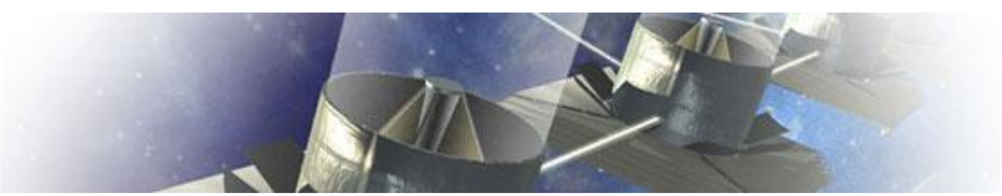
System Modeling and Dynamics of the Draganflyer XP Quad Rotor UAV

An AA/EE449 project in
collaboration with the UW DSSL

Andy Bradford
Andrew Nelson
Justin Palm

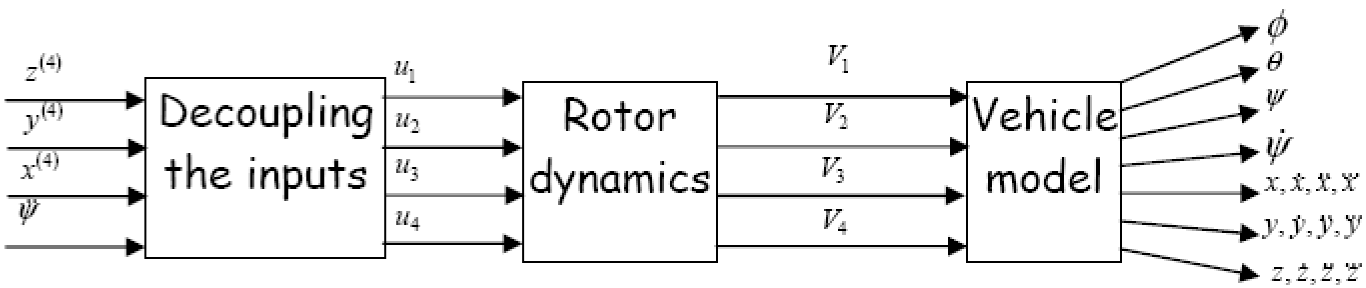
HUSKIES



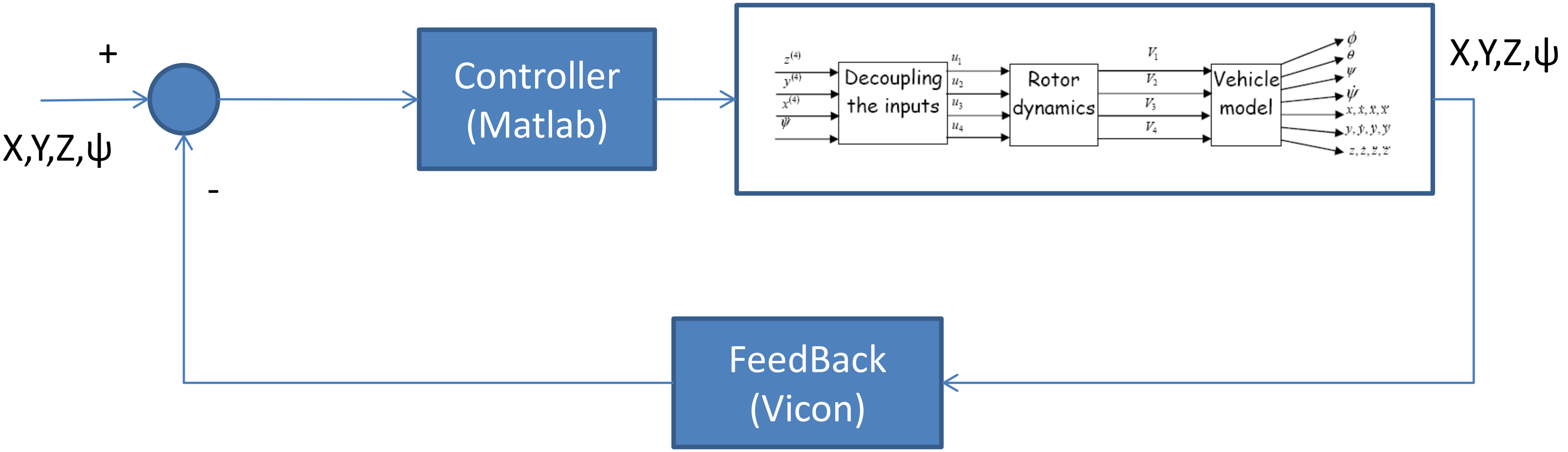
Distributed Space
Systems Lab

Quadrotor Dynamic Modeling

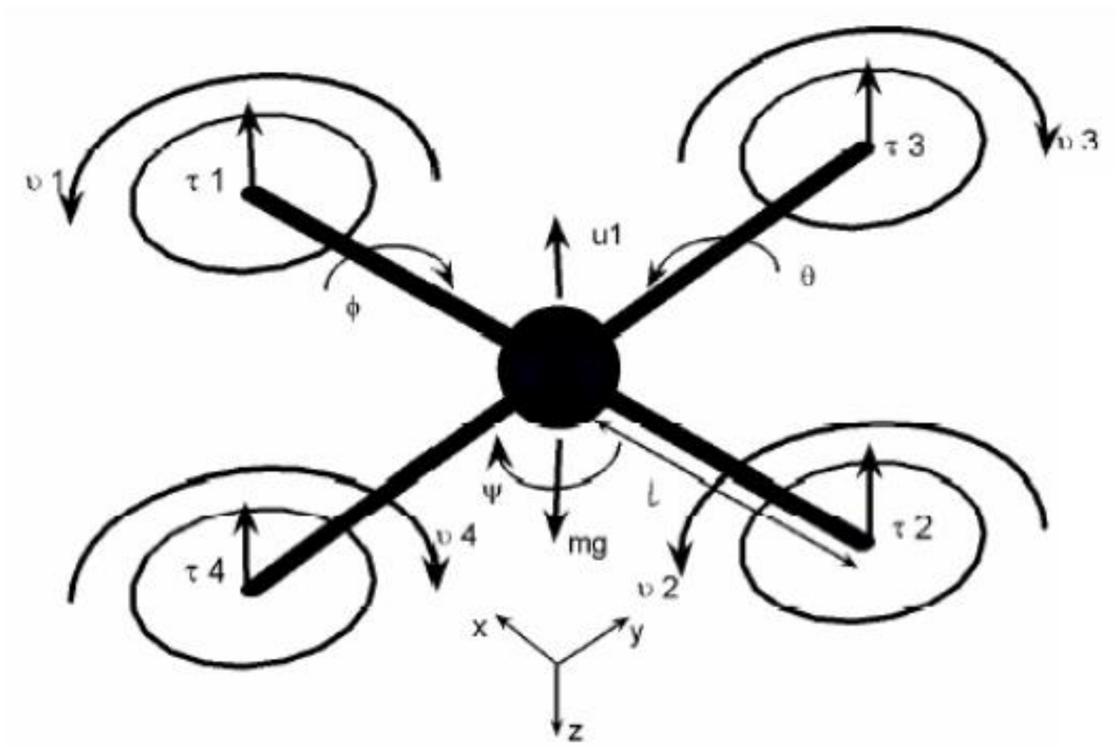
High level Plant Model



Top Level System Diagram



Free Body Diagram and Coordinate Frames

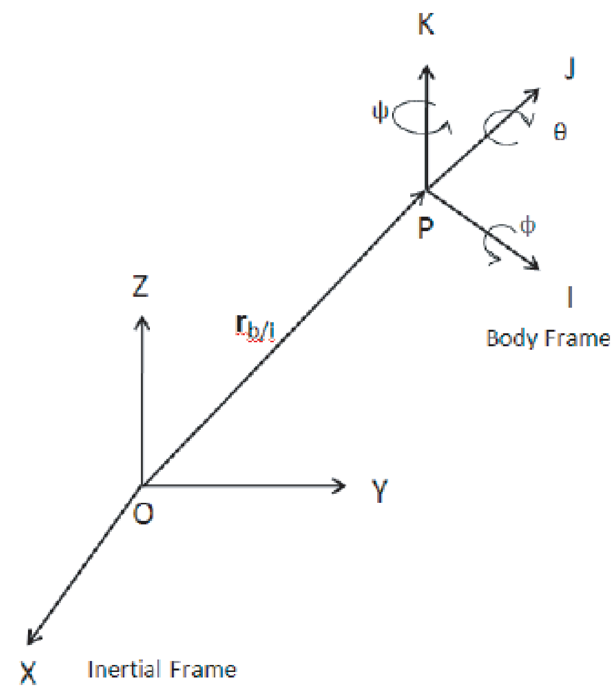


The total thrust: $u_1 = \tau_1 + \tau_2 + \tau_3 + \tau_4$

The rolling moment: $u_2 = l(\tau_3 - \tau_4)$

The pitching moment: $u_3 = l(\tau_1 - \tau_2)$

The yawing moment: $u_4 = \upsilon_1 + \upsilon_2 - \upsilon_3 - \upsilon_4$



Variables and Units

g	Gravitational acceleration	$(m.\sec^{-2})$
