Solution to Assignment 1

1 Q2

By creating a global variable that keeps the sum up of all the cell volume at each dt, total volume can be recorded. Refer to the attached gro script.

2 Q3

Monod equations for biomass and nutrient is as follows.

$$\dot{x} = \frac{rn}{a+n}x \tag{1}$$

$$\dot{n} = \frac{-1}{\gamma}\frac{rn}{a+n}x \tag{2}$$

The way gro implements growth is by discretizing the above ODE equation by dt. Therefore, discretizing the above set of ODEs yield

$$x(t+dt) = x(t)+\frac{rn(t)}{a+n(t)}x(t)dt \tag{3}$$

$$n(t+dt) = n(t)+\frac{-1}{\gamma}\frac{rn(t)}{a+n(t)}x(t)dt \tag{4}$$

Since the equation uses biomass, whereas gro uses volume, a density should be specified (This can be anything really, as long as it's consistent in your calculation). Let us assume that an E. coli cell is mostly water and thus have the density of $\rho \approx 1$ g/mL = 1 pg/fL). Then,

$$v(t+dt) = v(t)+\frac{1}{\rho}\frac{rn(t)}{a+n(t)}x(t)dt \tag{5}$$

gro updates the cell volume with the following equation

$$v(t+dt) = v(t)+kV\delta t, \tag{6}$$

where k is equal to the growth rate. Therefore, in order to implement the limited nutrient growth, at each dt, $k = rn/(a+n)$, must be computed to update the growth rate. The following figures show a sample trajectory of 1) total cell volume, 2) growth rate, and 3) nutrient concentration.

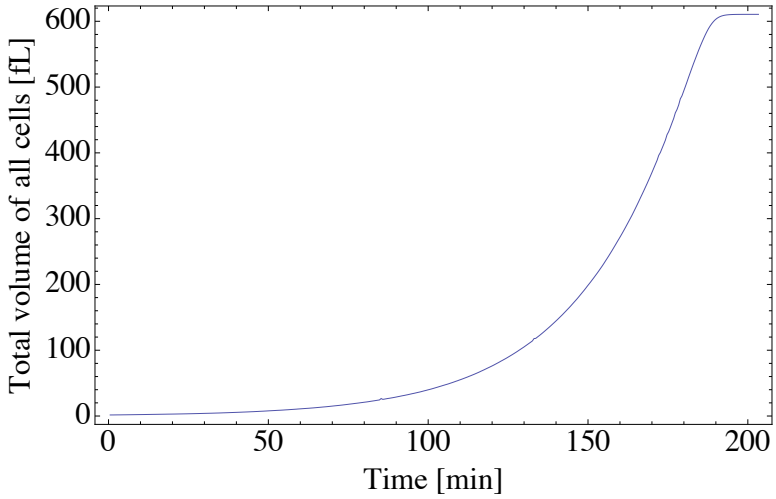

Fig. 1: Total cell volume vs time in nutrient limiting growth condition.

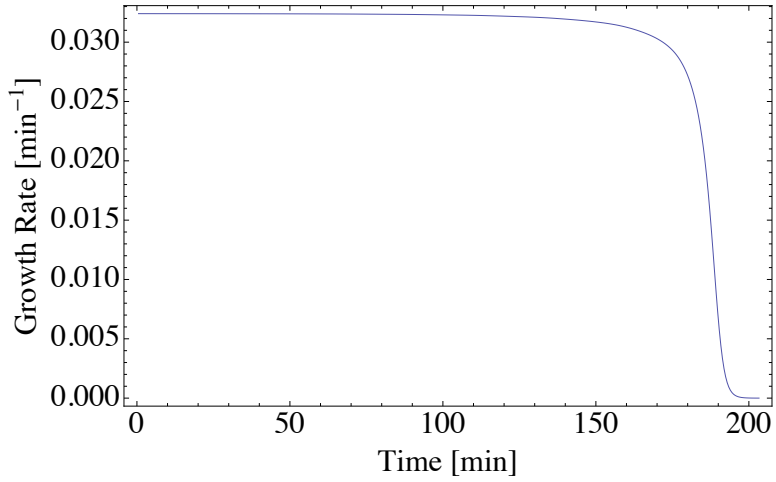

Fig. 2: Growth rate of E. coli vs time.

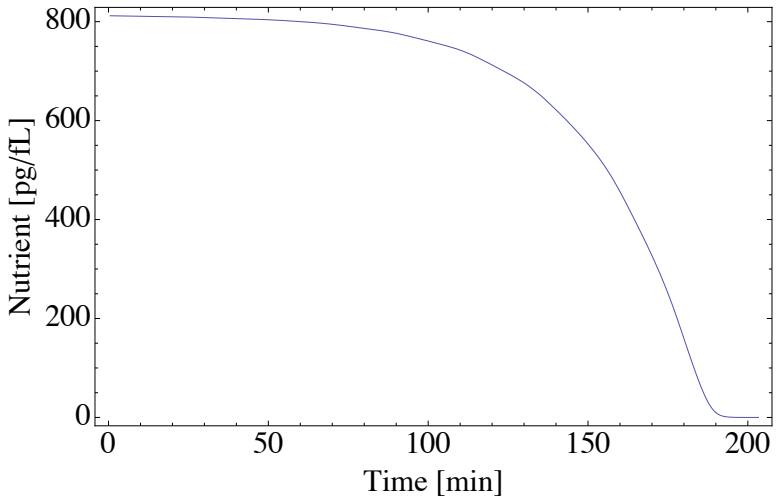

Fig. 3: Nutrient concentration vs. ti me.

```
include gro

n := 812;
gamma := 0.3;
vol := 0;
```

```
a := 40;
r := 0.034;
k := 0;

program p() := {

  true : {
    //compute the growth rate using Monod's heuristic model and set it as "growth_rate"
    k := r * n / ( a + n ),
    set ( "growth_rate", k ),
    //deplete the nutrient concentration accordingly
    n := n - k / gamma * dt,
    //sum the volume of the current cell to the total volume
    vol := vol + volume,
    message ( 2, "growth rate :"<> tostring(k) <> ", nutrient :" <> tostring(n) ),
  };

};

program main() := {

  p := [ t := 0, s := 0 ];
  // every 0.5 second output the global variables vol, k, and n.
  p.s > 0.5 : {
    print (p.t, ", ", vol, ", ", k, ", ", n, "\n" );
    p.s := 0;
  };

  true : {
    p.t := p.t + dt;
    p.s := p.s + dt;
    vol := 0;
  };
};


ecoli ( [], program p() );
```

Intro to Synthetic Biology

Assignment 1

Grade metric

1. (2 pts)
2. (3 pts)
    a. 2 pts : plots and description of approach in your report
    b. 1 pt : correct method
3. (5 pts)
    a. 2 pts : plots and description of approach in your report
    b. 3 pts : correctly applying the lecture material into gro for nutrient limited growth.