APPLICATION OF PATTERN RECOGNITION TO FORECAST CONGESTED CONDITIONS ON THE FREEWAY FOR USE IN RAMP METERING

VOLUME II

WA-RD 288.2
TNW 93-05.2

Final Technical Report
June 1993

Transportation Northwest
University Transportation Centers Program
Federal Region Ten

Washington State
Department of Transportation
Transit, Research, and
Intermodal Planning Division
in cooperation with the
United States Department of Transportation
Federal Highway Administration

TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO. WA-RD 288.2 / TNW 93-05.2	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
4. TITLE AND SUBTITLE APPLICATION OF PATTERN RECOGNITION TO FORECAST CONGESTED CONDITIONS ON THE FREEWAY FOR USE IN RAMP METERING — VOLUME II		5. REPORT DATE June 1993 6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) Nancy L. Nihan and Manish D. Babla		8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS Washington State Transportation Center (TRAC) University of Washington, JD-10 University District Building; 1107 NE 45th Street, Suite 535 Seattle, Washington 98105-4631		10. WORK UNIT NO. 11. CONTRACT OR GRANT NO. GC8719, Task 15
12. SPONSORING AGENCY NAME AND ADDRESS Washington State Department of Transportation Transportation Building, MS 7370 Olympia, Washington 98504-7370		13. TYPE OF REPORT AND PERIOD COVERED Final technical report 14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES

This study was conducted in cooperation with the U.S. Department of Transportation, Federal Highway Administration.

16. ABSTRACT

The current project addressed two major weak points of the existing WSDOT ramp control system. One weak point in the system is the fact that it reacts to the problem (congestion), rather than preventing the problem. The other weak point in the system is its reliance on detector data that may be in error. Both of these problems can be minimized by developing methods to accurately predict short-term traffic data. By predicting the onset of congestion early enough, the ramp metering system can act to prevent or delay occurrence of the problem. Also, if a detector has failed or is malfunctioning, the data from the detector can be estimated from short-term predictions based on neighboring detectors.

At the beginning of the current project, the researchers had hoped that the same model would provide a basis for both forecasting congestion (for predictive ramp control) and replacing erroneous data (predicting actual values). However, the best congestion or breakdown flow forecaster (the pattern recognition method) does not provide a basis for data prediction. The best method for filling in missing detector data turned out to be multivariate time series analysis.

Several pattern recognition and time series models were tested for further development. In both cases, the simpler models turned out to be the best choices, and in both cases, further model testing and development were recommended.

The research on both model types continues in follow-up studies that are expected to lead to incorporation of these models in the new TSMC computer system.

17. KEY WORDS ramp control, freeway management, traffic flow forecasts, forecast models, ramp metering		18. DISTRIBUTION STATEMENT No restrictions. This document is available to the public through the National Technical Information Service, Springfield, VA 22616	
19. SECURITY CLASSIF. (of this report) None	20. SECURITY CLASSIF. (of this page) None	21. NO. OF PAGES 98	22. PRICE

Final Technical Report
Research Project GC8719, Task 15
Freeway and Ramp Real Time Forecasting

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VOLUME II

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Federal Highway Administration

June 1993

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SUMMARY

Traffic congestion is one of the most important issues facing us today. We examined how traditional supply side solutions have failed to solve the problem, and why demand management strategies, such as ramp metering, are becoming more popular.

We analyzed the deficiencies in the ramp metering system currently employed by Washington State Department of Transportation (WSDOT), and briefly examined some past approaches of real-time control at the on-ramps.

This research aimed to develop a pattern recognition model capable of forecasting traffic conditions on the freeway 1 to 3 minutes ahead of time. Both parametric and non-parametric approaches to pattern recognition were reviewed, and a model based on the principles of statistical pattern recognition was developed.

The model was calibrated based on a.m. peak hour traffic data collected from south-bound I-5 over three working days obtained from the Traffic Systems Management System (TSMC) in Seattle, WA. Six statistical pattern recognition forecasting algorithms were developed to forecast traffic conditions 1 to 3 minutes ahead of time at the same section or at an upstream section.

The model was then tested on the same 3-day data obtained from the TSMC, and false positive and false negative rates were determined for each of the six algorithms. The developed statistical pattern recognition model was also tested for its efficacy in improving traffic performance on the freeway system by using INTRAS in a simulation of a.m. peak period traffic conditions on southbound I-5.

CHAPTER 1

INTRODUCTION

Combating urban freeway congestion is one of the most challenging tasks of our time. Congestion causes erratic stop-and-go driving, increased and unpredictable travel times, and lower average-travel speeds. Congestion also increases driver stress and results in higher rates of accidents and associated bodily injury and property damage. Congestion on freeways, because of the design for high traffic volumes at high speeds, also takes a tremendous toll in terms of lost travel time. All of these issues, when translated into dollars, indicate that relieving congestion on freeways would yield considerable monetary benefits.

Environmental damage, in terms of increased fuel consumption and a higher degree of air pollution, cannot easily be translated into monetary loss. Traffic congestion conflicts with our goal of providing a safe, efficient, reliable, and environmentally sensitive means of highway transportation. Relieving it, then, should be at the top of our agenda.

CONGESTION

When variations in roadway conditions, driver behavior, or traffic demand create a short-term increase in demand above capacity, congestion ensues. Congestion can be recurrent or non-recurrent. When variations are regular, which occurs daily in many urban areas during the a.m. and p.m. peak periods, this is called recurrent congestion. When the reduction in capacity is due to unusual circumstances, such as accidents or other incidents affecting traffic conditions, this is called non-recurrent congestion.

We are concerned, here, mainly with recurrent congestion, which is due to high demand exceeding the limited capacity of freeways during the rush hours. As delays increase exponentially at this high demand, it is desirable to keep the demand below capacity for efficient operation of the freeway system.

DEMAND AND SUPPLY SIDE SOLUTIONS

Traditionally, solutions to relieve congestion have been focused more on the supply side. High capacity additions, such as the construction of a new highway, or the addition of lanes to an existing highway, are both aimed at increasing the capacity in proportion to the demand.

However, results gained from past research indictate that though this approach is the most effective in terms of reducing congestion, it is financially, environmentally, and socially disruptive. It leads to continued low-density development, and the resulting reliance on single occupant vehicles. This supply side approach, in effect, exacerbates the problem by leading to increased traffic demand.

In addition, the near completion of the interstate freeway system, the federal government's stance of fiscal frugality, and the increasing participation of communities and environmental groups in transportation planning have made high-capacity additions to the nation's freeway system more difficult to accomplish.

Recent efforts have been focused on managing the demand side. The thrust is to manage the demand more effectively, both temporally and spatially, so that the freeway system operates more efficiently. This has focused our attention on Transportation System Management (TSM) alternatives directed at making improvements with small-scale changes on existing conditions.

Entrance ramp control is one TSM alternative aimed at effectively managing demand so that it does not exceed supply, thus, promoting efficient operation of the freeway system.

PROBLEM STATEMENT

The objective of this research was to develop a model, based on pattern recognition techniques, capable of forecasting traffic conditions on the freeway one or more minutes ahead of time. This information could then be used to promote a real-time, responsive ramp metering system that regulates demand at the on-ramps in anticipation of congestion, and thereby, delay, if not prevent, congestion on the main line.

OUTLINE

The remainder of this volume, in addition to discussing literature in the area of pattern recognition and its application to freeway surveillance and control, presents detailed methodology and results of the application of the Babla/Nihan statistical pattern recognition model to forecast freeway traffic conditions. The report then discusses the incorporation of this pattern recognition model in the ramp metering system currently used by the Washington State Department of Transportation (WSDOT) and the simulation of the integrated system using Integrate Traffic Simulation Software (INTRAS).

Chapter Two briefly examines the ramp metering system WSDOT currently uses, the problems associated with it, and past approaches toward developing a more responsive, real-time ramp metering system.

Chapter Three presents the research methodology used in testing the developed Babla/Nihan statistical pattern recognition model on a.m. peak-period traffic data, obtained from the Traffic Systems Management Center (TSMC), over a section of southbound I-5 in Seattle, Washington. It also describes the research design for the testing of WSDOT's ramp metering approach, and the application of the Davis/Nihan and Babla/Nihan models to WSDOT's ramp metering system to compare traffic performance by simulation of traffic using INTRAS.

Chapter Four discusses the degree to which the Babla/Nihan model predicts traffic conditions accurately, compared to the previously developed Davis/Nihan model. It also compares results from the simulation using INTRAS of WSDOT's approach, and application of the Davis/Nihan and Babla/Nihan models as forecasting models to WSDOT's approach.

Chapter Five analyzes the extent to which the objectives of the research effort were realized, and makes recommendations for future research and applications of the Babla/Nihan model of statistical pattern recognition.

Appendix A is a literature review of past applications of pattern recognition in freeway surveillance and control. A brief review of the science of pattern recognition is presented in Appendix B.

CHAPTER 2

WSDOT's RAMP CONTROL SYSTEM

Since 1981, the Washington State Department of Transportation (WSDOT) has been operating an integrated, traffic responsive on-ramp control system (Figure 1), to cope with the recurring traffic congestion problems on the Seattle region's portion of Interstate 5 running north of downtown Seattle. The ramp metering system is a computer-based, distributed intelligence system that consists of field located microprocessors and a centralized computer system. (1) It uses an on-line, centrally controlled algorithm that calculates the metering rates based on system-wide traffic conditions.

Loop detectors, located on the mainline and on the exit and entrance ramps of the I-5 freeway, collect real-time volume and lane occupancy data. These data are then input into a control algorithm on the TSMC central computers. Metering rates and timing intervals for each of the controllers are determined every 20 seconds.

The algorithm is called integrated and traffic responsive because metering rates are based on real-time, local, and system-wide capacity conditions. Thus, it not only considers the queueing conditions at the on-ramps, but also the interdependency of ramp operations in the calculation of the final metering rates.

The control algorithm computes two on-ramp entry rates, the local metering rate (LMR), and the bottleneck metering rate (BMR).

The local metering algorithm uses historical volume-occupancy relationships at the metered station to develop an occupancy-metering rate curve, so that the selected metering rates make up the difference between the estimated capacity and the real-time upstream volume. Then, based on the lane occupancy measurements immediately upstream of the given metered ramp, the local metering rate (LMR), is

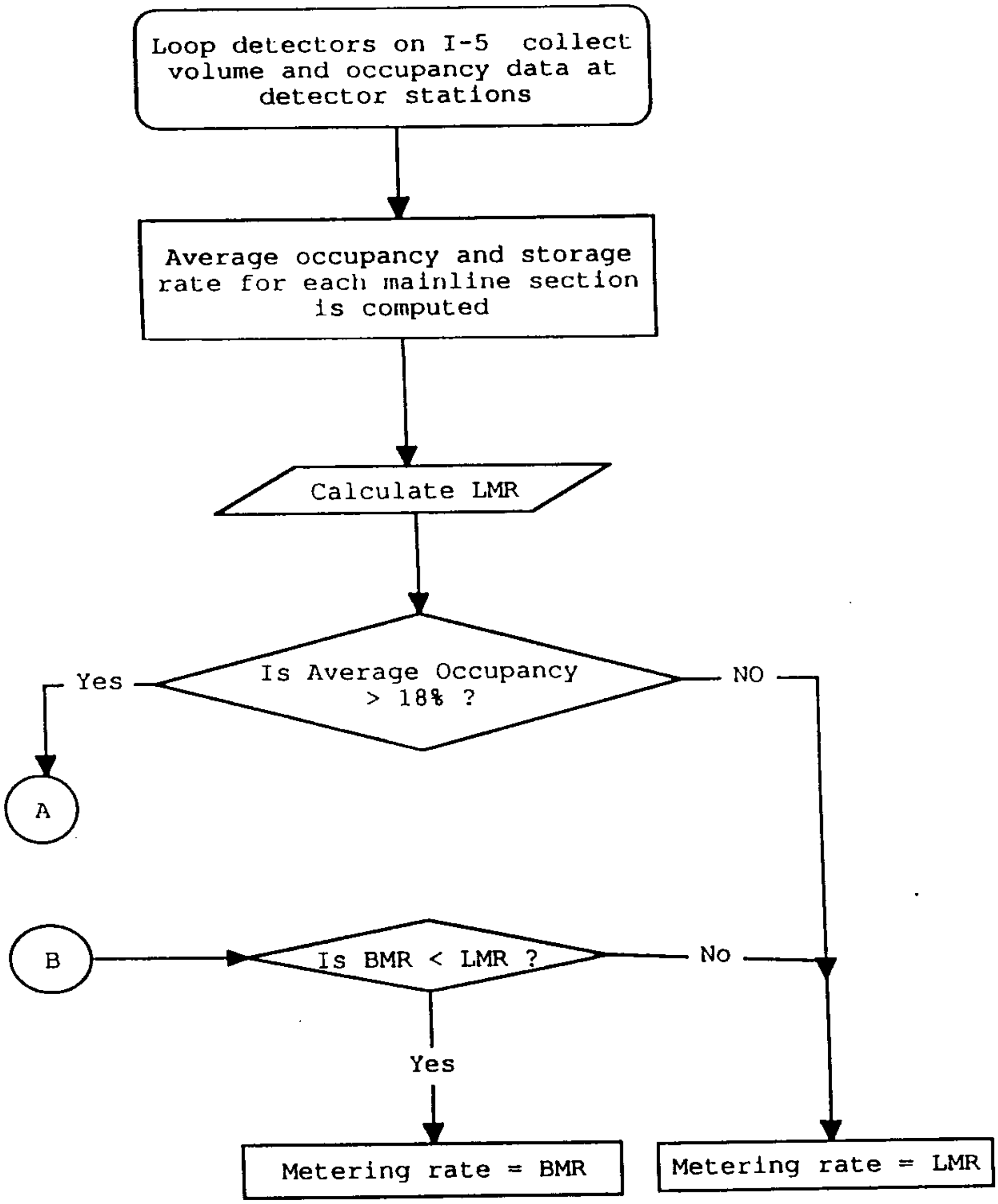

Figure 1 : WSDOT's current ramp metering system

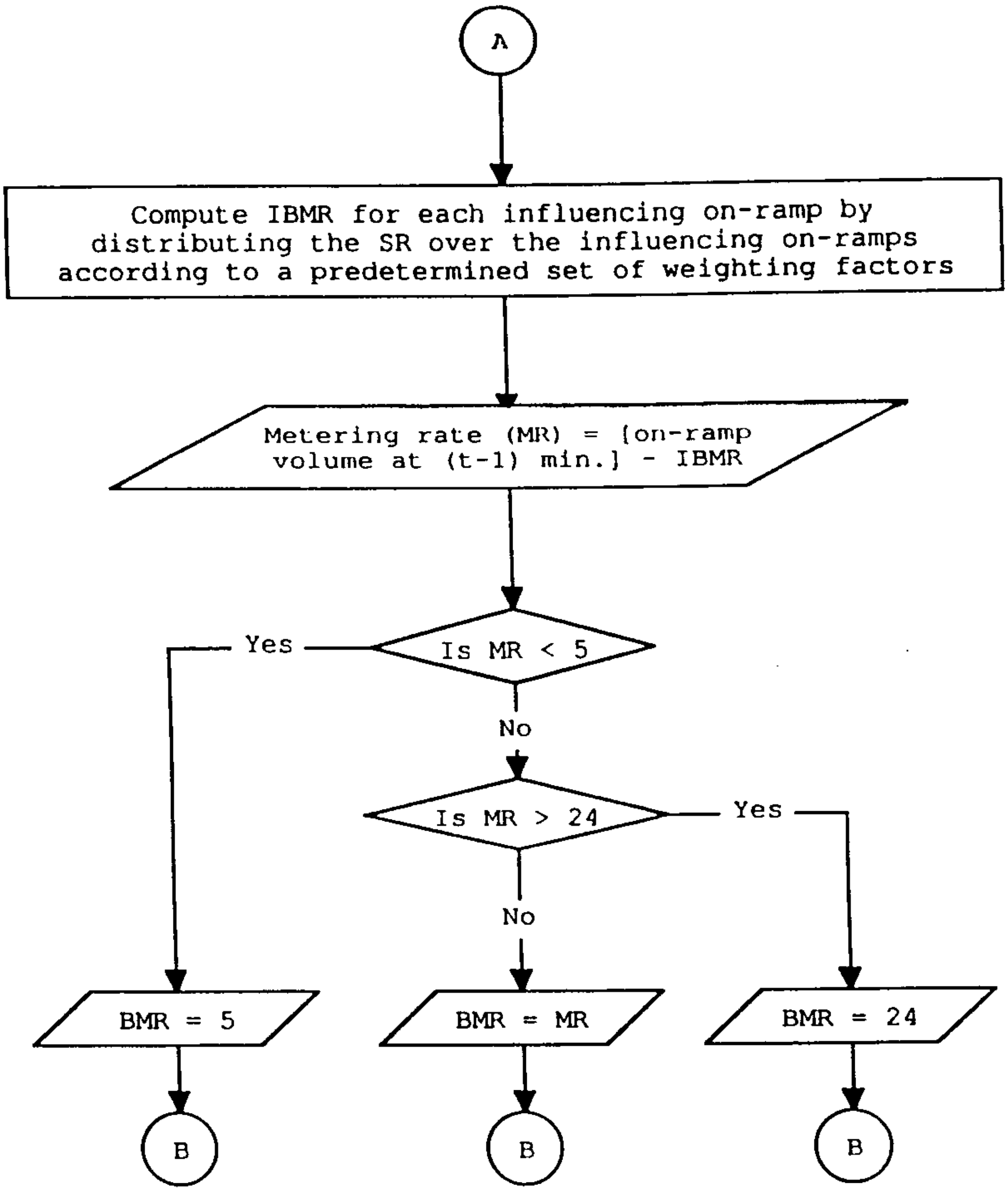

Figure 1 (Continued) : WSDOT's current ramp metering system

calculated based on straight-line interpolation of the occupancy-metering rate curve.

However, once the demand on the section of the freeway exceeds its capacity, causing queuing of vehicles, congestion occurs. The downstream detector station declares the section to be operating above capacity when the occupancies exceed an operator defined threshold of around 18 percent. The bottleneck metering algorithm then takes over, and the system calculates the upstream ramp reduction as the number of vehicles stored in the freeway section (storage rate, or SR), in the past minute.

This total reduction in upstream ramp volumes is then distributed as the reduction (IBMR), among all metered ramps that fall within the freeway section's area of influence, based on predetermined weighting factors. These weighting factors are based on the proximity of the ramp to the bottleneck, and the normal level of demand on the ramp.

When areas of influence for two or more sections overlap, and there is more than one bottleneck metering rate (BMR), for a ramp, then the most restrictive one is implemented. If the queues on the ramps reach beyond the queue and/or the advance queue detector (not shown in Figures 1 to 7) for a specified length of time, the metering rate is increased automatically to prevent any disruption in operation on the arterials.

PROBLEMS WITH WSDOT's APPROACH

WSDOT's approach to ramp control is responsive rather than anticipatory. The system does not react until after the bottleneck has been formed, and by that time restrictive conditions in the form of reductions in speed and traffic flow have already occurred on the main line. Metering rates that are too restrictive often result, and lead to the formation of excessive queues on the upstream ramps.

The system senses the unacceptable queuing, and increases the metering rate accordingly to clear the queue from the surface streets. This leads to further increased congestion on the freeway section, and the cycle repeats itself.

Thus, the algorithm is oriented more toward responding to the disorders rather than preventing them. An algorithm that would eliminate the congestion cycle outlined above, and still be based on real-time traffic conditions, is desired. A system that could forecast conditions on the freeway 1 or more minutes in advance would help eliminate these congestion conditions.

PAST APPROACHES TOWARD REAL-TIME CONTROL

Past research on forecasting freeway traffic conditions has been in the following four areas:

1. **The traffic flow forecaster**, based on the model presented by Papageorgiou, estimates the traffic speed from measurements of volume and lane occupancy, and then forecasts the volume at a downstream station given known volumes at upstream on-ramps and main line stations. (10) As the model requires an estimation of the Origin-Destination matrix, and cannot be used to forecast lane occupancy, we rejected using this model.

2. **Parametric regression methods**, based on the Box-Jenkins method, fit the time-series model to volume and lane-occupancy data, and then use the model to forecast future values given the knowledge of past and present values. (11) It was found that the forecast values tended to hover around the mean value of the time series and ignore extreme results, in which we are especially interested.

3. **Nonparametric regression methods**, or the nearest neighbor approach, are procedures based on hydrological forecasting methods. (12) Davis and Nihan developed a model that compares current measurements to an archive of past measurements, and bases its forecast on past occurrences most similar to

present conditions. (13) The non-parametric methods gave results only slightly superior to the parametric approach.