I_{xx}	Draganflyer X-pro's moment of inertia along x axis	$(kg.m^2)$
I_{yy}	Draganflyer X-pro's moment of inertia along y axis	$(kg.m^2)$
I_{zz}	Draganflyer X-pro's moment of inertia along z axis	$(kg.m^2)$
l	Arm length of the Draganflyer X-pro (from c.g. to tip)	(m)
m	Mass of the Draganflyer X-pro	(kg)
p	Rate of change of roll angle in body axes system	*(rad/sec)*
q	Rate of change of pitch angle in body axes system	*(rad/sec)*
Q_i	Torque generated by the ith rotor	*(N.m)*
r	Rate of change of yaw angle in body axes system	*(rad/sec)*
T_i	Thrust generated by the ith rotor	*(N)*
u	Airspeed along x axis in body axes system	*(m/sec)*
u_1, u1	Vertical thrust generated by the four rotors	*(N)*
u_2, u2	Rolling moment	*(N.m)*
u_3, u3	Pitching moment	*(N.m)*
u_4, u4	Yawing moment	*(N.m)*
v	Airspeed along y axis in body axes system	*(m/sec)*
V_i	Voltage applied on the ith rotor	*(Volts)*
w	Airspeed along z axis in body axes system	*(m/sec)*
x	x coordinate of the Draganflyer X-pro's c.g. (Earth axes)	*(m)*
y	y coordinate of the Draganflyer X-pro's c.g. (Earth axes)	*(m)*
z	z coordinate of the Draganflyer X-pro's c.g. (Earth axes)	*(m)*
ϕ	Roll angle of the Draganflyer X-pro (Euler angles)	*(rad)*
θ	Pitch angle of the Draganflyer X-pro (Euler angles)	*(rad)*
ψ	Yaw angle of the Draganflyer X-pro (Euler angles)	*(rad)*

Parameter Estimation

Thrust, *b*
  $\Omega = 21.093V + 38.942$
  $T = b(\Omega)^2$
  $b = 4.3248 \times 10^{-5}$

Torque, *k*
  $P = VI$
  $M = k\Omega^2$
  $k = VI/\Omega^3$
  $k = 5.96927 \times 10^{-8}$

Inertia, *I*
  I = [6.532*10^-3 0 0;
    0 6.6944*10^-3 0;
    0 0 1.2742*10^-2]
  I_rotor = 1.1998*10^-4

