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# ADAPTATION OF ANALYTICAL AND SEMI-ANALYTICAL NUMERICAL PHOTOGRAMMETRY METHODS INTO PRODUCTION ROUTINES IN HIGHWAYS PHOTOGRAMMETRY

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HR 503

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PREPARED FOR
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IN COOPERATION WITH
U.S. DEPARTMENT OF TRANSPORTATION
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# ADAPTATION OF ANALYTICAL AND SEMI-ANALYTICAL NUMERICAL PHOTOGRAMMETRY METHODS INTO PRODUCTION ROUTINES IN HIGHWAYS PHOTOGRAMMETRY

by

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16. Abstract

Semi-analytical, analytical, and analogical aerotriangulation methods, procedures, instructions, and recommendations are explained in detail.

Acceptable accuracies, required technical operator skill, operational time requirements, and instructions for adoption of semi-analytical aerotriangulation into the Photogrammetric Department are shown. Computer programs for space resection, linear transformation, and model connection procedures are included.

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# FOREWORD

This research administered by the Washington State Highways Department under the supervision of Mr. W.M. Foster, Assistant Director for Highway Development; Mr. J.H. Cooper, Roadway Development Engineer; Mr. D.A. Yates, Photogrammetric Engineer; Mr. G.A. Egge, Photogrammetrist. The mathematical analysis and computer programs, final report performed by Mr. C.Y. Hou, Photogrammetric Research Engineer.

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# CHAPTER I

# INTRODUCTION

## I-1. Introduction:

Aerotriangulation is a method for determining the ground positions of objects through photogrammetric measurement of aerial photographs with special emphasis on converting individual models of different scales to a common scale and connecting them to form a common strip. The common strip coordinates must then be adjusted and transformed to a ground coordinate system.

The three general methods used in the performance of numerical aerotriangulation, are namely:

1) Analogical 2) Analytical 3) Semi-analytical

Analogical method requires a stereoplotter which has bridging capability through the base in and base out design, such as the Wild A-7 Autograph, and Zeiss Stereoplanigraph C-8, Galileo-Santoni V. Strip coordinates are created by cantilever extension. The output from the instrumental operations consists of strip coordinates of all the measured points referred to the first model coordinates system. Absolute orientation and strip adjustment are performed mathematically.

The semi-analytical method is the numerical connection of each independent model to form a strip referred to the first model coordinate system by using any stereoplotters. Each model is independent of others, and each model has its own model coordinate system. Model connection, absolute orientation, and strip adjustment are performed mathematically.

In the analytical method, the basic measurements are image coordinates. The reconstitution of model and computation of unknown points is resolved mathematically.

The adjustment with the three methods - analogical, semi-analytical, analytical - has been the subject of numerous articles, and publications. The articles agree in general that practically the same accuracies can be obtained, (Wang [35], Hou [10]).

However, it is quite certain that there are considerable differences between one method and another regarding the application of the instruments, the required skill of the operators, and the cost of the work.

Therefore, the procedures of the three methods and various computer programs must be studied and selected, or developed and modified, to improve photogrammetric services to the Department.

I-2. The purpose and scope of the investigation:

The objectives of this research are:

1. To optimize semi- analytical procedures by combining strip formation with strip or block adjustment.

2. To prepare for future fully-analytical work by having all procedures and programs worked out prior to the time it is necessary to acquire a comparator.

3. Comparison of the three methods from the two points of view: Technical skill required of operators, and the time required for performance of the operations.

On the basis of this study, specific production routines of aero-triangulation to be used in the Photogrammetry Branch will be established in order to increase stereo-model production, reduce the man hours required for obtaining final results and to attain the best practical accuracies.

# CHAPTER II
# SEMI-ANALYTICAL AEROTRIANGULATION

## 2-1. Introduction:

The primary reason for using independent model aerotriangulation, also known as semi-analytical aerotriangulation, is to reduce the time and cost of obtaining control survey data on the ground by using "single model" photogrammetric instruments, such as Wild Autograph A8, B8, Santoni-Simplex II, Kern PG2, and Kelsh plotters.

In the independent model method, the coordinate system of the first model of a strip is retained as the origin of the coordinate system of the strip. Subsequently, each following model in succession is transformed to that system by mathematically connecting it to the preceding model, then applying the strip adjustment.

The procedure of semi-analytical aerotriangulation may be divided into the following outline:

1. Determination of the **perspective centers of the projectors and the approximate base value.**

2. Measuring all points in each model by independent model coordinate systems.

3. Mathematical model connection to form a strip.

4. Strip adjustment.

## 2-2. Determination of base value and perspective centers of projectors.

2-2-1. Introduction:

In most instruments such as Wild A8, B8, Santoni IIC, or Kelsh plotter, the perspective centers of the projectors are dependent upon the base setting, because of the potential variation of the element of common $\Phi$. Thus, the base value should be first determined.

The position of the projector centers in the model coordinate system can be determined by the following methods.

1. Space resection method.
2. Level method.

2-2-2. Space Resection Method.

2-2-2-1. Introduction.

The problem of space resection for grid plates consists of determining the three rectangular coordinates of the point in the air at which the grid plate was projected, and these coordinates are referred to the same axis as those used for model coordinates measuring. In order to determine the perspective center, the focal length of the plotter and three grid intersected points, which are not collinear, and whose projected model coordinates must be known. The coordinates of the grid plate are measured from the principal point as x, y, the points of the projected intersections can be recorded in model coordinates system as X, Y, Z. The computed space coordinates of the perspective center with direction cosines are refined by iterations until the residuals are negligible.

2-2-2-2. Mathematical Working Equation:

The position of the perspective center in model coordinate system is established by using the angles subtended at the pyramid as defined by the three grid (image) points a, b, c projected to model coordinates A, B, C and the inner perspective center of Xc, Yc, Zc, as shown in Figure 1.

Assume f is the principal distance for projector; $x_i$, $y_i$, are the image coordinates for grid points of a, b, c; $X_i$, $Y_i$, $Z_i$ are the model coordinates of projected grid points A, B, C, and three angles can be found as follows:

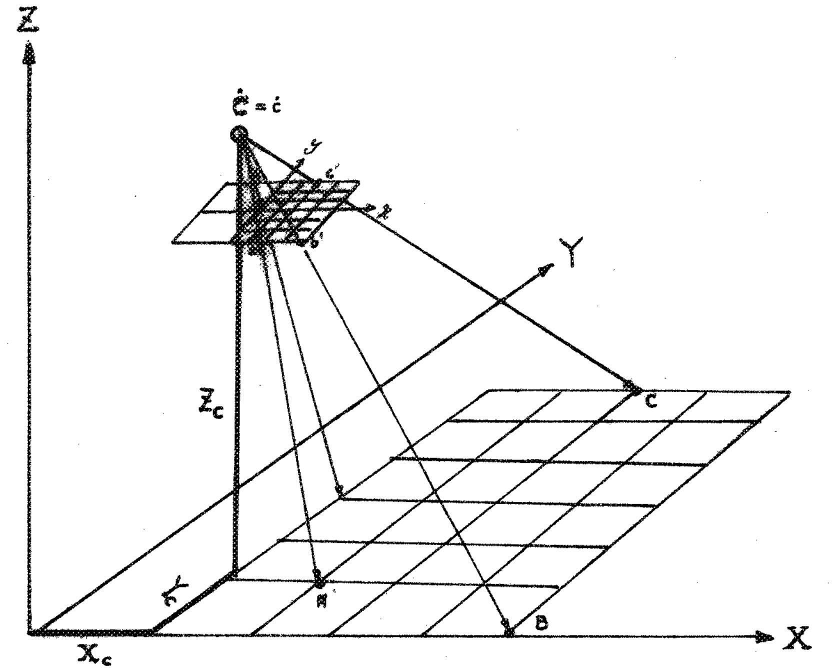


Fig. 1. Perspective center in semi-analytical aerotriangulation.

$$\cos XaĊ = \frac{x\dot{c}-xa}{a\dot{c}}$$

$$= \frac{-xa}{\sqrt{xz^2+ya^2+f^2}}$$

$$\cos YaĊ = \frac{y\dot{c}-ya}{a\dot{c}}$$

$$= \frac{-ya}{\sqrt{xa^2+ya^2+f^2}}$$

and

$$\cos zaĊ = \frac{z\dot{c}-za}{a\dot{c}}$$

$$= \frac{f}{\sqrt{xa^2+ya^2+f^2}} \qquad (1)$$

and the direction cosines in image system are:

$$\cos a\dot{c}b = \cos xa\dot{c}\cdot \cos xb\dot{c} + \cos ya\dot{c}.\cos yb\dot{c} + \cos za\dot{c}.\cos zb\dot{c}$$

$$= \frac{xa.xb + ya'yb + f^2}{(a\dot{c})\ (b\dot{c})}$$

and

$$\cos b\dot{c}c = \frac{xb.xc + yb\ .\ yc + f^2}{(b\dot{c})\ (c\dot{c})} \qquad (2)$$

$$\cos c\dot{c}a = \frac{xb.xa + yc.ya + f^2}{(c\dot{c})\ (a\dot{c})}$$

The direction cosines in model system are:

$$\cos XA\dot{C} = \frac{X\dot{C}-XA}{A\dot{C}}$$

$$\cos YA\dot{C} = \frac{Y\dot{C}-YA}{A\dot{C}}$$

$$\cos Z A\dot{C} = \frac{Z\dot{C}-ZA}{A\dot{C}}$$

and

$$\cos A\dot{C}B = \cos XA\dot{C} \cdot \cos XB\dot{C} + \cos YA\dot{C} \cdot \cos YB\dot{C} + \cos ZA\dot{C} \cos ZB\dot{C}$$

$$= \frac{(X\dot{C}-XA)\ (X\dot{C}-XB) + (Y\dot{C}-YA)\ (Y\dot{C}-YB) + (Z\dot{C}-ZA)\ (Z\dot{C}-ZY)}{\sqrt{(X\dot{C}-XA)^2 + (Y\dot{C}-YA)^2 + (Z\dot{C}-ZA)^2}\ \sqrt{(X\dot{C}-XB)^2 + (Y\dot{C}-YB)^2+(Z\dot{C}-ZB)^2}} \quad (3)$$

**Similarly,** the angles of cos BCC, cos CCA can be obtained.

Since the straight lines intersected in image and model space from perspective center C, the direction cosines in both systems should be equal. That is:

$$\cos a\dot{c}b = \cos A\dot{C}B$$
$$\cos b\dot{c}c = \cos B\dot{C}C \quad (4)$$
$$\cos c\dot{c}a = \cos C\dot{C}A$$

However, in the model space the coordinates of the perspective center are the approximate values. Thus equation (4) may be rewritten as:

$$\cos a\dot{c}b = (\cos A\dot{C}B\ (\text{approx})) + dA\dot{C}B$$

or

$$\cos a\dot{c}b = (\cos A\dot{C}B) + \frac{\partial \cos A\dot{C}B}{\partial X} dX + \frac{\partial \cos A\dot{C}B}{\partial Y} dY + \frac{\partial \cos A\dot{C}B}{\partial Z} dZ \quad (5)$$

and

$$\cos a\dot{c}b - (\cos A\dot{C}B) = V_1$$
$$\cos b\dot{c}c - (\cos A\dot{C}C) = V_2 \qquad (6)$$
$$\cos c\dot{c}a - (\cos C\dot{C}A) = V_3$$

In order to obtain a solution, the correction equations must first be linearized:

$$V_1 = \frac{\partial \cos A\dot{C}B}{\partial X} dX + \frac{\partial \cos A\dot{C}B}{\partial Y} dY + \frac{\partial \cos A\dot{C}B}{\partial Z} dZ$$
$$V_2 = \frac{\partial \cos B\dot{C}C}{\partial X} dX + \frac{\partial \cos B\dot{C}C}{\partial Y} dY + \frac{\partial \cos B\dot{C}C}{\partial Z} dZ \qquad (7)$$
$$V_3 = \frac{\partial \cos C\dot{C}A}{\partial X} dX + \frac{\partial \cos C\dot{C}A}{\partial Y} dY + \frac{\partial \cos C\dot{C}A}{\partial Z} dZ$$

where

$V_1$, $V_2$, $V_3$ = corrections for direction **cosines** in model space

$$\frac{\partial \cos A\dot{C}B}{\partial X} = \frac{(X-XA)}{(A\dot{C})}\left(\frac{1}{(B\dot{C})} - \frac{1}{(A\dot{C})}\cos a\dot{c}b\right) + \frac{X-XB}{(B\dot{C})}\left(\frac{1}{(A\dot{C})} - \frac{1}{(B\dot{C})}\cos a\dot{c}b\right)$$
$$= \cos XA\dot{C}\left(\frac{1}{(B\dot{C}} - \frac{1}{(A\dot{C})}\cos a\dot{c}b\right) + \cos XB\dot{C}\left(\frac{1}{(A\dot{C})} - \frac{1}{(B\dot{C})}\cos a\dot{c}b\right)$$

$$\frac{\partial \cos A\dot{C}B}{\partial Y} = \cos YA\dot{C}\left(\frac{1}{\dot{C}B} - \frac{1}{\dot{C}A}\cos a\dot{c}b\right) + \cos YB\dot{C}\left(\frac{1}{A\dot{C}} - \frac{1}{B\dot{C}}\cos a\dot{c}b\right)$$

$$\frac{\partial \cos A\dot{C}B}{\partial Z} = \cos ZA\dot{C}\left(\frac{1}{\dot{C}B} - \frac{1}{\dot{C}A}\cos a\dot{c}b\right) + \cos ZB\dot{C}\left(\frac{1}{\dot{C}A} - \frac{1}{\dot{C}B}\cos a\dot{c}b\right)$$

$$\frac{\partial \cos B\dot{C}C}{\partial X} = \cos XB\dot{C}\left(\frac{1}{\dot{C}c} - \frac{1}{\dot{C}B}\cos b\dot{C}c\right) + \cos XC\dot{C}\left(\frac{1}{\dot{C}B} - \frac{1}{\dot{C}c}\cos b\dot{C}c\right)$$

$$\frac{\partial \cos B\dot{C}c}{\partial Y} = \cos YB\dot{C} \left( \frac{1}{\dot{C}c} - \frac{1}{\dot{C}B} \cos b\dot{C}c \right) + \cos Yc\dot{C} \left( \frac{1}{\dot{C}B} - \frac{1}{\dot{C}c} \cos b\dot{C}c \right)$$

$$\frac{\partial \cos B\dot{C}c}{\partial Z} = \cos ZB\dot{C} \left( \frac{1}{\dot{C}c} - \frac{1}{\dot{C}B} \cos b\dot{C}c \right) + \cos Zc\dot{C} \left( \frac{1}{\dot{C}B} - \frac{1}{\dot{C}c} \cos b\dot{C}c \right)$$

$$\frac{\partial \cos c\dot{C}A}{\partial X} = \cos Xc\dot{C} \left( \frac{1}{\dot{C}A} - \frac{1}{\dot{C}c} \cos c\dot{C}a \right) + \cos XA\dot{C} \left( \frac{1}{\dot{C}c} - \frac{1}{\dot{C}A} \cos c\dot{C}a \right)$$

$$\frac{\partial \cos C\dot{C}A}{\partial Y} = \cos Yc\dot{C} \left( \frac{1}{\dot{C}A} - \frac{1}{\dot{C}c} \cos c\dot{C}a \right) + \cos YA\dot{C} \left( \frac{1}{\dot{C}c} - \frac{1}{\dot{C}A} \cos c\dot{C}a \right)$$

$$\frac{\partial \cos c\dot{C}A}{\partial Z} = \cos Zc\dot{C} \left( \frac{1}{\dot{C}A} - \frac{1}{\dot{C}C} \cos c\dot{C}a \right) + \cos ZA\dot{C} \left( \frac{1}{\dot{C}c} - \frac{1}{\dot{C}A} \cos c\dot{C}a \right)$$

and the exact position of the perspective center obtained by iterative method as equation (8):

$$\begin{aligned} X_c &= (Xc \text{ approx}) + dX_c \\ Yc &= (Yc \text{ approx}) + dY_c \\ Zc &= (Zc \text{ approx}) + dZ_c \end{aligned} \qquad (8)$$

Equation (7) is solved for the differential corrections dXc, dYc, dZc), which are added to the approximations Xc', Yc', Zc' to form new approximations. The iteration is continued until such time as the corrections become **negligible.**

A computer program has been written by the author. The flow diagram and output of the tested data are shown in Figure 2 and Table 1. The program listing and instruction can be found in the appendix.

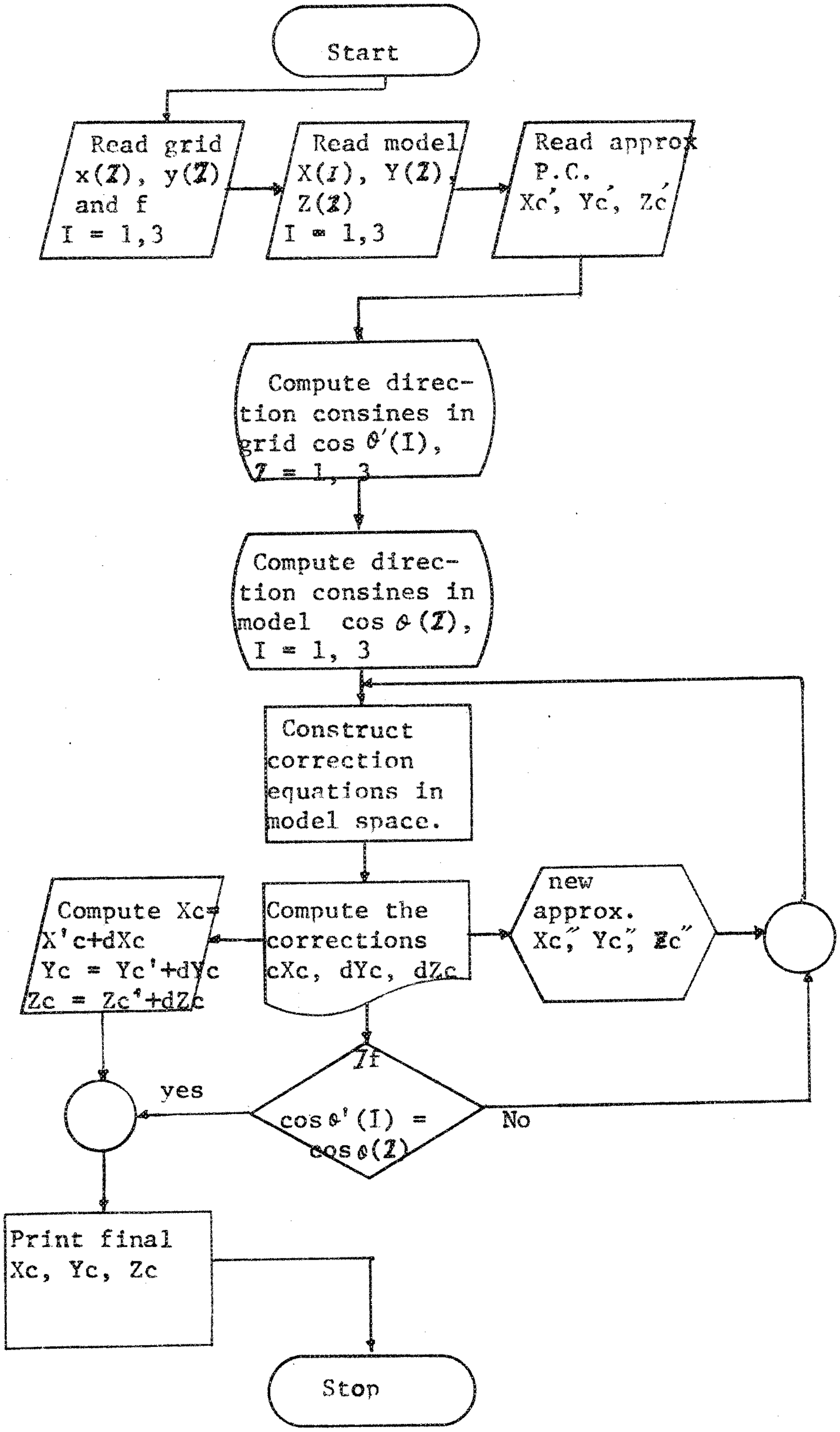



Figure 2. Flow diagram for determination of perspective center by using space resection method.

| | | | |
|---|---|---|---|
| ANGLE OF COS. | | | First Iteration |
| 0.484673 | 0.85951 | | |
| 0.375878 | 0.878892 | | |
| 0.593828 | 0.947065 | | |
| CORREECTIONS OF PERSPECTIVE CENTER | | | |
| -0.682491 | -21.4791 | -16.0345 | |
| PERSPECTIVE CENTER | | | |
| 49.3175 | 28.5209 | 33.9655 | |
| ANGLE OF COS. | | | Second Iteration |
| 0.484673 | 0.768323 | | |
| 0.375878 | 0.709479 | | |
| 0.593828 | 0.83919 | | |
| CORREECTIONS OF PERSPECTIVE CENTER | | | |
| 0.58496 | 0.856128 | -10.9373 | |
| PERSPECTIVE CENTER | | | |
| 49.9025 | 29.3771 | 23.0282 | |
| ANGLE OF COS. | | | Third Iteration |
| 0.484673 | 0.57681 | | |
| 0.375878 | 0.477763 | | |
| 0.593828 | 0.677127 | | |
| CORREECTIONS OF PERSPECTIVE CENTER | | | |
| 8.59907E-2 | 0.554099 | -2.80108 | |
| PERSPECTIVE CENTER | | | |
| 49.9885 | 29.9312 | 20.2272 | |
| ANGLE OF COS. | | | Fourth Iteration |
| 0.484673 | 0.492485 | | |
| 0.375878 | 0.383987 | | |
| 0.593828 | 0.601006 | | |
| CORREECTIONS OF PERSPECTIVE CENTER | | | |
| 1.27859E-2 | 0.070139 | -0.225046 | |
| PERSPECTIVE CENTER | | | |
| 50.0012 | 30.0013 | 20.0021 | |
| ANGLE OF COS. | | | Fifth Iteration |
| 0.484673 | 0.484729 | | |
| 0.375878 | 0.375934 | | |
| 0.593828 | 0.59388 | | |
| CORREECTIONS OF PERSPECTIVE CENTER | | | |
| 1.597E-4 | 7.02023E-4 | -1.61829E-3 | |
| PERSPECTIVE CENTER | | | |
| 50.0014 | 30.002 | 20.0005 | |
| ANGLE OF COS. | | | Final Output |
| 0.484673 | 0.484673 | | |
| 0.375878 | 0.375878 | | |
| 0.593828 | 0.593828 | | |
| CORREECTIONS OF PERSPECTIVE CENTER | | | |
| 6.55489E-7 | -2.86234E-7 | -1.46642E-8 | |
| PERSPECTIVE CENTER | | | |
| 50.0014 | 30.002 | 20.0005 | |

Table 1. Computer output for determinations of perspective center by using space resection method with iterative solution.

