Problem #2 Pg 54 Take for X a random set ...

$$Y_x(t) = \sum_{\sigma=1}^{s} \delta(t-\tau_\sigma)$$

$$P_X(x) = Q_s(\tau_1, \tau_2, \ldots, \tau_s)$$

$$\langle Y(t) \rangle = \int_{-\infty}^{\infty} Y_x(t)\, P_X(x)\, dx$$

$$\langle Y(t) \rangle = \sum_{s=1}^{\infty} \frac{1}{s!} \underbrace{\int_{-\infty}^{\infty} \sum_{\sigma=1}^{s} \delta(t-\tau_\sigma)\, Q_s(\tau_1, \tau_2, \ldots, \tau_s)\, d\tau_1\, d\tau_2 \ldots d\tau_s}_{f}$$

For $s=1$

$$f = \int_{-\infty}^{\infty} \delta(t-\tau_1)\, Q_1(\tau_1)\, d\tau_1 = Q_1(t)$$

For $s=2$

$$f = \int_{-\infty}^{\infty} \left[\delta(t-\tau_1) + \delta(t-\tau_2)\right] Q_2(\tau_1,\tau_2)\, d\tau_1\, d\tau_2$$

$$= \int_{-\infty}^{\infty} \delta(t-\tau_1)\, Q_2(\tau_1,\tau_2)\, d\tau_1\, d\tau_2 + \int_{-\infty}^{\infty} \delta(t-\tau_2)\, Q_2(\tau_1,\tau_2)\, d\tau_1\, d\tau_2$$

$$= \int_{-\infty}^{\infty} Q_2(t,\tau_2)\, d\tau_2 + \int_{-\infty}^{\infty} Q_2(\tau_1, t)\, d\tau_1$$

$$= \int_{-\infty}^{\infty} Q_2(t,\tau_2)\, d\tau_2 + \int_{-\infty}^{\infty} Q_2(t,\tau_2)\, d\tau_2$$

$$= 2\int_{-\infty}^{\infty} Q_2(t,\tau_2)\, d\tau_2$$

For $s=3$

$$f = \int_{-\infty}^{\infty} \left[\delta(t-\tau_1) + \delta(t-\tau_2) + \delta(t-\tau_3)\right] Q_3(\tau_1,\tau_2,\tau_3)\, d\tau_1\, d\tau_2\, d\tau_3$$

$$= \int_{-\infty}^{\infty} Q_3(t,\tau_2,\tau_3)\, d\tau_2\, d\tau_3 + \int_{-\infty}^{\infty} Q_3(\tau_1, t, \tau_3)\, d\tau_1\, d\tau_3 +$$

$$\int_{-\infty}^{\infty} Q_3(\tau_1,\tau_2,t)\, d\tau_1\, d\tau_2$$

$$= 3\int_{-\infty}^{\infty} Q_3(t,\tau_2,\tau_3)\, d\tau_2\, d\tau_3$$

$$\therefore \langle Y(t) \rangle = \sum_{s=1}^{\infty} \frac{1}{(s-1)!} \int_{-\infty}^{\infty} Q_s(t, \tau_2, \dots, \tau_s)\, d\tau_2\, d\tau_3 \dots d\tau_s$$

$$\langle Y(t_1) Y(t_2) \rangle = \sum_{s=1}^{\infty} \frac{1}{s!} \underbrace{\int_{-\infty}^{\infty} \left[\sum_{\sigma=1}^{s} \delta(t_1 - \tau_\sigma) \right] \left[\sum_{\sigma=1}^{s} \delta(t_2 - \tau_\sigma) \right] Q_s(\tau_1 \dots \tau_s)\, d\tau_1 \dots d\tau_s}_{g}$$

For $s=1$

$$g = \int_{-\infty}^{\infty} \delta(t_1 - \tau_1)\, \delta(t_2 - \tau_1)\, Q_1(\tau_1)\, \delta\tau_1 = 0$$

For $s=2$

$$g = \int_{-\infty}^{\infty} [\delta(t_1 - \tau_1) + \delta(t_1 - \tau_2)][\delta(t_2 - \tau_1) + \delta(t_2 - \tau_2)]\, Q_2(\tau_1, \tau_2)\, d\tau_1\, d\tau_2$$

$$= \int_{-\infty}^{\infty} \delta(t_1 - \tau_1)\, \delta(t_2 - \tau_2)\, Q_2(\tau_1, \tau_2)\, d\tau_1\, d\tau_2 + \int_{-\infty}^{\infty} \delta(t_1 - \tau_2)(t_2 - \tau_1)\, Q_2(\tau_1, \tau_2)\, d\tau_1\, d\tau_2$$

$$= 2 Q_2(t_1, t_2)$$

Similarly, for $s = 3$

$$g = 6 \int_{-\infty}^{\infty} Q_3(t_1, t_2, \tau_3)\, d\tau_3$$

$$\therefore \langle Y(t_1) Y(t_2) \rangle = \sum_{s=2}^{\infty} \frac{1}{(s-2)!} \int_{-\infty}^{\infty} Q_s(t_1, t_2, \tau_3, \dots, \tau_s)\, d\tau_3 \dots d\tau_s$$