Parameter	Symbol	Value	Units
Mass	m	0.48	kg
Thrust parameter	$\tilde{b}$	4.3248×10^{-5}	$kg \cdot m$
Torque constant	k	5.96927×10^{-8}	$N \cdot m \cdot s^2$
Inertia matrix	I	$\begin{bmatrix} 6.532 \times 10^{-3} & 0 & 0 \\ 0 & 6.6944 \times 10^{-3} & 0 \\ 0 & 0 & 1.2742 \times 10^{-2} \end{bmatrix}$	$kg \cdot m^2$
Rotor inertia	I_r	1.1998×10^{-4}	$kg \cdot m^2$
Vector to motor 1	$\mathrm{r}_{m_1/b}$	$\begin{bmatrix} 0.2319 & 0 & d_1 \end{bmatrix}^T$	m
Vector to motor 2	$\mathrm{r}_{m_2/b}$	$\begin{bmatrix} 0 & 0.2319 & d_1 \end{bmatrix}^T$	m
Vector to motor 3	$\mathrm{r}_{m_3/b}$	$\begin{bmatrix} -0.2319 & 0 & d_1 \end{bmatrix}^T$	m
Vector to motor 4	$\mathrm{r}_{m_4/b}$	$\begin{bmatrix} 0 & -0.2319 & d_1 \end{bmatrix}^T$	m

Equations of Motion

$$\mathbf{v}^b = \left[\begin{array}{ccc} u & v & w \end{array} \right]^T$$

$$\boldsymbol{\omega}_{b/i}^b = \left[\begin{array}{ccc} p & q & r \end{array} \right]^T$$

$$\boldsymbol{\Phi} = \left[\begin{array}{ccc} \phi & \theta & \psi \end{array} \right]^T$$

$$\mathbf{r}_{b/i}^i = \left[\begin{array}{ccc} X & Y & Z \end{array} \right]^T$$

$$\mathbf{x} = \left[\begin{array}{cccc} \mathbf{v}^b & \boldsymbol{\omega}_{b/i}^b & \boldsymbol{\Phi} & \mathbf{r}_{b/i}^i \end{array} \right]^T$$

$$\frac{d}{dt}\mathbf{x} = \frac{d}{dt} \left[\begin{array}{c} \mathbf{v}^b \\ \boldsymbol{\omega}_{b/i}^b \\ \boldsymbol{\Phi} \\ \mathbf{r}_{b/i}^i \end{array} \right]$$

$$\mathbf{F} = m \left.\frac{d}{dt}\right|_i \mathbf{v}$$

$$\mathbf{M} = \left.\frac{d}{dt}\right|_i \mathbf{H}$$

$$\left.\frac{d}{dt}\right|_i \mathbf{v} = \left.\frac{d}{dt}\right|_b \mathbf{v} + \boldsymbol{\omega}_{b/i}^b \times \mathbf{v}^b$$

Equations of Motion Continued

$$\mathbf{F} = m\left(\frac{d}{dt}\bigg|_b \mathbf{v} + \boldsymbol{\omega}_{b/i}^{b} \times \mathbf{v}^{\,b}\right)$$

$$\frac{d}{dt}\mathbf{v}^{\,b} = \frac{1}{m}\mathbf{F}^{\,b} - \boldsymbol{\omega}_{b/i}^{b} \times \mathbf{v}^{\,b},$$

$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix}^b = \frac{1}{m}\mathbf{F}^{\,b} - \begin{bmatrix} p \\ q \\ r \end{bmatrix}^b \times \begin{bmatrix} u \\ v \\ w \end{bmatrix}^b .$$

$$\mathbf{F}_g^{\,i} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}^i$$

$$T = \sum_{j=1}^{4} T_j$$

$$T_j = \tilde{b}\Omega_j^2.$$

$$\begin{aligned} \mathbf{F}_g^{\,b} &= C_{b/i}(\phi, \theta, \psi)\,\mathbf{F}_g^{\,i} \\ &= mg\begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} \end{aligned}$$

$$\mathbf{F}_T^b = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix}$$

$$\begin{aligned} \mathbf{M}^{\,b} &= \frac{d}{dt}\bigg|_i I\boldsymbol{\omega}_{b/i}^b \\ &= I\dot{\boldsymbol{\omega}}_{b/i}^b + \boldsymbol{\omega}_{b/i}^b \times I\boldsymbol{\omega}_{b/i}^b \\ &= \frac{d}{dt}\bigg|_b I\boldsymbol{\omega}_{b/i}^b + \boldsymbol{\omega}_{b/i}^b \times I\boldsymbol{\omega}_{b/i}^b \end{aligned}$$

$$\dot{\boldsymbol{\omega}}_{b/i}^b = I^{-1}\mathbf{M}^{\,b} - I^{-1}\left(\boldsymbol{\omega}_{b/i}^b \times I\boldsymbol{\omega}_{b/i}^b\right)$$