2-2-3. Level method.

2-2-3-1. Introduction.

The coordinates of the perspective Xc, Yc, Zc may also be determined by the level method. The instrument coordinates of three points projected from the diapositive or grid plates are measured in at least two different horizontal planes in the model space when the diapositives or grid plates are perpendicular to the Z axis and Kappa = 0.0. These measurements are carried out monocularly. The X, Y coordinates of the perspective center may be measured directly from the principal points, and Zc may be computed from the coordinates of the other two points located in the y direction through the principal point at two levels. (See figure 3 and 4).

2-2-3-2. Computation of the Z values of perspective center.

According to Figure 3, the z - values can be mathematically obtained as follows.

$$Zc = \frac{dy1}{dy} \cdot dz \qquad (9)$$

where

$dy = dy_1 - dy_2$

and $Zc$ = perspective center Z value

$dy1$ = y distance between the two images measured at readings in lower level

$dy_2$ = y distance between the same two images measured at readings in higher level

$dz$ = Z differential settings of two levels

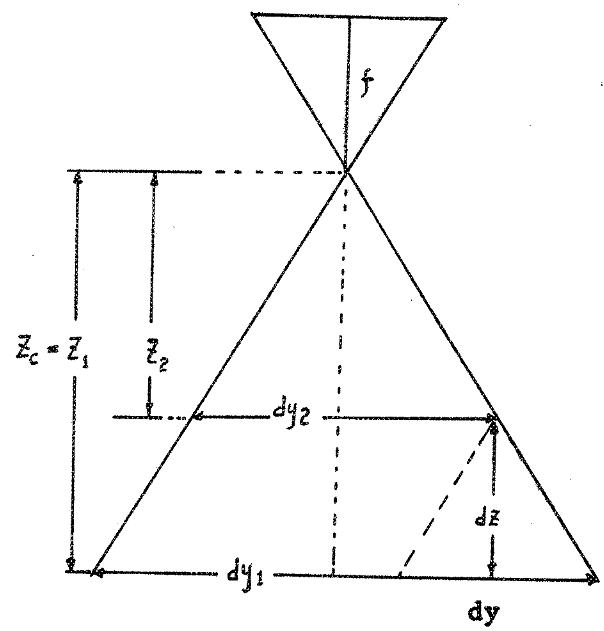



Fig. 3. Determination of Z-value for perspective center.

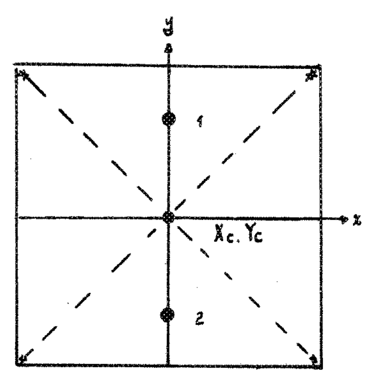



Fig. 4. Location of the points for Z-value computation.

2-2-4. Numerical testing of the two methods of perspective center determination.

Two tests have been performed by using the above mentioned methods and the final results are shown in Table 2.

| | Space Resection (mm) | Level Method (mm) |
|---|---|---|
| **Test - I** | | |
| X | 7439.50 | 7439.48 |
| Y | 1358.49 | 1358.48 |
| Z | 350.14 | 350.13 |
| **Test - II** | | |
| X | 7439.46 | 7439.42 |
| Y | 1358.40 | 1358.40 |
| Z | 350.15 | 350.10 |

Table 2. Comparison of the perspective center determined by space resection and level methods.

From Table 2, it can be seen that the comparison of methods result in similar accuracies, however, the level method is much simpler to utilize. Thus, the level method has been selected for use in the process. The detailed operational instruction for this method will be given in a later section.

2-3. Mathematical model connection to form a strip.

The coupling of the individual models into a strip of the first model coordinate system is accomplished by using the triple overlap area of the photographs and the projector centers of the models as shown in Fig. 4 and 5.

If x, y, z of second model coordinate system are rotated around their axes through the angles of w, $\varphi$, k respectively to the X,Y,Z coordinate system of first model at the common projecter center, the transformed coordinates of X, Y, Z may be obtained by a three dimensional transformation as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = C \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} \quad (10)$$

where

$M_{11} = \cos \varphi \cos k$

$M_{12} = \cos \varphi \sin k$

$M_{13} = -\sin \varphi$

$M_{21} = \sin w \sin \varphi \cos k - \cos w \sin k$

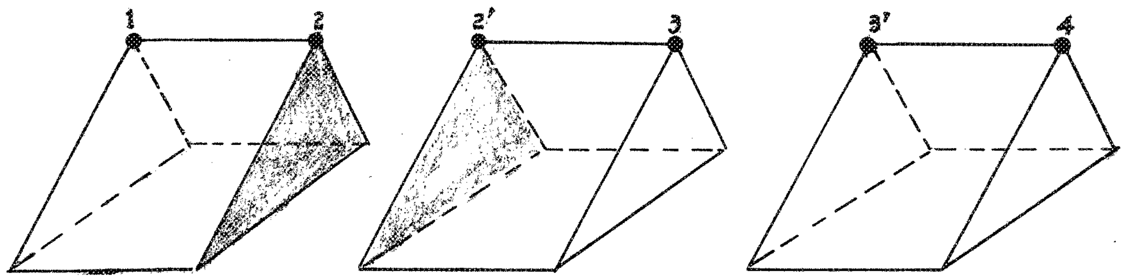



Fig. 5a. Model connection from the common model points and projector centers.

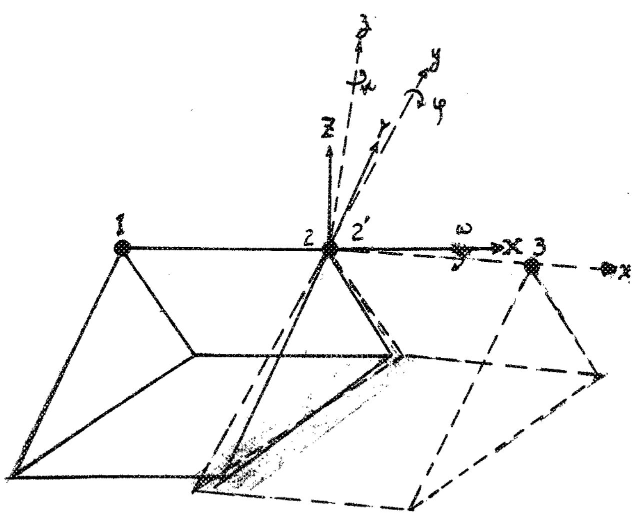



Fig. 5b. Model connection at common projector center 2 and 2'.

$M_{22}$ = sin w sin w sin y sin k + cos k cos w

$M_{23}$ = cos y sin w

$M_{31}$ = cos w sin y cos k + sin w sin k

$M_{32}$ = cos w sin y sin k - sin w sin k

$M_{33}$ = cos w cos y

The equation of (10) is not a linear, thus the equation must first be linearized.

Equation (10) may be rewritten as:

$$X = Xo + dX$$
$$Y = Yo + dY \qquad (11)$$
$$Z = Zo + dZ$$

and

$$dX = \frac{\partial X}{\partial w} dw + \frac{\partial X}{\partial y} dy + \frac{\partial X}{\partial k} d\kappa$$
$$dY = \frac{\partial Y}{\partial w} dw + \frac{\partial Y}{\partial y} dy + \frac{\partial Y}{\partial k} d\kappa \qquad (12)$$
$$dZ = \frac{\partial z}{\partial w} dw + \frac{\partial Z}{\partial y} dy + \frac{\partial Z}{\partial K} d\kappa$$

in which the script (0) indicates estimated values, the d's are corrections to the initial approximations.

After linearization by Taylor's expansion, and by neglecting all second and higher order terms it becomes:

$$\begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & C_2 & c_3 \end{bmatrix} \cdot \begin{bmatrix} dw \\ dy \\ dk \end{bmatrix} \qquad (13)$$

Where

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial w} \\ \frac{\partial Y}{\partial w} \\ \frac{\partial Z}{\partial w} \end{bmatrix} = c \begin{bmatrix} 0 & 0 & 0 \\ M_{31} & M_{32} & M_{33} \\ -M_{21} & -M_{22} & -M_{32} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

futhermore

$$\begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial \varphi} \\ \frac{\partial Y}{\partial \varphi} \\ \frac{\partial Z}{\partial \varphi} \end{bmatrix} = c \begin{bmatrix} -\sin\varphi \cos\kappa & -\sin\varphi \sin\kappa & -\cos\varphi \\ \sin w \cos\varphi \cos\kappa & \sin w \cos\varphi \sin\kappa & -\sin w \sin\varphi \\ \cos w \cos\varphi \cos k & \cos w \cos\varphi \sin k & -\cos w \sin\varphi \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and

$$\begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial \kappa} \\ \frac{\partial Y}{\partial K} \\ \frac{\partial Z}{\partial K} \end{bmatrix} = c \begin{bmatrix} -M_{12} & M_{11} & 0 \\ -M_{22} & M_{21} & 0 \\ -M_{32} & M_{31} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The equation (13) is solved for the differential corrections ($dw$, $d\varphi$, $d\kappa$) which are added to the approximations ($dw_p$, $d\varphi_p$, $d\kappa_0$) to form new approximations. The iteration is continued until such time as the corrections become negligible.

If the number of given points are then unknowns, we then have the least square condition to obtain the most probable values. Equation (13) may be assumed as observations equations and written in matrix form as equation (14).

$$V = AX-L \qquad (14)$$

where

$$V = \begin{bmatrix} dX_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ dZ_n \end{bmatrix} ; \quad A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} ; \quad X = \begin{bmatrix} dw \\ d\varphi \\ dk \end{bmatrix} ; \quad L = \begin{bmatrix} X-Xo \\ Y-Y_o \\ Z-Z_o \end{bmatrix}$$

The normal equation is

$$NX = U \qquad (15)$$

The most probable values can be found as equation (16).

$$X = N^{-1}U \qquad (16)$$

The final transformation of points to the first model coordinates system from second model can be computed by use of the equation (10).

The model connection can also be obtained on a four dimensional coordinate transformation (Schut program). The advantage of this method is the nonlinear coordinate transformation reduced to the solution of linear equation.

The connection of one model to the other requires that the coordinates of the common projection center and those of at least two model points be known in both models.

For the computation of the rotation, in both models the origin of the coordinate system is shifted to the common projection center and the vectors from this projection center to common model points are reduced to unit length. The mathematical equations can be found in reference (Schut [1]).

During the preparation of this research, the author undertook a restricted survey of the computer programs on semi-analytical aero-triangulation. (See listings of references) On the basis of this survey, the Schut[1] computer program has been selected and modified for future testings and processing for utilization. The modified computer program can be found in Appendix, and the operational instructions will be given in a later section.

2-4. Strip Adjustment.

After the numerical model connection of the independent models to form a strip, the strip coordinates must be transformed to a ground coordinate system.

The linear transformation may be employed, which consists of scale change and rotation as shown in equation (17).

$$E_T = a_{11}x + a_{12}y + E_o$$

$$N_T = a_{21}x + a_{22}y + N_o \qquad (17)$$

$$H_T = a_{31}x + a_{32}y + a_{33}z + H_o$$

In which E.N are horizontal ground control coordinates and H is elevation; x, y, z are strip coordinates; $E_o$, $N_o$, and $H_o$ are the mean ground control points in E, N, and H. $a_{11},..,a_{33}$ are coefficients of the transformation as unknowns.

Since the instrument coordinates are transformed to ground coordinates, the entire strip becomes systematically deformed horizontally and vertically.

The systematical errors may be adjusted by using the polynomial equation as equation (18).

$$E = a_o + a_1x + a_2y + a_3xy + a_4x^2 + . . .$$

$$N = b_o + b_1x + b_2y + b_3xy + b_4x^2 + . . . \qquad (18)$$

$$H = c_o + c_1x + c_2y + c_3xy + c_4x2 + . . .$$

and the final ground coordinates of all points can be obtained by equation (19)

$$E = E_T + dE$$

$$N = N_T + dN \qquad (19)$$

$$H = H_T + dH$$

where

E, N, H = final ground coordinates

$E_T$, $N_T$, $H_T$ = ground coordinates after linear transformation

$dE, dN, dH$ = correction of the systematic error due to aerotriangulation

$a_o$, $a_1 \ldots a_4$ are certain constants

x, y are the coordinates of the particular point in the strip

The transformed adjustments used by the Washington State Highway Department of Photogrammetry for strip and block adjustments has been programmed to utilize I.B.M. (model 360/50) computer (Schut [2]) and G.E., Time-share computer services (Hou [3]).

2-5. Computer program of model connection and strip adjustment.

The numerical model connection computer program and strip adjustment are performed in two steps. These steps are performed one after the other without operator intervention.

As a first stage, each model is transformed to the first model coordinates sytems, then the strip adjustment is performed in the second stage.

The computer input and output data are shown in Appendix.

**2-6. Technical instructions for performing semi-analytical aerotriangulation on department's plotters.**

**A detailed description of operations instruction for semi-analytical aerotriangulation will be given in 9 parts, and accompanied by a practical example.**

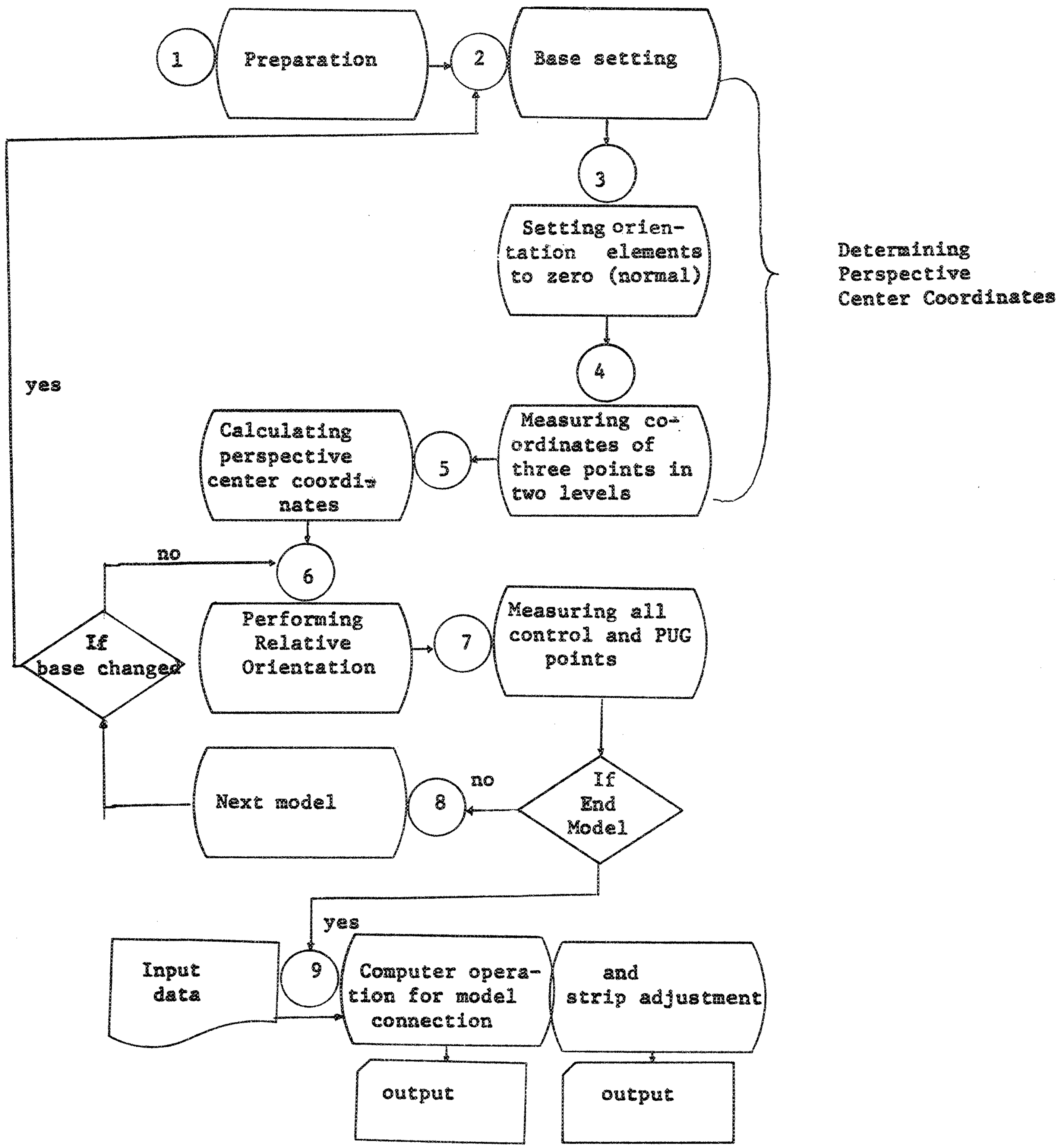



Figure 6. Operations instruction flow chart.

1. Preparation:

To introduce the semi-analytical aerotriangulation, one flight line of a current mapping/cross section project of "Toll Plaza to SR 405" was chosen.

The strip of seven models was photographed at flight height of 1500 feet with a Wild RC-8 wide angle camera. Five horizontal points, and 23 vertical points were surveyed and targeted as shown in Figure 7.

The preparation of the aerial triangulation was performed on paper prints by selecting the appropriate position and number of pass points (PUG points) in the triple overlap areas. This selection was done by visual inspection of the models under a stereoscope. The minimum three pass points were located in such a way that point 1 should be close to the nadir point. The other two pass points were on the y axis approximately plus and minus 8 cm from the nadir point. All ground control points and pass points were recorded on the strip sketch.

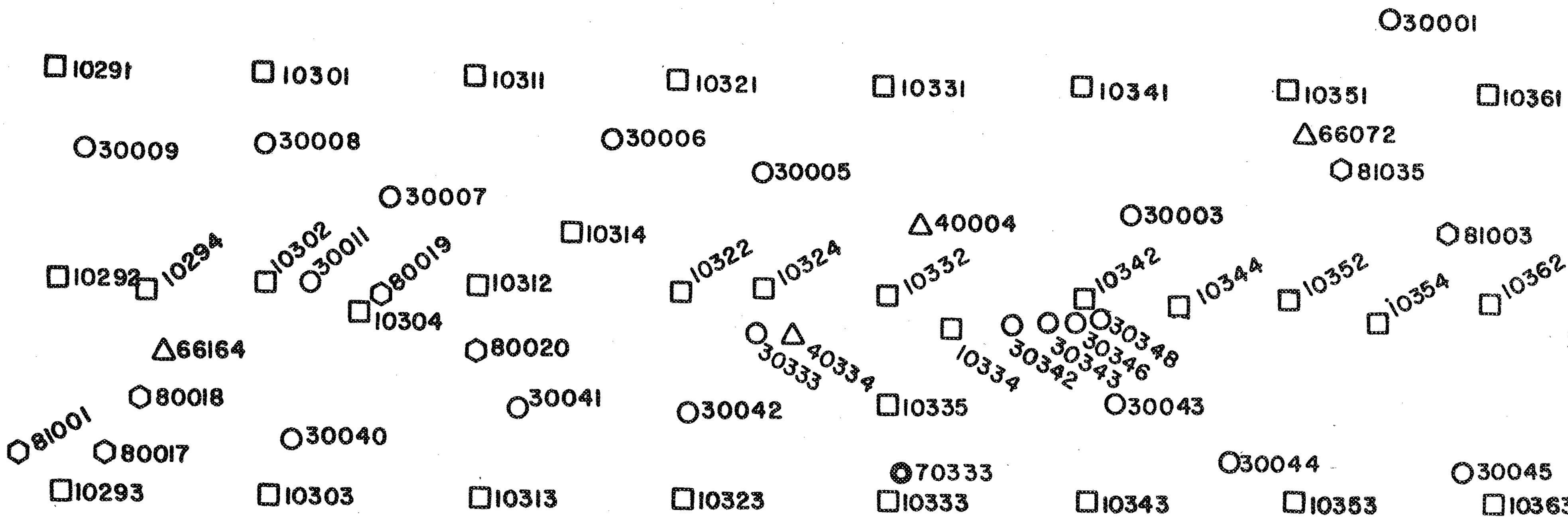
The selection of the pass points was followed by the precise marking of the points on the diapositives by a means of a Wild P.U.G. with drills of 60 micron diameters.

