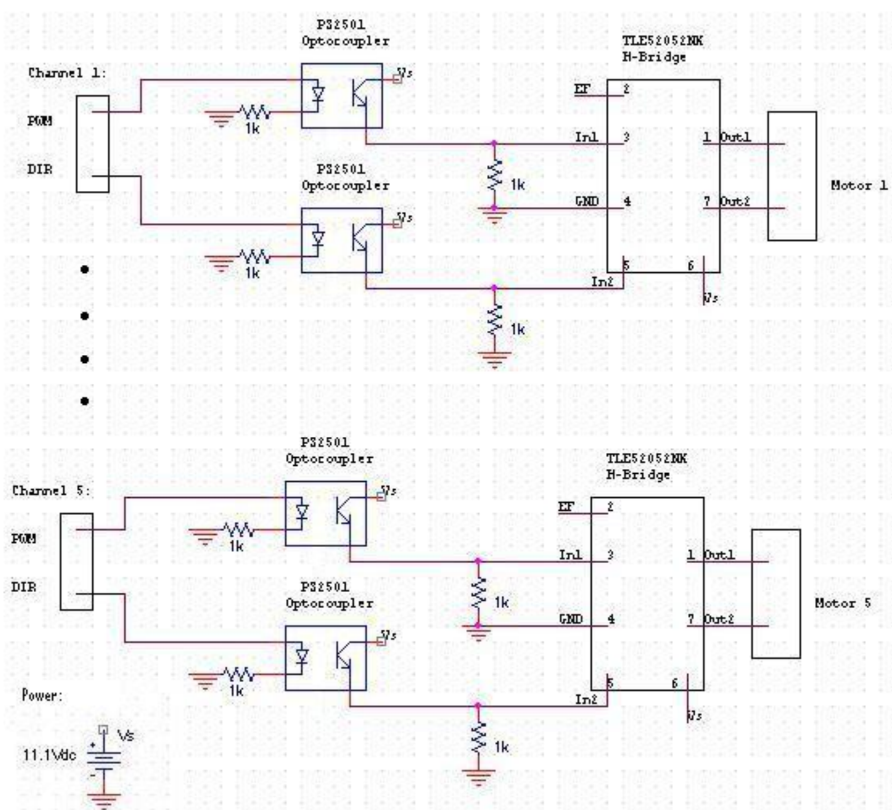

Figure 8. Power system schematic

Formation Flying Blimps MS2 Task List

	Task	Members	Deadline	
1	Identify and order necessary structural components, motors and propellers	All	4/9	☒
2	Finalize motor configuration	Boardman, Walker, Wintermute	4/9	☒
3	Thrust test motors for plant model	Boardman, Walker, Wintermute	4/28	☐
4	Develop blimp and gondola CAD model	Boardman, Walker	5/7	☐
5	Final blimp and gondola design	Boardman, Odland, Wintermute	5/14	☐
6	Construct blimps	All	5/21	☐
Software				
1	Develop VICON camera system interface	Boardman, Odland, Wintermute	4/16	☒
2	Research and define software system architecture between VICON camera system, central computer and microcontroller	All	4/23	☒
3	Establish WiFi communication between Microcontroller and Central Computer, validate with simple task test	Boardman, Odland, Walker	4/30	☒
4	Develop Microcontroller to circuit board communication	Bui, Odland, Wintermute	5/7	☐
5	Develop control law code	Bui, Boardman, Odland	5/19	☐
6	Implement control law onto microcontroller	Bui, Odland, Walker	5/21	☐
Controls				
1	Develop Plant Model	Bui, Walker, Wintermute	4/16	☒
2	Develop Waypoint Tracking Control Law	Bui, Walker, Wintermute	4/30	☐
3	Develop coordinated task and formation control law	Boardman, Odland, Walker,	5/7	☐

		Wintermute		
4	Develop Simulink Model for control law validation	Odland, Walker, Wintermute	5/14	☐
Power	**Task**	**Members**	**Deadline**	
1	Identify and order necessary electrical components	Bui, Walker	4/9	☒
2	Research Microcontroller– actuator interface, battery configurations	Bui, Boardman, Odland	4/30	☒
3	Preliminary circuit board design, validate via breadboard	Bui, Walker	5/7	☐
4	Final circuit board design, order PCB	Bui, Walker	5/14	☐
All				
1	Troubleshoot and integration	All	5/28	
2	Initial Flight Test	All	6/1	
3	Final Flight Test	All	6/4	

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Use of Resources:

1. Thesis paper on similar project for formation flying blimp vehicles. This was used in validate our plant model with other research projects.
2. Paper detailing the model determination portion of a project attempting a similar objective: autonomous waypoint tracking of a blimp vehicle. This was also used to validate our plant model with other research projects.
3. This was the textbook for our Introduction to Control Systems course, and provided the linearization procedure for state space representations, and the explicit equations for the controllability and observability matrices.
4. This text book derived the transform matrices between body and intertial reference frames for vehicles in flight.
5. Used this reference to better understand the formal definitions of observability and controllability and the derivations behind the corresponding matrices. This source will be used throughout the remainder of the course as its discussion of state space control techniques is quite detailed.
6. This paper provided an overall discussion of the methods behind minimal realization of linear systems, and will be used when analyzing other operating points.