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ANALYTICAL MODELING OF FOUNDATIONS FOR SEISMIC ANALYSIS OF BRIDGES

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16. ABSTRACT <p>The response of bridges when subjected to seismic excitation may be significantly influenced by the dynamic properties of their foundations. With current design practice, foundation elements are typically considered as elastic springs without consideration of material and radiation damping.</p> <p>The objectives of this research were to identify general foundation models that are suitable for modeling soil-structure interaction in seismic bridge analysis, to modify an existing nonlinear seismic bridge analysis computer program to include a new element capable of representing such models, and to conduct a parametric study to assess the effect of the increased energy dissipation mechanisms on the response of bridge substructures.</p> <p>For spread footing foundations, three different models were identified and applied to a typical two-column bridge bent. For pile foundations, four models were derived and applied to a five-column bent. The seismic response for each model was compared with conventional elastic and fixed-base models. Several soil stiffness values and earthquake records were considered for analysis. Maximum values of displacement, plastic hinge rotation, and cumulative plastic hinge rotations were noted and compared.</p> <p>It was concluded that the use of the foundation models can produce an important change in the bridge response when compared to that of the fixed-base model, depending on the frequency content of the earthquake and the stiffness of the soil. The effects of radiation damping were observed to be insignificant for foundations on stiff soil, but important for those on soft soil. In addition, the performance of the simpler damped foundation models was found to be quite similar to that of the more complex models.</p>			
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**ANALYTICAL MODELING OF FOUNDATIONS
FOR SEISMIC ANALYSIS OF BRIDGES**

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ANALYTICAL MODELING OF FOUNDATIONS FOR SEISMIC ANALYSIS OF BRIDGES

SUMMARY

The response of bridges when subjected to seismic excitation may be significantly influenced by a number of mechanisms that are not currently incorporated into typical analysis methods. In particular, the dynamic interaction of the bridge foundations with the founding soil is the subject of the research presented in this report.

The objectives of the research were to determine foundation models suitable for use with the seismic analysis of highway bridges, to modify the nonlinear seismic bridge analysis computer program, NEABS, to include a new element capable of representing the foundation models, and to conduct a parametric study to assess the performance of the foundation models relative to conventional fixed-base analysis in terms of their effects on the seismic response of two bridge bents.

Four models applicable to modeling spread footing foundations and four models applicable to modeling pile foundations were identified. A new element to represent these foundation models and other nonlinear soil reaction mechanisms, the "Discrete Foundation" element, was developed and implemented within NEABS.

Two bents from Washington bridges were selected to be the subjects of the parametric study, one supported on the spread footing models, the other on pile foundation models. Various seismic records were applied to the bents in the transverse direction and the response of each bent was studied.

It was concluded that inclusion of the foundation models can produce a

significant difference in the predicted bridge response, depending on the frequency content of the earthquake and the stiffness of the soil. In some cases, the fixed-base bent models resulted in column demands that were unconservative compared with those obtained using the foundation models. Relatively simple models were found to be adequate for modeling both spread footing and pile foundations. NEABS, equipped with the Discrete Foundation element, is capable of being used to further explore the effects of soil-foundation interaction on the seismic response of bridges.

CONCLUSIONS AND RECOMMENDATIONS

From the parametric study that was conducted, the following conclusions may be drawn:

1. The enhancement of a fixed-base model to include the foundation flexibility has a dramatic influence on the column demands for strong earthquakes. This seems to be a result of variations in the natural frequencies of the system and the actual effect depends on the frequency content of the earthquake. For the spread footing models and the El Centro records, increased column demands were noted for the flexible foundation while, for the Olympia records, the intermediate foundation was critical. Similar results were noted for the pile foundation models. Thus, no conclusion can be drawn regarding whether one foundation is more critical than another. However, the results indicated that a fixed-base model could easily underpredict column demands for an earthquake analysis. One should note that, in order to evaluate the effect of foundation properties on bridge response in a consistent manner, no attempt was made to alter the earthquake records on the basis of an

assumed soil layer. To include such effects, a separate analysis to obtain free field motion at the site must be performed.

2. The addition of concentrated dampers to model radiation damping had a significant effect only when the foundation was soft. As expected, the energy absorption of the dampers acted to reduce cumulative demand on the columns. Neglecting the radiation damping would probably have little effect on the response of similar structures when founded on soil of high or intermediate stiffness. For soil of low stiffness, however, the use of elastic foundations alone could lead to a somewhat conservative prediction of inelastic demand. If damping is added, the simple, three parameter model produced results that were in close agreement with those of the more complex models.
3. In the one test using a simplified secant pile head stiffness value chosen to approximate a bilinear pile head stiffness model, the predicted responses were in close agreement.
4. Although foundation models for spread footings are well developed, similar models for pile foundations are not. Several approaches were employed to obtain an equivalent pile head model for the Mercer Slough bridge, based upon test pile results. In addition, a Winkler pile foundation model was developed on the basis of soil properties. The latter model, which had a much more flexible effective lateral pile head stiffness value, predicted significantly higher column demands.