2. Setting the base value in plotter:

The base value can be obtained as:

$$bx = \frac{s'}{s} D.$$

Figure 7. STRIPSKETCH




Drum value x = 40000 y = 48000

Digitizer Setting x = 20000 y = 20000

in which: bx = base in mm

s' = photo scale ratio denominator

s = model scale ratio denominator

p = distance between two principal points scaled on photo image (first photo) in mm

As example of this test:

Given: photo scale 1" = 250' or 1:3000 = s'

model scale 1:1000 = s

D = 82 mm.

Compute: $bx = \frac{s'}{s} \cdot D = \frac{3000}{1000} \cdot 80$

$= 240$ mm.

Thus the base should be set at 240 mm.

3. Orientation elements setting zero:

All orientation elements should be set in zero positions. That is set common phi ($\Phi$) to zero (100g), set swing ($\kappa$), tips ($\varphi$), tilts (w) to centered positions with level in x and y directions on the carriers.

4. Measurement of three points in two levels.

a. Set the focal length of the camera on the two projectors.

b. Insert grid plates in both carriers (or diapositives of the first model, if the pass [PUG] points lie in y axis. through the principal points).

c. Connect the digitizer to the plotter. (Set resolution switches to 4x, if Wang Digitizer is used).

d. Drive x, y wheels to an exact x, y drum value near left projector center, index the digitizer x, y to 2000.00, 2000.00 for Wild A7, (250,000 500,000 in Wang Digitizer with Wild A8)., and record the exact x, y drum values in strip sketch.

e. Set the z to the lowest reference line of the z (glass scale for Wild A8). Preset zero in digitizer.

f. Check the digitizer coordinate systems. The coordinate systems x, y, z must be right-handed, x must increase to the right, direction of the bridging strip. Y must increase towards the top of the photo. Z must be positive unward.

g. Drive x, y wheels to left projector center (in this situation equals principle point) and record with digitizer the x, v coordinates of the perspective center of left projector (should be close to x = 200000, y = 200000 for Wild A7, 250000, 500000 for Widl A8 with Wang Digitizer). Record the other two points perpendicular to the x direction through principal point, as shown in Figure 8, point identifications are 11111111, 11111113, 11111115, (Column 2 to 9 for punched cards format). See Table 3.

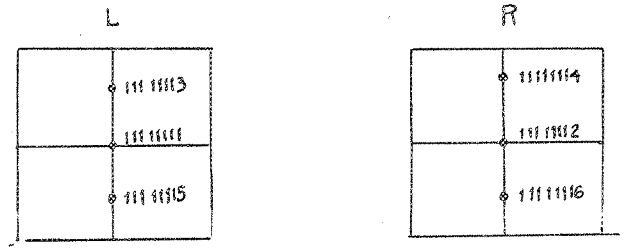



Figure 8. Point identification of perspective centers and pass (or PUG) points perpendicular to the x direction through center points. Digitizer z set 000000.

h. Drive x, y wheels to right projector center and record with digitizer the x, y coordinates of the perspective center of right projector, and record other two points as shown in Fig. 8. Point identifications are 11111112 for right perspective center, and 11111114, 11111116 for the two intersection points. (see Table 3)

i. Turn Z disc until z reads 100000 on digitizer and record same points as in steps g and h. Point identifications change to 22211121, 22211123, 22211125 for left projector and 22211122, 22211124, 22211126 for right projector as shown in Figure 9. (See Table 3).

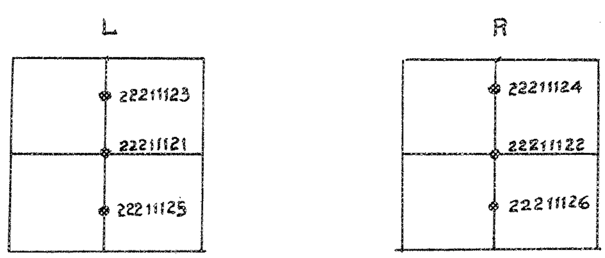



Figure 9. Point identifications for identical points as shown in Figure 8 for z setting 100000.

Grid Plates : Base = 240.00mm

| Point No. | X | Y | Z | Remark |
|---|---|---|---|---|
| Left | | | | Lowest Z set |
| 11111111 | 1894.87 | 2002.03 | 0480.00 | |
| 11111113 | 1894.87 | 2317.00 | 0480.00 | |
| 11111115 | 1894.87 | 1687.10 | 0480.00 | |
| Right | | | | |
| 11111112 | 2105.01 | 2002.07 | 0480.00 | |
| 11111114 | 2105.01 | 2317.01 | 0480.00 | |
| 11111116 | 2105.01 | 1687.14 | 0480.00 | |
| Left | | | | Higher z set |
| 22211121 | 1894.87 | 2002.03 | 0680.00 | |
| 22211123 | 1894.87 | 2185.68 | 0680.00 | |
| 22211125 | 1894.87 | 1818.38 | 0680.00 | |
| Right | | | | |
| 22211122 | 2105.01 | 2002.07 | 0680.00 | |
| 22211124 | 2105.01 | 2185.68 | 0680.00 | |
| 22211126 | 2105.01 | 1818.37 | 0680.00 | |

Table 3. Measured three points in two levels.

5. Determination of the perspective centers.

a. Compute the z-values for left projector:

According equation (9)

$$Z_c = \frac{dy1}{dy} \cdot dz$$

and

$$dy = dy_1 - dy_2$$

As an example in this test:

$dy_1$ = y11111113-y 11111115 = 231700

−168710

62990 = $dy_1$

$dy_2$ = y22211123 − y22211125 = 218568

−181838

36730 = $dy_2$

$dy = dy_1 - dy_2$ = 62990
36730
26260 = dy

$dz = Z_{higher} - Z_{1ower}$ = 68000
48000
20000 = dz

$$Z_c(\text{left}) = \left(\frac{62990}{26260} \times 20000\right) + Z_{1ower} \text{ reading}$$

$$= 95974$$

b, compute the Z − value for right projector

$dy_1$ = 611111114 − y11111116 = 231701
−168714
62987

$$dy_2 = y_{22211124} - y_{22211126} = \begin{array}{r} 218568 \\ -181837 \\ \hline 36731 \end{array} = dy_2$$

$$dy = dy_1 - dy_2 = \begin{array}{r} 62987 \\ -36731 \\ \hline 26256 \end{array} = dy$$

$$dz = Z_{higher} - Z_{lower} = 20000$$

$$Z_{c_{right}} = \left( \frac{62987}{26256} \times 20000 \right) + 48000$$

$$= 95979$$

After small computation of the Z-values, the perspective center of left projector card can be retained as point No. 11111111, x, y coordinates as recorded in Table 3, and change z value to 95974 under same format (column 28-36). (See input data)

Following the same procedure for perspective center of right projector the card can be retained as point No. 11111112, x, y coordinates as recorded in Table 3 and change z value to 95979 under same format. (See input data).

Then both cards can be duplicated for use as perspective centers in each model. (If base value is not changed.)

6. Relative orientation:

a. Proceed with standard procedures of relative orientation by removal of Y-parrallares with $\kappa$, $\varphi$, w motions). Perform this operation with precision. For the purpose of relative orientation, the quick release mechanism can be used.

b. After relative orientation, drive x, y wheels to x, y drum values near left projector as recorded in step 4, and reindex x, y as 200000, 200000 (A8 should be equal 250000, 500000) and check z in lowest reference line equal to zero in digitizer.

c. Record all the angular drum readings (k, y, w).

7. Model points measurements.

Each point needs the model identification before the point identification. (Column 2-4 for model number, column 5 -9 for point number) For example, if left photo number is 29, then model identification is 029. (See figure 10)

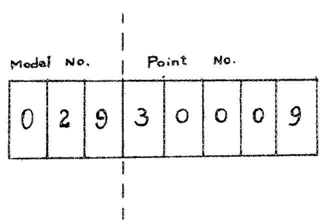



Figure 10. Model and point identification in digitizer.

a. Three or more (not over ten points) left-side pass (PUG points. The point identification should be set as follows. (Also see Figure 7, strip sketch.)

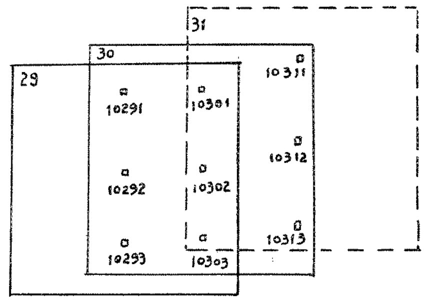



Figure 11. Pass (PUG) Points.

b. All remaining points within the model.

c. Three or more (not over 10 points) right-side pass PUG points (Such as 10301, 10302, 10303 in Figure 11).

8. Next model:

Change diapositives for second model, etc., and begin with step 6 (relative orientation) to 7 (measure model points).

If base is changed, repeat step 2 through 7.

9. Computer computation:

a. Input:

Input data for model connection and strip adjustment by using an IBM 2780 Remote Teleprocessing Terminal must be arranged in groups as follows, also shown in Figure 12.

(1) Computer job number card. (In computer room)

(2) Model connection program control deck.

(3) Job title card.

(4) Ground control cards.

(5) Equation card.

(6) First model cards:

1. -1 card.
2. Left projector center card. } As principal point for strip direction. Change point no. to 51111 and 51112.
3. Right projector center card. }
4. Left side PUG points.
5. All other points.
6. -2 card.
7. Right projector center card.
8. Right side PUG points.

(7) Second model cards.

1. -1 card.
2. Left projector center card.
3. Left side PUG points. (same points with same sequence as previous model right side)
4. All other points.
5. -2 card.
6. Right projector center card.
7. Right side PUG points.

(8) Next model, etc. (Same order as step (7) second model cards

(9) Blank card.

(10) Strip adjustment control deck.

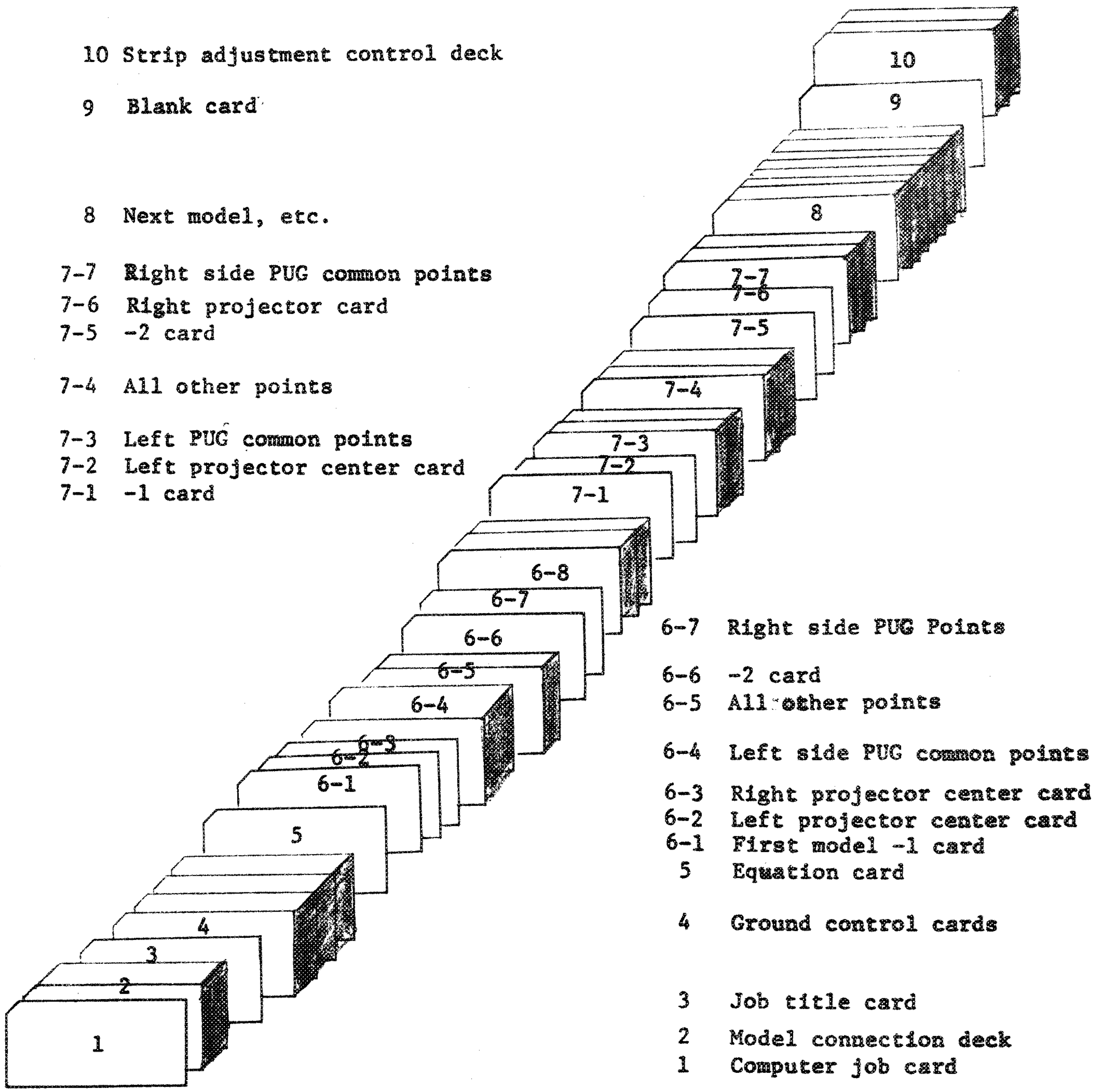



Figure 12. Input data for semi-analytical aerotriangulation.

Five input format must be punched as follows.

A. Job title card. (01 card)

Columns 1-6; Job-number, columns 10-11; punch 01, column 12; Model number, columns 27-32; date; columns 43-70; job title, column 73; flight number, columns 75-78; first photo number, columns 78-79; last photograph number, column 80; run number. (see sample in appendix)

B. Equation card.

Columns 1-4; column 1 punch -, columns 2-4 punch degree of corrections. (In this program, if the model number more than one, the degree of corrections for scale and azimuth, longitudinal bend, transversal tilt will automatically be adjusted by using a second order polynomial equation).

C. Divide card.

Columns 1-4; column 1 punch -, column 4 punch 1 or 2.

D. Ground control cards.

Columna 5-9; point number, columns 10-18; East, columns 19-27; North, columns 28-36; elevation.

E. Instrument card.

Columns 2-4; model number; 5-9; point number, 10-18; x, 19-27; y, 28-36 z.

All ground control point identifications should be numerated between 20000 to 69999. Below or over these numbers will be assumed as transformed points only.

b. Output for model connection and strip adjustment.

After data input to computer, the output of the computations will be listed as follows:

(1) List of coordinates of all points in a strip.

(2) Adjusted residuals of ground control points and coordinates of all points in ground coordinate system.

The numerical listings of input data and output results of the model connections and strip adjustment can be found in Appendix.

2-7. Results of semi-analytical aerotriangulation.

Four test strips using a Wild Autograph A-7, and an A-8, have been triangulated and evaluated in terms of accuracy, operator skill, and time required for performance of the operations.

All results are shown in Table 4 and Table 5, and Figure 7.

As can be seen from Table 4, the accuracies from semi-analytical aerotriangulation of the tested strips, when compared the results of the analogical method show only a negligible difference. However, the time for performance of the semi-analytical aerotriangulation operations are much faster than the analogical solution, which presently is employed in our Department.

2-8. Conclusion.

The results obtained in various tests and in practical work show that the semi-analytical method should be used for aerotriangulation operations in our Department. The advantage of this method may be illustrated as follows:

| Test Strip No. | Models | Flight Height(ft) & Method | R.M.S.E. in x adjusted points | R.M.S.E. in x check points (15) | R.M.S.E. in y adjusted points | R.M.S.E. in y check points (15) | R.M.S.E. in z adjust point | R.M.S.E. in z check point (13) | Operators | Time of measurement hrs | Remark |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 7 | 1500 Analogical | 0.14 | | 0.09 | | 0.11 | | A | 8 | Wild A7 |
| 1 | 7 | 1500 Semi-Analy | 0.09 | | 0.05 | | 0.08 | | A | 5 | Wild A7 |
| 2 | 5 | 1500 Analogical | 0.09 | | 0.11 | | 0.11 | | A | 6 | Wild A7 |
| 2 | 5 | 1500 Semi-Analy | 0.01 | | 0.00 | | 0.11 | | B | 3 | Wild A8 |
| 3 * | 6 | 1500 Analogical | 0.06 | 0.16 | 0.05 | 0.09 | 0.06 | 0.20 | C | 5 | Wild A7 |
| 3 * | 6 | 1500 Semi-Analy | 0.05 | 0.12 | 0.04 | 0.10 | 0.06 | 0.22 | C | 4 | Wild A7 |
| 4 | 6 | 1500 Analogical | 0.14 | | 0.09 | | 0.12 | | C | 5 | Wild A7 |
| 4 | 6 | 1500 Semi-Analy | 0.39 | | 0.18 | | 0.13 | | B | 3 | Wild $A8_2$ |
| 4 | 6 | 1500 Semi-Analy | 0.09 | | 0.08 | | 0.12 | | C | 4 | Wild A7 |

Table 4. Comparison of results for semi-analytical method and analogical method.

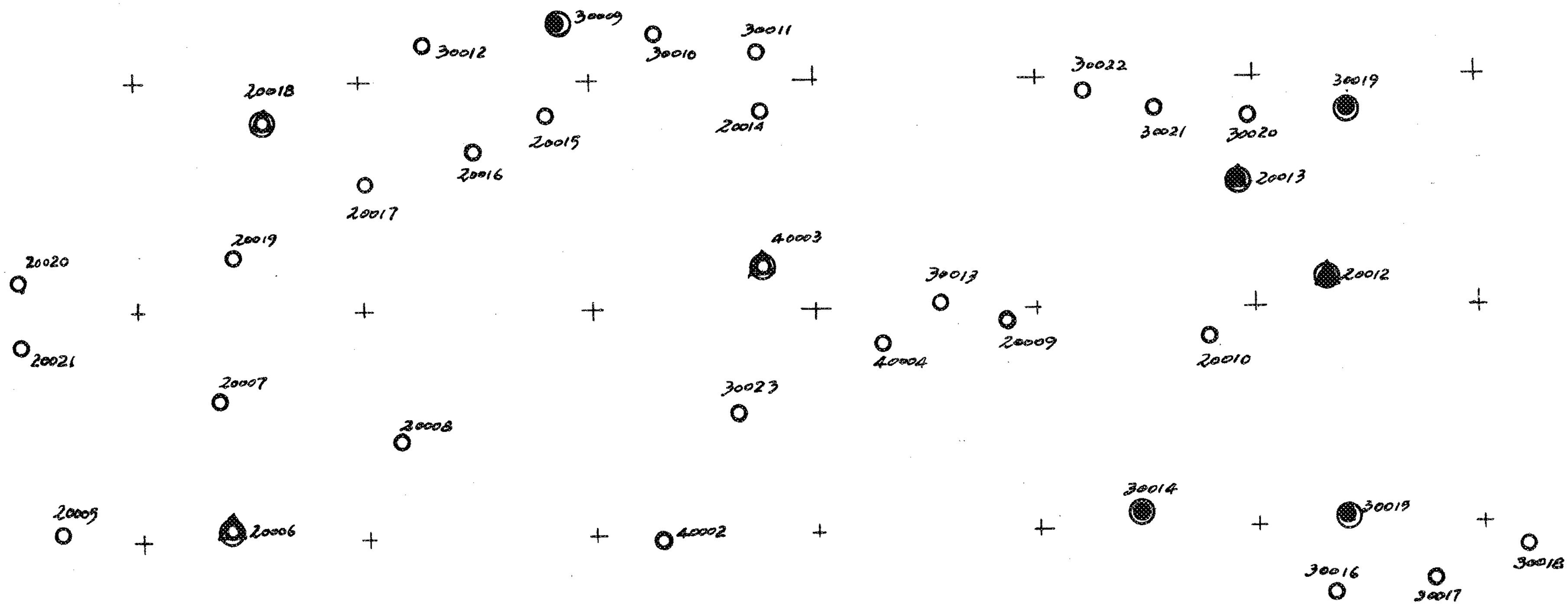
*See Table 5 and Figure 7.

| Point No. | X | | | Y | | | Z | | |
|---|---|---|---|---|---|---|---|---|---|
| | Dep. | Ind* | Ind. | Dep. | Ind* | Ind. | Dep. | Ind* | Ind. |
| 20005 | -0.01 | -0.09 | -0.23 | 0.07 | -0.05 | 0.13 | | | |
| 20007 | -0.15 | 0.18 | -0.17 | -0.06 | -0.04 | 0.08 | | | |
| 20008 | +0.21 | 0.12 | 0.07 | 0.11 | 0.17 | 0.21 | | | |
| 20009 | -0.17 | 0.03 | -0.05 | -0.01 | 0.07 | -0.11 | | | |
| 20010 | -0.02 | 0.15 | 0.15 | 0.00 | -0.05 | 0.04 | | | |
| 20013 | 0.18 | 0.00 | 0.01 | -0.05 | 0.00 | 0.06 | | | |
| 20014 | 0.10 | 0.09 | 0.00 | -0.05 | -0.04 | -0.08 | | | |
| 20015 | 0.24 | -0.07 | -0.11 | -0.13 | -0.04 | -0.07 | | | |
| 20016 | 0.15 | 0.15 | 0.19 | 0.15 | 0.12 | -0.07 | | | |
| 20017 | 0.17 | 0.07 | 0.20 | 0.16 | 0.08 | 0.06 | | | |
| 20019 | 0.10 | 0.07 | 0.11 | 0.17 | 0.15 | 0.22 | | | |
| 20020 | -0.09 | 0.11 | 0.14 | 0.00 | 0.18 | 0.15 | | | |
| 20021 | 0.19 | 0.25 | 0.22 | 0.06 | 0.19 | 0.32 | | | |
| 30010 | | | | | | | -0.07 | -0.29 | -0.15 |
| 30011 | | | | | | | -0.15 | -0.06 | -0.12 |
| 30012 | | | | | | | -0.03 | -0.29 | -0.29 |
| 30013 | | | | | | | -0.25 | 0.14 | 0.10 |
| 30016 | | | | | | | 0.19 | -0.02 | 0.01 |
| 30017 | | | | | | | 0.07 | -0.10 | -0.08 |
| 30018 | | | | | | | 0.09 | -0.12 | -0.11 |
| 30020 | | | | | | | -0.23 | -0.39 | 0.36 |
| 30021 | | | | | | | -0.32 | 0.29 | 0.23 |
| 30022 | | | | | | | -0.51 | 0.22 | 0.14 |
| 30023 | | | | | | | -0.09 | 0.13 | 0.17 |
| 40002 | 0.03 | 0.04 | 0.03 | 0.03 | 0.08 | 0.15 | 0.02 | 0.08 | 0.09 |
| 40004 | 0.25 | 0.05 | 0.02 | 0.03 | 0.00 | 0.07 | 0.02 | 0.06 | 0.05 |
| R.M.S.E. | ±0.16 | ±0.12 | ±0.14 | ±0.09 | ±0.10 | ±0.14 | ±0.22 | ±0.20 | ±0.20 |

Table 5. Comparison of the residuals for the check points between dependent model and independent model aerotriangulation.