4. **The statistical pattern recognition approach** aims to develop a method to classify traffic data in the form of input-output difference (or the storage rate), and lane occupancy measurements at the main line station at different lags into two categories, those which precede uncongested and congested conditions at the same, or at an upstream section. (14)

STATISTICAL PATTERN RECOGNITION APPROACH

Using the Boxplot feature of Minitab, Davis and Nihan identified two sets of variables, one for lightly- and one for heavily-congested traffic, as viable for developing a decision rule. This decision rule was based on statistical pattern recognition for discriminating between uncongested and congested traffic conditions on the freeway. Next, they developed a forecasting model consisting of decision rules based on thresholds assigned to the above lagged variables. (14)

Nihan and Davis calibrated and tested the above statistical pattern recognition model on data from a section of the I-5 freeway north of Seattle. They found that for the lightly congested data, the model correctly predicted 92 percent of the intervals. The model falsely predicted congested conditions (false positive) 5 percent of the time, and was not able to forecast congestion (false negative) 36 percent of the time. However, for highly congested traffic, it correctly predicted only 63 percent of the intervals, with a false positive rate of 7 percent, and a false negative rate of 73 percent. They also discovered that there was a trade-off function between the false positive and the false negative rates.

Jihong Kim described the implementation of the Davis/Nihan statistical pattern recognition forecasting model in the existing metering system of the Washington State Department of Transportation (Figure 2), and its application on a section of the I-5 freeway between 195th & 205th Streets in Northeast Seattle. (15)

However, a minimum metering rate of 5 veh/min. and a maximum of 24 veh/min. is always implemented. (This is not shown in Figure 2.)

Nihan and Davis expressed confidence in this preliminary approach of applying statistical pattern recognition techniques to predict congestion for real-time ramp metering, and recommended investigation into other, more detailed pattern recognition approaches.

Our objective was to improve the false negative rate from its present value of 36 percent for lightly congested traffic and 73 percent for heavily congested traffic, with no significant adverse effect on the false positive rates.

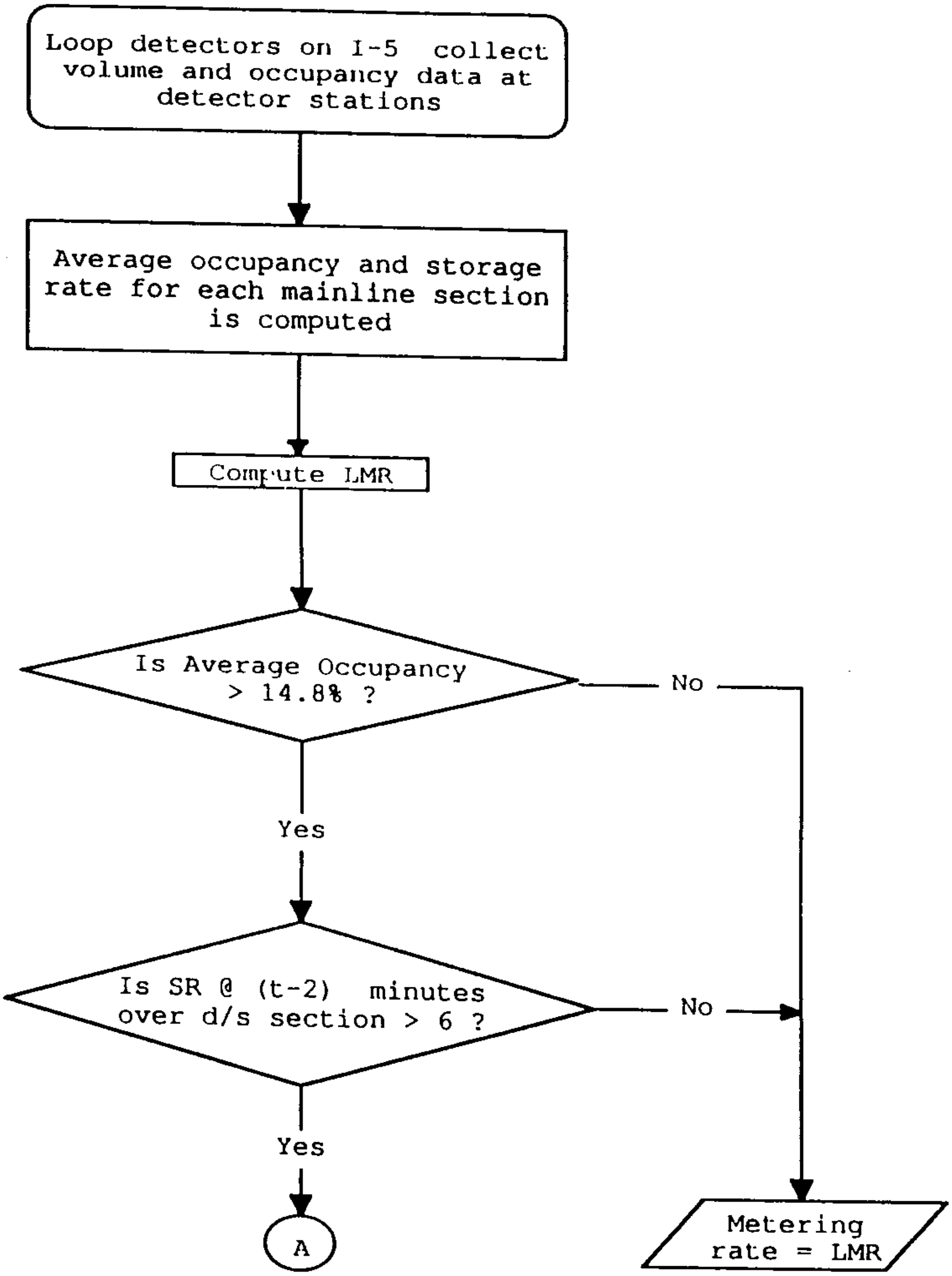

Figure 2 : Application of Davis/Nihan model of statistical pattern recognition to WSDOT's current ramp metering system

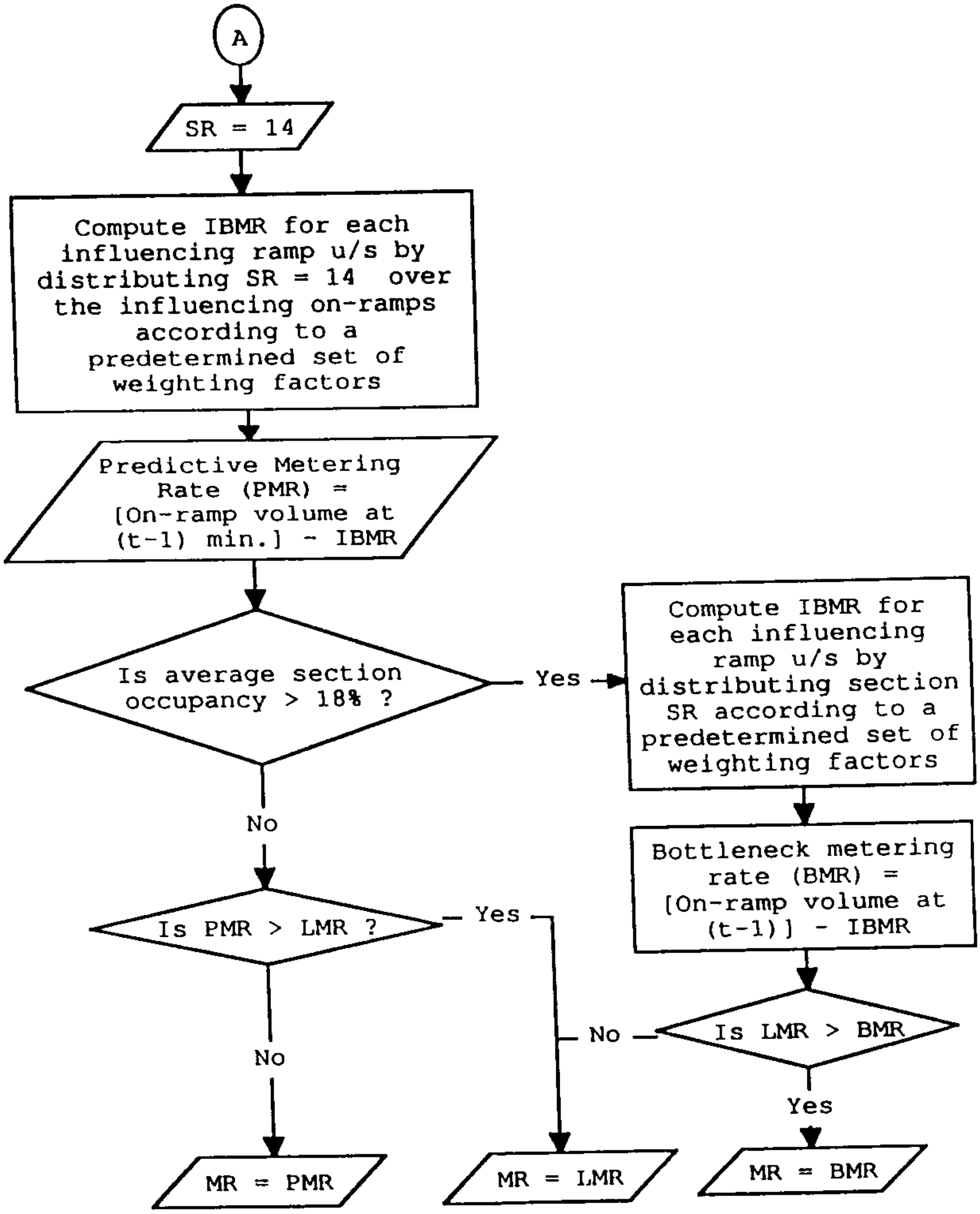

Figure 2 (Continued) : Application of Davis/Nihan model of statistical pattern recognition to WSDOT's current ramp metering system.

CHAPTER 3

RESEARCH METHODOLOGY

A model, based on one of the pattern recognition approaches described in Chapter 2, was calibrated and tested on a.m. peak hour traffic data over a section of southbound I-5 freeway north of Seattle. These data were obtained from the TSMC in Seattle, Washington. The calibrated model was then incorporated in WSDOT's current ramp metering algorithm, and using INTRAS software, the system was simulated to represent a.m. peak hour traffic conditions over southbound I-5 freeway.

The objective was two-fold: to determine the accuracy with which the developed pattern recognition algorithm would forecast both uncongested and congested traffic conditions on the freeway, and to determine the improvement in overall system performance achieved by incorporating the developed pattern recognition algorithm into WSDOT's current ramp metering system.

The researchers' next step was to select traffic variables to represent traffic conditions on the freeway. They also had to select a pattern recognition approach, (several of which are described in Chapter 2), to be used to forecast uncongested and congested traffic conditions on the freeway. The data obtained would then be used in the ramp metering system.

SELECTION OF THE TRAFFIC VARIABLES

For calibration and testing of the pattern recognition algorithm, one-minute traffic data from two adjoining sections of southbound I-5 (Figure 3) were collected by the TSMC through loop detectors located over I-5, from 205th St. N.E. to 185th St. N.E., north of Seattle. Congestion occurs daily over this section during the a.m. peak periods when traffic flows south toward downtown Seattle. The data were collected over the main line stations at 205^{th} St. N.E., 195^{th} St. N.E., and 185^{th} St. N.E., and over the HOV and normal lanes at the 205^{th} St. N.E. on-ramp.

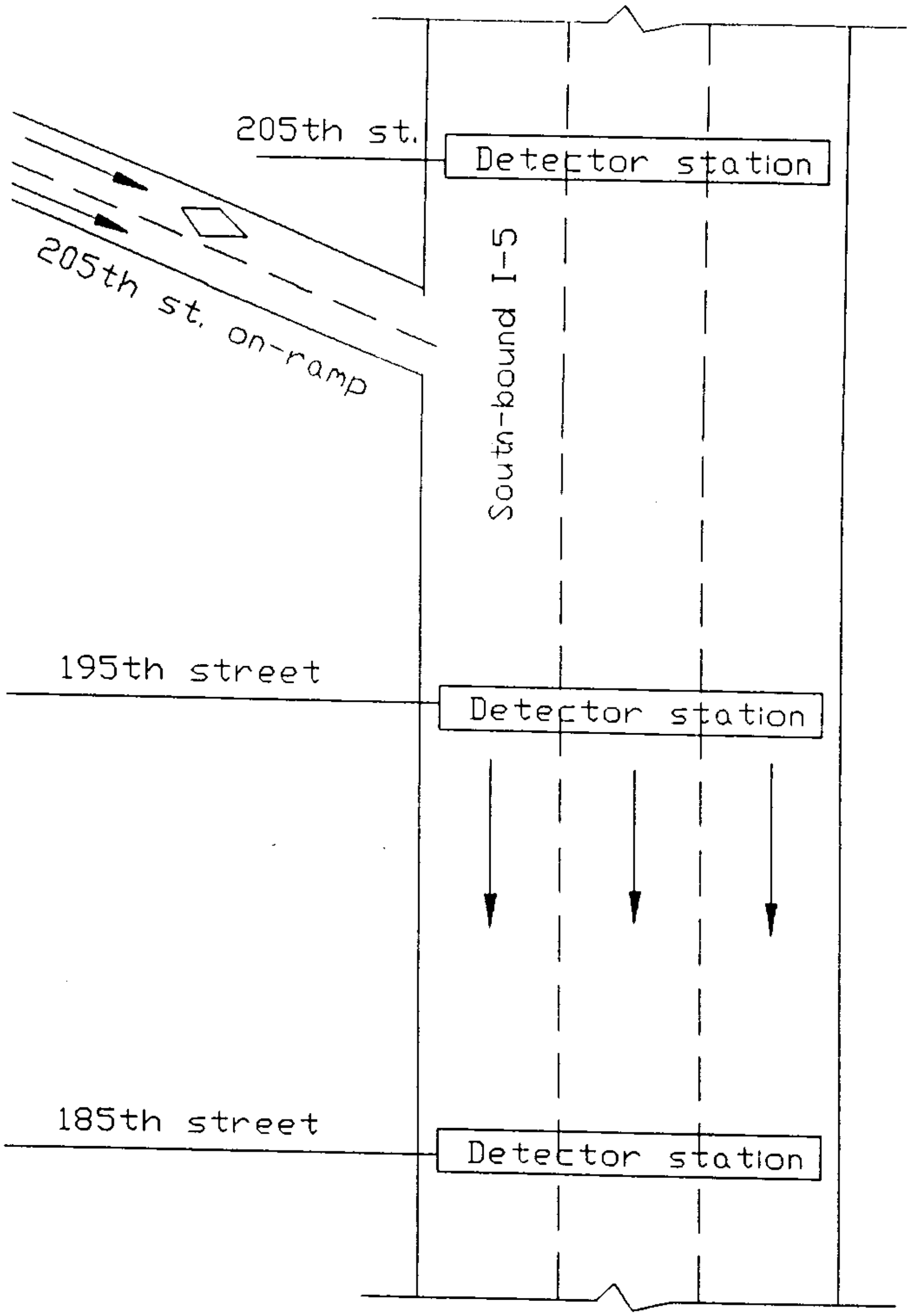

Figure 3: Section of south-bound I-5 used for collecting traffic data for calibration and testing of the Babla/Nihan statistical pattern recognition model.

The traffic data collected are in the form of one-minute volume and occupancy at the detector stations (TABLE 1). A total of four weekdays of traffic data were collected from Tuesday, March 27 to Friday, March 30, 1990.

Since the traffic data of March 27 over the on-ramp at 205th St. N.E. were found to have errors, we were only able to use traffic data on March 28 and March 30, from 6:00 a.m. to 9:25 a.m., and on March 29, from 6:00 a.m. to 8:15 a.m. A sample of the traffic data on March 28, from 7:11 to 7:40 a.m., is shown in Table 1.

As stated above, the data collected by the TSMC are in the form of one-minute volume and occupancy at main line and on-ramp detector stations. Input-output difference, which is also equal to the number of vehicles stored over a section, was derived from the volumes at stations upstream and downstream of the section and from volumes at on-ramps as follows:

Section Storage rate (SR) = (upstream mainline volume) +

(sum of volumes over all on-ramps feeding into section) -

(downstream mainline volume)

Also, average occupancy over a section was calculated as the average of the occupancies over detector stations immediately upstream and downstream of the main line section. A sample of the average occupancies and the storage rates of the data in Table 1 is shown in Table 2.

When vehicles are stored in a section, that section is more likely to operate under congested conditions. Because the objective was to classify the set of traffic variables into those representing traffic conditions before both uncongested and congested traffic conditions, a bivariate set, consisting of the storage rate (SR) and average occupancy over a section, was selected to represent the freeway traffic conditions.

TABLE 1

SAMPLE OF AM PEAK PERIOD TRAFFIC DATA

TIME (AM.)	MAIN LINE 185TH ST. N.E. VOL	OCC	MAIN LINE 195TH ST. N.E. VOL	OCC	MAIN LINE 205TH ST. N.E. VOL	OCC	ON-RAMP @ 205TH VOLUME	HOV ON-RAMP @ 205TH VOLUME
7:11	95	13.3	98	19.1	96	24.5	7	10
7:12	106	13.3	102	22.9	90	25.7	6	9
7:13	103	13.0	101	19.2	94	22.6	5	3
7:14	89	17.9	108	18.3	92	24.7	6	0
7:15	60	32.8	107	20.2	81	25.2	3	0
7:16	91	21.4	95	29.6	104	20.2	6	0
7:17	103	19.8	67	28.9	105	20.2	5	0
7:18	102	14.5	80	29.7	103	22.4	8	6
7:19	100	13.6	93	21.4	73	31.6	4	0
7:20	104	12.7	111	20.3	48	38.4	7	0
7:21	109	13.7	96	18.8	77	28.9	6	0
7:22	87	14.3	115	23.8	101	21.4	7	3
7:23	90	30.1	122	22.7	103	15.9	7	3
7:24	97	18.5	99	24.8	99	14.5	7	0
7:25	97	14.3	71	32.4	109	14.1	4	3
7:26	108	15.3	88	25.0	100	15.0	9	0
7:27	111	15.0	103	18.9	74	28.8	6	6
7:28	105	15.6	101	13.2	51	37.4	10	3
7:29	67	25.0	102	13.5	80	24.5	7	0
7:30	88	26.8	110	17.8	92	19.3	8	0
7:31	97	17.0	89	29.2	116	16.4	5	0
7:32	106	14.3	84	23.1	114	18.8	6	7
7:33	110	16.4	99	22.9	98	24.6	5	3
7:34	100	20.0	106	17.6	54	27.8	7	0
7:35	101	23.4	100	16.1	76	20.8	7	0
7:36	98	20.5	106	20.8	101	17.2	4	2
7:37	99	16.6	88	18.9	95	16.2	8	1
7:38	113	17.3	96	17.1	99	19.2	6	3
7:39	99	14.5	111	18.7	86	27.4	7	3
7:40	114	20.0	110	15.6	80	15.9	5	6

These traffic variables then form the input vector, which then enabled us to forecast traffic on the freeway 1 or more minutes ahead of time.

TABLE 2

STORAGE RATE AND AVERAGE OCCUPANCY FOR AM PEAK PERIOD TRAFFIC DATA

TIME (A.M.)	MAINLINE BETWEEN STORAGE RATE	SECTION 185 & 195 AVERAGE OCCUPANCY	MAINLINE BETWEEN STORAGE RATE	SECTION 195 & 205 AVERAGE OCCUPANCY	CONGESTION INDICATOR
7:11	3	16.20	15	21.80	2
7:12	-4	18.10	3	24.30	2
7:13	-2	16.10	1	20.90	2
7:14	19	18.10	-10	21.50	1
7:15	47	26.50	-23	22.70	1
7:16	4	25.50	15	24.90	2
7:17	-36	24.35	43	24.55	2
7:18	-22	22.10	37	26.05	2
7:19	-7	17.50	-16	26.50	1
7:20	7	16.50	-56	29.35	1
7:21	-13	16.25	-13	23.85	1
7:22	28	19.05	-4	22.60	1
7:23	32	26.40	-9	19.30	1
7:24	2	21.65	7	19.65	2
7:25	-26	23.35	45	23.25	2
7:26	-20	20.15	21	20.00	2
7:27	-8	16.95	-17	23.85	1
7:28	-4	14.40	-37	25.30	1
7:29	35	19.25	-15	19.00	1
7:30	22	22.30	-10	18.55	1
7:31	-8	23.10	32	22.80	2
7:32	-22	18.70	43	20.95	2
7:33	-11	19.65	7	23.75	2
7:34	6	18.80	-45	22.70	1
7:35	-1	19.75	-17	18.45	1
7:36	8	20.65	1	19.00	2
7:37	-11	17.75	16	17.55	1
7:38	-17	17.20	12	18.15	2
7:39	12	16.60	-15	23.05	1
7:40	-4	17.80	-19	15.75	1

NOTES: For congestion indicator,
1 represents uncongested traffic condition on section between 205th and 195th streets, and
2 represents congested traffic condition on section between 205th and 195th streets.