Equations of Motion Continued

$$\frac{d}{dt}\begin{bmatrix} p \\ q \\ r \end{bmatrix} = I^{-1}\mathbf{M}^{b} - I^{-1}\left(\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}\right)$$

$$\mathbf{M}^{b} = \mathbf{M}_T^b + \mathbf{M}_R^b + \mathbf{M}_G^b.$$

$$\mathbf{M}_T^b = \sum_{j=1}^{4} \mathbf{r}_{m_j/b}^{\,b} \times \mathbf{F}_{T_j}^b$$

$$\mathbf{M}_G^b = \sum_{j=1}^{4} I_r \left(\tilde{\mathbf{\Omega}}_j \times \omega_{b/i}^b\right)$$

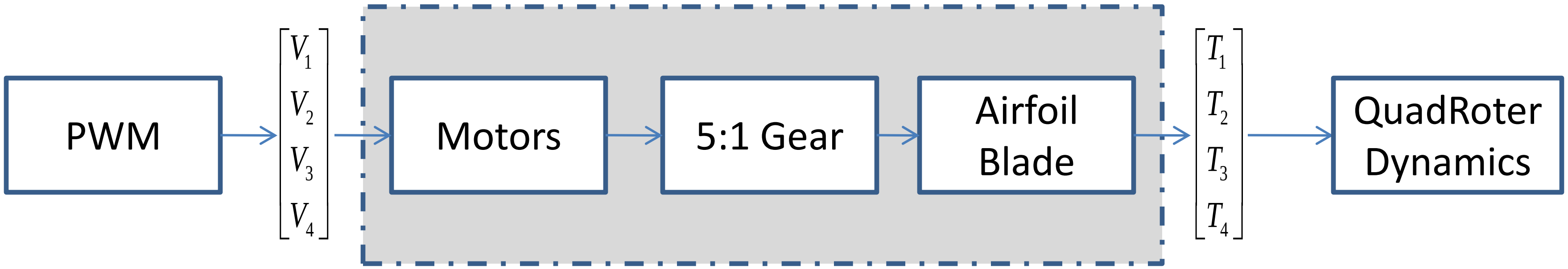
$$\mathbf{M}_R^b = \begin{bmatrix} 0 \\ 0 \\ k\left(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right) \end{bmatrix}$$