* Perspective centers are determined by space resection methods.

Figure 13. Seattle test strip.




+ PUG point
○ Check points
Control points (H & V)
Control points (H only)
Control points (V only)

1. Most experienced stereoplotter operators can perform the semi-analytical aerotriangulation procedures without any additional special instrument training.

2. Total time for aerotriangulation operations will be reduced as much as 40% in comparison to the present method, thus reducing total cost of the photogrammetric services for any production work.

3. The recorded relative orientation elements determined in aerotriangulation using stereo plotter (Wild A-7) can be reset for mapping or cross-sectioning in Wild A-8, or Santoni IIc, thus reducing the time of relative orientation operations which in turn reduces the total cost of the photogrammetric services.

4. The Wild Autograph A-8 has the capabilities to perform the aerotriangulation procedures, thus A-8 may assist the A-7 during overloads in the bridging schedules.

5. Operation Instructions using the suggested method as developed will be very helpful in the field of photogrammetry.

# CHAPTER III

# ANALYTICAL AEROTRIANGULATION

## 3-1. Introduction.

Analytical photogrammetry is not a new concept, having been thoroughly developed by mathematicians prior to 1930's. However, the laborious computations inspired development of analogical instruments which make a simple graphic solution. The speed with which present day computers perform otherwise laborious computations has revitalized the mathematical analytical methods which permit corrections of systematic errors not correctable through the mechanical analogical systems.

## 3-2. Data measurements.

### 3-2-1. Preparation.

Photo coordinates may be measured monoscopically or stereoscopically when utilizing the AP/2C stereo-comparator. Use as a monocomparator requires that all nine pass points (PUG points) must be drilled into diapositives by means of a point transfer device. Use as a stereo-comparator completes the measurements of a point on both plates of a stereo pair of photographs simultaneously, thus only three pass points are required on each diapositive. All pass points and ground control points should be recorded on the strip sketch.

### 3-3-2. Stereo-comparator measurements. (Analytical plotter AP/2C)

The analytical plotter (AP/C) can be used for the photo-coordinates measurement in three different ways: as a plotter, a monocomparator, or as a stereo-comparator.

The method of using it as a stereo-comparator is divided into two parts:

A. Procedure for reading in the main program.

B. Procedure for measuring photo-coordinates.

A. Procedure for reading in the main program. (See Fig.15)

1. Turn on the format switch, viewer panel switch, teletypewriter switch (local), and paper tape switch.
2. Press "computer on."
3. Press "servos off."
4. Press "reader on."
5. Take the "main program" tape from program tape box.
6. Place the tape in reader head with the small tape feed holes over the tape feed wheel.
7. Turn input switch on "Load D."
8. Turn input control switch on "reader."
9. Turn display switch on "C".
10. Turn mode switch on "BP1".
11. Turn overflow switch on "run."
12. Press "state" key.
13. Press "logic" key.
14. Press "start" key.
15. If the reader stops, turn input switch on "normal."
16. Press "start" key.
17. When it stops, turn input switch on "Load D."
18. Press state, logic, and start keys.
19. Turn input switch on "normal."

20. Press "start" key.

21. When the overflow lamp glows, press "state" key.

22. Turn mode on "run."

23. Press "start" key.

24. Push "tape read" button on viewer panel to read in the last part of the program tape.

25. Press "reader off" and "serves on."

26. Press "computer program button 1" on. (Printed out only the point no. $x_1$, $y_1$, $x_2$, $y_2$.)

B. <u>Procedure for measuring photo-coordinates</u>. (See Fig.14)

1. Push "point coord." button on.

2. Push photo point "X" button on.

3. Push "photo 2" button on.

4. Place the photogrpahs on the carriages.

5. Record with teletype writer the $X_1$, $Y_1$, $X_2$, and $Y_2$ coordinates of the fiducial marks.

   5-1 Drive XY wheels to fiducial mark 1 (left).

   5-2 Push photo point "Y" button on.

   5-3 Clear Y-parallax with incremental input control.

   5-4 Push photo point "X" button on.

   5-5 Clear X-parallax with incremental input control.

   5-6 Press "tape punch" button on. The photo-coordinates of fiducial mark 1 will be printed and punched in teletypewriter.

   5-7 Same procedure for fiducial mark 2 (right) 3 and 4 as #5-1 to 5-5.

6. Record the coordinates of control points (same procedure as #5-1 to 5-6 can be used).

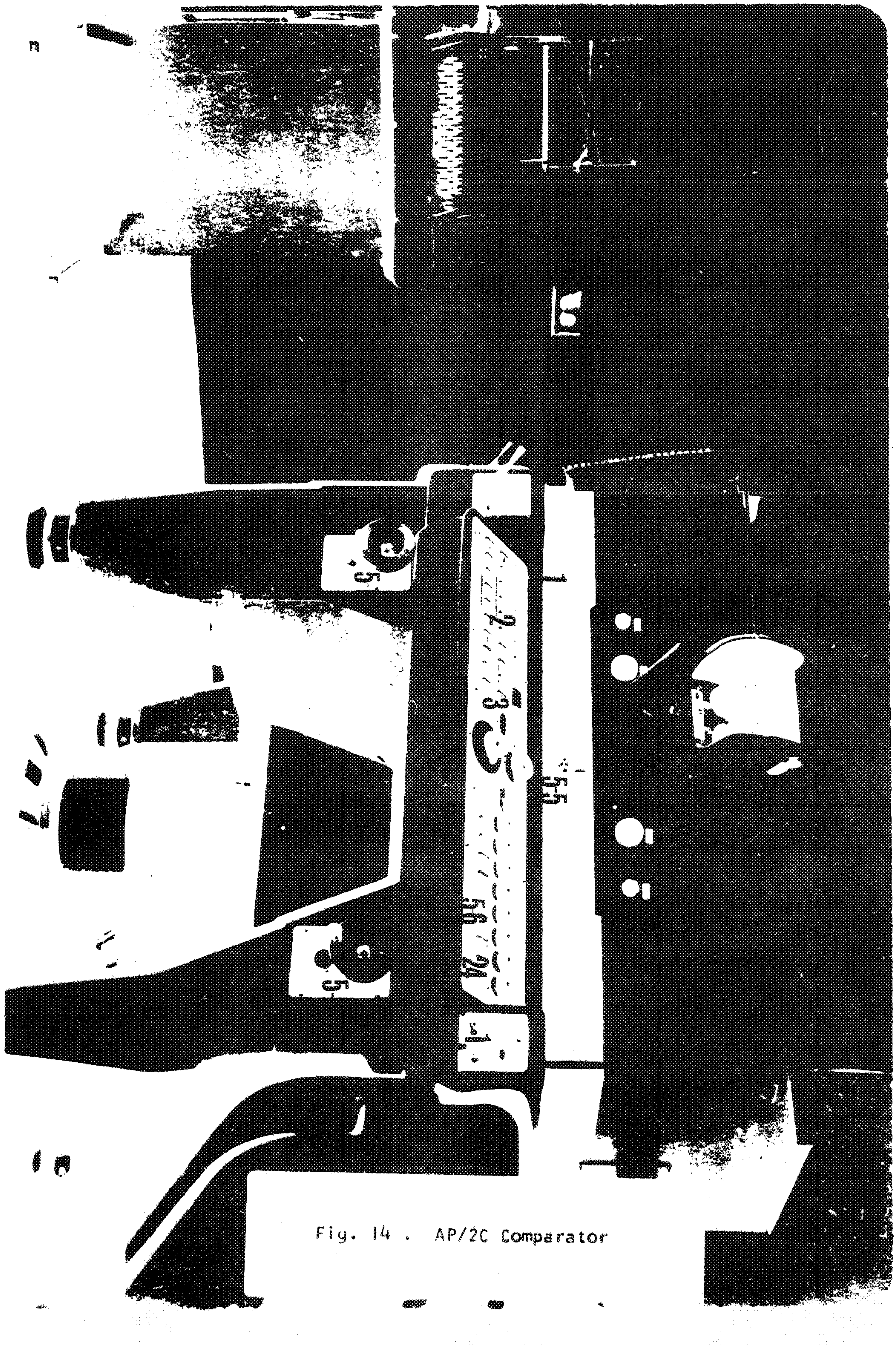


Fig. 14 . AP/2C Comparator

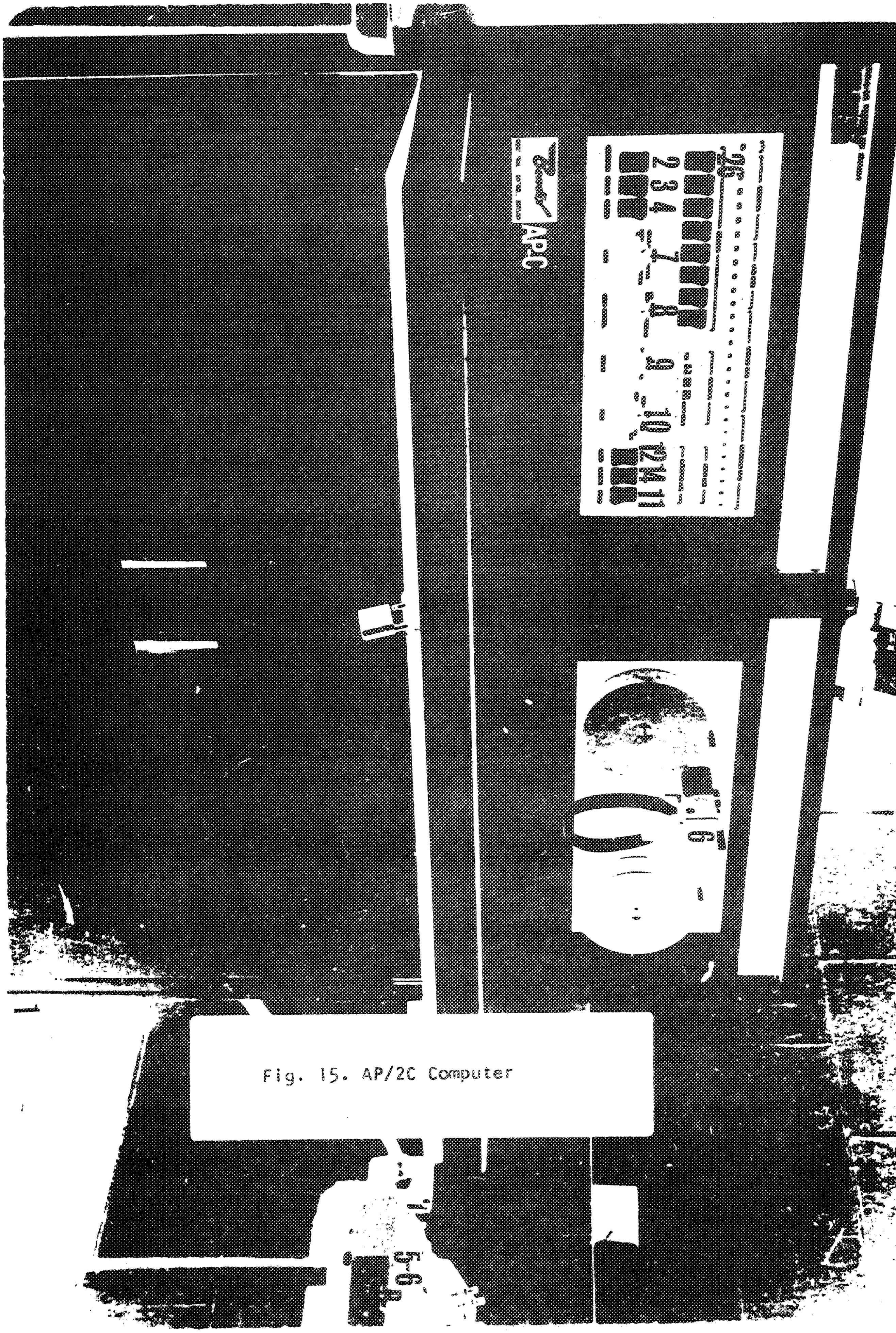


Fig. 15. AP/2C Computer

7. Record the coordinates of object as #5-1 to 5-6.

   After all diapositives have been recorded, press "computer stop," "servos off", turn mode switch on "AP1", and press "computer off." Viewer panel switch off, teletypewriter switch off, paper tape switch off and turn format switch off.

3-3. Data processing.

3-3-1. Linear transformation of the comparator coordinates to photo coordinates.

Analytical aerotriangulation requires photo coordinates with the origin at the prinicipal point. These photo coordinates (x,y) must be obtained by using a linear transformation in order to change the scale and rotation from comparator-to photo-coordinates system.

According to Figure 16 turning the E.N. comparator coordinates system by the angle E and displacing it parallel to the turned position by the distances $x_m$ and $y_m$, we obtain a new system is obtained in which the coordinates of point p will be:

$$x_t = x_m + mE \cos E + mN \sin E$$
$$y_t = ym + mN \cos \mathcal{E} - mE \sin \mathcal{E} \qquad (20)$$

where

$x_t$, $y_t$ are the transformed coordinates of point p with photo-coordinates system. E, N are the coordinates of point p expressing the comparator system. m is the change of scale in both systems.

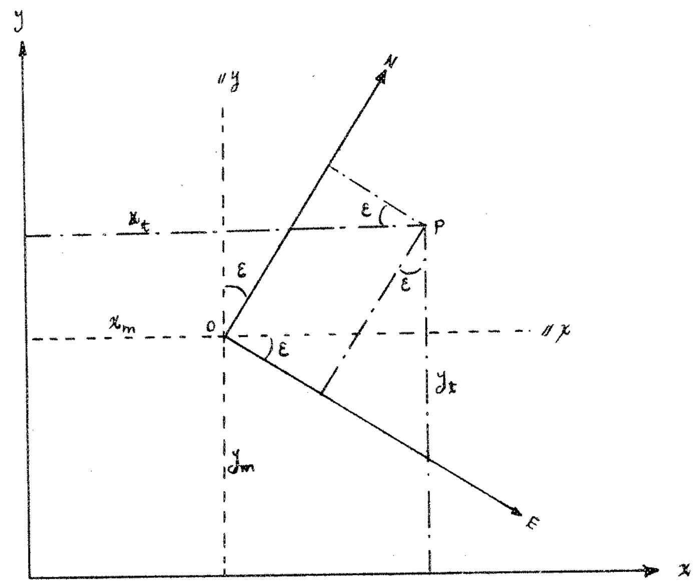



Figure 16. Coordinates Transformation

Assume

$$o = m \cos E \qquad (21)$$
$$p = m \sin E$$

Then equation (20) can be written in new form:

$$x_t = x_m + E\ o + N\ p \qquad (22)$$
$$y_t = y_m + N\ o + E\ p$$

It is a fact that the transformed coordinates $x_t$, $y_t$ are not the same as the initial coordinates x, y. The difference between both values can be assumed as error equation:

$$V_x = x_m + E\ o + N\ p - x \qquad (23)$$
$$V_y = y_m + N\ o - E\ p - y$$

in which x and y are the calibrated coordinates of the fiducial marks, Vx, Vy display the errors between the transformed and initial coordinates.

The principal point of a photograph in both coordinates system can be obtained by equation (24) with the coordinates of four fiducial marks.

$$x_s = \frac{[x]}{4}; \quad y_s = \frac{[y]}{4}; \quad E_s = \frac{[E]}{4}; \quad N_s = \frac{[N]}{4} \qquad (24)$$

The error equation can be rewritten in the form:

$$Vx = xm + (E-Ex)\ o + (N-Ns)\ p - (x-xs) \qquad (25)$$
$$Vy = ym + (N-Ns)\ o - (E-Es)\ p - (y-ys)$$

or written in matrix form as:

$$V = AX - L \qquad (26)$$

where

$$V = \begin{bmatrix} v_{x1} \\ v_{x1} \\ . \\ . \\ . \\ . \\ v_{x4} \\ v_{y4} \end{bmatrix} ; A = \begin{bmatrix} 1 & 0 & E_1-E_s & N_1 - N_s \\ 0 & 1 & N_1-N_s & -(E_1-E_s) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 1 & 0 & E_4-E_s & N_4-N_s \\ 0 & 1 & N_4-N_s & -(E_4-E_s) \end{bmatrix} ; X = \begin{bmatrix} x_m \\ y_m \\ \\ \\ \\ \\ o \\ p \end{bmatrix} ; L = \begin{bmatrix} x_1 - x_s \\ y_1 - y_s \\ . \\ . \\ . \\ . \\ x_4-x_s \\ y_4-y_s \end{bmatrix} \qquad (27)$$

By applying least squares condition, X (the unknowns) are chosen that for equal weights the sum of the squares of the errors is a minimum; thus: $V^tV = (X^tA^t-L^t)\ (AX-L) = X^tA^tAX - 2L^tAX - L^tL$ (28)

For minimization, the partial differentials of this function with respect to each independent variable must be zero. The partial derivative matrix is:

$$\frac{\partial [V^tV]}{\partial X} = A^tAx - A^tL = o \qquad (29)$$

Applying the following notation:

$$A^tA = N \text{ and } A^tL = U \qquad (30)$$

then the normal equation is:

$$NX = U \qquad (31)$$

The unknowns can be found by computing the inverse of N thus:

$$X = N^{-1}U \qquad (32)$$

All points in each photo should be transformed to the photo coordinates system by using equation (22). A computer program of linear transformation has been written by the author. The program listing can be found in the appendix.

3-3-2. Correction of the systematic film distortion.

A very important factor in photographs is the distortion of film due to dimensional change, which may be uniform or nonuniform; the systematic distortions may be corrected for scale by linear transformation, but the nonuniformity of dimensional change in different directions of areas of the film will result in noncorrectable errors when only four fiducial marks have been measured. For example, a flight strip at 3000 feet of seven models for "Outlook to Grandview" has been bridged and adjusted and the residuals are shown in Table 6 and Figure 11.

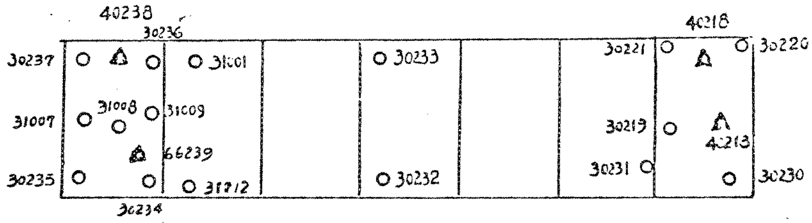
All residuals of the adjusted points in this flight are somewhat large because of the nonsystematic film distortion.

A "flash plate" comparison was used for detecting nonuniform size changes of film.

First, the coordinates of the four corner fiducial marks of the flash plate and eight diapositives were measured and recorded. Then, each example was used to fit the calibrated positions of our fiducial marks as shown in equation (20).

The deformation of four fiducial marks of the flash plate and eight diapositives are shown in Table 7 and displayed in Figure 18. The solid lines indicate the calibrated positions, and the dashed lines indicate the residual deformation of the film.