SELECTION OF THE PARAMETRIC / STATISTICAL APPROACH

The objective was to classify the bivariate set of the input-output difference and occupancy over a section of the freeway into two classes: those preceding uncongested traffic conditions and those preceding congested traffic conditions.

The researchers had the choice of employing either the parametric approach, or any of the non-parametric approaches (including the statistical approach), described in Chapter 3. Tou states that the statistical approach is often the yardstick to which the performance of other pattern recognition algorithms is compared. (21). He also states that non-parametric approaches are useful only when no assumptions can be made about the underlying distribution or characterizing parameters. Sing-Tze-Bow suggests using the parametric approach when the samples can be assumed to arise from a multivariate normal gaussian distribution. (22)

If the set of bivariate traffic data are assumed to have arisen from a multivariate normal gaussian distribution, the parametric approach should be used.

On the basis of the definition of congested freeway conditions used by WSDOT in the implementation of the bottleneck metering algorithm, and in earlier research on the development of the statistical pattern recognition algorithm by Davis (14), the researchers concluded that an occupancy equal to, or above, 18 percent along with a positive storage rate, represented congested conditions over a freeway section. Any other traffic data, then, represented uncongested traffic conditions. The 1-minute bivariate set of traffic data, over the section between 205th St. N.E. and 195th St. N.E., were thus identified as representing either uncongested or congested traffic conditions (see Table 2).

We know from traffic theory that congestion in the form of a shock wave always proceeds upstream from the affected main line section. Since our objective was to forecast traffic conditions on the freeway 1 or more minutes in advance, lagged observations from 1 to 3 minutes at the same section between 205th St. N.E. and 195th St. N.E., and at a downstream section between 195th St. N.E. and 185th

St. N.E. were then sorted into classes representing conditions preceding congested or uncongested conditions.

A sample of the data in Table 1 is shown in the form of two classes representing traffic conditions preceding uncongested and congested traffic conditions, respectively.

Table 3 presents traffic data at 1 minute preceding uncongested or congested traffic conditions at the same section between 205th St. N.E. and 195th St. N.E. (LG1), and

Table 4 presents traffic data at 2 minutes preceding uncongested or congested traffic conditions at the same section (LG2).

TABLE 3

TRAFFIC DATA AT 1 MIN. PRECEDING UNCONGESTED AND CONGESTED TRAFFIC AT SAME SECTION (LG1)

TRAFFIC AT 1 MIN. PRECEDING UNCONGESTED TRAFFIC		TRAFFIC AT 1 MIN. PRECEDING CONGESTED TRAFFIC	
STORAGE RATE	AVERAGE OCCUPANCY	STORAGE RATE	AVERAGE OCCUPANCY
1	20.90	15	21.80
-10	21.50	3	24.30
37	26.05	-23	22.70
-16	26.50	15	24.90
-56	29.35	43	24.55
-13	23.85	-9	19.30
-4	22.60	7	19.65
21	20.00	45	23.25
-17	23.85	-10	18.55
-37	25.30	32	22.80
-15	19.00	43	20.95
7	23.75	-17	18.45
-45	22.70	16	17.55
1	19.00	12	18.15
-15	23.05		

TABLE 4

TRAFFIC DATA AT 2 MIN. PRECEDING UNCONGESTED AND CONGESTED TRAFFIC AT SAME SECTION (LG2)

TRAFFIC AT 2 MIN. PRECEDING UNCONGESTED TRAFFIC		TRAFFIC AT 2 MIN. PRECEDING CONGESTED TRAFFIC	
STORAGE RATE	AVERAGE OCCUPANCY	STORAGE RATE	AVERAGE OCCUPANCY
3	24.30	15	21.80
1	20.90	-10	21.50
43	24.55	-23	22.70
37	26.05	15	24.90
-16	26.50	-4	22.60
-56	29.35	-9	19.30
-13	23.85	7	19.65
45	23.25	-15	19.00
21	20.00	-10	18.55
-17	23.85	32	22.80
-37	25.30	-45	22.70
43	20.95	1	19.00
7	23.75	-17	18.45
16	17.55		
12	18.15		

Table 5 presents traffic data at 3 minutes preceding uncongested or congested traffic conditions at the same section, between 205th St. N.E. and 195th St. N.E. (LG3),

Table 6 presents traffic data between 195th St. N.E. and 185th St. N.E. at 1 minute preceding uncongested or congested traffic conditions at an upstream section (LG1DS),

Table 7 presents traffic data between 195th St. N.E. and 185th St. N.E. at 2 minutes preceding uncongested or congested traffic conditions at an upstream section (LG2DS), and

Table 8 presents traffic data between 195th St. N.E. and 185th St. N.E. at 3 minutes preceding uncongested or congested traffic conditions at an upstream section (LG3DS).

TABLE 5

TRAFFIC DATA AT 3 MIN. PRECEDING UNCONGESTED AND CONGESTED TRAFFIC AT SAME SECTION

TRAFFIC AT 3 MIN. PRECEDING UNCONGESTED TRAFFIC		TRAFFIC AT 3 MIN. PRECEDING CONGESTED TRAFFIC	
STORAGE RATE	AVERAGE OCCUPANCY	STORAGE RATE	AVERAGE OCCUPANCY
15	21.80	1	20.90
3	24.30	-10	21.50
15	24.90	-23	22.70
43	24.55	-13	23.85
37	26.05	-4	22.60
-16	26.50	-9	19.30
-56	29.35	-37	25.30
7	19.65	-15	19.00
45	23.25	-10	18.55
21	20.00	7	23.75
-17	23.85	-17	18.45
32	22.80	43	20.95
-45	22.70		
1	19.00		
16	17.55		

TABLE 6

TRAFFIC DATA AT 1 MIN. PRECEDING UNCONGESTED AND CONGESTED TRAFFIC AT AN U/S SECTION

TRAFFIC AT 1 MIN. PRECEDING UNCONGESTED TRAFFIC U/S		TRAFFIC AT 1 MIN. PRECEDING CONGESTED TRAFFIC U/S	
STORAGE RATE	AVERAGE OCCUPANCY	STORAGE RATE	AVERAGE OCCUPANCY
-2	16.10	3	16.20
19	18.10	-4	18.10
-22	22.10	47	26.50
-7	17.50	4	25.50
7	16.50	-36	24.35
-13	16.25	32	26.40
28	19.05	2	21.65
-20	20.15	-26	23.35
-8	16.95	22	22.30
-4	14.40	-8	23.10
35	19.25	-22	18.70
-11	19.65	-1	19.75
6	18.80	-11	17.75
8	20.65		
-17	17.20		
12	16.60		

TABLE 7

TRAFFIC DATA AT 2 MIN. PRECEDING UNCONGESTED AND CONGESTED TRAFFIC AT AN U/S SECTION

TRAFFIC AT 2 MIN. PRECEDING UNCONGESTED TRAFFIC U/S

STORAGE RATE	AVERAGE OCCUPANCY
-4	18.10
-2	16.10
-36	24.35
-22	22.10
-7	17.50
7	16.50
-13	16.25
-26	23.35
-20	20.15
-8	16.95
-4	14.40
-22	18.70
-11	19.65
-1	19.75
-11	17.75
-17	17.20

TRAFFIC AT 2 MIN. PRECEDING CONGESTED TRAFFIC U/S

STORAGE RATE	AVERAGE OCCUPANCY
3	16.20
19	18.10
47	26.50
4	25.50
28	19.05
32	26.40
2	21.65
35	19.25
22	22.30
-8	23.10
6	18.80
8	20.65

TABLE 8

TRAFFIC DATA AT 3 MIN. PRECEDING UNCONGESTED AND CONGESTED TRAFFIC AT AN U/S SECTION

TRAFFIC AT 3 MIN. PRECEDING UNCONGESTED TRAFFIC U/S

STORAGE RATE	AVERAGE OCCUPANCY
3	16.20
-4	18.10
4	25.50
-36	24.35
-22	22.10
-7	17.50
7	16.50
2	21.65
-26	23.35
-20	20.15
-8	16.95
-8	23.10
6	18.80
8	20.65
-11	17.75

TRAFFIC AT 3 MIN. PRECEDING CONGESTED TRAFFIC U/S

STORAGE RATE	AVERAGE OCCUPANCY
-2	16.10
19	18.10
47	26.50
-13	16.25
28	19.05
32	26.40
-4	14.40
35	19.25
22	22.30
-11	19.65
-1	19.75
-22	18.70

TABLE 9

PROOF THAT THE BIVARIATE TRAFFIC DATA COMES FROM A BIVARIATE NORMAL DISTRIBUTION

DAY	PREDICTION INTERVAL	TRAFFIC DATA FROM SECTION	PATTERN CLASS	% OF BIVARIATE TRAFFIC VECTORS WITH $X^2 < 5.99$
MARCH 28	1 MIN.	1	1	92
			2	96
		2	1	95
			2	88
	2 MIN.	1	1	94
			2	98
		2	1	92
			2	94
	3 MIN.	1	1	96
			2	92
		2	1	95
			2	94
MARCH 29	1 MIN.	1	1	96
			2	96
		2	1	99
			2	92
	2 MIN.	1	1	94
			2	96
		2	1	97
			2	96
	3 MIN.	1	1	93
			2	98
		2	1	97
			2	94
MARCH 30	1 MIN.	1	1	94
			2	94
		2	1	93
			2	98
	2 MIN.	1	1	95
			2	95
		2	1	94
			2	92
	3 MIN.	1	1	95
			2	97
		2	1	95
			2	95

NOTES: Section 1 is mainline between 205th St. N.E. & 195th St. N.E.
Section 2 is mainline between 195th St. & 185th St. N.E.
Pattern classes 1 & 2 represent traffic data preceding uncongested and congested traffic.

It was then decided to adopt the parametric approach if these traffic data could be assumed to arise from a bivariate normal gaussian distribution. For this, the chi-square value was computed for each set of bivariate traffic data from the expression

$$X^2(1 - \alpha)df = [1 / (1 - p^2)] *$$
$$\{ [(x_1 - \mu_1)^2 / \sigma_1{}^2] + [(x_2 - \mu_2)^2 / \sigma_2{}^2] -$$
$$[2 * p * (X_1 - \mu_1) * (X_2 - \mu_2) / (\sigma_1 * \sigma_2)] \}$$

Thus, if 95 percent of the chi-squared values were less $X^2{}_{0.95(2)} = 5.99$, then the distribution can be assumed to be bivariate normal.

As shown in Table 9, approximately 95 percent of the X^2 values were less than 5.99 for each of the two pattern classes at lags of 1, 2, and 3 minutes for forecasting traffic conditions at the same, and at an upstream section. Hence, the patterns from each of the two pattern classes can be assumed to arise from a bivariate normal gaussian distribution, and so the researchers selected the statistical pattern recognition approach to forecast freeway traffic conditions 1 to 3 minutes ahead of time.

On the basis of statistical considerations, it is then possible to derive a decision rule that is optimal since, on an average basis, its use yields the lowest probability of committing classification errors.

TEST ON TSMC DATA

Based on the parametric approach developed in Chapter 3, we assume class w_1 to represent traffic conditions 1 to 3 minutes preceding uncongested traffic conditions, and class w_2 to represent traffic conditions 1 to 3 minutes preceding congested traffic conditions upstream or at the same freeway section. Also, $\underline{x}$ represents the bivariate vector consisting of traffic data in the form of storage rate and average occupancy at the same section or at a section downstream of the section at which we are attempting to forecast congested conditions.

Then, from equations (8) and (9), developed in Chapter 3, we assign the traffic condition as preceding uncongested traffic conditions if the risk of deciding class w_1 is less than the risk of deciding class w_2, mathematically expressed as

$$\{L_{11}-L_{12})\, p(w_1) \exp[-1/2(\underline{x}-m_1)C_1^{-1}(\underline{x}-m_1)]/ (2\ \pi)^{2/2}\, |C_1|^{1/2}\} +$$
$$\{L_{21}-L_{22})\, p(w_2) \exp[-1/2(\underline{x}-m_2)C_2^{-1}(\underline{x}-m_2)]/ (2\ \pi)^{2/2}\, |C_2\, 1|^{1/2}\} < 0 \qquad (8)$$

and assign the traffic conditions as preceding congested traffic condition, if the risk of deciding class w_2 is less than the risk of deciding class w_1, mathematically expressed as

$$\{L_{11}-L_{12})\, p(w_1) \exp[-1/2(\underline{x}-m_1)C_1^{-1}(\underline{x}-m_1)]/(2\ \pi)^{2/2}|C_1|^{1/2}\} +$$
$$\{L_{21}-L_{22})\, p(w_2) \exp[-1/2(\underline{x}-m_2)C_2^{-1}(\underline{x}-m_2)]/(2\ \pi)^{2/2}|C_2|^{1/2}\} > 0 \qquad (9)$$

Based on traffic data over the a.m. peak period for the 3 days, the a priori probability $p(w_i)$, and the mean and covariance matrices for each of the two pattern classes were calculated. The a priori probability for the two classes was calculated as follows:

$p(w_1) = N_1 / (N_1 + N_2)$, and

$p(w_2) = N_2 / (N_1 + N_2)$,

where N_1 and N_2 are the total number of bivariate vectors in classes one and two respectively.

The aggregate "a priori" probability and the mean and covariance matrices for each of the two pattern classes were then calculated from the 3-day bivariate traffic data for 1-minute, 2-minute, and 3-minute lags to predict traffic conditions upstream and at the same section. These values are summarized in Table 10.

Since the same data set was used for both the calibration of the statistical pattern recognition model and testing the developed model, the researchers needed to get a more extensive data set from the TSMC and divide that data set into two parts. For example, if 1-minute traffic data for the test sections had been obtained over 15 a.m. peak periods, then ten or more of these values could have been used exclusively for calibrating the model, and with the help of parameters obtained from the calibration, the developed model could have been tested over the remaining a.m. peak periods. The calibration and the test sets would then have been completely independent. However, because of practical difficulties experienced in getting such extensive traffic data from the TSMC, the researchers decided to limit the testing to the same three data sets used for the calibration of the model.

The option, then, was to either calibrate the statistical pattern recognition model on the basis of 2 of the 3 days from which data were obtained, and then test it on the third day, or use the same three data sets for calibration as well as testing.

If the former approach had been chosen, the problem of having the same data set for both calibration and testing could have been avoided. However, estimates of the parameters obtained would not have been as efficient as they were when all the three data sets were used for the calibration process.

This problem is not a serious one, since estimates of the statistical parameters, in the form of the mean and covariance matrices of traffic data, were obtained over three a.m. peak periods from an average of a (3*180/2) 270 bivariate set of vectors containing traffic data. The parameters obtained during the calibration process were then treated as "global averages" for uncongested and congested traffic conditions over the test sections. Thus, the algorithms obtained from these parameters could be tested on any data set (including the same three data sets used for calibration of the model).

TABLE 10

AGGREGATE VALUES OF PARAMETERS FOR EACH PATTERN CLASS FOR EACH RUN

LAG (MIN.)	MAIN LINE SECTION	PATTERN CLASS	MEAN VECTOR	INVERSE OF COVARIANCE MATRIX C^{-1}		$\lvert C \rvert$
1	1	1	-2.10	0.0046	0.0016	99.94
			17.84	0.0016	0.0224	
		2	5.74	0.0032	-.0006	60.00
			22.80	-.0006	0.0861	
	2	1	0.42	0.0075	0.0005	91.43
			16.42	0.0005	0.0395	
		2	0.22	0.0045	0.0009	57.20
			21.63	0.0009	0.0682	
2	1	1	2.62	0.0035	-.0011	115.93
			17.97	-.0011	0.0214	
		2	-3.27	0.0050	0.0069	46.41
			22.40	0.0069	0.1032	
	2	1	-3.22	0.0086	0.0056	60.13
			16.87	0.0056	0.0359	
		2	7.14	0.0061	-.0028	43.90
			20.69	-.0028	0.0867	
3	1	1	3.20	0.0036	-.0013	115.59
			18.14	-.0013	0.0216	
		2	-4.27	0.0055	0.0078	48.00
			22.04	0.0078	0.0903	
	2	1	-2.19	0.0069	0.0034	68.61
			17.13	0.0034	0.0326	
		2	5.09	0.0064	-.0059	42.75
			20.15	-.0059	0.0906	

NOTES: Section 1 is main line between 205th St. N.E. & 195th St. N.E.
Section 2 is main line between 195th St. N.E. & 185th St. N.E.
Pattern classes 1 & 2 represent traffic data preceding uncongested and congested traffic.
Mean vector for each pattern class is a 2*1 matrix
Inverse of the covariance matrix, C^{-1}, for each pattern class is a 2*2 matrix.
$\lvert C \rvert$ is the determinant of the covariance matrix for each pattern class.

The decision in equations (8) and (9) above, can be performed with knowledge of the loss functions L_{11}, L_{12}, L_{21}, and L_{22}. Then, L_{11} and L_{22} represent the loss of making a correct classification, and L_{21} and L_{12} represent the loss of making an incorrect classification. Tou states that in most pattern recognition systems, the loss is assigned as "nil" for both correct and erroneous decisions. (21) Thus, the loss function for two classes is expressed as $L_{ij} = 1 - d_{ij}$,

where $d_{ij} = h_i$ when $i = j$, $(0 < h_i < 1)$ and

$d_{ij} = 0$ when $i = j$.

Also, often a negative loss or positive gain can be assigned to correct classifications, and zero loss can be assigned to misclassifications. Thus, in our case, L_{11} and L_{22} can vary between zero and -1, and L_{12} and L_{21} can vary between +1 and zero, respectively.

Then $(L_{11} - L_{12})$ will be approximately -1, and $(L_{21} - L_{22})$ will be approximately +1. The recommended values for these loss functions are -1 and +1, representing a zero loss of correct decisions and a unit loss misclassification (22).

But this would not have to be the case in our research. The loss might be zero for forecasting an uncongested condition correctly, or the loss could be negative -- in other words, it could be a positive gain for correctly forecasting congested conditions.

In addition, the loss resulting from misclassifying an uncongested condition could be either greater or even less than the loss from misclassifying a congested condition. Misclassifying a congested condition would adversely affect our objective, to correctly predict congested conditions ahead of time, but misclassifying an uncongested condition would hurt the credibility of the ramp metering system that used the pattern recognition algorithm.

For this reason, $(L_{11} - L_{12})$ and $(L_{21} - L_{22})$ were tested over a range of values from -1 to +1 in increments of 0.1. Thus, a total of 21*21 = 441 combinations of these loss functions were explored for each of the six runs (Table 10). The algorithm developed to test the developed statistical pattern recognition algorithm over this range of loss functions can be found in Appendix C.

TEST BY SIMULATION USING INTRAS

INTRAS is a microscopic freeway simulation program that represents a "real world" traffic network (24). Knowledge of freeway operations and surveillance systems is incorporated into this detailed traffic simulation. INTRAS is used for studying freeway incident detection and control strategies, including ramp metering and diversion.

On the basis of the input, the simulated surveillance system produces output analogous to that generated by an on-line system. Vehicles traversing the freeway and ramp links move with respect to the cars they follow, lane changing, and the vehicle generation component developed for INTRAS. Point processing procedures process each individual detector's output to generate local estimates of traffic flow parameters.

The algorithm's accuracy in predicting traffic conditions, uncongested or congested, on the freeway was determined earlier by tests on the a.m. peak-hour traffic data from the TSMC. The researchers then decided to use INTRAS to simulate WSDOT's current ramp metering system, the statistical pattern recognition model developed by Davis (14), and the statistical pattern recognition model developed in this research (Babla/Nihan model).

The objective was to evaluate the performance of the statistical pattern recognition forecasting approach (Babla/Nihan model) in reducing congestion and improving traffic flow on the section of I-5, and compare it to the ramp metering system currently used by WSDOT and the Davis/Nihan statistical pattern recognition model.

The section of I-5 freeway over which the performance of the statistical approach was evaluated is shown in Figure 4. The network geometries of the southbound I-5 freeway are represented in the form of links, which are unidirectional roadway segments having identical geometric characteristics, such as number of lanes, grade, width of lanes, and free-flow speed. Nodes are placed at the intersections of geometric discontinuities and at the intersections of cross streets and ramp links with the freeway.

In addition to the information in Figure 4, information on the mean desired free-flow speeds on the main line (55 mph.) and ramps (35 mph.), pavement type (asphalt), the radius of curvature, and superelevation of freeway and ramps were also input to INTRAS. It was also assumed that 20 percent of the traffic consisted of high-performance passenger cars, 1 percent of inter-city buses, 6 percent of trucks, and the remaining 73 percent were low-performance passenger cars. Also, 26 percent of the traffic was assigned to the rightmost lane, and 38 percent and 36 percent, to the second and third lanes, respectively. Accleration and auxiliary lanes are not shown in Figure 4.

After the geometries of the section of I-5 southbound freeway have been input (Figure 4), INTRAS requires equivalent hourly volumes for each time slice over which the simulation is to be carried out.