Eulers Navigation Equations

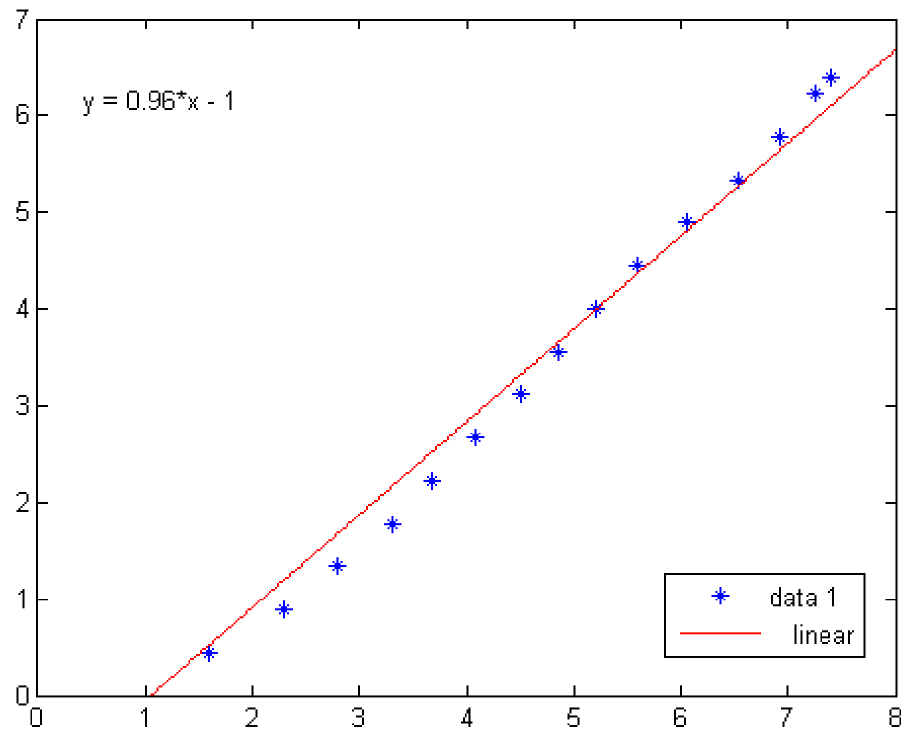
$$\frac{d}{dt}\mathbf{\Phi} \;=\; H\left(\mathbf{\Phi}\right)\boldsymbol{\omega}^{b}_{b/i} \;=\; \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\frac{d}{dt}\mathbf{r}^{i}_{b/i} \;=\; C_{i/b}\mathbf{v}^{\,b} \;=\; \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ -c\phi s\psi + s\phi s\theta c\psi & c\phi c\psi + s\phi s\theta s\psi & s\phi c\theta \\ s\phi s\psi + cosphis\theta c\psi & -s\phi c\psi + c\phi s\theta s\psi & c\phi c\theta \end{bmatrix}^{T} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Determining Voltage-Thrust Relationship



Voltage (V)