The deformation of the photographs taken during this flight was




▲ Horizontal & vertical control point

○ Vertical control point

Figure 17. Strip sketch of "Outlook to Grandview"

| POINT | RESIDUALS EAST | NORTH | ELEVATION |
|---|---|---|---|
| 30219 | | | -0.12 |
| 30220 | | | 0.29 |
| 30221 | | | 0.76 |
| 30230 | | | -0.57 |
| 30231 | | | 0.07 |
| 30232 | | | 0.02 |
| 30233 | | | -0.21 |
| 30234 | | | -0.07 |
| 30235 | | | 0.97 |
| 30236 | | | -0.33 |
| 30237 | | | -0.61 |
| 40217 | -0.51 | -0.58 | -0.53 |
| 40218 | 0.31 | 0.64 | 0.27 |
| 40238 | -0.08 | 0.46 | -0.07 |
| 66239 | 0.28 | -0.52 | 0.12 |
| R.M.S.E. | 0.33 | 0.56 | 0.44 |

Table 6. Residuals in feet after strip adjustment due to the errors of the film distortion.

larger than that of the directly exposed photographic glass plate. It is not possible to determine from the deformation, whether it was caused during the film handling in the aerial camera or during the film processing. However, if the film distortions are approximately the same size as the glass plate as photo No. 3, 4, 5, 6, 7, and 8, this film may be used in the analogue plotter. When large distortion occurs when comparing to the flash plate the film should be rejected as in photo No. 1 and 2.

| FILM NO | POINT NO | ERROR-X | ERROR-Y | REMARK |
|---|---|---|---|---|
| 0 | 1 | .010 | .008 | |
| 0 | 2 | -.011 | -.012 | |
| 0 | 3 | .015 | .011 | (FLASH [illegible]) |
| 0 | 4 | -.014 | -.007 | |
| 1 | 1 | -.040 | .022 | |
| 1 | 2 | .017 | -.027 | |
| 1 | 3 | -.012 | .004 | |
| 1 | 4 | .035 | .001 | |
| 2 | 1 | .008 | .004 | |
| 2 | 2 | .024 | -.016 | |
| 2 | 3 | -.011 | .049 | |
| 2 | 4 | -.021 | -.036 | |
| 3 | 1 | .002 | .026 | |
| 3 | 2 | -.039 | -.046 | |
| 3 | 3 | .059 | .009 | |
| 3 | 4 | -.022 | .010 | |
| 4 | 1 | .009 | .018 | |
| 4 | 2 | -.036 | -.042 | |
| 4 | 3 | .060 | .015 | |
| 4 | 4 | -.033 | .009 | |
| 5 | 1 | .003 | .020 | |
| 5 | 2 | -.043 | -.049 | |
| 5 | 3 | .073 | .010 | |
| 5 | 4 | -.033 | .020 | |
| 6 | 1 | .012 | .009 | |
| 6 | 2 | -.048 | -.047 | |
| 6 | 3 | .087 | .011 | |
| 6 | 4 | -.050 | .028 | |
| 7 | 1 | .009 | .010 | |
| 7 | 2 | -.041 | -.038 | |
| 7 | 3 | .069 | .006 | |
| 7 | 4 | -.037 | .023 | |
| 8 | 1 | .005 | .016 | |
| 8 | 2 | -.041 | -.044 | |
| 8 | 3 | .069 | .006 | |
| 8 | 4 | -.033 | .021 | |

**Table 7.** Residuals in mm after linear transformation of the four fiducial marks of a test strip

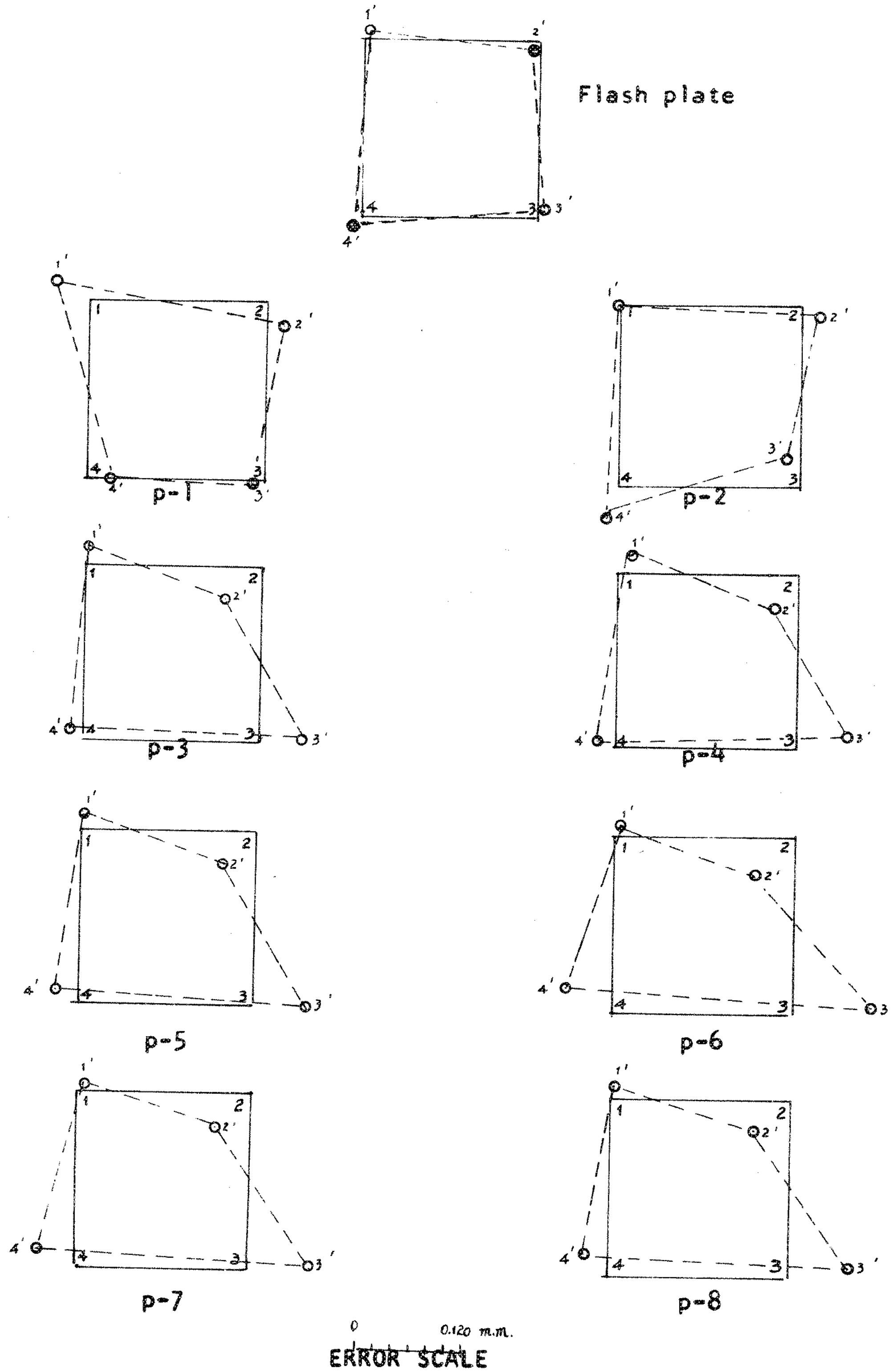



Fig. 18. Film distortions after linear transformation of the four fidcial marks of a test flight strip

3-3-3. Lens distortion.

Camera calibration data may contain tabular values of lens distortion, $\Delta r$, as a function of radial distance r from the principal point. The distance is given by the formula:

$$r = \sqrt{x^2 + y^2} \qquad (33)$$

The distortion values may be stored in the computer as a table or a curve of $\Delta r$ at increments of r. If the lens distortion table is not given, then the values of the distortion can be determined by fitting an nth order polynomial to the distortion curve.

$$\Delta r = ko\ r + k1r^3 + k2r^5 \cdots \qquad (34)$$

where

ko, k1, k2 are the coefficients of the radial distortion curve.

r is the radial distance from the principal point.

$\Delta r$ is radial lens distortion.

The corrected image coordinates are:

$$x_d = x - \Delta x$$
$$y_d = y - \Delta y \qquad (35)$$

and

$$\Delta x = \frac{x . \Delta r}{r}$$
$$\Delta y = \frac{y . \Delta r}{r} \qquad (36)$$

where

$x_d$, $y_d$ are corrected image coordinates with the original the principal point.

x, y are photo coordinates with the original at the principal point.

$\Delta x$, $\Delta y$ are radial lens correction in x, y components.

Thus, the equation for the photo coordinates x, and y corrected for lens distortion are:

$$x_d = x - \Delta x = x - \frac{x}{r}\Delta r = x\left(1 - \frac{\Delta r}{r}\right) \qquad (37)$$

$$y_d = y - \Delta y = y - \frac{y}{r}\Delta r = y\left(1 - \frac{\Delta r}{r}\right)$$

The computed numerical lens distortion value for Wild RC8 camera No. 15UAg. 274 are given in Table 8.

3-3-4. Corrections of earth curvature and refraction.

The effect of the curvature of the earth along the length of the strip can be applied to x and y directions as shown in previous report. Hou [6]:

The x direction is:

$$dx = \frac{xHr^2}{2Rf^2}$$

and in y direction is

$$dy = \frac{yHr^2}{2Rf^2} \qquad (38)$$

Where dx, dy are the correction of the earth curvature in photo coordinates, f is focal length, H denoted as flight height, R is radius of earth.

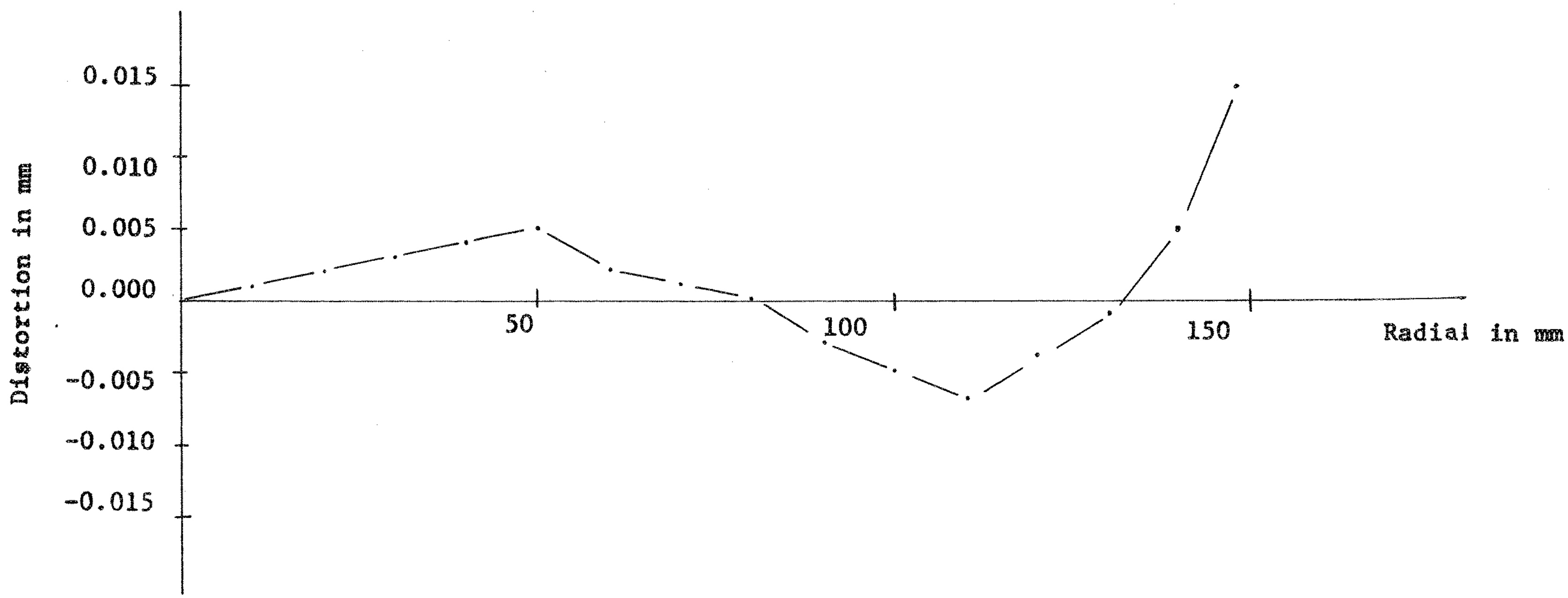



Fig. 19. Lens distortion curve for Wild RC8 camera No. 15 UAg. 274

TABLE 8. TABLE FOR LES DISTORTION FOR WILD RC8 CAMERA-15.JAN.274

| RADIUS MM | DISTORTION MICRON | RADIUS MM | DISTORTION MICRON |
|---|---|---|---|
| 0. | 0 | | |
| 1. | 0.148898 | 51. | 3.55338 |
| 2 | 0.29759 | 52 | 3.48105 |
| 3 | 0.445871 | 53. | 3.40215 |
| 4 | 0.593535 | 54 | 3.31671 |
| 5 | 0.740379 | 55 | 3.22477 |
| 6. | 0.886199 | 56 | 3.1264 |
| 7. | 1.03079 | 57. | 3.02165 |
| 8. | 1.17395 | 58. | 2.9106 |
| 9. | 1.31548 | 59. | 2.79334 |
| 10. | 1.45519 | 60. | 2.66996 |
| 11. | 1.59286 | 61. | 2.54057 |
| 12 | 1.72831 | 62 | 2.40528 |
| 13 | 1.86134 | 63 | 2.26421 |
| 14 | 1.99177 | 64 | 2.11752 |
| 15 | 2.11939 | 65 | 1.96534 |
| 16. | 2.24403 | 66 | 1.80783 |
| 17. | 2.3655 | 67. | 1.64517 |
| 18. | 2.48361 | 68. | 1.47754 |
| 19. | 2.5982 | 69 | 1.30513 |
| 20. | 2.70908 | 70. | 1.12816 |
| 21 | 2.81607 | 71. | 0.94683 |
| 22 | 2.91902 | 72. | 0.761384 |
| 23 | 3.01776 | 73 | 0.572062 |
| 24 | 3.11212 | 74 | 0.37912 |
| 25 | 3.20196 | 75 | 0.182827 |
| 26 | 3.2871 | 76 | -1.65337E-2 |
| 27 | 3.36742 | 77. | -0.218668 |
| 28. | 3.44276 | 78. | -0.423268 |
| 29 | 3.51298 | 79. | -0.63001 |
| 30. | 3.57795 | 80. | -0.838559 |
| 31. | 3.63755 | 81. | -1.04857 |
| 32 | 3.69164 | 82 | -1.25967 |
| 33 | 3.7401 | 83 | -1.47148 |
| 34 | 3.78284 | 84 | -1.68362 |
| 35 | 3.81973 | 85 | -1.89567 |
| 36 | 3.85068 | 86 | -2.10722 |
| 37 | 3.87559 | 87 | -2.31783 |
| 38. | 3.89437 | 88. | -2.52704 |
| 39. | 3.90695 | 89 | -2.73439 |
| 40. | 3.91324 | 90. | -2.9394 |
| 41. | 3.91318 | 91. | -3.14157 |
| 42 | 3.90671 | 92 | -3.34038 |
| 43 | 3.89377 | 93 | -3.5353 |
| 44 | 3.87431 | 94 | -3.72579 |
| 45 | 3.8483 | 95 | -3.91129 |
| 46. | 3.8157 | 96 | -4.09121 |
| 47 | 3.77649 | 97 | -4.26497 |
| 48. | 3.73066 | 98. | -4.43194 |
| 49. | 3.6782 | 99 | -4.5915 |
| 50. | 3.6191 | 100 | -4.743 |

CONTINUING TABLE 8

| RADIUS MM | DISTORTION MICRON |
|---|---|
| 101 | -4.88578 |
| 102 | -5.01915 |
| 103 | -5.14242 |
| 104 | -5.25487 |
| 105 | -5.35576 |
| 106 | -5.44434 |
| 107 | -5.51983 |
| 108 | -5.58145 |
| 109 | -5.62839 |
| 110 | -5.65981 |
| 111 | -5.67487 |
| 112 | -5.6727 |
| 113 | -5.65242 |
| 114 | -5.61312 |
| 115 | -5.55386 |
| 116 | -5.47372 |
| 117 | -5.37171 |
| 118 | -5.24686 |
| 119 | -5.09815 |
| 120 | -4.92455 |
| 121 | -4.72502 |
| 122 | -4.49849 |
| 123 | -4.24387 |
| 124 | -3.96004 |
| 125 | -3.64587 |
| 126 | -3.30019 |
| 127 | -2.92185 |
| 128 | -2.50962 |
| 129 | -2.0623 |
| 130 | -1.57862 |
| 131 | -1.05734 |
| 132 | -0.497154 |
| 133 | 0.103251 |
| 134 | 0.74521 |
| 135 | 1.43008 |
| 136 | 2.15926 |
| 137 | 2.93414 |
| 138 | 3.75616 |
| 139 | 4.62677 |
| 140 | 5.54746 |
| 141 | 6.51972 |
| 142 | 7.54509 |
| 143 | 8.62512 |
| 144 | 9.76139 |
| 145 | 10.9555 |
| 146 | 12.2091 |
| 147 | 13.5238 |
| 148 | 14.9013 |
| 149 | 16.3434 |
| 150 | 17.8516 |
| 151 | 19.4279 |

COEFF. FOR LES DISTORTION,K0,K1,K2

1.48932E-4
-3.42813E-8
1.46451E-12

RESIDUALS OF CURVE FITTING

0
4.55182E-4
7.09069E-4
5.77941E-4
-8.67738E-5
-1.38093E-3
6.69933E-4
1.28119E-4
-8.38606E-4
6.05399E-5
2.56931E-4
1.34011E-3
-9.24646E-4
-5.78732E-4
5.47336E-4
-9.88182E-5

The corrections to the photogrammetric refraction for large scale photographs (flight height 300 feet - 12,000 feet) can be neglected as shown in previous report Hou [6].

3-3-5. Space resection and intersection for Relative orientations and model connections.

Analytical aerotriangulation requires computations consisting of space resections and intersections.

Space resections are used to determine the location of both camera stations (R and L). The space resection program is based on the well known collinearity equations as follows:

$$x = \left[ \frac{(X-X_o)m_{11} + (Y-Y_o)m_{12} + (Z-Z_o)m_{13}}{(X-X_o)m_{31} + (Y-Y_o)m_{32} + (Z-Z_o)m_{33}} \right] C$$

$$y = \left[ \frac{(X-X_o)m_{21} + (Y-Y_o)m_{22} + (Z-Z_o)m_{23}}{(X-X_o)m_{31} + (Y-Y_o)m_{32} + (Z-Z_o)m_{33}} \right] C \ldots\ldots\ldots\ldots\ldots (39)$$

where x and y are the image coordinates (right or left depending upon the camera station in question) of the control point whose space coordinate is X, Y and Z. $X_o$, $Y_o$, and $Z_o$ are the space coordinates of camera station (right or left initially estimated). C is the focal length of the camera; m's are elements of rotational M orthogonal matrix consisting of direction cosines or of the exterior ($\varphi$, w, k) orientation elements of the presently unknown camera station. The above equations can be written in the form of collinearity condition using determinant notation, namely:

$$\begin{vmatrix} x & c \\ M_1X^* & M_3X^* \end{vmatrix} = 0 = Fx$$

$$\begin{vmatrix} x & c \\ M_2X^* & M_3X^* \end{vmatrix} = 0 = Fy \qquad (40)$$

where

$$M_i = [m_{i1}, m_{i2}, m_{i3}]$$

$$X^* = \begin{bmatrix} X-X_o \\ Y-Y_o \\ Z-Z_o \end{bmatrix}$$

if the points are more than the unknowns, the least square adjustment is used. The observation equations are:

$$Vx = Fx + dFx$$

$$Vy = Fy + dFy \qquad (41)$$

where F's are composed of initially estimated values and

$$dFx = \frac{\partial Fx}{\partial X_o} \Delta X_o + \frac{\partial Fx}{\partial Y_o} \Delta Y_o + \frac{\partial Fx}{\partial Z_o} \Delta Z_o$$

$$+ \frac{\partial Fx}{\partial w} \Delta w + \frac{\partial Fx}{\partial \varphi} \Delta \varphi + \frac{\partial Fx}{\partial k} \Delta \kappa \qquad (42)$$

This is where $\Delta$'s are the corrections to the estimated values which theoretically should be $dX_o$, $dY_o$, etc. A similar equation is formed for the dFy.

From the observation equations, the normal equations are formed, and the $\Delta$'s are computed. These are then regarded as corrections and added to the originally estimated values. The process as can be seen is based on Newton's method of iteration, and continues until the $\Delta$'s are zero.

The positions of the camera are then determined through these resections. Then the points observed on the surface of the point are determined by space intersections. The same mathematical concept has been used as presented before with the only difference in the coordinates of camera stations ($X_o$, $Y_o$, $Z_o$ - Equation (39) now known as X, Y, and Z space coordinates, computed from observed points. Equation (41) remains the same as as a general notation but Equation (42) modifies to the following formula:

$$dFx = \frac{\partial Fx}{\partial X} dX + \frac{\partial Fx}{\partial Y} dY + \frac{\partial Fx}{\partial Z} dZ, \text{ etc.} \qquad (43)$$

A modified computer program of Schut [5] for space resection and intersection for relative orientation and model connection has been employed in analytical aerotriangulation.

The input data for these computations were first obtained from linear transformation from comparator coordinates to photo coordinates. The comparator coordinates were measured on the University of Washington's AP/2c analytical plotter. The instrument was used as a stereo comparator and an instruction for operations has been written by the author.

The operator only eliminates the x, y parallaxes for an observation.

In this way it is possible to observe and record up to 250 points per hour.

The comparator coordinates, recorded on punched paper tape, are transformed by a time-sharing computer system according to section 3-3-1. Subsequent programs for correcting film and lens distortion, for relative orientation and model connections are automatically linked to the initial program.

3-3-6. Strip adjustment.

After the relative orientation and model connection, the strip coordinates can be transformed to a ground coordinate system and adjusted with few known ground control points. The transformed adjustments used by the Washington State Highway Department of Photogrammetry for strip and block adjustments has been programmed to utilize I.B.M. (model 360/50) computer (Schut [2]) and G.E. Time-share computer services (Hou [3]).

The input and output data can be found in appendix.

3-4. Conclusion.

Using the same test strip, similar accuracies were obtained from semi-analytical, analogical and (analytical-aerotriangulation methods using University of Washington's Stereo-comparitor AP/2C).

Since a comparator is not available in our department, a comparison of time between analytical and analogical methods are unobtainable. However, in analytical aerotriangulation it requires only the image coordinate measurements. Relative, absolute orientation, model connection and adjustments are then performed by computer.