INTRODUCTION

RESEARCH OBJECTIVES

The objectives of this research were to review current techniques for

modeling soil-structure interaction in seismic structural analysis, to develop a computational tool with which these techniques may be implemented, and to evaluate the effect of the foundation properties on the seismic response of bridges.

To achieve the objectives, the following tasks were accomplished:

1. A literature review was conducted and the findings were summarized.
2. The computer program, NEABS (Nonlinear Earthquake Analysis of Bridge Systems) (1), is a public-domain dynamic bridge analysis program that is capable of modeling nonlinearities. It was modified to include discrete dampers and hysteretic springs for foundation modeling.
3. The modified version of NEABS was used to evaluate the effect of various foundation models and soil stiffness values on the seismic response of two typical bridge bents founded on spread footing and pile foundations, respectively.

BACKGROUND

The response of bridges when subjected to seismic excitation may be significantly influenced by the dynamic characteristics of the foundation (2, 3, 4). For example, the interaction of the bridge superstructure with the abutments has been the cause of significant damage in past earthquakes (4, 5). Although damage to other types of foundation elements, such as spread footings and piles, has been shown to be minimal, their performance can have an important effect on the structural behavior (6), especially when the founding soil is soft (7).

Although research has shown that a significant amount of seismic energy is dissipated through the material and radiation damping associated with bridge supports and surrounding soil (8), these soil-structure interaction effects are not considered in detail in current design practice (9) and little emphasis has been

placed upon studying the role of foundations in the seismic analysis of bridges (4, 10). Current design guidance is simplistic in that the foundation elements are considered as linear springs (4, 11). The effect of gaps and the material nonlinearity of the soil at abutments are approximated by manually varying the spring constants such that the soil strength is not exceeded. However, important additional nonlinearities at abutments result from the force developed in the abutment key (3) and the energy loss due to impact during expansion joint gap closure (12). Barenberg and Foutch (13) have reported that the elastic method is unconservative for abutments.

The influence of foundations is typically included through the use of translational and rotational springs. However, nonlinearities can arise from several sources, such as inelastic soil behavior and connection details at pile caps (6). Other important considerations include soil stiffness degradation that occurs during cyclic loading (14), loss of strength in the soil due to liquefaction, the influence of pile group behavior, and radiation damping. In addition, hysteretic damping may be included intentionally through the use of base isolation techniques (15, 16, 17).

In order to properly represent hysteretic material damping and viscous radiation damping, Spyrakos (9) has recommended that a general, nonlinear, spring-damper model be used to represent the translational and rotational properties of piles, footings, and abutments. However, most computer software that is available for the dynamic analysis of bridges only has the capability to perform elastic analyses. Energy dissipation is included through proportional damping in which a damping coefficient is associated with certain modes of vibration. Concentrated dampers and hysteretic springs, such as those that would be required to accurately model foundations, are not available for this type of analysis. Therefore, there is a need, not only to develop bridge analysis

software that can properly model foundation effects, but also to evaluate the importance of these effects for various types of bridges and foundations.

Soil-Structure Interaction

Soil-structure interaction refers to the effect that the founding soil has on the dynamic response of a structure and, conversely, the effect of the structure on the soil motion. The influence on the structural response often includes an amplification of the translational motion, the introduction of a rocking component for an embedded foundation, an increase in the flexibility of the system, and the addition of damping from hysteretic action of the soil (hysteretic damping) and radiation of energy away from the structure in the form of outward-propagating soil waves (radiation damping).

Two general approaches are available for rationally incorporating soil-structure interaction effects into structural analysis (18). In the "direct method," the structure and a portion of the founding soil are both incorporated into a finite element mesh. This is the simplest approach conceptually, but a number of drawbacks, including the need for a large model, energy absorbing boundaries, and detailed soil properties, make its use prohibitive for all but the most extreme analysis demands.

A simpler, more efficient approach is the substructure method. Here, the structure and the soil are analyzed separately. A simplified model is constructed that can approximate the behavior of the soil at the foundation. This simplified model is then coupled with the structure at the supports, and the structure is analyzed.

The foundation model is typically composed of one or more springs or spring/damper combinations arranged in series and/or parallel for each degree-of-freedom. They are chosen on the basis of the assumed foundation behavior,

which is obtained either experimentally or analytically.

The most common analytical model is one in which the soil domain is considered to be a homogeneous, elastic half-space. The frequency domain solution for the dynamic response of a rigid disk on an elastic half-space has been derived and extended for footings of various other shapes and depths of embedment. One should note that the disk/half-space solution is frequency dependent. For nonlinear dynamic analysis, which must be conducted in the time domain, various foundation models have been proposed that reproduce the analytical foundation response for certain ranges of loading frequencies. Four such models, consisting of combinations of linear springs, masses, and dampers, are shown in Fig. 1. For a comprehensive review, one may refer to works by Wolf (19) and Richart (20).