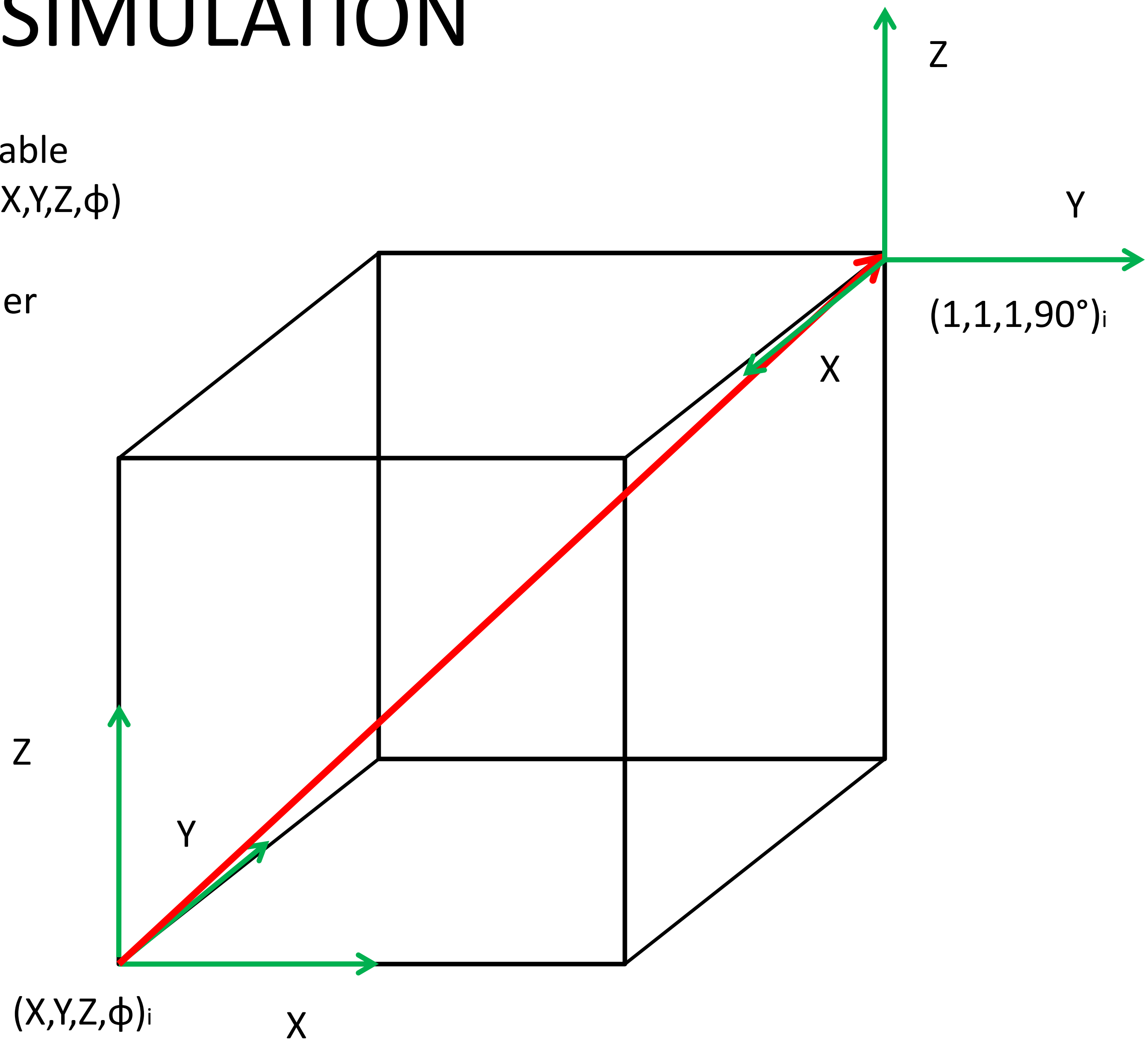


Thrust (N)