Savings in time using a comparator rather than an analogical instrument would be noticably greater.

Linear transformation for detection of film distortion may be considered for adoption in semi-analytical or analogical aerotriangulation used in our photogrammetric procedures.

# CHAPTER IV

## CONCLUSIONS AND RECOMMENDATIONS

On the basis of this study, several findings in aerotriangulation indicate a recommendation for their adoption into the Photogrammetry Branch which will result in increased stereo model production, better accuracies, and saving in processing time.

1. Using the same test strip, similar accuracies are obtained from semi-analytical, analogical, and analytical-aerotriangulation methods. Since a comparator is not available in our Department, a comparison of time between analytical and analogical methods are unobtainable. However, in analytical aerotriangulation it requires only the image coordinates measurement. Relative, absolute orientation, model connection, and adjustments are then performed by computer. Savings in time using a stereo comparator rather than an analogical instrument would be greater.

2. The plotters in our Department such as Wild Autograph A-7 and A-8 could be used as mono-comparators for analytical aerotriangulation. However, it requires a minimum of 9 pass points (PUG Points) which must be drilled into the diapositives by means of a point transfer devise. In comparison, a stereo-comparator requires only three pass points per diapositive. The probable errors by points transfer, plus the additional time required

to pug and measure the additional points utilizes much more time over that of a stereo-comparator. Thus, the analogical instrument is not recommended for use as a mono comparator in production work.

3. Linear transformation of comparator coordinates to photo coordinates system in analytical aerotriangulation can be considered for adoption in semi-analytical or analogical aerotriangulation for detection of film distortions.

4. The results obtained in various tests and in practical work show that the semi-analytical method should be used for aerotriangulation operations in our Department. The advantages of this method can be illustrated as follows:

a. Total time for aerotriangulation operations will be reduced to 40% in comparison to present method, thus reducing total cost of the photogrammetric services for any production work.

b. Recorded orientation elements in aerotriangulation using stereo plotter (Wild A-7) can be reset for mapping or cross-sectioning in Wild A-8 or Santoni IIc. Thus, reducing the time of relative orientation operations which in turn reduces the total cost of the photogrammetric services.

c. The Wild Autograph A-8 has the capabilities to perform the aerotriangulation procedures during overloads of Wild A-7 in the bridging schedules.

d. Most experienced stereo-operator can perform the semi-analytical aerotriangulation procedures without any additional special training.

5. The position of the projector centers in the model coordinate system for semi-analytical aerotriangulation can be determined by using the simple level method, which results in similar accuracies, when compared to the space resection method.

6. In order to eliminate the operation errors during processing of semi-analytical aerotriangulation, the following facts must be taken into consideration.

a. After using the quick release mechanism in A-7 or A-8 for relative orientation, one must orient drive x,y wheels to x,y drum values of left projector as recorded. Then reindex x,y in the digitizer as recorded.

b. The digitizer coordinate x must increase in direction of bridging of the strip to right. y must increase towards the top of the photo, Z should be set on zero to the lowest reference line of the Z scale in instrument.

# APPENDIX A

Determination of the perspective center by use of space resection method.

1. Input and Output data.
2. Computer program listings (BASIC).

# INPUT & OUTPUT

```
SPACE*        13:21PST     01/09/73

                   WASHINGTON STATE HIGHWAYS DEPARTMENT
NUMERICAL DETERMINATION OF SPACE RESECTION FOR OBLIQUE PHOTOGRAPH


PHOTO NO. & FOCAL LENGTH? RIGHT-1,152.36

ENTER PHOTO COORDINATES FOR 4 POINTS,X1,Y1,X2,Y2,X3,Y3,X4,Y4
USE 0 FOR 4TH WHEN USING 3 POINTS
? 100,100,0,-100,100,-----100,100,0,0

ENTER GROUND COORDINATES 4 POINTS,EAST,NORTH,ELEV. IN SAME
ORDER AS PHOTO COORDINATES,USE 0 FOR 4TH WHEN USING 3 POINTS.
? 7669.19,1588.25,0,7439.54,1128.58,0,7209.63,1588.15,0,0,0,0

ENTER APPROXIMATE COORDINATES OF CAMERA POSITION,E,N,ELEV.
? 7400,1360,300

             COORDINATES OF CAMERA POSITION   PHOTO NO:RIGHT-1 .
POINTS.        EAST          NORTH            FLIGHT-ALTITUDE  REMARK.

1,2,3 .         7439.5         1358.49 .        350.139

                  ORIENTATION ANGLES (DEGREES).
POINTS.        DEPRESSION     SWING            AZIMUTH.

1,2,3           89.988 .       18.4009 .        198.386



USED    .25 UNITS.
BYE .
0000.60. CRU    0000.13  TCH    0002.10. KC.

OFF AT 13:25PST 01/09/73.
```

```
050 PRINT "                    WASHINGTON STATE HIGHWAYS DEPARTMENT"
100 PRINT "NUMERICAL DETERMINATION OF SPACE RESECTION FOR OBLIQUE PHOTO"
101 REM MATHEMATIC ANALYSIS AND COMPUTER PROGRAM BY C.Y.HOU
102 PRINT
103 PRINT
105 PRINT "PHOTO NO. & FOCAL LENGTH";
110 INPUT A$,F
120 PRINT "PHOTO COORDINATES FOR X(I),Y(I),WHERE I=1 TO 3"
130 INPUT X1,Y1,X2,Y2,X3,Y3
140 PRINT "GROUND COORDINATES FOR E(I),N(I),H(I), WHERE I=1 TO 3"
150 INPUT E1,N1,H1,E2,N2,H2,E3,N3,H3
200 PRINT "APPROXIMATE COORDINATES OF PERSPECTIVE CENTER (EP,NP,HP)"
210 INPUT E8,N8,H8
220 PRINT
230 LET D1=SQR((X1-X2)↑2+(Y1-Y2)↑2+(F)↑2)
240 LET D2=SQR((X1-X3)↑2+(Y1-Y3)↑2+(F)↑2)
250 LET D3=SQR((X2-X3)↑2+(Y2-Y3)↑2+(F)↑2)
260 LET L7=SQR((E1-E2)↑2+(N1-N2)↑2+(H1-H2)↑2)
270 LET L8=SQR((E1-E3)↑2+(N1-N3)↑2+(H1-H3)↑2)
280 LET L9=SQR((E2-E3)↑2+(N2-N3)↑2+(H2-H3)↑2)
300 LET H4=(F/D1)*L7
310 LET H5=(F/D2)*L8
320 LET H6=(F/D3)*L9
330 LET H8=(H4+H5+H6)/3
350 LET D4=SQR((X1)↑2+(Y1)↑2+(F)↑2)
360 LET D5=SQR((X2)↑2+(Y2)↑2+(F)↑2)
370 LET D6=SQR((X3)↑2+(Y3)↑2+(F)↑2)
410 LET I1=-X1/D4
415 LET I2=-X2/D5
416 LET I3=-X3/D6
418 LET J1=-Y1/D4
419 LET J2=-Y2/D5
420 LET J3=-Y3/D6
423 LET K1=F/D4
424 LET K2=F/D5
425 LET K3=F/D6
430 LET A=(I1*I2)+(J1*J2)+(K1*K2)
440 LET B=(I2*I3)+(J2*J3)+(K2*K3)
445 LET C=(I3*I1)+(J3*J1)+(K3*K1)
446 FOR R=1 TO 20
450 LET L4=SQR((E8-E1)↑2+(N8-N1)↑2+(H8-H1)↑2)
451 LET L5=SQR((E8-E2)↑2+(N8-N2)↑2+(H8-H2)↑2)
452 LET L6=SQR((E8-E3)↑2+(N8-N3)↑2+(H8-H3)↑2)
455 LET E4=E8-E1
456 LET E5=E8-E2
```

```
457 LET E6=E8-E3
458 LET N4=N8-N1
459 LET N5=N8-N2
460 LET N6=N8-N3
463 LET Z4=H8-H1
464 LET Z5=H8-H2
465 LET Z6=H8-H3
470 LET I4=E4/L4
471 LET I5=E5/L5
472 LET I6=E6/L6
474 LET J4=N4/L4
475 LET J5=N5/L5
476 LET J6=N6/L6
477 LET K4=Z4/L4
478 LET K5=Z5/L5
479 LET K6=Z6/L6
481 LET A1=(I4*I5)+(J4*J5)+(K4*K5)
482 LET A2=(I5*I6)+(J5*J6)+(K5*K6)
483 LET A3=(I6*I4)+(J6*J4)+(K6*K4)
495 IF A=A1 THEN 40000
496 LET V1=A-A1
497 LET V2=B-A2
498 LET V8=C-A3
500 LET I7=(E4*E5)+(N4*N5)+(Z4*Z5)
510 LET J7=(E5*E6)+(N5*N6)+(Z5*Z6)
520 LET K7=(E6*E4)+(N6*N4)+(Z6*Z4)
530 LET A4=A*L4*L5
540 LET A5=B*L5*L6
550 LET A6=C*L6*L4
560 LET W1=I7-A4
570 LET W2=J7-A5
580 LET W8=K7-A6
581 LET W1=W1*(-1)
582 LET W2=W2*(-1)
583 LET W8=W8*(-1)
600 LET G1=L4/L5
610 LET G2=L5/L4
620 LET G3=L6/L5
630 LET G4=L6/L4
640 LET G5=L4/L6
650 LET G6=L5/L6
660 LET G7=(1-G1*A)
670 LET G8=(1-G2*A)
```

```
680 LET G9=(1-G6*B)
690 LET P1=(1-G3*B)
700 LET P2=(1-G4*C)
710 LET P3=(1-G5*C)
720 LET O1=(G7*E5)+(G8*E4)
730 LET B1=(G7*N5)+(G8*N4)
740 LET C1=(G7*Z5)+(G8*Z4)
750 LET O2=(G9*E6)+(P1*E5)
760 LET B2=(G9*N6)+(P1*N5)
770 LET C2=(G9*Z6)+(P1*Z5)
780 LET O3=(P2*E4)+(P3*E6)
790 LET B3=(P2*N4)+(P3*N6)
800 LET C3=(P2*Z4)+(P3*Z6)
830 LET O4=O2*O1
840 LET O5=O2*B1
850 LET O6=O2*C1
860 LET O7=O2*W1
870 LET B4=O1*O2
880 LET B5=O1*B2
890 LET B6=O1*C2
900 LET B7=O1*W2
910 LET X4=O1*O3
915 LET X5=O1*B3
920 LET X6=O1*C3
925 LET X7=O1*W3
930 LET Y4=O3*O1
935 LET Y5=O3*B1
940 LET Y6=O3*C1
945 LET Y7=O3*W1
950 LET F2=O5-B5
955 LET F3=O6-B6
960 LET F4=O7-B7
965 LET F6=X5-Y5
970 LET F7=X6-Y6
975 LET F8=X7-Y7
985 LET X8=F4*F7
986 LET Y8=F3*F8
990 LET X9=F2*F7
995 LET O8=F3*F6
1000 LET B8=F2*F8
1100 LET B9=F4*F6
1200 LET T9=X9-O8
1300 LET T2=(X8-Y8)/T9
```

```
 1400 LET T3=(B8-B9)/T9
1500 LET T6=-(B1*T2)
1600 LET T7=-(C1*T3)
1700 LET T1=(T6+T7+W4)/O1
19001 LET M1=E8+T1
19003 LET M2=N8+T2
19004 LET M3=H8+T3
20000LET E8=M1
20001 LET N8=M2
20003 LET H8=M3
20050 NEXT K
40000 PRINT "PERSPECTIVE CENTER COORDINATES"
40001 PRINT "EAST";M1,"NORTH";M2,"ELEVATION";M3
99999 END

READY.
```

# APPENDIX B

Instruction for Semi-analytical aerotriangulation.

1. Instruction

2. Input
    1. Ground control points
    2. Single model coordinates

3. Output
    1. Strip coordinates
    2. Adjustment

4. Computer program listings

## INPUT DATA

```
//PH#30136 JOB (,
//          65SA,,2,500),'65SA SEMI-ANALYSIS  ',
//          CLASS=L,MSGLEVEL=1,TIME=1
//P65242  EXEC PGM=P65242,REGION=36K
//STEPLIB    DD   DSNAME=REMOTE.LODLIB(P65242),DISP=SHR
//FT01F001    DD  DDNAME=SYSIN
//FT03F001    DD  SYSOUT=A,DCB=(RECFM=FA,LRECL=133,BLKSIZE=133)
//FT05F001    DD  DSN=&&FILE05,UNIT=SYSDA,SPACE=(CYL,(1,1)),
//             DCB=(RECFM=FB,LRECL=80,BLKSIZE=80),DISP=(,PASS)
//FT06F001    DD  SYSOUT=A,DCB=(RECFM=FA,LRECL=133,BLKSIZE=133)
//FT09F001    DD  DSN=&&FILE09,UNIT=SYSDA,SPACE=(CYL,(1,1)),
//             DCB=(RECFM=FB,LRECL=80,BLKSIZE=80),DISP=(,PASS)
//SYSUDUMP    DD  SYSOUT=A
//SYSIN       DD  *
71-373   017             032973          TOLL PLAZA TO SR405-RESEARCH   5 29361
    30012                         2837
    30013166168895 23804498       2859
    30001                        17538
    30003                        14502
    30005                         9566
    30006                         5902
    30007                         5223
    30008                         9005
    30009166354686 23822118       6207
    30011                         5437
    30040                        11506
    30041                         5425
    30042                         8332
    30043                        17160
    30044                        19313
    30045                        21975
    30333                        10404
    30342                        13514
    30343                        13833
    30346                        14391
    30348                        14905
    66164166302920 23738898       7944
    40001165333876 23576099      10523
    40334166501455 23552468      10796
    66072166689327 23392044      17783
    70004166566406 23527397      13505
-111
-  1
 11151111    189487    200203    95974
 11151112    210501    200207    95979
 02910291    192970    215475   062895
 02910292    192225    199212   061063
 02910294    202729    197251   061113
 02930009    196493    208931   061597
 02966164    204045    189284   061720
 02980017    196179    183073   061202
 02980018    200210    186614   061127
 02981001    188558    179750   060793
-  2
 11111112    210501    200207    95979
 02910301    214486    217618   062506
```

| | | | |
|---|---|---|---|
| 02910302 | 213951 | 200430 | 061478 |
| 02910303 | 211996 | 180372 | 062931 |
| 02930008 | 213916 | 210174 | 062105 |
| 02930040 | 214616 | 184158 | 062362 |
| - 1 | | | |
| 11111111 | 189487 | 200203 | 95974 |
| 03010301 | 193154 | 218165 | 061416 |
| 03010302 | 192530 | 200446 | 060357 |
| 03010303 | 190473 | 179766 | 061876 |
| 03030008 | 192538 | 210493 | 061006 |
| 03030040 | 193180 | 183665 | 061250 |
| 03010304 | 203472 | 196376 | 059798 |
| 03030011 | 198160 | 200014 | 059987 |
| 03080019 | 205271 | 197311 | 059843 |
| - 2 | | | |
| 11111112 | 210501 | 200207 | 95979 |
| 03010311 | 211362 | 217103 | 060292 |
| 03010312 | 211902 | 201513 | 059568 |
| 03010313 | 211113 | 180236 | 060605 |
| 03030007 | 206860 | 205855 | 059821 |
| 03030041 | 215816 | 188421 | 059479 |
| 03080020 | 210793 | 197648 | 059862 |
| - 1 | | | |
| 11111111 | 189487 | 200203 | 95974 |
| 03110311 | 191479 | 218984 | 056402 |
| 03110312 | 191878 | 201673 | 055587 |
| 03110313 | 190684 | 178051 | 056696 |
| 03130007 | 186340 | 206565 | 055760 |
| 03130041 | 196040 | 187084 | 055571 |
| 03180020 | 190589 | 197395 | 055880 |
| 03110314 | 205422 | 205402 | 055884 |
| - 2 | | | |
| 11111112 | 210501 | 200207 | 95979 |
| 03110321 | 214457 | 221412 | 056623 |
| 03110322 | 213971 | 200277 | 056660 |
| 03110323 | 211420 | 181112 | 056447 |
| 03130006 | 207931 | 213481 | 055991 |
| 03130042 | 212479 | 187026 | 056272 |
| - 1 | | | |
| 11111111 | 189487 | 200203 | 95974 |
| 03210321 | 193063 | 221695 | 056117 |
| 03210322 | 193112 | 200298 | 056180 |
| 03210323 | 191021 | 180825 | 055977 |
| 03230006 | 186668 | 213509 | 055466 |
| 03230042 | 191942 | 186844 | 055797 |
| 03210324 | 202053 | 199672 | 056566 |
| 03230005 | 201678 | 210350 | 056372 |
| 03230333 | 201281 | 198558 | 056456 |
| 03240334 | 204262 | 198705 | 056562 |
| 03210333 | 210069 | 176381 | 059151 |
| 03270004 | 218927 | 207955 | 057357 |
| - 2 | | | |
| 11111112 | 210501 | 200207 | 95979 |
| 03210331 | 214835 | 221815 | 058101 |
| 03210332 | 213766 | 202537 | 057167 |

```
03210335   210028   185896   057502
03240004   218927   207955   057357
03270333   216880   180562   058706
-  1
11111111   189487   200203    95974
03310331   192150   220144   061075
03310332   192433   202395   060293
03310335   190083   186892   060664
03340004   196813   207699   060460
03370333   196717   182453   061811
03310334   201962   199208   060510
03330342   201733   197718   060420
03330343   204430   197223   060500
-  2
11111112   210501   200207    95979
03310341   214247   220348   061972
03310342   213731   198437   060871
03310343   212274   182823   061228
03330043   217092   190712   061265
03330346   209935   196181   060642
03330348   215352   195309   060770
-  1
11111111   189487   200203    95974
03410341   192806   220239   062219
03410342   193089   198489   061167
03410343   192201   182943   061539
03430043   196697   190948   061611
03430346   189407   196116   060908
03430348   194808   195444   061091
03410344   203600   198193   061483
03430003   198478   205715   061094
03430044   212324   186348   062283
-  2
11111112   210501   200207    95979
03410351   215184   218224   061578
03410352   214977   198917   062391
03410353   215661   185488   062412
03466072   216572   215556   062120
-  1
11111111   189487   200203    95974
03510351   192760   215051   067648
03510352   192768   199182   068321
03510353   193451   188145   068313
03566072   193930   212874   068075
03510354   203135   198512   068447
03510361   209567   217188   068030
03510362   210354   198633   068396
03510363   208866   186328   068762
03530001   201928   220619   068049
03530045   207129   187072   068712
03581002   201679   208196   067734
03581003   208623   204036   067993
03581035   200263   208485   067740

//        EXEC   JBKY65A7
//BACKSPCE.SYSUT1   DD   DSN=&&FILE05,DISP=(OLD,DELETE)
```

```
//                  DD   DSN=&&FILE09,DISP=(OLD,DELETE)
/*
```

INVOICE NO. 71-373 TOLL PLAZA TO SR405-RESEARCH FLIGHT NO. 5 PHOTO NO. 29-36 RUN 1

| MODEL IDENT | POINT IDENT | STRIP COORDINATES X | Y | Z | RESIDUALS DX | DY | DZ |
|---|---|---|---|---|---|---|---|
| 33 | 10335 | 2665.49 | 1880.46 | 628.27 | -0.02 | -0.00 | -0.00 |
| 33 | 40004 | 2744.11 | 2070.06 | 626.31 | 0.01 | -0.00 | 0.01 |
| 33 | 70333 | 2724.29 | 1833.87 | 638.16 | 0.01 | -0.00 | 0.00 |
| 33 | 10334 | 2785.89 | 1986.73 | 625.95 | | | |
| 33 | 30342 | 2782.61 | 1972.96 | 625.08 | | | |
| 33 | 30343 | 2807.48 | 1966.29 | 625.52 | | | |
| 111 | 11112 | 2870.27 | 1988.13 | 957.97 | | | |
| 34 | 10341 | 2916.94 | 2175.18 | 639.09 | -0.00 | -0.01 | 0.04 |
| 34 | 10342 | 2895.45 | 1970.62 | 628.03 | -0.00 | 0.00 | -0.00 |
| 34 | 10343 | 2870.07 | 1825.57 | 630.98 | 0.00 | -0.04 | -0.03 |
| 34 | 30043 | 2921.11 | 1895.79 | 631.11 | 0.01 | -0.00 | 0.01 |
| 34 | 30346 | 2858.21 | 1952.39 | 626.21 | -0.00 | 0.01 | -0.01 |
| 34 | 30348 | 2908.23 | 1940.13 | 626.81 | -0.01 | 0.00 | 0.01 |
| 34 | 10344 | 2993.90 | 1956.26 | 628.88 | | | |
| 34 | 30003 | 2954.00 | 2032.58 | 626.62 | | | |
| 34 | 30044 | 3062.95 | 1835.34 | 634.10 | | | |
| 111 | 11112 | 3067.63 | 1965.06 | 953.78 | | | |
| 35 | 10351 | 3124.76 | 2131.67 | 628.46 | 0.00 | -0.01 | 0.01 |
| 35 | 10352 | 3101.72 | 1950.49 | 635.25 | -0.01 | 0.00 | 0.04 |
| 35 | 10353 | 3093.37 | 1823.55 | 634.55 | 0.00 | -0.03 | -0.05 |
| 35 | 66072 | 3134.96 | 2105.06 | 633.17 | -0.00 | 0.01 | 0.00 |
| 35 | 10354 | 3219.06 | 1927.69 | 635.91 | | | |
| 35 | 10361 | 3319.73 | 2131.47 | 631.72 | | | |
| 35 | 10362 | 3301.62 | 1918.53 | 634.79 | | | |
| 35 | 10363 | 3266.68 | 1780.24 | 638.39 | | | |
| 35 | 30001 | 3237.56 | 2181.79 | 632.72 | | | |
| 35 | 30045 | 3247.94 | 1791.27 | 637.99 | | | |
| 35 | 81002 | 3216.54 | 2040.39 | 628.39 | | | |
| 35 | 81003 | 3289.74 | 1982.75 | 630.60 | | | |
| 35 | 81035 | 3200.81 | 2045.75 | 628.58 | | | |