The researchers decided to use INTRAS to simulate the traffic conditions over the a.m. peak hour from 6:30 a.m. to 7:30 a.m. Equivalent hourly volumes in 15-minute time slices were input into INTRAS (Table 11) by averaging historical weekday data over the same period obtained previously from the TSMC.

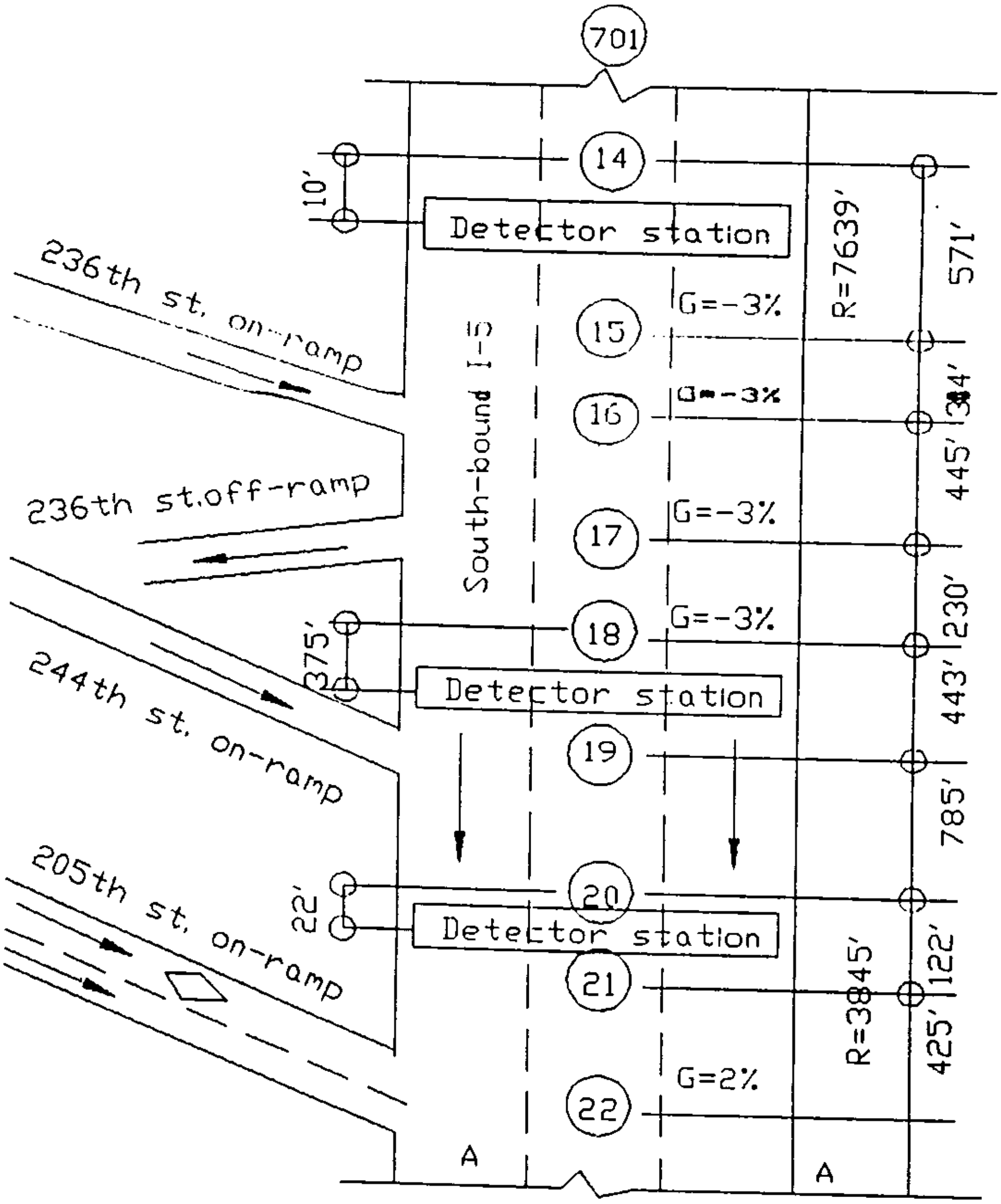

Figure 4: Section of south-bound I-5 for testing WSDOT approach, and the incorporation of Davis/Nihan and Babla/Nihan statistical pattern recognition models in the WSDOT approach, to ramp metering by simulation using INTRAS.

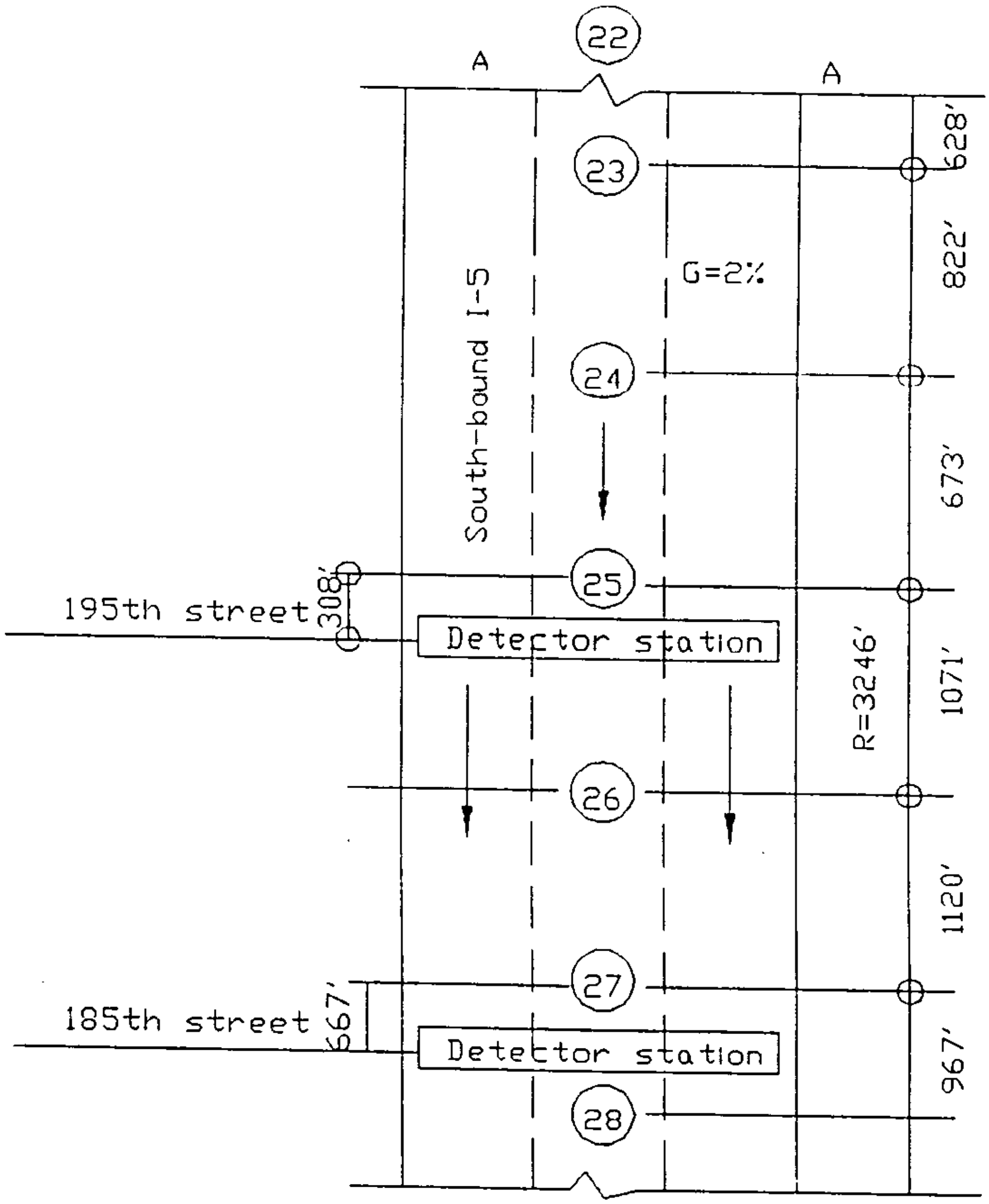

Figure 4 (Continued): Section of south-bound I-5 for testing WSDOT approach, and the incorporation of Davis/Nihan and Babla/Nihan statistical pattern recognition models in the WSDOT approach, to ramp metering by simulation using INTRAS.

TABLE 11

TEST VOLUMES INPUT FOR SIMULATION TO INTRAS

TIME (AM.)	MAIN LINE VOLUME IN VEH/HR	VOLUME ON 236TH ON-RAMP IN VEH/HR	VOLUME ON 244TH ON-RAMP IN VEH/HR	VOLUME ON 205TH ON-RAMP IN VEH/HR
6:30 - 6:45	5000	400	375	375
6:45 - 7:00	4900	450	425	425
7:00 - 7:15	4800	475	450	450
7:15 - 7:30	4700	550	500	500

Three percent of the mainline volume flowing into an upstream section was considered to be leaving through off-ramps.

Four simulation runs were executed using the data:

1. with WSDOT's current ramp metering system (Figure 5),
2. with the Davis/Nihan statistical pattern recognition model (Figure 6), and
3. with the Babla/Nihan statistical pattern recognition approach developed in this research (Figure 7).

A minimum metering rate of 5 veh/ln and a maximum of 24 veh/ln was always implemented (not shown in figures).

INTRAS gave measures of effectiveness (MOEs) for each of the simulation runs. The MOEs output by INTRAS included average speed, vehicle-miles of travel, and vehicle-minutes of delay on the freeway and on ramps. It also gave information on the fuel consumption, and the total number of vehicles served by the mainline and by on-ramps.

These values were then compared to determine the effectiveness of the forecasting algorithm in improving freeway traffic performance.

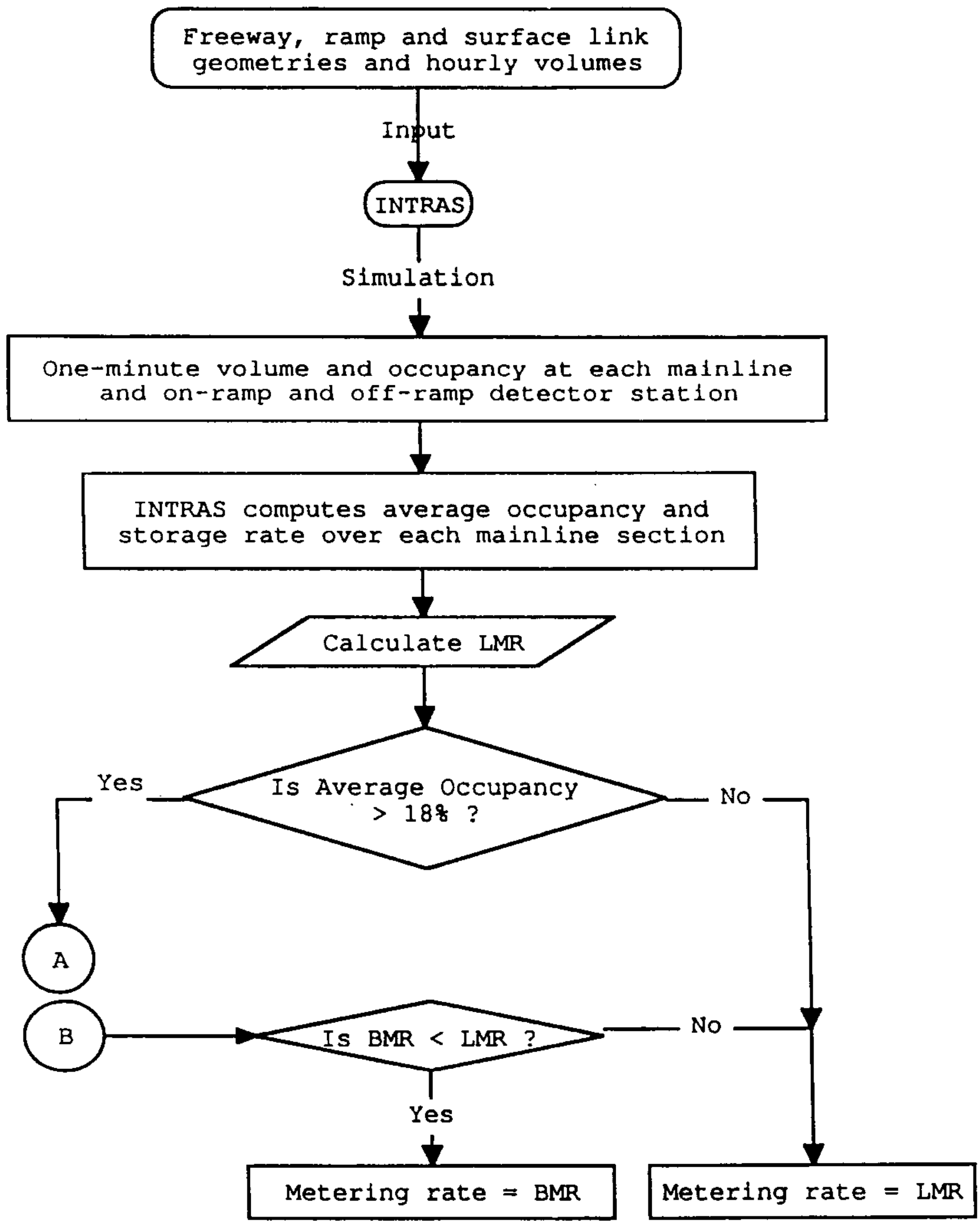

Figure 5: Test of WSDOT's current approach to ramp metering by simulation of traffic conditions on I-5 using INTRAS

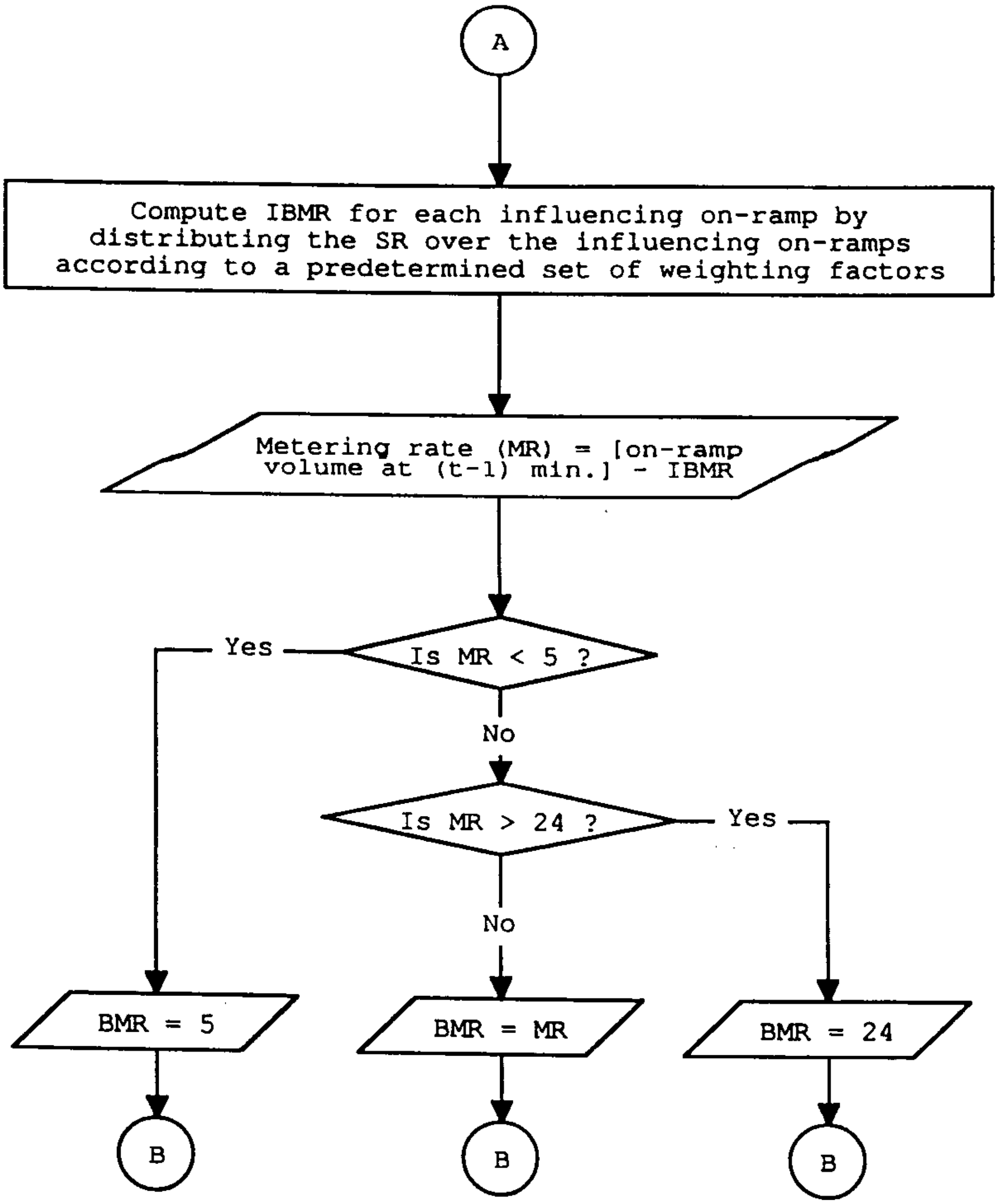

Figure 5 (Continued) : Test of WSDOT's current approach to ramp metering by simulation of traffic conditions on I-5 using INTRAS.

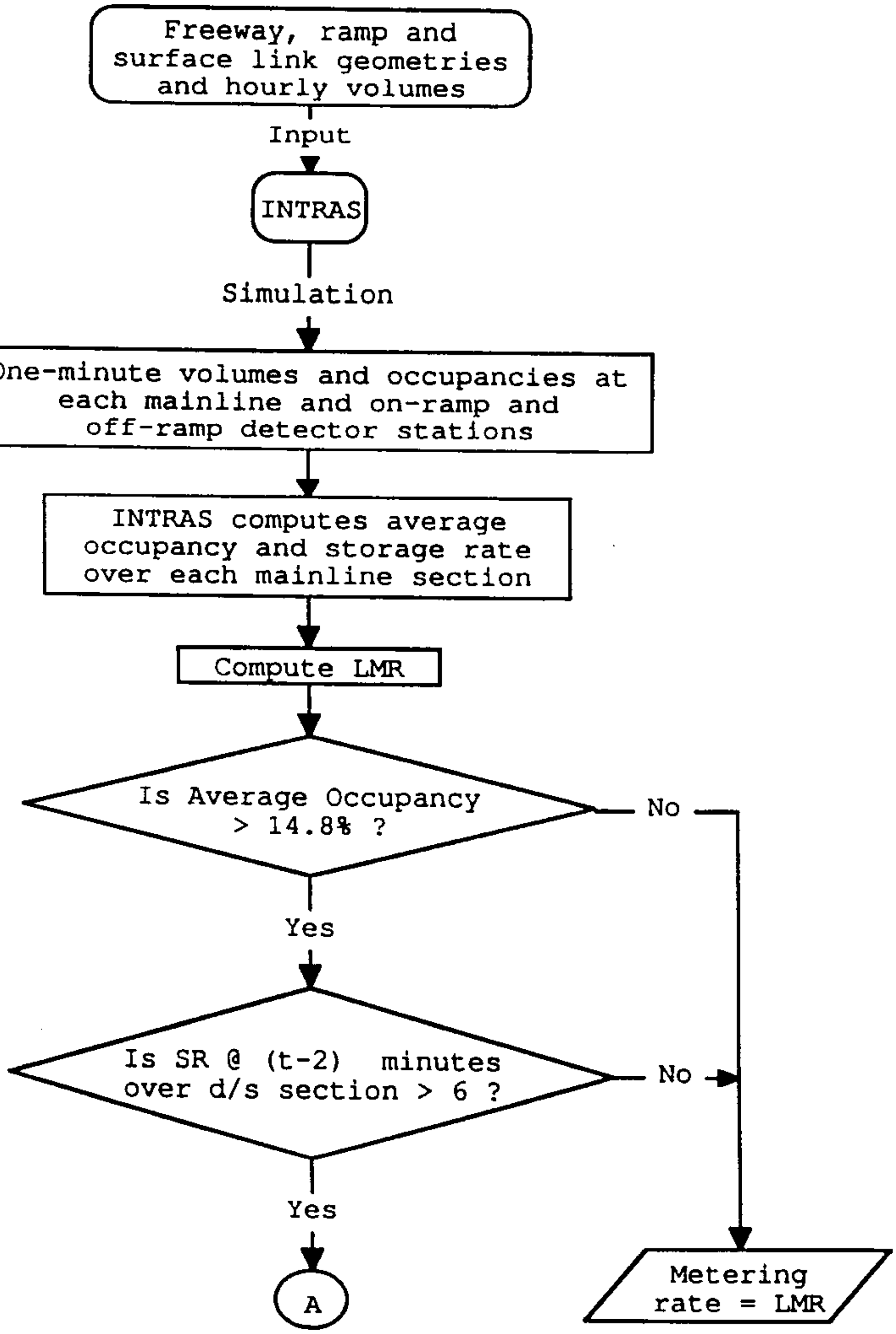

Figure 6: Test of Davis/Nihan model of statistical pattern recognition by simulation of traffic conditions on I-5 using INTRAS.

