#2. Pg 5 Construct a distribution ...

Consider the density function of a random variable X

$$P_X(x) = \begin{cases} \frac{1}{a} x^{-k} & \text{for } 1 \le x \le \infty \\ 0 & \text{else} \end{cases} \qquad \text{—— (1)}$$

where $a > 0$ and is a normalization constant.

The n^{th} moment μ_n of X is

$$\mu_n = \frac{1}{a}\int_1^\infty x^n P_X(x)\,dx = \frac{1}{a}\int_1^\infty x^{-(k-n)}\,dx$$

The n^{th} moment of X, μ_n exists if and only if

$$k - n > 1$$

$$\Rightarrow \quad k > n+1$$

Clearly, if the n^{th} moment exists, so do all lower-order moments.

By choosing k in (1) such that

$$n+1 < k \le n+2$$

the distributron

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has moments μ_n exist up to but not beyond a precribed value n.