**OUTPUT**

WASHINGTON STATE HIGHWAY COMMISSION --INDEPENDENT MODELS TRIANGULATION-- DATE 03/29/73 PAGE 1

INVOICE NO. 71-373 TOLL PLAZA TO SR405-RESEARCH FLIGHT NO. 5 PHOTO NO. 29-36 RUN 1

| MODEL IDENT | POINT IDENT | STRIP COORDINATES X | Y | Z | RESIDUALS DX | DY | DZ |
|---|---|---|---|---|---|---|---|
| 111 | 51111 | 1894.87 | 2002.03 | 959.74 | | | |
| 111 | 51112 | 2105.01 | 2002.07 | 959.79 | | | |
| 29 | 10291 | 1929.70 | 2154.75 | 628.95 | | | |
| 29 | 10292 | 1922.25 | 1992.12 | 610.63 | | | |
| 29 | 10294 | 2027.29 | 1972.51 | 611.13 | | | |
| 29 | 30009 | 1964.93 | 2089.31 | 615.97 | | | |
| 29 | 66164 | 2040.45 | 1892.84 | 617.20 | | | |
| 29 | 80017 | 1961.79 | 1830.73 | 612.02 | | | |
| 29 | 80018 | 2002.10 | 1866.14 | 611.27 | | | |
| 29 | 81001 | 1885.58 | 1797.50 | 607.93 | | | |
| 111 | 11112 | 2105.01 | 2002.07 | 959.79 | | | |
| 30 | 10301 | 2144.86 | 2176.19 | 625.05 | 0.00 | 0.01 | -0.01 |
| 30 | 10302 | 2139.50 | 2004.31 | 614.79 | -0.01 | 0.01 | 0.01 |
| 30 | 10303 | 2119.95 | 1803.70 | 629.33 | -0.01 | -0.02 | 0.02 |
| 30 | 30008 | 2139.17 | 2101.76 | 621.04 | 0.01 | 0.02 | -0.01 |
| 30 | 30040 | 2146.17 | 1841.58 | 623.63 | 0.01 | 0.00 | 0.01 |
| 30 | 10304 | 2245.81 | 1965.17 | 610.92 | | | |
| 30 | 30011 | 2194.16 | 2000.30 | 611.99 | | | |
| 30 | 80019 | 2263.22 | 1974.29 | 611.61 | | | |
| 111 | 11112 | 2308.81 | 2002.74 | 962.77 | | | |
| 31 | 10311 | 2321.63 | 2166.44 | 616.70 | -0.01 | -0.00 | 0.01 |
| 31 | 10312 | 2327.44 | 2015.26 | 609.83 | -0.01 | 0.01 | -0.01 |
| 31 | 10313 | 2320.29 | 1808.83 | 619.92 | 0.00 | -0.03 | 0.01 |
| 31 | 30007 | 2278.39 | 2057.23 | 611.55 | 0.01 | 0.02 | -0.01 |
| 31 | 30041 | 2365.79 | 1888.39 | 609.65 | -0.01 | 0.01 | 0.04 |
| 31 | 80020 | 2316.77 | 1977.73 | 612.54 | 0.01 | 0.00 | -0.02 |
| 31 | 10314 | 2445.24 | 2049.70 | 611.67 | | | |
| 111 | 11112 | 2492.36 | 2005.65 | 961.74 | | | |
| 32 | 10321 | 2521.99 | 2190.76 | 617.47 | -0.01 | -0.02 | 0.05 |
| 32 | 10322 | 2520.65 | 2006.11 | 618.10 | -0.00 | 0.00 | 0.01 |
| 32 | 10323 | 2500.98 | 1838.31 | 616.60 | 0.00 | -0.05 | -0.04 |
| 32 | 30006 | 2466.06 | 2120.62 | 612.38 | 0.01 | 0.01 | 0.03 |
| 32 | 30042 | 2509.41 | 1890.14 | 614.94 | 0.00 | -0.02 | -0.03 |
| 32 | 10324 | 2597.76 | 1999.98 | 620.77 | | | |
| 32 | 30005 | 2595.39 | 2092.14 | 619.10 | | | |
| 32 | 30333 | 2591.00 | 1990.43 | 619.88 | | | |
| 32 | 40334 | 2616.74 | 1991.46 | 620.57 | | | |
| 32 | 10333 | 2665.20 | 1798.36 | 642.51 | | | |
| 32 | 70004 | 2744.09 | 2070.06 | 626.30 | | | |
| 111 | 11112 | 2673.67 | 2003.95 | 960.20 | | | |
| 33 | 10331 | 2709.97 | 2189.99 | 633.01 | -0.01 | 0.00 | 0.01 |
| 33 | 10332 | 2699.11 | 2023.74 | 625.06 | 0.01 | 0.00 | -0.01 |

PROCESSING ENDED AT EOD

INVOICE NO. 71-373 TOLL PLAZA TO SR405-RESEARCH FLIGHT NO. 5 PHOTO NO. 29 - 36 RUN 1

DEGREE OF CORRECTION FOR SCALE AND AZIMUTH, LONGITUDINAL BEND, TRANSVERSAL TILT - 222

| POINT IDENTIFICATION | | EASTING | NORTHING | ELEVATION | R E S I D U A L S | | |
|---|---|---|---|---|---|---|---|
| | | | | | EAST | NORTH | ELEVATION |
| | | **C O N T R O L** | **P O I N T S** | | | | |
| 29 | 30009 | 1663546.86 | 238221.18 | 62.07 | 0.12 | -0.02 | -0.08 |
| 29 | 66164 | 1663029.20 | 237388.98 | 79.44 | -0.13 | -0.05 | -0.06 |
| 30 | 30008 | 1664082.60 | 237608.86 | 90.05 | | | 0.11 |
| 30 | 30040 | 1663135.69 | 236852.37 | 115.06 | | | -0.09 |
| 30 | 30011 | 1663859.95 | 237119.15 | 54.37 | | | 0.18 |
| 31 | 30007 | 1664308.01 | 236966.01 | 52.23 | | | 0.07 |
| 31 | 30041 | 1663926.35 | 236166.94 | 54.25 | | | 0.06 |
| 32 | 30006 | 1665070.64 | 236446.88 | 59.02 | | | -0.18 |
| 32 | 30042 | 1664336.54 | 235638.51 | 83.32 | | | 0.02 |
| 32 | 30005 | 1665328.39 | 235886.85 | 95.66 | | | -0.10 |
| 32 | 30333 | 1664938.39 | 235617.45 | 104.04 | | | -0.02 |
| 32 | 40334 | 1665014.55 | 235524.68 | 107.96 | 0.02 | 0.09 | 0.04 |
| 33 | 40004 | 1665664.49 | 235272.87 | 134.93 | | | |
| 33 | 30342 | 1665412.17 | 234856.96 | 135.14 | | | 0.06 |
| 33 | 30343 | 1665457.32 | 234745.87 | 138.33 | | | 0.05 |
| 34 | 30043 | 1665515.15 | 234126.03 | 171.60 | | | -0.02 |
| 34 | 30346 | 1665548.34 | 234518.44 | 143.91 | | | 0.01 |
| 34 | 30348 | 1665643.47 | 234298.26 | 149.05 | | | -0.10 |
| 34 | 30003 | 1666115.35 | 234388.15 | 145.02 | | | -0.02 |
| 34 | 30044 | 1665689.66 | 233429.51 | 193.13 | | | 0.05 |
| 35 | 66072 | 1666893.27 | 233920.44 | 177.83 | 0.00 | -0.03 | 0.05 |
| 35 | 30001 | 1667466.48 | 233755.25 | 175.38 | | | 0.02 |
| 35 | 30045 | 1666046.50 | 232618.92 | 219.75 | | | -0.03 |
| | | | | ROOT MEAN SQUARE ERROR | 0.09 | 0.05 | 0.08 |
| | | **T R A N S F O R M E D** | **P O I N T S** | | | | |
| 29 | 10291 | 1663691.47 | 238536.45 | 118.28 | | | |
| 29 | 10292 | 1663065.90 | 238106.69 | 40.53 | | | |
| 29 | 10294 | 1663288.01 | 237661.23 | 46.89 | | | |
| 29 | 80017 | 1662577.22 | 237506.66 | 55.90 | | | |
| 29 | 80018 | 1662822.02 | 237456.20 | 51.91 | | | |
| 29 | 81001 | 1662239.60 | 237696.47 | 36.20 | | | |
| 111 | 11112 | 1663623.23 | 237473.80 | 1671.63 | | | |
| 30 | 10301 | 1664375.14 | 237796.89 | 105.41 | | | |
| 30 | 10302 | 1663721.38 | 237333.68 | 65.75 | | | |

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INVOICE NO. 71-373 TOLL PLAZA TO SR405-RESEARCH FLIGHT NO. 5 PHOTO NO. 29 - 36 RUN 1

DEGREE OF CORRECTION FOR SCALE AND AZIMUTH, LONGITUDINAL BEND, TRANSVERSAL TILT - 222

| POINT IDENTIFICATION | | EASTING | NORTHING | ELEVATION | RESIDUALS EAST | RESIDUALS NORTH | RESIDUALS ELEVATION |
|---|---|---|---|---|---|---|---|
| 30 | 10303 | 1662921.40 | 236843.74 | 142.55 | | | |
| 30 | 10304 | 1663874.51 | 236828.50 | 52.83 | | | |
| 30 | 80019 | 1663957.31 | 236789.46 | 56.13 | | | |
| 111 | 11112 | 1664198.15 | 236718.77 | 1691.63 | | | |
| 31 | 10311 | 1664835.18 | 237112.32 | 72.31 | | | |
| 31 | 10312 | 1664289.87 | 236665.81 | 47.82 | | | |
| 31 | 10313 | 1663503.14 | 236113.06 | 104.59 | | | |
| 31 | 80020 | 1664120.54 | 236600.19 | 61.93 | | | |
| 31 | 10314 | 1664748.76 | 236324.98 | 58.34 | | | |
| 111 | 11112 | 1664724.57 | 236045.18 | 1692.35 | | | |
| 32 | 10321 | 1665488.32 | 236436.40 | 80.86 | | | |
| 32 | 10322 | 1664798.89 | 235922.74 | 92.76 | | | |
| 32 | 10323 | 1664120.37 | 235524.29 | 93.33 | | | |
| 32 | 10324 | 1664992.87 | 235619.22 | 107.91 | | | |
| 32 | 10333 | 1664434.11 | 234803.32 | 221.16 | | | |
| 32 | 70004 | 1665664.06 | 235273.97 | 135.05 | 0.37 | -1.03 | -0.17 |
| 111 | 11112 | 1665227.68 | 235367.04 | 1690.97 | | | |
| 33 | 10331 | 1666013.92 | 235736.97 | 159.17 | | | |
| 33 | 10332 | 1665366.01 | 235309.77 | 129.94 | | | |
| 33 | 10335 | 1664739.54 | 235032.20 | 150.83 | | | |
| 33 | 70333 | 1664732.01 | 234683.38 | 201.06 | | | |
| 33 | 10334 | 1665472.53 | 234883.52 | 138.68 | | | |
| 111 | 11112 | 1665721.46 | 234592.54 | 1687.67 | | | |
| 34 | 10341 | 1666540.72 | 234927.20 | 194.84 | | | |
| 34 | 10342 | 1665720.76 | 234431.49 | 152.71 | | | |
| 34 | 10343 | 1665110.92 | 234118.18 | 172.76 | | | |
| 34 | 10344 | 1665944.27 | 234025.59 | 160.60 | | | |
| 111 | 11112 | 1666190.58 | 233794.85 | 1675.77 | | | |
| 35 | 10351 | 1666963.25 | 234032.81 | 154.29 | | | |
| 35 | 10352 | 1666226.16 | 233609.39 | 194.13 | | | |
| 35 | 10353 | 1665731.46 | 233283.43 | 196.88 | | | |
| 35 | 10354 | 1666471.56 | 233109.68 | 202.27 | | | |
| 35 | 10361 | 1667510.82 | 233308.75 | 176.01 | | | |
| 35 | 10362 | 1666669.78 | 232777.36 | 200.30 | | | |
| 35 | 10363 | 1666058.29 | 232518.34 | 222.77 | | | |
| 35 | 81002 | 1666882.59 | 233435.50 | 161.56 | | | |
| 35 | 81003 | 1666874.60 | 233001.82 | 177.19 | | | |

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INVOICE NO. 71-373 TOLL PLAZA TO SR405-RESEARCH FLIGHT NO. 5 PHOTO NO. 29 - 36 RUN 1

DEGREE OF CORRECTION FOR SCALE AND AZIMUTH, LONGITUDINAL BEND, TRANSVERSAL TILT - 222

| POINT IDENTIFICATION | EASTING | NORTHING | ELEVATION | RESIDUALS EAST | RESIDUALS NORTH | RESIDUALS ELEVATION |
|---|---|---|---|---|---|---|
| 35 81035 | 1666858.25 | 233508.97 | 161.65 | | | |

END OF JOB

```
HASP-II***HASP-II........END JOB  74......10.29.15 AM 29 MAR 73....ROOM 65SA....PH#30136....65SA SEMI-ANAL
HASP-II***HASP-II........END JOB  74......10.29.15 AM 29 MAR 73....ROOM 65SA....PH#30136....65SA SEMI-ANAL
HASP-II***HASP-II........END JOB  74......10.29.15 AM 29 MAR 73....ROOM 65SA....PH#30136....65SA SEMI-ANAL
HASP-II***HASP-II........END JOB  74......10.29.15 AM 29 MAR 73....ROOM 65SA....PH#30136....65SA SEMI-ANAL
HASP-II***HASP-II........END JOB  74......10.29.15 AM 29 MAR 73....ROOM 65SA....PH#30136....65SA SEMI-ANAL
HASP-II***HASP-II........END JOB  74......10.29.15 AM 29 MAR 73....ROOM 65SA....PH#30136....65SA SEMI-ANAL
```

```
//        65SA,,2,500),'65SA SEMI-ANALYSIS  ',
//        CLASS=L,MSGLEVEL=1,REGION=100K,TIME=1
//P65242 EXEC FORTGCL,PARM.FORT='LOAD,NODECK,LINECNT=54,NAME=P65242'
//FORT.SYSUDUMP DD SYSOUT=A
//FORT.SYSIN   DD *
C      INDEPENDENT MODELS STRIP-TRIANGULATION.
C      N.R.C. PROGRAM----G.H. SCHUT
C      MOD. PROM. BY C.Y. HOU FROM SCHUT PROGRAM
C      WASHINGTON STATE HIGHWAYS DEPARTMENT, PHOTOGRAMMETRY BRANCH , FEB.1973
C      TEST FOR WILD-B8.
       DOUBLE PRECISION AX(22),AY(22),AZ(22),AF(11),S(13),
     1      XP,YP,ZP,XQ,YQ,ZQ,X1,Y1,Z1,X2,Y2,Z2,D1,D2,F,
     2      S11,S12,S13,S14,S21,S22,S23,S24,S31,S32,S33,S34,S44,
     3      G1,G2,G3,G4,A,B,C,D,PROJ
       DIMENSION LS(22),LP(22),DATE(3),PHOTO(2),TITLE(7)
       INTEGER PAGE
       EQUIVALENCE (S11,S(1)),(S12,S(4)),(S13,S(7)),(S14,S(10)),
     1          (G1,S21,S(2)),(S22,S(5)),(S23,S(8)),(S24,S(11)),
     2      (G2,S31,S(3)),(G3,S32,S(6)),(S33,S(9)),(S34,S(12)),
     3          (G4,S(13))
     1 FORMAT(A6,3X,A2,I1,14X,3A2,10X,7A4,2X,A2,A3,A2,A1)
     2 FORMAT(4H-008)
     3 FORMAT(I4,I5,3I9)
     4 FORMAT(4H-222)
     5 FORMAT(37H1 WASHINGTON STATE HIGHWAY COMMISSION,3X,
     2        36H--INDEPENDENT MODELS TRIANGULATION--,3X,5HDATE ,A2,
     3        2(1H/,A2),4X,4HPAGE,I3/1H0,10X,12HINVOICE NO. ,A6,4X,7A4,
     4        14H    FLIGHT NO. ,A2,13H    PHOTO NO. ,A3,1H-,A2,6H  RUN ,
     5        A2/15H0  MODEL  POINT,11X,17HSTRIP COORDINATES,25X,
     6        9HRESIDUALS/15H0   IDENT   IDENT,8X,1HX,11X,1HY,11X,1HZ,
     7        12X,2HDX,10X,2HDY,10X,2HDZ///)
     6 FORMAT(I4,I5,3F9.5)
     7 FORMAT(1H ,2I7,3P6F12.2)
     8 FORMAT (14HOERROR AT CARD 2I7)
     9 FORMAT(80X)
       XYZ = 0.0
       PAGE = 1
       READ  (1,1) PROJ,ID,MOD,DATE,TITLE,FLT,PHOTO,RUN
       WRITE (5,1) PROJ,ID,MOD,DATE,TITLE,FLT,PHOTO,RUN
       WRITE (9,2)
       DO 15 I=1,100
       READ  (1,3) LA,LB,LC,LD,LE
       IF (LA.NE.0) GO TO 17
    15 WRITE (5,3) LA,LB,LC,LD,LE
    17 IF (MOD.EQ.1) GO TO 18
       WRITE (5,4)
       GO TO 20
    18 WRITE (5,3) LA
    20 LINE = 0
       WRITE (3,5) DATE,PAGE,PROJ,TITLE,FLT,PHOTO,RUN
       PAGE = PAGE+1
C
C      READ GROUND CONTROL,IF ANY
       IF (XYZ) 900,100,840
   100 J = 1
       MA = 1
       MB = 1
```

```
      MC = 1
      GO TO 801
  101 MBB = 3
      IF (J-1) 999, 110, 201
C
C     ALTERNATIVELY,READ FIRST POINT OF FIRST MODEL
  110 MB = 2
      MC = 2
      GO TO 801
C
C     READ TIE POINT WITH GROUND OR WITH PRECEDING MODEL
  200 MA=1
      MBB=4
  201 IF (J-12) 202, 202, 990
  202 K=J-1
      MC=3
      XP=AX(1)
      YP=AY(1)
      ZP=AZ(1)
      DO 203 L=1,13
  203 S(L)=0.
      GO TO 801
  204 IF (J-K-1) 999,205,210
  205 XQ=AX(K+1)
      YQ=AY(K+1)
      ZQ=AZ(K+1)
      GO TO 800
C
C     REDUCED COORDINATES,DISTANCES
  210 JK=J-K
      X1=AX(JK)-XP
      Y1=AY(JK)-YP
      Z1=AZ(JK)-ZP
      X2=AX(J)-XQ
      Y2=AY(J)-YQ
      Z2=AZ(J)-ZQ
      D1=DSQRT(X1*X1+Y1*Y1+Z1*Z1)
      D2=DSQRT(X2*X2+Y2*Y2+Z2*Z2)
      X1=X1/D1
      Y1=Y1/D1
      Z1=Z1/D1
      X2=X2/D2
      Y2=Y2/D2
      Z2=Z2/D2
      AF(JK)=D1/D2
C     CONTRIBUTIONS TO THE NORMAL EQUATION
      G4=G4+(X1+X2)**2+(Y1+Y2)**2+(Z1+Z2)**2
      S11=S11+X1*X2
      S22=S22+Y1*Y2
      S33=S33+Z1*Z2
      S14=S14+Z1*Y2
      S24=S24+X1*Z2
      S34=S34+Y1*X2
      G1=G1+Z2*Y1
      G2=G2+X2*Z1
```

```
      G3=G3+Y2*X1
      IF (JK-K) 800, 300, 999
C
C     COMPLETE THE NORMAL EQUATION
  300 S44=G4-4.*(S11+S22+S33)
      S11=G4-4.*S11
      S22=G4-4.*S22
      S33=G4-4.*S33
      S12=-2.*(G3+S34)
      S13=-2.*(G2+S24)
      S23=-2.*(G1+S14)
      S14=2.*(G1-S14)
      S24=2.*(G2-S24)
      S34=2.*(G3-S34)
C     SOLVE THE NORMAL EQUATIONS
      S22=S22-S12*S12/S11
      S23=S23-S13*S12/S11
      S24=S24-S14*S12/S11
      S33=S33-S13*S13/S11-S23*S23/S22
      S34=S34-S14*S13/S11-S24*S23/S22
      S44=S44-S14*S14/S11-S24*S24/S22
      IF (S44-S33) 301,301,302
  301 D=1.
      C=-S34/S33
      GO TO 303
  302 C=1.
      D=-S34/S44
  303 B=-(S24*D+S23*C)/S22
      A=-(S14*D+S13*C+S12*B)/S11
C     COMPUTE THE TRANSFORMATION MATRIX
      S11=D*D+A*A-B*B-C*C
      S22=D*D-A*A+B*B-C*C
      S33=D*D-A*A-B*B+C*C
      S21=2.*(A*B+C*D)
      S12=2.*(A*B-C*D)
      S13=2.*(A*C+B*D)
      S31=2.*(A*C-B*D)
      S32=2.*(B*C+A*D)
      S23=2.*(B*C-A*D)
      F=0.
      G4=K-1
      DO 304 L=2,K
  304 F=F+AF(L)/G4
      G4=F/(D*D+A*A+B*B+C*C)
      DO 305 L=1,9
  305 S(L)=S(L)*G4
C
C     TRANSFORM COMMON PROJECTION CENTRE AND TIES
      J=1
      MC=4
      LS(1)=LS(K+1)
      GO TO 820
  401 MB=MBB
      J=K+1
  402 J=J+1
      IF (J-2*K) 810,810,410
C     TRANSFORM ALL FOLLOWING POINTS, TILL -2 CARD IS MET
```

```
  410 J=1
      MA=2
      MB=2
      GO TO 801
  411 GO TO (401,801, 402, 402),MB
C
C     READ TIE POINTS WITH NEXT MODEL
  500 MB=1
      MC=5
      GO TO 801
C
C     SUBROUTINES
C     READ A CARD
  800 J=J+1
  801 READ  (1,6) LS(J),LP(J),AX(J),AY(J),AZ(J)
      IF (LS(J)) 902, 901, 802
  802 GO TO (811,810),MA
C     TRANSFORM
  810 X2=AX(J)-XO
      Y2=AY(J)-YO
      Z2=AZ(J)-ZO
      AX(J)=S11*X2+S12*Y2+S13*Z2+XP
      AY(J)=S21*X2+S22*Y2+S23*Z2+YP
      AZ(J)=S31*X2+S32*Y2+S33*Z2+ZP
      JK=J-K
  811 GO TO (900, 820, 831, 830),MB
C     PRINT AND PUNCH
  820 WRITE (3,7) LS(J),LP(J),AX(J),AY(J),AZ(J)
      GO TO 835
  830 AX(J)=0.5*(AX(J)+AX(JK))
      AY(J)=0.5*(AY(J)+AY(JK))
      AZ(J)=0.5*(AZ(J)+AZ(JK))
  831 DX=AX(J)-AX(JK)
      DY=AY(J)-AY(JK)
      DZ=AZ(J)-AZ(JK)
      IF (LP(J)-LP(JK)) 832,833,832
  832 LP(J)=0
  833 WRITE (3,7) LS(J),LP(J),AX(J),AY(J),AZ(J),DX,DY,DZ
  835 LINE = LINE+1
      IF (LINE.LT.42) GO TO 840
      XYZ = 1.0
      GO TO 20
  840 JX=100000.*AX(J)+0.5
      JY=100000.*AY(J)+0.5
      JZ=100000.*AZ(J)+0.5
      IF (LP(J).GE.20000.AND.LP(J).LT.70000) GO TO 845
      WRITE (9,3) LS(J),LP(J),JX,JY,JZ
      GO TO 900
  845 WRITE (5,3) LS(J),LP(J),JX,JY,JZ
C     SHUNT
  900 GO TO (800,801,204,411,800),MC
  901 GO TO (800,990,990,999,990),MC
  902 IF (LS(J)+2) 905,904,903
  903 GO TO (101,990,990,990,200),MC
  904 GO TO (990,500,990,500,990),MC
```

```
  905 GO TO (990,990,990,100,990),MC
C
  990 WRITE (3,8) LS(J),LP(J)
C     END OF ALL COMPUTATION
  999 WRITE (9,9)
      END FILE 5
      END FILE 9
      CALL EXIT
      END
/*
//LKED.SYSLMOD DD DSN=REMOTE.LODLIB(P65242),DISP=(OLD,KEEP),
//             SPACE=,DCB=BLKSIZE=
//LKED.SYSIN DD *
     NAME P65242(R)
//TEST       EXEC PGM=*.P65242.LKED.SYSLMOD,TIME=1
//FT01F001     DD DDNAME=SYSIN
//FT03F001     DD SYSOUT=A,DCB=(RECFM=FA,LRECL=133,BLKSIZE=133)
//FT05F001     DD DSN=&&FILE05,UNIT=SYSDA,SPACE=(CYL,(1,1)),
//            DCB=(RECFM=FB,LRECL=80,BLKSIZE=80),DISP=(,PASS)
//FT06F001     DD SYSOUT=A,DCB=(RECFM=FA,LRECL=133,BLKSIZE=133)
//FT09F001     DD DSN=&&FILE09,UNIT=SYSDA,SPACE=(CYL,(1,1)),
//            DCB=(RECFM=FB,LRECL=80,BLKSIZE=80),DISP=(,PASS)
//SYSUDUMP     DD SYSOUT=A
//SYSIN        DD *
**********INPUT DATA***************
/*
//         EXEC  JBKY65A7
//BACKSPCE.SYSUT1  DD  DSN=&&FILE05,DISP=(OLD,DELETE)
//                 DD  DSN=&&FILE09,DISP=(OLD,DELETE)
/*
```