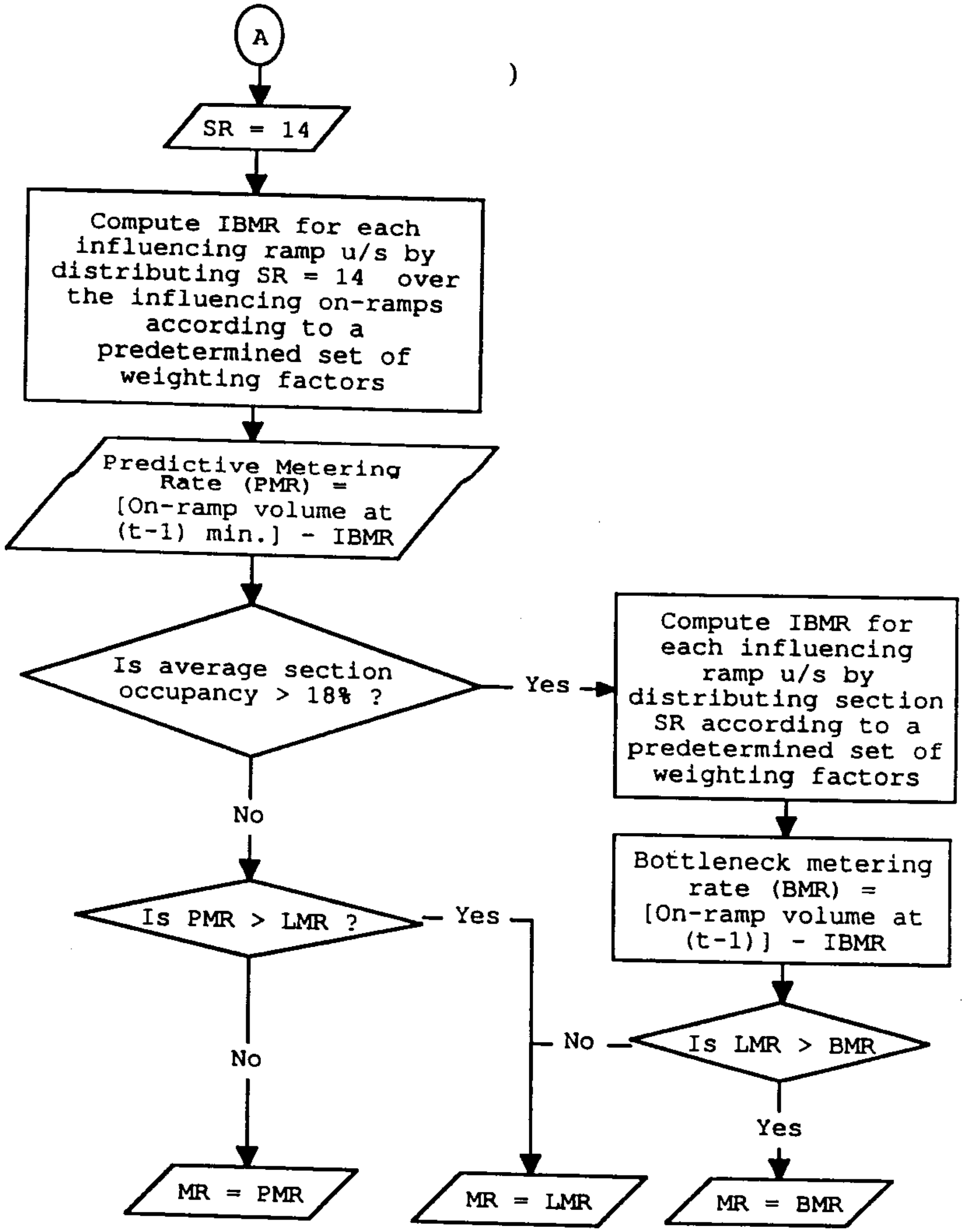

Figure 6 (Continued) : Test of Davis/Nihan model of statistical pattern recognition by simulation of traffic conditions on I-5 using INTRAS,

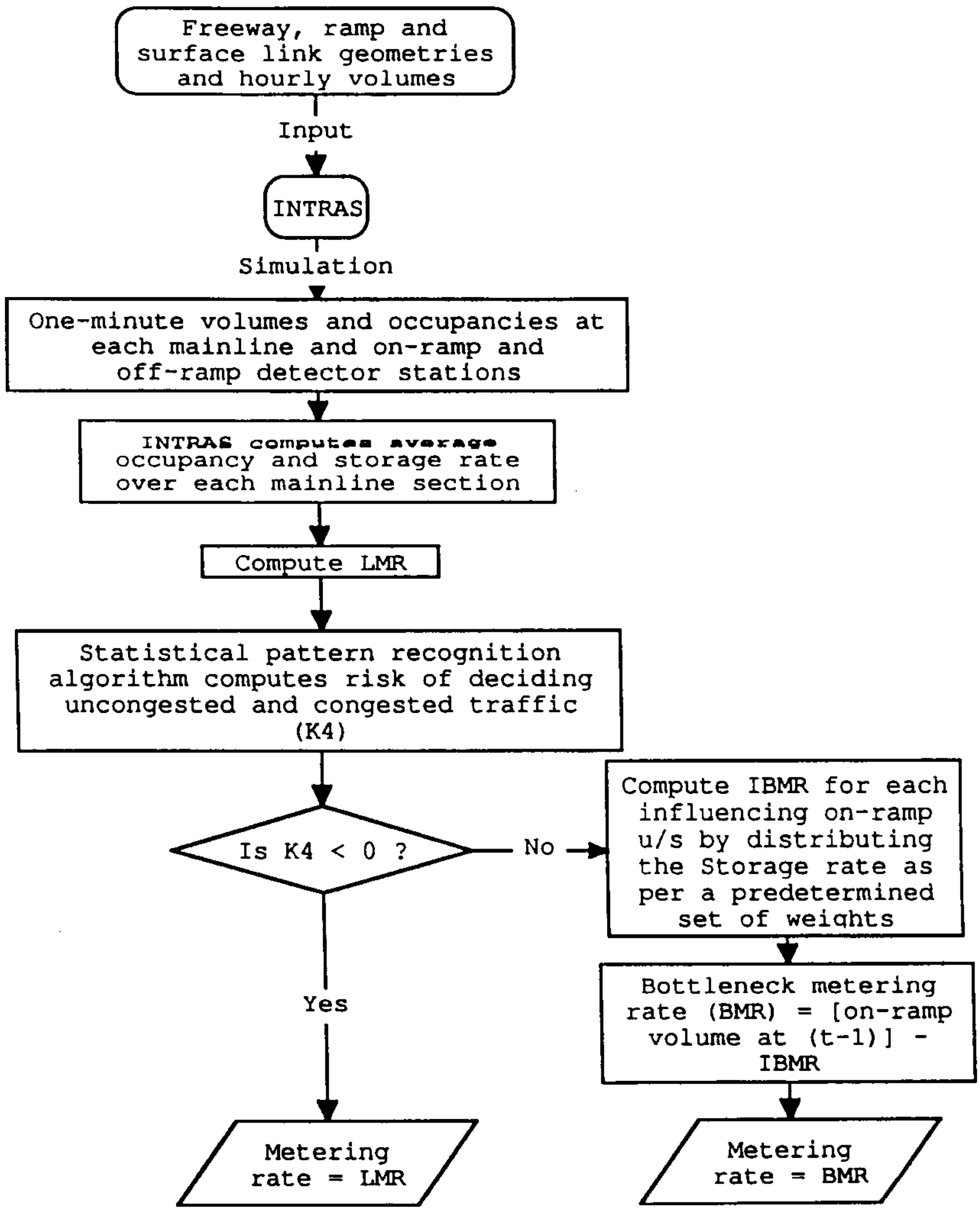

Figure 7: Test of Babla/Nihan model of statistical pattern recognition by simulation of traffic conditions on I-5 using INTRAS.

CHAPTER 4

RESULTS

TEST ON TSMC DATA

The statistical pattern recognition model was tested on the a.m. peak period traffic data. The Fortran program (Appendix C) was tested on preclassified traffic data representing conditions prior to uncongested and congested traffic conditions for accuracy in the prediction of traffic conditions. This test was performed at lags of 1, 2, and 3 minutes (Tables 3 through 8) at the same section, and at a section upstream.

The results were obtained in the form of false positive and false negative rates for each pair of loss functions in the range from -1 to +1, for each of the six runs of the program. A sample output of 'LG3DS' for the Fortran program shown in Appendix C can be found in Appendix D.

A trade-off was observed between the false positive and false negative rates for various pairs of loss functions. Hence, it was decided to keep the false positive rates at an average of ten percent, and lower the false negative rates as much as possible. This ensured that we would improve upon the existing conditions of zero percent false positive and 100 percent false negative rates (as we were not predicting congestion).

Thus, we would still improve upon the existing conditions by detecting congested conditions ahead of time from zero percent to as high as possible, while not increasing the false negative rates to a level that would discredit the system. The best results for each of the six runs are recorded in Table 12.

The accuracy of our predictions, in terms of lower false positive and false negative rates, seemed to improve with an increase in the time of prediction. Also, the results at the same lag are better when the prediction is made for traffic conditions at a section upstream than for traffic conditions at the same section.

TABLE 12

BEST RESULTS FOR TEST OF BABLA/NIHAN MODEL ON TSMC DATA

Run	Loss functions		Day 1		Day 2		Day 3	
	k1	k2	% False +ve	% False -ve	% False +ve	% False -ve	% False +ve	% False -ve
LG1	-1.0	0.5	36.6	76.5	9.0	71.4	4.5	80.6
LG2	-0.8	0.5	14.4	68.0	12.5	52.1	9.3	60.0
LG3	-1.0	0.8	15.8	58.3	6.6	53.2	10.6	36.2
LG1DS	-0.9	0.3	31.7	100.0	7.5	69.4	5.4	85.5
LG2DS	-1.0	1.0	7.6	40.0	14.1	31.3	11.1	60.0
LG3DS	-1.0	1.0	9.6	50.0	14.8	46.8	7.7	58.6

Notes:
1. Run 'LG1' is for prediction of traffic condition at same section 1 minute ahead of time
2. Run 'LG2' is for prediction of traffic condition at same section 2 minutes ahead of time
3. Run 'LG3' is for prediction of traffic condition at same section 3 minutes ahead of time
4. Run 'LG1DS' is for prediction of traffic condition at a section u/s 1 minute ahead of time
5. Run 'LG2DS' is for prediction of traffic condition at a section u/s 2 minute ahead of time
6. Run 'LG3DS' is for prediction of traffic condition at a section u/s 3 minute ahead of time

Thus, prediction for traffic conditions at an upstream section 3 minutes ahead of time (LG3DS) gave us the best results for the 3-day TSMC data. Also, prediction of traffic conditions at an upstream section 1 minute in advance (LG1DS) for day one is even worse than the existing condition, with a false positive rate of 31 percent and a false negative rate of 100 percent. These results are compared with results reported by the application of the Davis/Nihan model in Table 13. The percent correct rates were reported as an average of correct predictions over all 3 days.

Assuming that the a.m. peak hour data from SB I-5 freeway represented highly congested data, the Babla/Nihan model of statistical pattern recognition has given better results than the earlier approach by Davis and Nihan.

The Davis/Nihan and Babla/Nihan models (LG3DS) were then tested for their efficacy in improving freeway traffic performance. The researchers incorporated both models into WSDOT's current ramp-metering approach and a simulation of traffic conditions on southbound I-5 using INTRAS software.

TEST BY SIMULATION USING INTRAS

The Davis/Nihan and Babla/Nihan statistical pattern recognition models were then incorporated as forecasting algorithms into the ramp control logic currently used by WSDOT, and the system simulated using INTRAS. The simulation results were then compared with each other and with a simulation of the same traffic conditions using WSDOT's current ramp-metering approach.

TABLE 13

RESULTS FROM APPLICATION OF DAVIS/NIHAN AND BABLA/NIHAN MODELS ON TSMC DATA

TYPE OF DATA	PERCENT CORRECT	PERCENT FALSE POSITIVES	PERCENT FALSE NEGATIVES
Davis / Nihan model applied to lightly congested data	92	5	36
Davis / Nihan model applied to highly congested data	68	7	73
Babla / Nihan model (LG3DS) applied to a.m. peak data of southbound I-5	75	10	52

The results from the three simulation runs are summarized in Table 14. Figure 8 shows that the average speed on the main line freeway improves considerably with the application of the Davis/Nihan model, but this is achieved at the expense of considerable delay on the ramps and surface links (Figure 9). In comparison, the Babla/Nihan model achieves a smaller increase in main line freeway speed over the current WSDOT approach, but with only a very small increase in delays at ramps and surface links.

This not only results in higher average speeds system-wide (Figure 8) and reduced total delay system-wide (Figure 9) for the Babla/Nihan model than the other two approaches, but also results in increased mileage in terms of miles traveled per gallon of fuel consumed (Figure 10).

The main line freeway volume (Table 14) at 1,568 veh/ln/hr for the Davis/Nihan model is 39 veh/ln/hr greater than the main line volumes resulting from the application of the Babla/Nihan model. Only 866 vehicles at the three on-ramps were allowed access during the test hour by application of Davis/Nihan model, whereas 1,189 vehicles were allowed access by application of the Babla/Nihan approach. Thus, the Babla/Nihan model allowed access to 323 more vehicles at the on-ramps during the test period at the expense of decreasing the mainflow by 39 veh/ln/hr. Compared to the WSDOT approach, the Babla/Nihan forecasting model increased the main line flow by 77 veh/ln/hr by restricting access to a total of only 75 vehicles at the three on-ramps during the test hour.

In contrast to the two other approaches, only the total vehicle-miles of travel (Figure 12) on the freeway and the total vehicle miles of travel system-wide were higher with the Davis/Nihan model. However, as can be seen in Figure 12, the Babla/Nihan model achieves about 70 percent of the increase in vehicle-miles on the freeway, and about 80 percent of the increase in the total vehicle-miles traveled system-wide

TABLE 14

RESULTS FROM APPLICATION OF DAVIS/NIHAN AND BABLA/NIHAN MODELS BY SIMULATION USING INTRAS

	WSDOT EXISTING ALGORITHM	DAVIS/NIHAN ALGORITHM	BABLA/NIHAN ALGORITHM
Average speed on freeway in mph	23.8	28.0	25.3
Average speed system-wide, mph	20.2	19.6	20.9
Total veh-miles traveled on freeway	7726	8340	8132
Total veh-miles traveled system-wide	8100	8604	8488
Veh-min. of delay on freeway	10970	8757	10327
veh-min. of delay system-wide	14985	16778	14854
Freeway volume in veh/ln/hr	1452	1568	1529
Total number of vehicles output from all on-ramps	1264	866	1189
Total gallons of fuel consumed system-wide	1678	1779	1722
Veh-miles traveled per gallon of fuel consumed system-wide	4.83	4.84	4.93

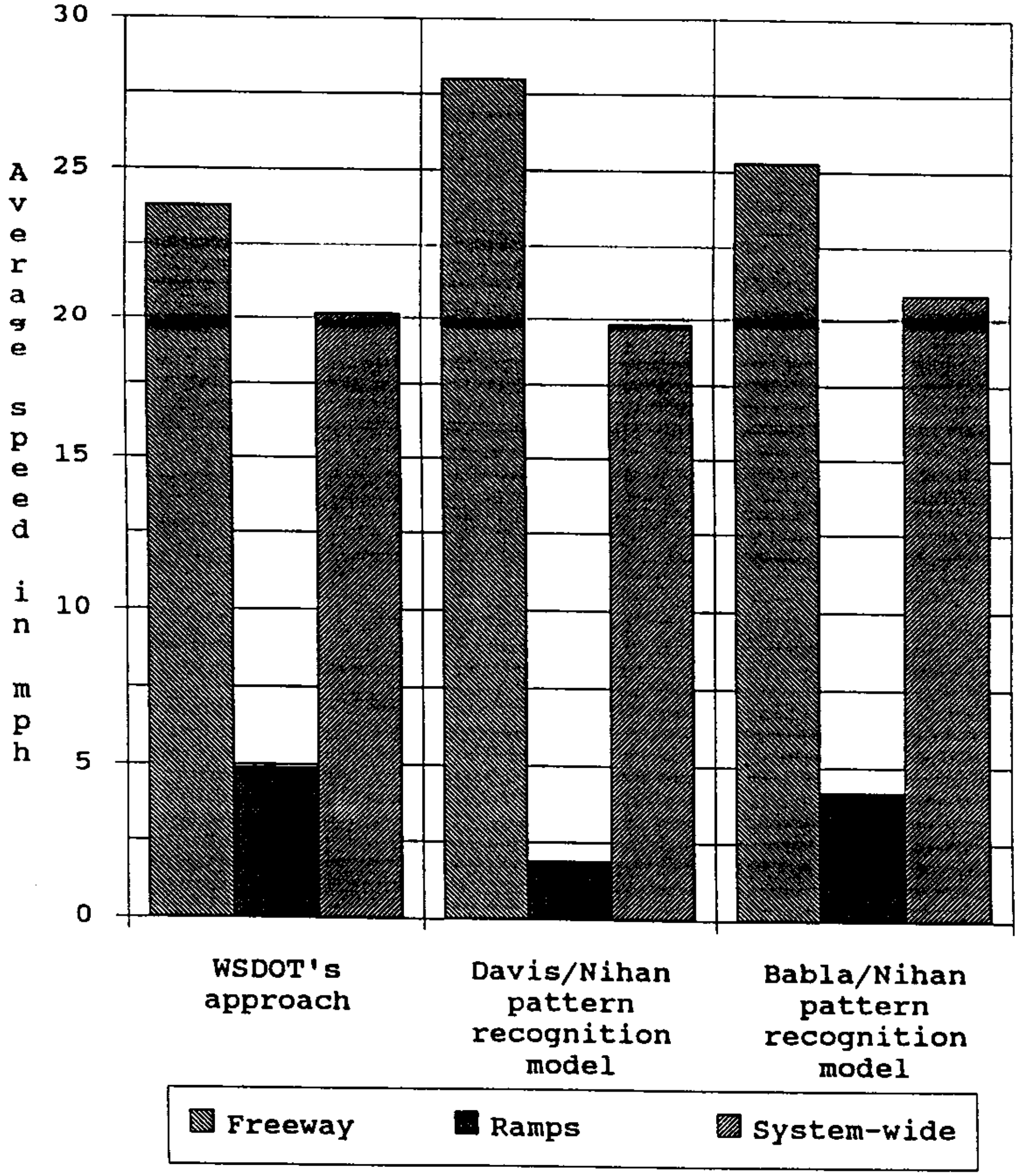

Figure 8: Average speed over the freeway, ramps, and system-wide for different approaches to ramp metering determined by simulation using INTRAS.

of the improvement achieved by using the Davis/Nihan model. The WSDOT approach is the least effective approach of the three.

These results should, however, be interpreted with caution, as the section of I-5 over which the simulation was carried out is only a small part of the section of I-5 that runs through metropolitan Seattle. So any savings in main line vehicle delay achieved by higher average mainline vehicle speeds has been under represented. In addition, the number of vehicles delayed at the on-ramps would decrease if the entire section of I-5 running through metropolitan Seattle were included because these vehicles would be able to redistribute over the on-ramps not presently included in the system, thereby reducing the queueing at on-ramps.

The application of the Davis/Nihan model gives a higher average main line speed and more vehicles delayed at on-ramps. It appears that it compares unfavorably with the Babla/Nihan model, but by not including the entire section of I-5 running through metropolitan Seattle in this research, this comparison is inconclusive.

Also, the volumes that were simulated with INTRAS were obtained from the operation of the current WSDOT system (Table 11). These volumes are the result of ramp controls through local and bottleneck metering rates, and do not reflect the demand at the on-ramps. Thus, we were comparing the ability of the three approaches (WSDOT's current approach, the Davis/Nihan model, and the Babla/Nihan model) to forecast traffic conditions on the freeway by measuring traffic that had already been affected by ramp controls. This lessens the validity of our results.

However, preliminary results from these tests do indicate that the developed Babla/Nihan model performs better than the existing system, and significantly better than the Davis/Nihan forecasting model.