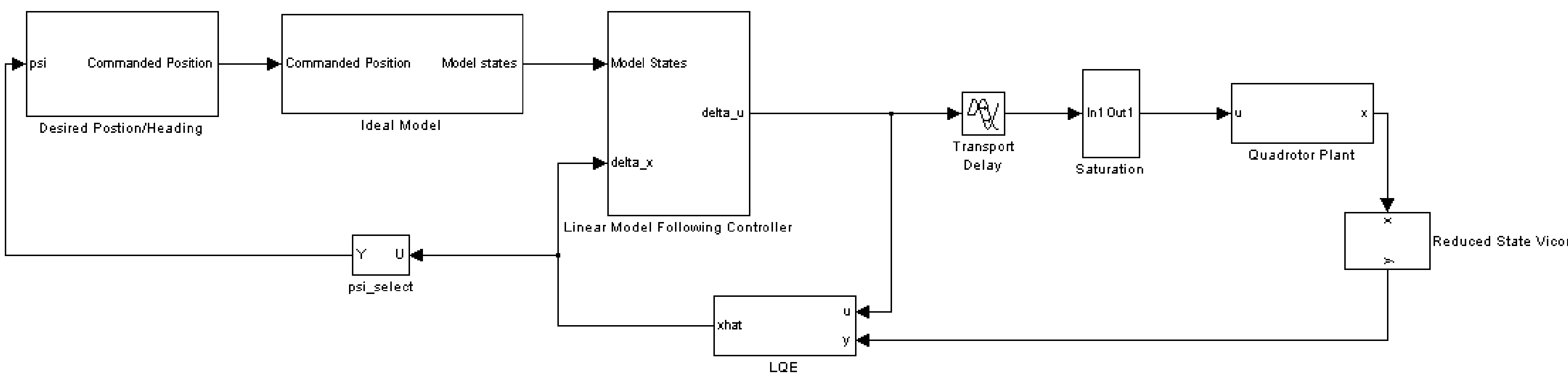
MATLAB SIMULATION

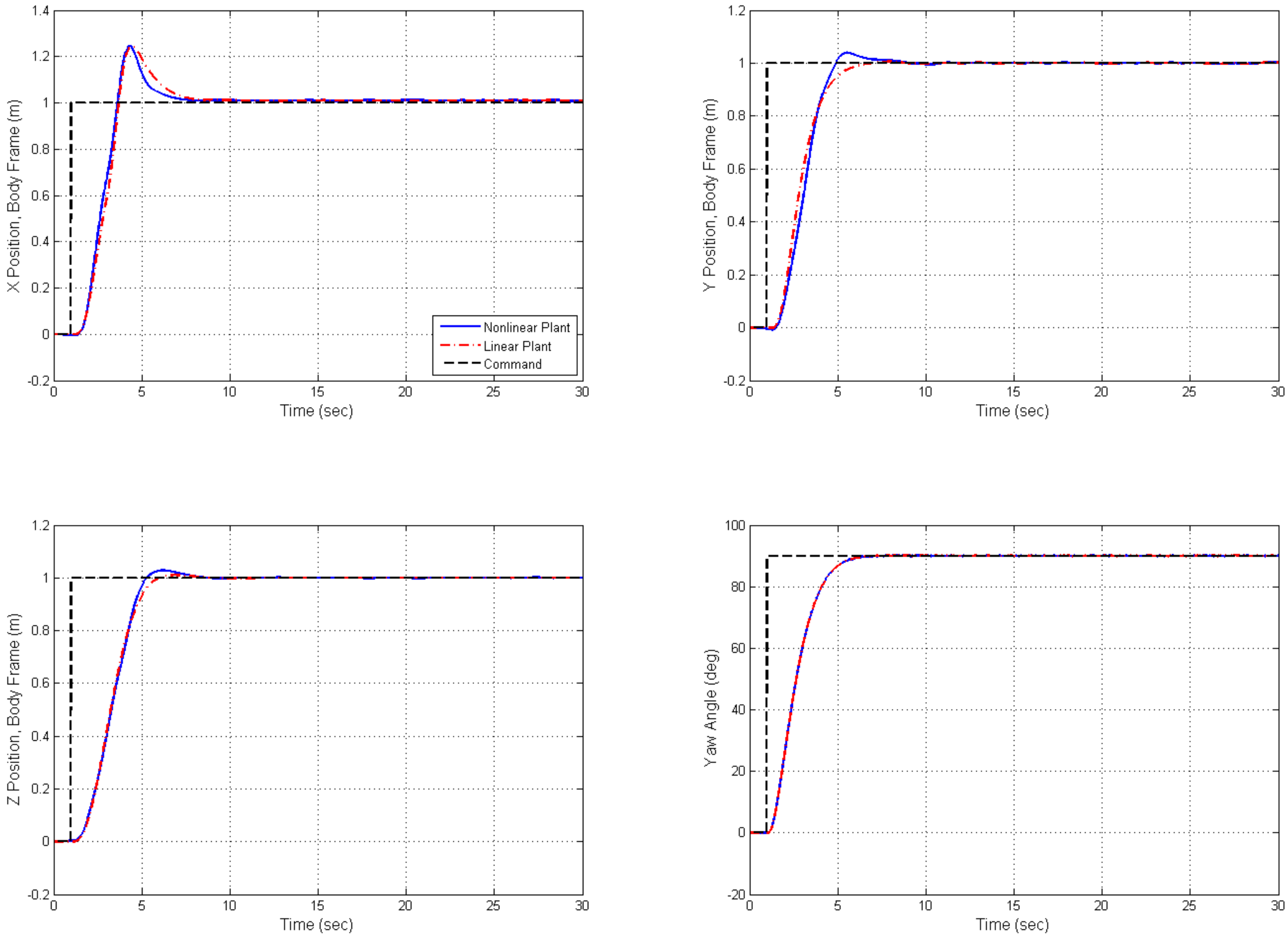
Step Input from stable hovering positon (X,Y,Z,ϕ)

Using LQR controller



Linearized Simulink Simulation Model





X Position, Body Frame (m)
Y Position, Body Frame (m)
Z Position, Body Frame (m)
Yaw Angle (deg)
Time (sec)
Nonlinear Plant
Linear Plant
Command

Open Loop Testing

Results

- Quadrotor highly unstable
- Motors not spinning at the same rate
    - Will need Calibration
- Implement feedback in phases
    - Yaw control first

Bibliography

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