# APPENDIX C

Linear transformation for comparator coordinates to photo coordinates system and film distortion computation.

1. Input (recorded from AP/2C - comparator coordinates)
2. Output
    1. Film distortion
    2. Transformed photo coordinates
    3. Computer program listings ( BASIC )

# INPUT

| | | | | |
|---|---|---|---|---|
| 0102002 | -0178866 | 0197626 | -0193838 | 0196784 |
| 0102002 | -0178812 | 0197550 | -0193832 | 0196784 |
| 0102002 | -0178828 | 0197528 | -0193834 | 0196780 |
| 0102002 | -0178820 | 0197528 | -0193838 | 0196782 |
| 0102003 | 0033182 | 0195110 | 0018192 | 0195822 |
| 0102003 | 0033184 | 0195124 | 0018194 | 0195820 |
| 0102004 | 0030754 | -0016920 | 0017200 | -0016246 |
| 0102004 | 0030756 | -0016924 | 0017200 | -0016250 |
| 0102001 | -0181266 | -0014492 | -0194824 | -0015266 |
| 0102001 | -0181268 | -0014498 | -0194822 | -0015266 |
| | | | | |
| 2000500 | -0083992 | 0001280 | -0183402 | 0008566 |
| ~~2000500~~ | ~~-0083990~~ | [illegible] | [illegible] | [illegible] |
| 2000500 | -0083990 | 0001286 | -0183406 | 0008566 |
| 1003100 | -0070744 | 0001556 | -0170760 | 0009148 |
| 1003100 | -0070744 | 0001558 | -0170788 | 0009146 |
| 1003100 | -0070744 | 0001558 | -0170772 | 0009132 |
| 2000600 | -0028788 | 0002238 | -0131032 | 0010704 |
| 2000600 | -0028794 | 0002234 | -0131032 | 0010700 |
| 1004100 | 0013592 | 0003478 | -0091220 | 0012782 |
| 1004100 | 0013636 | 0003444 | -0091218 | 0012790 |
| 1004100 | 0013650 | 0003448 | -0091222 | 0012790 |
| 1004100 | 0013616 | 0003430 | -0091216 | 0012782 |
| 7004100 | 0021424 | 0005910 | -0083970 | 0015204 |
| 7004100 | 0021434 | 0005882 | -0083962 | 0015210 |
| 7004100 | 0021438 | 0005910 | -0083968 | 0015204 |
| 2000800 | 0027806 | 0028284 | -0077646 | 0036506 |
| 2000800 | 0027828 | 0028294 | -0077642 | 0036506 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 2000800 | 0027832 | 0028300 | -0077648 | 0036504 |
| | | | | |
| 7004200 | 0022408 | 0026500 | -0082808 | 0034710 |
| 7004200 | 0022402 | 0026498 | -0082804 | 0034702 |
| 2000700 | -0031620 | 0038198 | -0134386 | 0045020 |
| 2000700 | -0031612 | 0038202 | -0134388 | 0045024 |
| 7003100 | -0061546 | 0022248 | -0162730 | 0029100 |
| 7003100 | -0061566 | 0022260 | -0162730 | 0029106 |
| 7003100 | -0061544 | 0022252 | -0162720 | 0029104 |
| 2002100 | -0088790 | 0047806 | -0190284 | 0053532 |
| 2002100 | -0088802 | 0047814 | -0190294 | 0053532 |
| 2002000 | -0091418 | 0068592 | -0193928 | 0074128 |
| 2002000 | -0091416 | 0068584 | -0193930 | 0074138 |
| 2001900 | -0045098 | 0081898 | -0147962 | 0087968 |
| 2001900 | -0045104 | 0081902 | -0147958 | 0087964 |
| 1004200 | 0009326 | 0065234 | -0095586 | 0071970 |
| 1004200 | 0009342 | 0065214 | -0095584 | 0071974 |
| 1004200 | 0009430 | 0065232 | -0095584 | 0071978 |
| 2001800 | -0028748 | 0116470 | -0129918 | 0123170 |
| 2001800 | -0028752 | 0116468 | -0129922 | 0123168 |
| 2001700 | 0008804 | 0131140 | -0091046 | 0138322 |
| 2001700 | 0008808 | 0131134 | -0091042 | 0138328 |
| 200~~170~~0/16 | 0040302 | 0139462 | -0058028 | 0146908 |
| 200~~170~~0/16 | 0040296 | 0139468 | -0058022 | 0146914 |

# OUTPUT

RUN

LINEAR 11:42PST 04/24/73

LINEAR TRANSFORMATION FOR FILM DISTORTION COMPUTATION

| POINT-NO | X | Y |
|---|---|---|
| FILM DISTORTION | | |
| 1 | 6.10352E-3 | -7.96509E-3 |
| 2 | 1.30615E-2 | 7.04956E-3 |
| 3 | -0.012085 | 1.19934E-2 |
| 4 | -6.95801E-3 | -1.10686E-2 |

TRANSFORMATION PHOTO COORDINATES

| MOD-NO | POINT-NO | X | Y |
|---|---|---|---|
| PRINP.P | 00000 | 9106.01 | 1006. |
| 11 | 10031 | 9023.951 | 924.512 |
| 11 | 10032 | 9021.463 | 1030.011 |
| 11 | 10033 | 9019.395 | 1093.899 |
| 11 | 10041 | 9103.462 | 928.520 |
| 11 | 10042 | 9098.824 | 987.677 |
| 11 | 10043 | 9090.721 | 1092.216 |
| 1 | 20005 | 9011.314 | 923.872 |
| 1 | 20006 | 9063.665 | 926.251 |
| 1 | 20008 | 9116.920 | 952.292 |
| 1 | 20007 | 9060.153 | 960.545 |
| 1 | 20021 | 9004.226 | 968.798 |
| 1 | 20020 | 9000.488 | 989.373 |
| 1 | 20018 | 9064.260 | 1038.701 |
| 1 | 20017 | 9285.113 | 1054.868 |
| 1 | 20016 | 9136.033 | 1062.772 |
| 1 | 30012 | 9123.229 | 1078.848 |
| 0 | 0 | 9194.721 | 916.153 |

USED .68 UNITS

LINEAR 04/24/73

```
100 FILES B1
101 PRINT "LINEAR TRANSFORMATION FOR FILM DISTORTION COMPUTATION"
102 REM MATHEMATIC, AND COMPUTER PROGRAM BY C.Y.HOU
103 REM WASHINGTON STATE HIGHWAYS,PHOTOGRAMMETRY BRANCH
104 REM PROGRAM FOR LINEAR TRANS. FOR ANALY. PHOTOG. AND FILM DIST.
110 DIM N(100),M(100),O(100),A(100),A$(100),B(100),C(100),J(100)
112 DIM W(100)
113 DIM K(100)
115 DIM B$(100),C$(100)
116 DIM K(100),L(100)
117 DIM F(100),G(100)
120 DIM I(100),X(100),Y(100),Z(100),V(100),U(100),T(100),R(100)
170 LET M=4
171 LET O=50
180 FOR I= 1 TO M
190 READ A(I),B(I),C(I)
191 LET B(I)=B(I)*0.01
192 LET C(I)=C(I)*0.01
200 LET X9=X9+B(I)
210 LET Y9=Y9+C(I)
220 NEXT I
230 LET X8=X9/M
240 LET Y8=Y9/M
250 FOR I= 1 TO M
260 LET X(I)=B(I)-X8
270 LET Y(I)=C(I)-Y8
280 NEXT I
300 FOR I= 1 TO M
310 READ #1,B$(I),C$(I),U(I),V(I)
311 LET U(I)=U(I)*0.001
312 LET V(I)=V(I)*0.001
320 LET X7=X7+U(I)
330 LET Y7=Y7+V(I)
340 NEXT I
350 LET X6=X7/M
360 LET Y6=Y7/M
370 FOR I= 1 TO M
380 LET W(I)=U(I)-X6
390 LET Z(I)=V(I)-Y6
400 NEXT I
410 FOR I=1 TO M
420 LET E1=E1+(W(I)*X(I))
430 LET F1=F1+(Z(I)*Y(I))
440 LET G1=G1+(W(I)↑2+Z(I)↑2)
450 LET H1=H1+(W(I)*Y(I))
460 LET M1=M1+(Z(I)*X(I))
470 NEXT I
480 LET N9=E1+F1
490 LET O9=M1-H1
500 LET P=N9/G1
```

```
LINEAR      04/24/73

510 LET Q=09/G1
511 PRINT "POINT-NO","X","Y"
520 LET R1=X8-P*X6-Q*Y6
530 LET R2=Y8-P*Y6+Q*X6
532 PRINT "FILM DISTORTION"
540  FOR I=1 TO M
550 LET S=R1+P*U(I)+Q*V(I)
560 LET T=R2+P*V(I)-Q*U(I)
570 LET K(I)=S-B(I)
580 LET L(I)=T-C(I)
590 PRINT A(I),K(I),L(I)
600 NEXT I
605 PRINT "TRANSFORMATION PHOTO COORDINATES"
606 PRINT "MOD-NO","POINT-NO","X","Y"
607 PRINT"PRINP,P","00000",X8,Y8
610 FOR J= 1 TO O
611 IF END #1 THEN 710
620 READ #1,F(J),G(J),Z(J),K(J)
621 LET Z(J)=Z(J)*0.001
622 LET K(J)=K(J)*0.001
630 LET S=R1+P*Z(J)+Q*K(J)
640 LET T=R2+P*K(J)-Q*Z(J)
649:  ###          ######          ######.###          ######.###
650 PRINT USING 649, F(J),G(J),S,T
690 NEXT J
700 DATA 1,900000,90000,2,900000,111200,3,921202,111201,4,921200,90000
710 END
```

# APPENDIX D

Input format for analytical aerotriangulation.

Input data for analytical aerotriangulation:

Input data for analytical aerotriangulation by using an IBM 2780 Remote Teleprocessing Terminal or a G.E. Time-sharing computer system must be arranged in groups as follows.

(1) Computer job number card.

→(2) Linear transformation program control deck.

(3) Job title card.

(4) Ground control deck — same format as semi-analytical program

(5) Equation card.

(6) Correction cards for RC8 camera No. 15 VAG 274

(7) Analytical aerotriangulation control deck.

(8) First model cards.

1. 4 fiducial marks cards (left, right)

2. 6 PUG points for relative orientation (left, right) *

3. Other points

(9) Next model, etc.

Same order as step (8).

(10) Blank card.

(11) Strip adjustment control deck.

*Format (I4, I5, 4I7). See input data.

# INPUT

```
//PH#40209 JOB (,
//          65FA,,3),'65FA ANALYTIC TRI-AN',
//          CLASS=L,MSGLEVEL=1,REGION=100K,TIME=1
//P65248  EXEC  FORTGCL,PARM.FORT='LOAD,NODECK,LINECNT=54,NAME=P65248'
//FORT.SYSIN DD *
*****ANALYTICAL PROGRAM CONTROL DECK*****
//LKED.SYSLMOD  DD DSN=&&GOSET(P65248)
//LKED.SYSIN    DD  *
    NAME P65248(R)
//T65248    EXEC PGM=*.P65248.LKED.SYSLMOD,TIME=1
//FT01F001    DD DDNAME=SYSIN
//FT02F001    DD SYSOUT=B
//FT03F001    DD SYSOUT=A,DCB=(RECFM=FA,LRECL=133,BLKSIZE=133)
//FT06F001    DD SYSOUT=A,DCB=(RECFM=FA,LRECL=133,BLKSIZE=133)
//SYSABEND DD SYSOUT=A
//SYSIN       DD *
HR-503   01                                          CAPITOL TEST ANALYTIC. AP/2C
    20006163392553 20654618     13833
    20018163281366 20666228      6579
    20012163315020 21061790
    20013163277508 21047779
    30014                       23466
    30015                       24911
    30019                         954
    40003163317332 20849133     19608
    20005163391217 20600720
    20009163330057 20949622
    20010163331785 21027011
    30012                        1229
    30023                       18848
    30013                       16500
    30021                        1068
    30020                        1006
    20016163263190 20741052
    20007163357805 20655030
    20008163370920 20709798
    20014163258215 20863061
    20015163253417 20779790
    20017163269508 20706313
    20019163314476 20645991
    20021163344616 20599991
    30010                        1249
    30011                        1200
    30016                       28630
    30017                       29215
    30018                       29507
    30022                        1216
    40004163337617 20885305     16847
    70033                         903
    20020163323802 20599182
    30009                        1344
    40002163390852 20793016     19200
-111
   3      152360 100000 100000  88000     500           1                 RCA 0
*****LENS DISTORTION CORRECTION DECK*****
```

```
11      091060101006000910601010060000   6
111003191106299173029023951 924512
1110032910611410238929021463103001 1
111003391062191084694901939510938 99
11100419194658 9201569103462 928520
11100429189743 9818749098824 987677
1110043917474210841229090721109 2216
11200059097089 9168719011314 923872
11200069152263 9184619063665 926251
11200089208563 9451679116920 952292
11200079149018 9543789060153 960545
11200219091758 9633259004226 968798
11200209088892 9840739000488 989373
11200189150989103265490642601038701
11200179188359104775192851131054868
11200169219744105643891360331062772
11300129206812107183291232291078848
```

See appendix C, output.

```
*****STRIP ADJUSTMENT PROGRAM CONTROL DECK*****
/*
```

# OUTPUT

| ITERATION | A1 | A2 | A3 | dby | dbz |
|---|---|---|---|---|---|
| 1 | -0.0554393178 | -0.0323533697 | -0.0007225313 | 0.0314834229 | 0.0089627682 |
| 2 | 0.0000497588 | 0.0018488117 | -0.0005543734 | 0.0021303304 | -0.0021254837 |
| 3 | -0.0000004860 | -0.0000002916 | 0.0000006273 | -0.0000067925 | 0.0000063747 |

| MODEL | ORIENTATION MATRIX | | |
|---|---|---|---|
| 11 | 0.9995349240 | 0.0020682746 | -0.0304246288 |
| 11 | -0.0003809072 | 0.9984662684 | 0.0553621339 |
| 11 | 0.0304924697 | -0.0553247973 | 0.9980026934 |

| MODEL | POINT | X | Y | Z | RESIDUAL |
|---|---|---|---|---|---|
| 11 | 0 | 200000 | 400000 | 600000 | |
| 11 | 0 | 288000 | 402957 | 600602 | |
| 11 | 10031 | 204857 | 306713 | 439757 | -1 |
| 11 | 10032 | 200116 | 420088 | 428888 | 3 |
| 11 | 10033 | 200234 | 488142 | 429334 | -2 |
| 11 | 10041 | 290026 | 312822 | 445286 | 1 |
| 11 | 10042 | 285338 | 375410 | 444716 | -2 |
| 11 | 10043 | 276652 | 487125 | 430093 | 1 |
| 11 | 20005 | 190517 | 305244 | 438046 | |
| 11 | 20006 | 247816 | 309493 | 442501 | |
| 11 | 20008 | 304096 | 338223 | 445375 | |
| 11 | 20007 | 244607 | 346434 | 441965 | |
| 11 | 20021 | 184861 | 354640 | 438117 | |
| 11 | 20020 | 181820 | 376674 | 438136 | |
| 11 | 20018 | 248579 | 428765 | 435408 | |
| 11 | 20016 | 326226 | 455984 | 430922 | |
| 11 | 30012 | 312024 | 473162 | 430695 | |

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INVOICE NO. HR-503 CAPITOL TEST ANALYTIC. AP/2C FLIGHT NO. PHOTO NO. - RUN

DEGREE OF CORRECTION FOR SCALE AND AZIMUTH, LONGITUDINAL BEND, TRANSVERSAL TILT - 111

| POINT IDENTIFICATION | | EASTING | NORTHING | ELEVATION | RESIDUALS EAST | RESIDUALS NORTH | RESIDUALS ELEVATION |
|---|---|---|---|---|---|---|---|
| | | CONTROL POINTS | | | | | |
| 11 | 10032 | 1632850.91 | 206201.46 | 10.53 | | | |
| 11 | 10042 | 1633344.99 | 206957.20 | 151.60 | | | |
| 11 | 20005 | 1633912.17 | 206007.20 | 103.19 | 0.20 | -0.05 | |
| 11 | 20006 | 1633925.53 | 206546.18 | 138.33 | -0.10 | -0.06 | -0.00 |
| 11 | 20008 | 1633709.20 | 207097.98 | 157.54 | 0.01 | -0.01 | |
| 11 | 20007 | 1633578.05 | 206550.30 | 131.82 | -0.20 | -0.11 | |
| 11 | 20021 | 1633446.16 | 205999.91 | 102.02 | 0.16 | 0.04 | |
| 11 | 20020 | 1633238.02 | 205991.82 | 101.44 | -0.07 | 0.08 | |
| 11 | 20018 | 1632813.66 | 206662.28 | 65.79 | 0.01 | 0.11 | -0.00 |
| 11 | 30012 | 1632458.19 | 207294.65 | 12.29 | | | 0.00 |
| | | | ROOT MEAN SQUARE ERROR | | 0.13 | 0.07 | 0.00 |

T R A N S F O R M E D P O I N T S

END OF JOB

# APPENDIX E

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