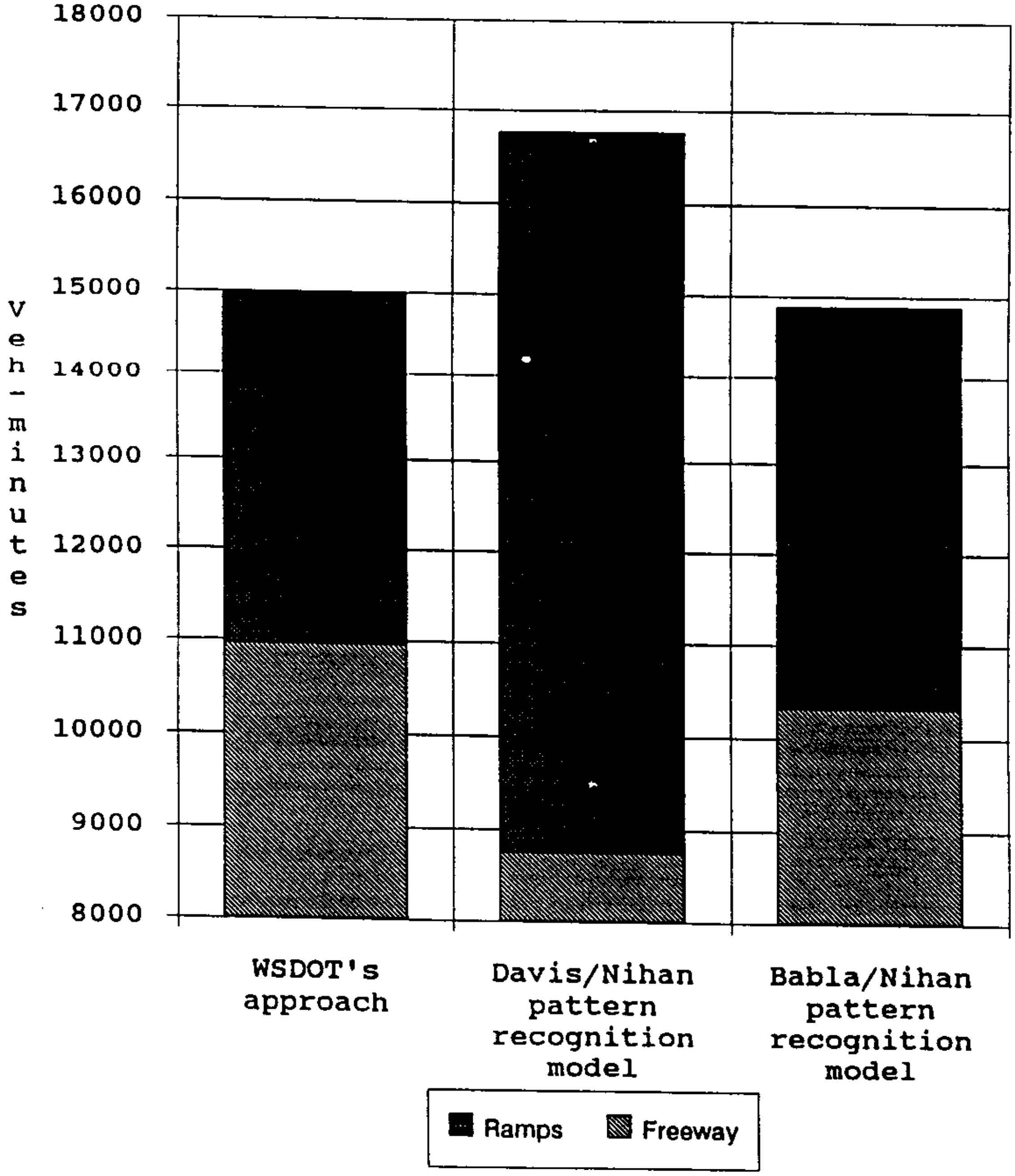

Figure 9: Total veh-minutes of delay on the freeway and at ramps for different approaches to ramp metering determined by simulation using INTRAS.

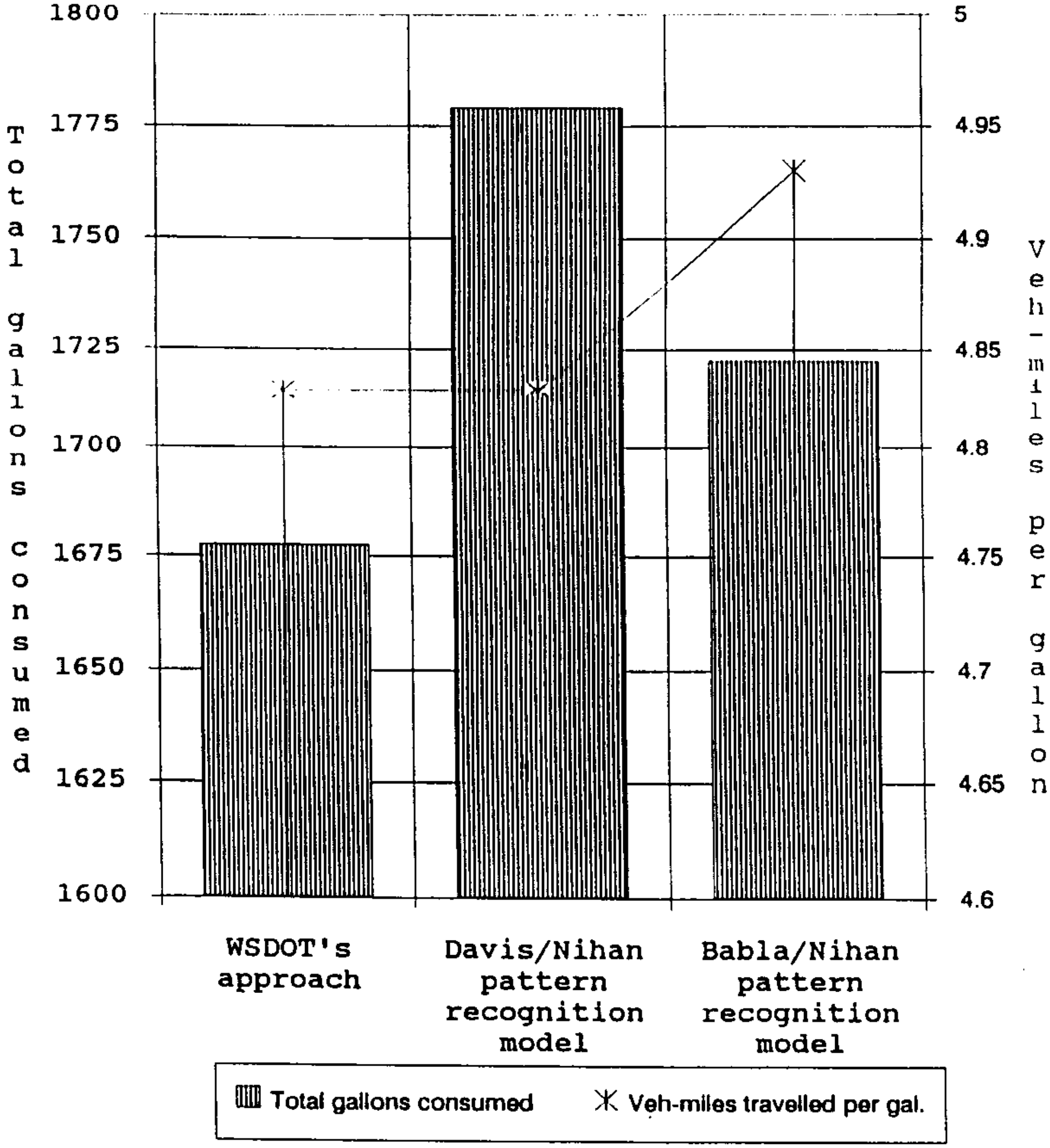

Figure 10: Fuel consumption system-wide during the test hour for different approaches to ramp metering determined by simulation using INTRAS.

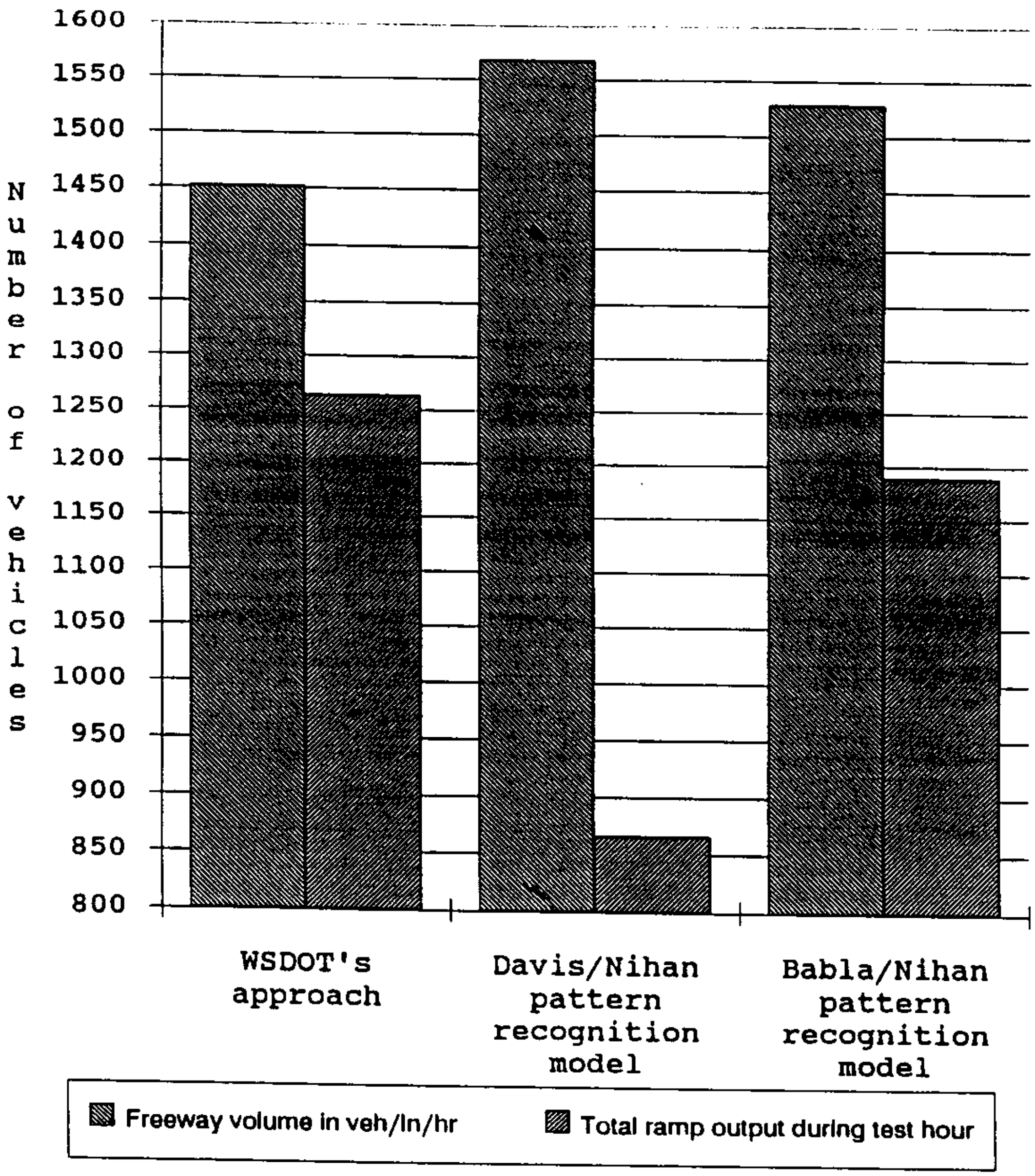

Figure 11: Mainline freeway volume in veh/ln/hr and total number of vehicles output from all ramps during the test hour for different approaches to ramp metering determined by simulation using INTRAS.

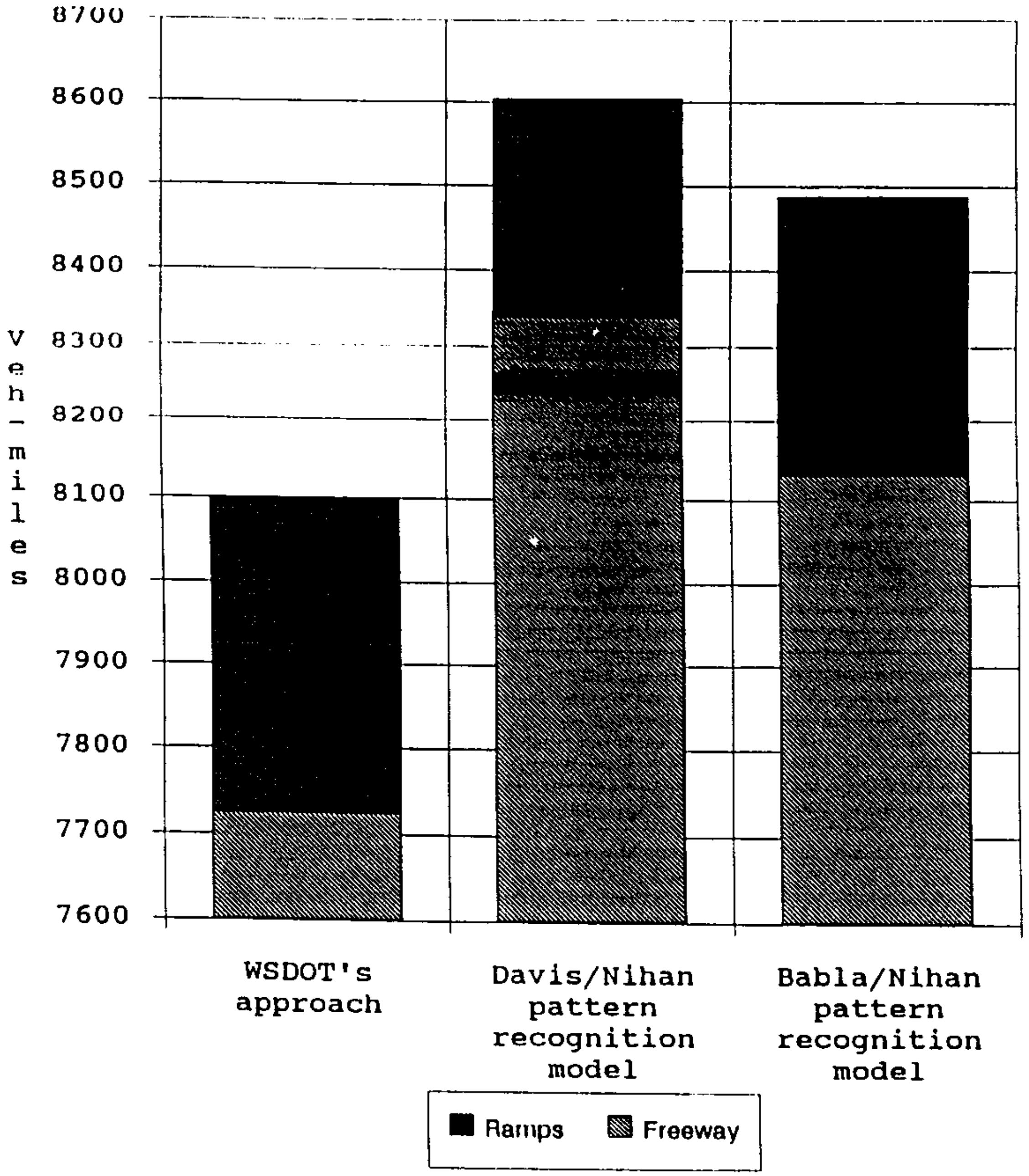

Figure 12: Total veh-miles travelled on the freeway and ramps for different approaches to ramp metering determined by simulation using INTRAS.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

The results of testing the forecasting capability of the model on preclassified traffic data (Tables 3 through 8) indicated that the accuracy of the forecasts improved as the "lag," or the time available for forecasting, increased from 1 to 3 minutes (Table 12). For the same "lag", forecasting for traffic conditions at an upstream location gave better results than forecasting for traffic conditions at the same section.

Finally, of the six algorithms tested, an algorithm forecasting traffic conditions 3 minutes ahead of time for an upstream section (LG3DS) gave the best results for all three days. It correctly detected traffic conditions 75 percent of the time, with a congestion detection rate of 52 percent and falsely predicted congestion 10 percent of the time.

Thus, from tests on the 3-day TSMC data, it appears that the Babla/Nihan model has a better forecasting ability than the Davis/Nihan model.

The results of simulating the models using INTRAS indicated that the Babla/Nihan model of statistical pattern recognition had the highest system-wide average speed, the lowest system-wide total delay, and the highest gas mileage in terms of vehicle-miles traveled per gallon consumed. Compared to the Davis/Nihan model, it allowed 323 more vehicles at the on-ramps to flow into the freeway during the test hour at the expense of restricting flow on the mainline by only 39 veh/ln/hr.

However, as was noted earlier, the simulation results should be interpreted with caution since the section over which the simulation was carried out was a small area. In addition, we applied metering to volumes which were already the result of ramp controls.

Our objective to develop a model based on pattern recognition techniques that could forecast traffic conditions ahead of time, has been successful, as the Babla/Nihan model was able to accurately predict traffic condition at an upstream section 3 minutes ahead of time 75 percent of the time. Also, the false positive rate of 10 percent is reasonable, considering that we were able to accurately predict congestion 52 percent of the time (Table 13).

Preliminary results from the incorporation of the Babla/Nihan forecasting model in WSDOT's current ramp metering system, and simulation of the system under I-5 peak hour traffic conditions using INTRAS, indicated that incorporation of the Babla/Nihan forecasting model resulted in considerable improvements in freeway traffic performance.

These results indicate that future refining of the Babla/Nihan statistical pattern recognition model is merited, as is incorporating it on-line at the TSMC central computers.

RECOMMENDATIONS FOR FUTURE RESEARCH

The researchers recommend that the above results be viewed as preliminary. Any application of the Babla/Nihan model to traffic surveillance systems should be preceded by intensive data gathering and extensive validation of the parameters in the model.

Peak hour traffic data over at least 15 to 20 days should be used for calibration of the model to ensure that the estimated parameters are closer to their real values. This should be followed by testing the model over an independent set of peak hour traffic data to ensure proper validation. The parameters obtained from this extensive research over 15 or 20 data sets could then be used as input for simulation of the I-5 a.m. peak period conditions using INTRAS.

The simulation run should also include the entire section of southbound I-5 running through metropolitan Seattle. The volumes input for simulation should further be calibrated so that they reflect the actual demand at the on-ramps, and not the volumes resulting from subsequent metering.

The results from this refined simulation run would then accurately indicate the merit of incorporating the developed Babla/Nihan forecasting model on-line on the TSMC central computers.

The results of this research, while promising, indicate that investigation into other pattern recognition techniques is warranted -- not only for forecasting traffic conditions on the freeway, but for other situations in transportation engineering, like incident detection, where the problem is to find more than one solution.

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APPENDIX A

LITERATURE REVIEW

RAMP METERING

Ramp metering has come a long way since the days when a policeman stopped traffic at freeway entrances and released vehicles one at a time. (1) The effectiveness of this technique was evaluated, and, consequently, the policeman was replaced with automation. Today, all traffic on the freeway, including traffic entering through on-ramps, is managed through a central surveillance center. An evaluation of the use of microcomputers as traffic-responsive ramp controllers has been done by B.C. Fong. (2)

In their study on peak period traffic volumes on the I-5 freeway in metropolitan Seattle, Nihan and Davis reported decreased mainline volumes, resulting from ramp controls acting to keep the freeway volumes at or below capacity, decreased travel times, and a flattening of the morning and evening peaks, due to the shifting of some trips to off-peak times. (3)

Ramp control methods include ramp closure, pre-timed or fixed-time metering, locally actuated metering, centralized interconnected metering, and system-wide ramp metering. (4) Fixed-time metering methods have preset fixed metering rates according to the time of day, based on historical upstream demand, and downstream capacity data. However, the repetitive nature of recurrent congestion cannot be predicted, and requires real-time control through traffic responsive ramp metering. Locally actuated, centralized interconnected, and system-wide ramp metering techniques fall under the broad category of real-time, traffic-responsive ramp metering.

APPLICATIONS OF PATTERN RECOGNITION TO FREEWAY SURVEILLANCE

Research analogous to this research includes the development and evaluation of algorithms aimed at detecting traffic incidents from freeway surveillance data. (5) The algorithms were based on data obtained from the Los Angeles and Minneapolis freeway surveillance systems, and consisted of aggregated 1-minute occupancy rates and volumes on the freeway. The algorithms aimed to identify an incident by the specific patterns they produced.

Payne classified the traffic conditions into three types: incident-free, incident occurred, and incident continuing conditions. Thus, in addition to algorithms signaling the occurrence of an incident, they also signaled its continuation and termination. All of his algorithms are based upon occupancy data. Identification of an incident involved binary decisions, comparing a feature value to threshold values at each stage of a tree-structure. The thresholds were functions of occupancy; at the section, and at upstream and downstream sections. They were determined through extensive data-collection of traffic conditions during incident and incident-free conditions.

The researchers concluded that the performance of any specific algorithm differed among facilities, performing best when the calibration of the thresholds was done on data from the implementing facility. Further, in choosing threshold values, they observed a trade-off between the detection rate and the false alarm rate. Thus, improvement in the detection rate could only be achieved with an increase in the cost associated with false alarms. Calibration of thresholds was then used to achieve acceptable values of the false alarm rates for a reasonable detection rate.

Tsai and Case applied a pattern recognition approach to an existing incident detection system on the Queen Elizabeth Freeway in Ontario. (6) Their aim was to improve the false alarm rates while maintaining acceptable detection rates. Thus, once an incident was detected, the purpose of the pattern recognition algorithm was to distinguish between true and false alarms, based on their different duration rate characteristics. Using the pattern recognition approach, they managed to reduce the false alarm rate by 33 percent, from its previous value of 0.09 percent to 0.06 percent.

Collins applied a computer-based algorithm, PATREG, to identify the traffic disturbances following an incident. (7) The PATREG algorithm monitored the average traffic speed in each lane between a pair of upstream and downstream detector stations, using a pattern recognition technique. It indicated the occurrence of an incident when the calculated traffic speeds fell outside the predetermined upper and lower threshold values of speed determined for those lanes.

Peter Bohnke and Elmar Pfannerstill have explored the idea of identifying individual vehicles, or their platoons through the characteristic wave-form patterns each vehicle produces when passing over an induction loop detector, for a more efficient traffic management and route guidance system. (8)

APPENDIX B

SCIENCE OF PATTERN RECOGNITION

Recognition is regarded as the basic attribute of human beings and all living organisms, and pattern is the description of an object. A human being is superior, partly, because of his/her superior pattern recognition abilities. We practice pattern recognition at every instant in our daily lives in the form of recognition of concrete items through our senses, and the recognition of abstract items through conceptual pattern recognition.

Pattern recognition is used in many disciplines - medical diagnosis, language translation, and statistics, to name a few. (16, 17) Interest in this area is still growing at a rapid rate, with interdisciplinary study and research in such areas as engineering, computer science, information science, statistics, physics, chemistry, linguistics, psychology, biology, physiology, and medicine.

Chen defines pattern recognition "as a science in which we take advantage of the large storage and processing abilities of the computer coupled with its ability to work in high dimensions to perform highly complex recognition tasks which heretofore have been performed primarily by humans." (18)

The primary task in pattern recognition is pattern classification, which consists primarily of classifying the data into two or more pattern classes. (19) The pattern recognition problem consists mainly of representation of the input data measured from the object to be recognized, extraction of features characteristic of the input data, reduction in dimensionality of the input vectors, and finally, the derivation of optimum decision functions to identify and classify the patterns.

Our problem, then, is to derive a decision function for classifying the traffic data from loop detectors on the freeway into those preceding uncongested traffic conditions and those preceding congested traffic conditions. The loop detector data are available in the form of volume and occupancy at main line, on-ramp, and off-ramp detector stations. Thus, volume, occupancy, speed, and input-output difference over a section of the freeway can be used as input vectors.

The three different approaches to pattern recognition can be classified as: heuristic, syntactic, and mathematical methods. (20) Heuristic methods use ad-hoc decision rules, as in character recognition, based on human experience and intuition; syntactic methods use the relationships between sub-patterns, as in chromosome identification and picture recognition; and mathematical methods employ classification rules derived from a mathematical framework for the purpose of classification of the data in pattern recognition.

Because of its utilization of the clustering concept to represent input data in space, the mathematical approach was the obvious choice for this research project.

The mathematical approach to pattern recognition can be statistical (parametric) or deterministic (non-parametric). The statistical approach defines the discriminant function as a class of probability densities defined by a relatively small number of parameters. (The parameters here refer to the mean and the covariance matrices used to derive decision functions to separate the pattern classes.)

When no assumptions can be made about the underlying distribution or the characterizing parameters, the non-parametric approach is used. Despite its label, the non-parametric approach consists of parameters of a multivariate polynomial decision function to separate the pattern classes.

PARAMETRIC / STATISTICAL APPROACH

The pattern classes are assumed to arise from a multivariate normal Gaussian distribution, the parameters being the mean and the covariance matrix.

Tou uses the parametric approach to pattern recognition in his research. (21) Assuming that two pattern classes exist, the a priori probabilities $p(w_1)$ and $p(w_2)$ are calculated from the population of labeled samples. The a priori probabilities refer to our expectations based on past experience and have no relation to the current data.

On the basis of the labeled samples of the means ($\underline{m}_1$ and $\underline{m}_2$) and the covariance matrices (C_1 and C_2) for the two pattern classes, the probability density function of $\underline{x}$ that comes from class w_i, $p(\underline{x}/w_i)$, are estimated from the expression,

$p(x/w_1) = \{ \exp[-1/2 (\underline{x} - m_1) C_1^{-1} (\underline{x} - m_1)] \} / \{ (2\pi)^{2/2} |C_1|^{1/2} \}$, and

$p(x/w_2) = \{ \exp[-1/2 (\underline{x} - m_2) C_2^{-1} (\underline{x} - m_2)] \} / \{ (2\pi)^{2/2} |C_2|^{1/2} \}$,

where C is the covariance matrix given by $C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

and $\underline{m}_1$ and $\underline{m}_2$ are the mean vectors of classes w_1 and w_2.

If L_{ij} represents the average loss of deciding that class w_j is true when, in fact, the sample pattern actually belongs to class w_i, $p(w_i/\underline{x})$ represents the probability that $\underline{x}$ comes from class w_i, and $R_i(\underline{x})$ is the expected loss in assigning $\underline{x}$ to class w_i, then

$$R_1(\underline{x}) = L_{11}\, p(w_1/\underline{x}) + L_{21}\, p(w_2/\underline{x}), \text{ and} \qquad (1)$$

$$R_2(\underline{x}) = L_{12}\, p(w_1/\underline{x}) + L_{22}\, p(w_2/\underline{x}). \qquad (2)$$

Using Bayes formula, the probability of $\underline{x}$ coming from w_i, also called the a posteriori probability,

$$p(w_i/\underline{x}) = [\, p(w_i)\, p(\underline{x}/w_i)\,] / p(\underline{x}), \qquad (3)$$

equations (1) and (2) become,

$$R_1(\underline{x}) = L_{11} \{ [\, p(w_1)\, p(\underline{x}/w_1)\,] / p(\underline{x}) + L_{21} \{ [\, p(w_2)\, p(\underline{x}/w_2)\,] / p(\underline{x}), \text{ and} \qquad (4)$$

$$R_2(\underline{x}) = L_{12} \{ [\, p(w_1)\, p(\underline{x}/w_1)\,] / p(\underline{x}) + L_{22} \{ [\, p(w_2)\, p(\underline{x}/w_2)\,] / p(\underline{x}), \qquad (5)$$

Since $[1/p(\underline{x})]$ is common in the determination of both $R_1(\underline{x})$ and $R_2(\underline{x})$, it is dropped, and equations (4) and (5) become

$$R_1(\underline{x}) = L_{11}\, p(w_1)\, p(\underline{x}/w_1) + L_{21}\, p(w_2)\, p(\underline{x}/w_2), \qquad (6)$$

$$R_2(\underline{x}) = L_{12}\, p(w_1)\, p(\underline{x}/w_1) + L_{22}\, p(w_2)\, p(\underline{x}/w_2)\,, \qquad (7)$$

Now, $\underline{x}$ is to be assigned to the class where it has the minimum risk of being misclassified. Thus, the decision rule for minimum probability of misclassification of $\underline{x}$ becomes

assign $\underline{x}$ to class w_1 if $R_1(\underline{x}) < R_2(\underline{x})$,

assign $\underline{x}$ to class w_2 if $R_1(\underline{x}) > R_2(\underline{x})$.

Substituting the probability density functions in the above equations, the decision rule then becomes

assign $\underline{x}$ to class w_1 if

$$\{L_{11}-L_{12})\, p(w_1) \exp[-1/2(\underline{x}-m_1)C_1^{-1}(\underline{x}-m_1)]/\,(2\ \pi)^{2/2}\,|C_1\ 1|^{1/2}\} +$$
$$\{L_{21}-L_{22})\, p(w_2) \exp[-1/2(\underline{x}-m_2)C_2^{-1}(\underline{x}-m_2)]/\,(2\ \pi)^{2/2}\,|C_2\ 1|^{1/2}\}$$
$$. \quad . \quad < 0 \qquad (8)$$

and assign $\underline{x}$ to class w_2 if

$$\{L_{11}-L_{12})\, p(w_1) \exp[-1/2(\underline{x}-m_1)C_1^{-1}(\underline{x}-m_1)]/(2\pi)^{2/2}|C_1|^{1/2}\} +$$
$$\{L_{21}-L_{22})\, p(w_2) \exp[-1/2(\underline{x}-m_2)C_2^{-1}(\underline{x}-m_2)]/(2\pi)^{2/2}|C_2|^{1/2}\}$$
$$. \quad . \quad > 0 \qquad (9)$$

NON-PARAMETRIC APPROACH

The deterministic (non-parametric) approach is based on mathematical classification rules that do not explicitly employ the statistical properties of pattern classes under consideration here.

Sing-Tze-Bow applies this non-parametric approach to two-dimensional data. (22) The general decision function separating the sample vectors into two pattern classes w_1 and w_2 is of the form

$$d(\underline{x}) = w_1\, f_1(\underline{x}) + w_2\, f_2(\underline{x}) + w_3 = 0.$$

where $w = (w_1,w_2,w_3)$ represents the weight vector, and

$x = (x_1,x_2,1)$ represents the augmented pattern vector.

The non-parametric approach of pattern classification can be determined by several classification methods: the minimum-distance, piecewise linear, nearest neighbor, or cluster analysis classification method.

Minimum Distance Classification Method

Tou discusses the minimum distance classification method using the concept of proximity of the patterns in euclidean space, as a measure of similarity between patterns. (21)

Considering two pattern classes w_1 and w_2, represented by prototypes z_1 and z_2, respectively, Tou defines the euclidean distance between the sample vector and the prototype as

$$D_i = \|\underline{x} - \underline{z}_i\| = [\ (\underline{x} - \underline{z}_i)'\ (\underline{x} - \underline{z}_i)\ ]^{1/2}.$$

The decision rule, then, is to assign $\underline{x}$ to the class w_i so that the distance

$D_i = \underline{x}'\underline{x} - 2\_[\underline{x}'\ \underline{z}_i\ - 1/2\ \underline{z}_i'\ \underline{z}_i]$ is a minimum.

Since $\underline{x}'\underline{x}$ is a constant for all pattern classes, the decision rule then changes in assigning $\underline{x}$ to class w_i, such that,

$D_i = \underline{x}'\ \underline{z}_i\ - 1/2\ \underline{z}_i'\ \underline{z}_i$ is a maximum.

Piecewise Linear Decision Boundaries

When the pattern classes have different prototypes in different regions of the feature space, piecewise linear discriminant functions may be used as an approximation of the non-linear quadratic boundaries. They are linear over subregions of the feature space, and the perpendicular bisector of a pair of prototypes in two different pattern classes in a feature space form the piecewise linear decision function between the two pattern classes over that feature space.

Nearest Neighbor Classification Method

The nearest neighbor (NN) approach classifies the sample vector into the class to which its nearest neighbor belongs. Like the minimum distance classification method, the NN approach also assumes that the distance between the sample vectors is an indication of the similarity between them. The K-nearest neighbor goes a step further and classifies the sample vector by sampling its K nearest neighbors.

Cluster Analysis

The cluster analysis approach includes the concept that patterns belonging to the same class show a greater degree of natural association than do sample patterns belonging to different pattern classes. This approach is especially useful when the number and nature of the pattern classes is unknown, or when the labeled samples for the pattern classes are not available.

Sing-Tze-Bow groups clustering algorithms (depending on whether or not a criterion function is used in the clustering process), into indirect/optimization and direct/constructive algorithms, respectively. (22) He also classifies them into agglomerative/bottom-up and divisive/top-down approaches. The approach used depends on whether the isolated patterns are coalesced or the individual pattern classes are subdivided according to some optimizing function, respectively. Many algorithms use a combination of these approaches.

Sing-Tze-Bow considers the minimization of the sum of squared distance. Considering two pattern classes, the first two sample patterns, $\underline{x}_1$ and $\underline{x}_2$, are assigned as cluster centers of two different pattern classes, w_1 and w_2. Based on euclidean distance, every other sample pattern is assigned, to the class whose cluster center it is nearest. The cluster centers are updated after a predetermined number of additions. After all the sample patterns have been assigned, the new cluster centers are computed, and the classification is performed once again. This process is continued until the updated cluster patterns stay unchanged. The patterns then belong to the classes they were assigned to in the final iteration, with the cluster centers representing prototypes of the two pattern classes.

TRAINING OF PATTERN CLASSIFIERS

The concept of training, in which the formulation of parameters of the decision function is extended into the classification stage, can be applied to both the parametric and the non-parametric approaches described above.

Sing-Tze-Bow designs the classifier by assuming the best possible values of the parameters/weights from the labeled patterns, and then modifying them with the information from the most recent data. (22) Thus, in the parametric approach the mean and the covariance functions are updated, and in the non-parametric parametric approach the weight vector is modified, after a predetermined number of sample patterns are added to the pattern class.

This training of the decision function only takes place during the design and updating processes. Once the algorithm yields acceptable results, the training is discontinued, and the algorithm is applied toward performing the task of assigning patterns to their respective classes. Training is used only when enough data are not available or when the cost of data collection is high.

APPENDIX C

FORTRAN PROGRAM FOR TEST OF MODEL ON TSMC DATA

```
VARIABLE IDENTIFICATION

COUNT1     - COUNTS NUMBER OF ROWS
COLUMN     - COLUMN POINTER
ROWNUM     - NUMBER OF ROWS IN INPUT FILE
I,J,K      - LOOP COUNTERS
NEG        - NUMBER OF NEGATIVES IN A COLUMN
COUNT2     -  -1.0 TO +1.0 COUNTER
COUNT3     -  -1.0 TO +1.0 COUNTER
K4         - SOLUTION TO MAIN EQUATION
M1         - MATRIX FORMED FROM INPUT FILE
M2         - [2X1] MATRIX INPUT BY USER
M3         - [2X1] MATRIX INPUT BY USER
P1,P2      - VALUES FOR MAIN EQUATION INPUT BY USER
MTEMP      - TRANSPOSE MATRIX OF M1- (M2 OR M3)
MTEMP1     - MATRIX OF M1- (M2 OR M3)
TEMP1      - 1st ELEMENT OF M1 MINUS 1st ELEMENT OF M2 OR M3
TEMP2      - 2nd ELEMENT OF M1 MINUS 2nd ELEMENT OF M2 OR M3
DNEW       -  -0.5 TIMES MATRIX CALCULATIONS RESULT (1st SET)
ENEW       -  -0.5 TIMES MATRIX CALCULATIONS RESULT (2nd SET)
C1INV      - [2X2] INVERSE MATRIX INPUT BY USER
C2INV      - [2X2] INVERSE MATRIX INPUT BY USER
C1DET      - DETERMINANT OF MATRIX INPUT BY USER
C2DET      - DETERMINANT OF MATRIX INPUT BY USER
C          - RESULTANT MATRIX FROM MATRIX MULTIPLY CALL
D          - RESULTANT MATRIX FROM MATRIX MULTIPLY CALL
E          - RESULTANT MATRIX FROM MATRIX MULTIPLY CALL
```

```
C     VARIABLE DECLARATION

      INTEGER   COUNT1, COLUMN, ROWNUM, I, J, M, NEG(441)

      REAL      COUNT2, COUNT3, K4, NEGPER, POSPER,
     \          M1(120,2), M2(2,1), M3(2,1), P1, P2,
     \          MTEMP(1,2), MTEMP1(2,1),
     \          TEMP1, TEMP2, DNEW, ENEW,
     \          C1INV(2,2), C2INV(2,2), C1DET, C2DET,
     \          C(1,2), D(1,1), E(1,1), RESULTS(120,441)

      CHARACTER*12 FILENAME

      EXTERNAL MRRRR
```

```
C    PROMPT FOR INPUT VARIABLES

        PRINT *, 'ENTER INPUT FILE NAME'
        READ *, FILENAME

        PRINT *, 'ENTER NUMBER OF ROWS IN FILE'
        READ *, ROWNUM

        PRINT *, 'ENTER M2 MATRIX'
        READ(*,800) M2(1,1), M2(2,1)
        PRINT *, M2(1,1), M2(2,1)

        PRINT *, 'ENTER M3 MATRIX'
        READ(*,800) M3(1,1), M3(2,1)
        PRINT *, M3(1,1), M3(2,1)

        PRINT *, 'ENTER P1 & P2'
        READ *, P1,P2
        PRINT*, P1,P2

        PRINT *, 'ENTER C1 INVERSE'
        READ(*,810) C1INV(1,1), C1INV(1,2), C1INV(2,1), C1INV(2,2)
        PRINT *, C1INV(1,1), C1INV(1,2), C1INV(2,1), C1INV(2,2)

        PRINT *, 'ENTER C2 INVERSE'
        READ(*,810) C2INV(1,1), C2INV(1,2), C2INV(2,1), C2INV(2,2)
        PRINT*, C2INV(1,1), C2INV(1,2), C2INV(2,1), C2INV(2,2)

        PRINT *, 'ENTER C1 DET & C2 DET'
        READ(*,820) C1DET, C2DET
        PRINT*, C1DET, C2DET


        OPEN(1,  FILE    =FILENAME,ACCESS='SEQUENTIAL',
     \           FORM    ='FORMATTED',STATUS='OLD')


C    READS THE INPUT FILE

        DO 30 I=1,ROWNUM

        READ(1,900,END=90) M1(I,1), M1(I,2)

        PRINT *, M1(I,1), M1(I,2)

 30  CONTINUE

C    INITIALIZE COUNTER VARIABLES

 90 COLUMN = 1

100  COUNT1 = 1
        COUNT2 = -1.00
        COUNT3 = -1.00
```

```
      WRITE(6,850)

150   NEG(COLUMN) = 0

      DO 200 M=1,ROWNUM

C     MATRIX MANIPULATIONS

C     MATRIX SUBTRACT

      TEMP1 = M1(M,1) - M2(1,1)
      TEMP2 = M1(M,2) - M2(2,1)

C     MATRIX TRANSPOSE

      MTEMP(1,1) = TEMP1
      MTEMP(1,2) = TEMP2

C     MATRIX NORMAL

      MTEMP1(1,1) = TEMP1
      MTEMP1(2,1) = TEMP2

C     IMSL MATRIX LIBRARY CALLS

      CALL MRRRR(1,2,MTEMP,1,2,2,C1INV,2,1,2,C,1)

      CALL MRRRR(1,2,C,1,2,1,MTEMP1,2,1,1,D,1)

      DNEW = D(1,1)
      DNEW = DNEW * -0.5

C     MATRIX SUBTRACT

      TEMP1 = M1(M,1) - M3(1,1)
      TEMP2 = M1(M,2) - M3(2,1)

C     MATRIX TRANSPOSE

      MTEMP(1,1) = TEMP1
      MTEMP(1,2) = TEMP2

C     MATRIX NORMAL
      MTEMP1(1,1) = TEMP1
      MTEMP1(2,1) = TEMP2

      CALL MRRRR(1,2,MTEMP,1,2,2,C2INV,2,1,2,C,1)

      CALL MRRRR(1,2,C,1,2,1,MTEMP1,2,1,1,E,1)

      ENEW = E(1,1)
      ENEW = ENEW * -0.5

C     MAIN EQUATION
```

```
      K4 =  ((COUNT2/(6.283*C1DET)) * EXP(DNEW)*P1) +
     \       ((COUNT3/(6.283*C2DET)) * EXP(ENEW)*P2)

      RESULTS(ROWNUM,COLUMN) = K4

      IF (K4.LT.0) THEN
        NEG(COLUMN) = NEG(COLUMN) + 1
      END IF

C       COUNTER TESTS

      IF (COUNT1.LT.ROWNUM) THEN
        COUNT1 = COUNT1 + 1
        GOTO 200

      ELSE IF (COUNT3.LT.1.0) THEN
        COUNT1 = 1
        COUNT3 = COUNT3 + 0.10
        GOTO 175

      ELSE IF (COUNT2.LE.1.0) THEN
        COUNT1 = 1
        COUNT2 = COUNT2 + 0.10
        COUNT3 = -1.00
        GOTO 175

      ELSE
        GOTO 200

      END IF

 175  NEGPER = (NEG(COLUMN)/REAL(ROWNUM)) * 100
      POSPER = 100 - NEGPER

      WRITE(6,875) COLUMN, COUNT2, COUNT3, NEGPER, POSPER
      COLUMN = COLUMN + 1
      GOTO 150

 200  CONTINUE

 800  FORMAT(F6.3,1X,F6.3)
 810  FORMAT(F6.4,1X,F6.4,1X,F6.4,1X,F6.4)
 820  FORMAT(F6.2,1X,F6.2)

 850  FORMAT('COLUMN',6X,'K2',6X,'K3',6X,'NEG(%)',6X,'POS(%)')
 875  FORMAT(2X,I3,7X,F4.1,7X,F4.1,7X,F5.1,7X,F5.1)

 900  FORMAT(F4.0,3X,F5.2)

      CLOSE(1)
      END
```

APPENDIX D

RESULTS OF FORTRAN PROGRAM TEST ON TSMC DATA

LOSS FUNCTIONS		2DAY FALSE		3DAY FALSE		4DAY FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve
-1.0	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-1.0	0.1	0.0	91.7	0.0	100.0	0.0	100.0
-1.0	0.2	1.8	85.4	1.6	93.6	0.0	96.6
-1.0	0.3	2.6	77.1	4.9	87.2	1.0	96.6
-1.0	0.4	3.5	70.8	6.6	85.1	1.0	91.4
-1.0	0.5	3.5	68.8	8.2	76.6	1.0	89.7
-1.0	0.6	3.5	66.7	11.5	70.2	1.9	79.3
-1.0	0.7	6.1	60.4	11.5	66.0	2.9	69.0
-1.0	0.8	7.0	58.3	13.1	59.6	4.8	65.5
-1.0	0.9	7.9	54.2	13.1	53.2	5.8	63.8

-1.0	1.0	9.6	50.0	14.8	46.8	7.7	58.6
-0.9	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.9	0.1	0.0	91.7	0.0	100.0	0.0	100.0
-0.9	0.2	2.6	85.4	1.6	89.4	0.0	96.6
-0.9	0.3	3.5	72.9	4.9	87.2	1.0	94.8
-0.9	0.4	3.5	68.8	8.2	78.7	1.0	89.7
-0.9	0.5	3.5	66.7	9.8	72.3	1.0	82.8
-0.9	0.6	6.1	62.5	11.5	68.1	2.9	75.9
-0.9	0.7	6.1	58.3	13.1	61.7	4.8	69.0
-0.9	0.8	7.9	56.3	13.1	53.2	5.8	63.8

LOSS		2DAY		3DAY		4DAY	
FUNCTIONS		FALSE		FALSE		FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve
-0.9	0.9	9.6	50.0	14.8	46.8	7.7	58.6
-0.9	1.0	11.4	47.9	21.3	44.7	10.6	50.0
-0.8	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.8	0.1	0.0	91.7	0.0	97.9	0.0	100.0
-0.8	0.2	2.6	85.4	1.6	89.4	0.0	96.6
-0.8	0.3	3.5	70.8	6.6	87.2	1.0	93.1
-0.8	0.4	3.5	68.8	8.2	76.6	1.0	89.7
-0.8	0.5	4.4	62.5	11.5	68.1	1.9	75.9
-0.8	0.6	6.1	58.3	13.1	63.8	4.8	69.0
-0.8	0.7	7.9	58.3	13.1	53.2	5.8	63.8
-0.8	0.8	9.6	50.0	14.8	46.8	7.7	58.6
-0.8	0.9	12.3	47.9	21.3	44.7	10.6	50.0
-0.8	1.0	14.0	41.7	24.6	42.6	15.4	48.3
-0.7	-1.0	0.0	100.0	0.0	100.0	0.0	100.0

-0.7	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.7	0.1	0.9	89.6	0.0	97.9	0.0	96.6
-0.7	0.2	2.6	77.1	4.9	87.2	1.0	96.6
-0.7	0.3	3.5	68.8	6.6	83.0	1.0	91.4
-0.7	0.4	3.5	66.7	9.8	70.2	1.0	82.8
-0.7	0.5	6.1	60.4	11.5	66.0	3.8	69.0
-0.7	0.6	7.0	58.3	13.1	59.6	4.8	63.8
-0.7	0.7	9.6	50.0	14.8	46.8	7.7	58.6
-0.7	0.8	12.3	45.8	21.3	44.7	11.5	50.0
-0.7	0.9	14.0	41.7	24.6	42.6	15.4	46.6
-0.7	1.0	15.8	39.6	31.1	38.3	20.2	39.7
-0.6	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	-0.9	0.0	100.0	0.0	100.0	0.0	100.0

LOSS FUNCTIONS		2DAY FALSE		3DAY FALSE		4DAY FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve-1.0
-0.6	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.6	0.1	1.8	85.4	0.0	95.7	0.0	96.6
-0.6	0.2	3.5	72.9	4.9	87.2	1.0	94.8
-0.6	0.3	3.5	68.8	8.2	76.6	1.0	89.7
-0.6	0.4	6.1	62.5	11.5	68.1	2.9	75.9
-0.6	0.5	7.0	58.3	13.1	59.6	4.8	63.8
-0.6	0.6	9.6	50.0	14.8	46.8	7.7	58.6
-0.6	0.7	13.2	45.8	23.0	44.7	12.5	50.0
-0.6	0.8	14.0	41.7	27.9	40.4	18.3	43.1
-0.6	0.9	17.5	37.5	32.8	38.3	21.2	34.5
-0.6	1.0	20.2	37.5	37.7	36.2	26.0	29.3
-0.5	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	-0.6	0.0	100.0	0.0	100.0	0.0	100.0

-0.5	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.5	0.1	1.8	85.4	1.6	93.6	0.0	96.6
-0.5	0.2	3.5	70.8	6.6	85.1	1.0	91.4
-0.5	0.3	3.5	66.7	11.5	70.2	1.9	79.3
-0.5	0.4	7.0	58.3	13.1	59.6	4.8	65.5
-0.5	0.5	9.6	50.0	14.8	46.8	7.7	58.6
-0.5	0.6	13.2	43.8	23.0	42.6	13.5	50.0
-0.5	0.7	15.8	41.7	27.9	38.3	20.2	41.4
-0.5	0.8	18.4	37.5	34.4	36.2	24.0	31.0
-0.5	0.9	23.7	29.2	41.0	34.0	29.8	29.3
-0.5	1.0	25.4	22.9	47.5	29.8	34.6	25.9
-0.4	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	-0.5	0.0	100.0	0.0	100.0	0.0	100.0

LOSS		2DAY		3DAY		4DAY	
FUNCTIONS		FALSE		FALSE		FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve
-0.4	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.4	0.1	2.6	85.4	1.6	89.4	0.0	96.6
-0.4	0.2	3.5	68.8	8.2	76.6	1.0	89.7
-0.4	0.3	6.1	58.3	13.1	63.8	4.8	69.0
-0.4	0.4	9.6	50.0	14.8	46.8	7.7	58.6
-0.4	0.5	14.0	41.7	24.6	42.6	15.4	48.3
-0.4	0.6	17.5	37.5	32.8	38.3	21.2	34.5
-0.4	0.7	23.7	35.4	39.3	34.0	27.9	29.3
-0.4	0.8	25.4	22.9	47.5	29.8	34.6	25.9
-0.4	0.9	32.5	20.8	50.8	21.3	38.5	25.9
-0.4	1.0	33.3	14.6	63.9	19.1	39.4	19.0
-0.3	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	-0.2	0.0	100.0	0.0	100.0	0.0	100.0

-0.3	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.3	0.1	3.5	72.9	4.9	87.2	1.0	94.8
-0.3	0.2	6.1	62.5	11.5	68.1	2.9	75.9
-0.3	0.3	9.6	50.0	14.8	46.8	7.7	58.6
-0.3	0.4	14.0	41.7	27.9	40.4	18.3	43.1
-0.3	0.5	20.2	37.5	37.7	36.2	26.0	29.3
-0.3	0.6	25.4	22.9	47.5	29.8	34.6	25.9
-0.3	0.7	33.3	18.8	54.1	19.1	39.4	24.1
-0.3	0.8	36.8	12.5	63.9	17.0	40.4	15.5
-0.3	0.9	39.5	8.3	68.9	17.0	41.3	12.1
-0.3	1.0	43.9	6.3	73.8	12.8	46.2	10.3
-0.2	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	-0.1	0.0	100.0	0.0	100.0	0.0	100.0

LOSS FUNCTIONS		2DAY FALSE		3DAY FALSE		4DAY FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve
-0.2	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.2	0.1	3.5	68.8	8.2	76.6	1.0	89.7
-0.2	0.2	9.6	50.0	14.8	46.8	7.7	58.6
-0.2	0.3	17.5	37.5	32.8	38.3	21.2	34.5
-0.2	0.4	25.4	22.9	47.5	29.8	34.6	25.9
-0.2	0.5	33.3	14.6	63.9	19.1	39.4	19.0
-0.2	0.6	39.5	8.3	68.9	17.0	41.3	12.1
-0.2	0.7	45.6	6.3	73.8	8.5	46.2	8.6
-0.2	0.8	52.6	4.2	78.7	4.3	51.9	6.9
-0.2	0.9	57.9	2.1	82.0	4.3	54.8	5.2
-0.2	1.0	59.6	2.1	82.0	4.3	58.7	3.4
-0.1	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	0.0	0.0	100.0	0.0	100.0	0.0	100.0
-0.1	0.1	9.6	50.0	14.8	46.8	7.7	58.6
-0.1	0.2	25.4	22.9	47.5	29.8	34.6	25.9

-0.1	0.3	39.5	8.3	68.9	17.0	41.3	12.1
-0.1	0.4	52.6	4.2	78.7	4.3	51.9	6.9
-0.1	0.5	59.6	2.1	82.0	4.3	58.7	3.4
-0.1	0.6	63.2	2.1	82.0	2.1	62.5	3.4
-0.1	0.7	64.9	2.1	86.9	2.1	64.4	3.4
-0.1	0.8	64.9	2.1	86.9	2.1	67.3	0.0
-0.1	0.9	65.8	0.0	86.9	2.1	70.2	0.0
-0.1	1.0	69.3	0.0	86.9	2.1	71.2	0.0
0.0	-1.0	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.9	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.8	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.7	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.6	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.5	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.4	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.3	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.2	0.0	100.0	0.0	100.0	0.0	100.0
0.0	-0.1	0.0	100.0	0.0	100.0	0.0	100.0
0.0	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.0	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.0	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.0	0.3	100.0	0.0	100.0	0.0	100.0	0.0

LOSS FUNCTIONS		2DAY FALSE		3DAY FALSE		4DAY FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve
0.0	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.0	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.0	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.0	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.0	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.0	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.0	1.0	100.0	0.0	100.0	0.0	100.0	0.0
0.1	-1.0	30.7	100.0	13.1	97.9	28.8	100.0
0.1	-0.9	34.2	100.0	13.1	97.9	29.8	100.0
0.1	-0.8	35.1	97.9	13.1	97.9	32.7	100.0
0.1	-0.7	35.1	97.9	13.1	97.9	35.6	96.6
0.1	-0.6	36.8	97.9	18.0	97.9	37.5	96.6
0.1	-0.5	40.4	97.9	18.0	95.7	41.3	96.6
0.1	-0.4	47.4	95.8	21.3	95.7	48.1	93.1
0.1	-0.3	60.5	91.7	31.1	83.0	58.7	87.9
0.1	-0.2	74.6	77.1	52.5	70.2	65.4	74.1
0.1	-0.1	90.4	50.0	85.2	53.2	92.3	41.4
0.1	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.1	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.1	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.1	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.1	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.1	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.1	0.6	100.0	0.0	100.0	0.0	100.0	0.0

0.1	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.1	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.1	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.1	1.0	100.0	0.0	100.0	0.0	100.0	0.0
0.2	-1.0	40.4	97.9	18.0	95.7	41.3	96.6
0.2	-0.9	42.1	97.9	18.0	95.7	45.2	94.8
0.2	-0.8	47.4	95.8	21.3	95.7	48.1	93.1
0.2	-0.7	54.4	93.8	26.2	91.5	53.8	91.4
0.2	-0.6	60.5	91.7	31.1	83.0	58.7	87.9
0.2	-0.5	66.7	85.4	36.1	80.9	60.6	81.0
0.2	-0.4	74.6	77.1	52.5	70.2	65.4	74.1
0.2	-0.3	82.5	62.5	67.2	61.7	78.8	65.5
0.2	-0.2	90.4	50.0	85.2	53.2	92.3	41.4
0.2	-0.1	96.5	31.3	91.8	23.4	99.0	10.3
0.2	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.2	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.2	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.2	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.2	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.2	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.2	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.2	0.7	100.0	0.0	100.0	0.0	100.0	0.0

LOSS FUNCTIONS		2DAY FALSE		3DAY FALSE		4DAY FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve-1.0
0.2	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.2	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.2	1.0	100.0	0.0	100.0	0.0	100.0	0.0
0.3	-1.0	56.1	93.8	26.2	87.2	53.8	89.7
0.3	-0.9	60.5	91.7	31.1	83.0	58.7	87.9
0.3	-0.8	63.2	87.5	36.1	83.0	59.6	84.5
0.3	-0.7	66.7	81.3	45.9	80.9	60.6	75.9
0.3	-0.6	74.6	77.1	52.5	70.2	65.4	74.1
0.3	-0.5	79.8	62.5	62.3	63.8	74.0	70.7
0.3	-0.4	86.0	58.3	72.1	59.6	81.7	56.9
0.3	-0.3	90.4	50.0	85.2	53.2	92.3	41.4
0.3	-0.2	93.9	37.5	88.5	31.9	97.1	24.1
0.3	-0.1	96.5	27.1	95.1	12.8	99.0	5.2
0.3	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.3	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.3	1.0	100.0	0.0	100.0	0.0	100.0	0.0

0.4	-1.0	66.7	85.4	36.1	80.9	60.6	81.0
0.4	-0.9	67.5	79.2	49.2	78.7	61.5	74.1
0.4	-0.8	74.6	77.1	52.5	70.2	65.4	74.1
0.4	-0.7	76.3	64.6	60.7	66.0	72.1	70.7
0.4	-0.6	82.5	62.5	67.2	61.7	78.8	65.5
0.4	-0.5	86.0	58.3	75.4	57.4	84.6	51.7
0.4	-0.4	90.4	50.0	85.2	53.2	92.3	41.4
0.4	-0.3	93.9	41.7	86.9	36.2	95.2	31.0
0.4	-0.2	96.5	31.3	91.8	23.4	99.0	10.3
0.4	-0.1	97.4	14.6	98.4	10.6	100.0	3.4
0.4	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.4	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.4	1.0	100.0	0.0	100.0	0.0	100.0	0.0
0.5	-1.0	74.6	77.1	52.5	70.2	65.4	74.1

LOSS FUNCTIONS		2DAY FALSE		3DAY FALSE		4DAY FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve-1.0
0.5	-0.9	76.3	70.8	59.0	66.0	70.2	70.7
0.5	-0.8	81.6	62.5	65.6	63.8	76.0	69.0
0.5	-0.7	84.2	58.3	72.1	61.7	79.8	58.6
0.5	-0.6	86.8	56.3	77.0	57.4	86.5	50.0
0.5	-0.5	90.4	50.0	85.2	53.2	92.3	41.4
0.5	-0.4	93.0	41.7	86.9	40.4	95.2	34.5
0.5	-0.3	96.5	33.3	88.5	29.8	98.1	20.7
0.5	-0.2	96.5	29.2	93.4	14.9	99.0	8.6
0.5	-0.1	98.2	14.6	98.4	6.4	100.0	3.4
0.5	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.5	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.5	1.0	100.0	0.0	100.0	0.0	100.0	0.0
0.6	-1.0	79.8	62.5	62.3	63.8	74.0	70.7
0.6	-0.9	82.5	62.5	67.2	61.7	78.8	65.5
0.6	-0.8	86.0	58.3	72.1	59.6	81.7	56.9
0.6	-0.7	86.8	54.2	77.0	55.3	87.5	50.0

0.6	-0.6	90.4	50.0	85.2	53.2	92.3	41.4
0.6	-0.5	93.0	41.7	86.9	40.4	95.2	36.2
0.6	-0.4	93.9	37.5	88.5	31.9	97.1	24.1
0.6	-0.3	96.5	31.3	91.8	23.4	99.0	10.3
0.6	-0.2	96.5	27.1	95.1	12.8	99.0	5.2
0.6	-0.1	98.2	14.6	100.0	4.3	100.0	3.4
0.6	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.6	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.6	1.0	100.0	0.0	100.0	0.0	100.0	0.0
0.7	-1.0	84.2	60.4	68.9	61.7	79.8	60.3
0.7	-0.9	86.0	58.3	75.4	57.4	84.6	53.4
0.7	-0.8	87.7	54.2	78.7	55.3	88.5	50.0
0.7	-0.7	90.4	50.0	85.2	53.2	92.3	41.4
0.7	-0.6	93.0	41.7	86.9	40.4	95.2	36.2

LOSS FUNCTIONS		2DAY FALSE		3DAY FALSE		4DAY FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve-1.0
0.7	-0.5	93.9	39.6	88.5	34.0	96.2	31.0
0.7	-0.4	96.5	33.3	90.2	29.8	99.0	17.2
0.7	-0.3	96.5	31.3	93.4	17.0	99.0	8.6
0.7	-0.2	97.4	22.9	95.1	12.8	99.0	3.4
0.7	-0.1	99.1	10.4	100.0	2.1	100.0	3.4
0.7	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.7	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.7	1.0	100.0	0.0	100.0	0.0	100.0	0.0
0.8	-1.0	86.0	58.3	75.4	57.4	84.6	51.7
0.8	-0.9	87.7	52.1	78.7	55.3	89.4	50.0
0.8	-0.8	90.4	50.0	85.2	53.2	92.3	41.4
0.8	-0.7	92.1	41.7	86.9	46.8	94.2	36.2
0.8	-0.6	93.9	41.7	86.9	36.2	95.2	31.0
0.8	-0.5	95.6	37.5	88.5	31.9	98.1	24.1
0.8	-0.4	96.5	31.3	91.8	23.4	99.0	10.3
0.8	-0.3	96.5	29.2	93.4	12.8	99.0	6.9

0.8	-0.2	97.4	14.6	98.4	10.6	100.0	3.4
0.8	-0.1	100.0	8.3	100.0	2.1	100.0	0.0
0.8	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.8	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.8	1.0	100.0	0.0	100.0	0.0	100.0	0.0
0.9	-1.0	88.6	52.1	78.7	55.3	89.4	50.0
0.9	-0.9	90.4	50.0	85.2	53.2	92.3	41.4
0.9	-0.8	92.1	43.8	86.9	46.8	94.2	36.2
0.9	-0.7	93.9	41.7	86.9	38.3	95.2	31.0
0.9	-0.6	93.9	37.5	88.5	31.9	97.1	24.1
0.9	-0.5	96.5	33.3	90.2	27.7	99.0	17.2
0.9	-0.4	96.5	31.3	91.8	21.3	99.0	10.3
0.9	-0.3	96.5	27.1	95.1	12.8	99.0	5.2
0.9	-0.2	97.4	14.6	98.4	10.6	100.0	3.4

LOSS FUNCTIONS		2DAY FALSE		3DAY FALSE		4DAY FALSE	
k2	k3	+ve	-ve	+VE	-ve	+VE	-ve-1.0
0.9	-0.1	100.0	8.3	100.0	0.0	100.0	0.0
0.9	0.0	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.1	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.2	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.3	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.4	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.5	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.6	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.7	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.8	100.0	0.0	100.0	0.0	100.0	0.0
0.9	0.9	100.0	0.0	100.0	0.0	100.0	0.0
0.9	1.0	100.0	0.0	100.0	0.0	100.0	0.0
1.0	-1.0	90.4	50.0	85.2	53.2	92.3	41.4
1.0	-0.9	92.1	45.8	86.9	46.8	94.2	36.2
1.0	-0.8	93.0	41.7	86.9	40.4	95.2	34.5
1.0	-0.7	93.9	39.6	88.5	34.0	97.1	31.0
1.0	-0.6	96.5	33.3	88.5	29.8	98.1	20.7
1.0	-0.5	96.5	31.3	91.8	23.4	99.0	10.3
1.0	-0.4	96.5	29.2	93.4	14.9	99.0	8.6
1.0	-0.3	97.4	22.9	95.1	12.8	99.0	3.4
1.0	-0.2	98.2	14.6	98.4	6.4	100.0	3.4
1.0	-0.1	100.0	8.3	100.0	0.0	100.0	0.0
1.0	0.0	100.0	0.0	100.0	0.0	100.0	0.0
1.0	0.1	100.0	0.0	100.0	0.0	100.0	0.0

1.0	0.2	100.0	0.0	100.0	0.0	100.0	0.0
1.0	0.3	100.0	0.0	100.0	0.0	100.0	0.0
1.0	0.4	100.0	0.0	100.0	0.0	100.0	0.0
1.0	0.5	100.0	0.0	100.0	0.0	100.0	0.0
1.0	0.6	100.0	0.0	100.0	0.0	100.0	0.0
1.0	0.7	100.0	0.0	100.0	0.0	100.0	0.0
1.0	0.8	100.0	0.0	100.0	0.0	100.0	0.0
1.0	0.9	100.0	0.0	100.0	0.0	100.0	0.0
1.0	1.0	100.0	0.0	100.0	0.0	100.0	0.0