PROCEDURES

The computer program, NEABS, was chosen as the means by which the methods that have been proposed to include the effects of soil-structure interaction in bridge analysis would be implemented. The source coding for NEABS is in the public domain, and it was obtained and modified. In order to apply the models mentioned above to represent the dynamic properties of bridge foundations, a new, discrete foundation element was added. Upon completion of a comprehensive set of numerical tests to verify that the new element performed properly, two sets of parametric studies were performed.

Description of NEABS

The computer program, NEABS, was originally developed by Tseng and

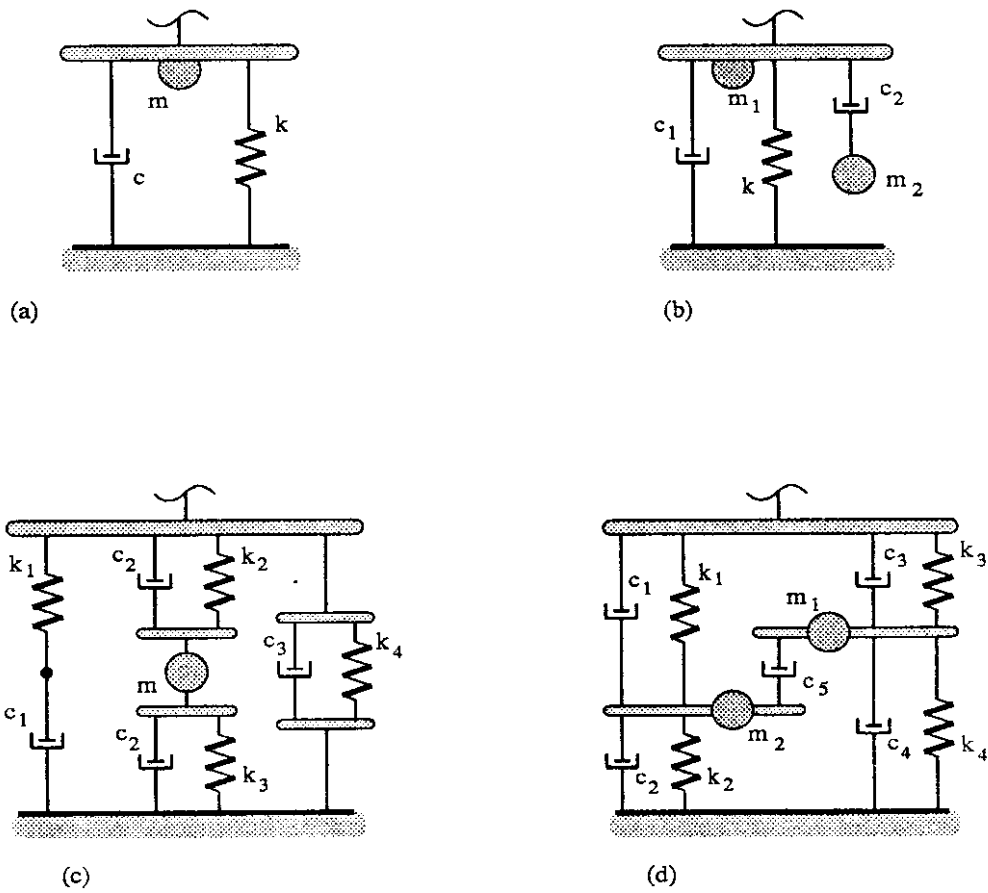


Figure 1. Discrete models of elastic half-space system: (a) three parameters; (b) five parameters; (c) nine parameters; (d) eleven parameters.

Penzien in 1973 (21) to study the seismic performance of long, multiple span bridges. Using the finite element method, NEABS idealizes a structure as a discrete system subject to nodal dynamic loadings and/or to prescribed support motions.

Four element types are available to model the structural members of a bridge. Deck sections and columns are modeled with a beam element that may be either elastic or elasto-plastic. In the case of the elasto-plastic beam, the ends are allowed to develop perfectly-plastic hinges. An elastic curved beam element is also available. Supports may be given elastic stiffnesses with a boundary spring element. A nonlinear expansion joint element is included which can model the opening and closing of the joint gap, the impact at gap closure, and elasto-plastic joint tie bars.

Lumped masses and mass moments of inertia may be assigned to structure nodes directly or specified through mass densities for both the straight and curved beam elements. Energy dissipation not included as yielding in the elasto-plastic elements is accomplished globally by using two-parameter Rayleigh viscous damping. With Rayleigh damping, the global damping matrix is assumed to be a linear combination of the global mass and stiffness matrices. For an elastic structure, this has the effect of assigning a unique damping ratio to each of the structure's modes of vibration.

Both static and dynamic nodal loadings may be prescribed, as well as support motion. Dynamic nodal loads and support motions are specified by supplying load and acceleration time histories, respectively.

The equations of motion are solved in the time domain to allow nonlinear response, using the Newmark method of direct time integration. Either constant or linear acceleration between time steps may be assumed. At each time step, the out-of-balance force vector from the previous time step is added to the

current applied equivalent force to minimize the accumulation of integration errors. In addition, the program will iterate and/or subdivide the time step used in the integration to ensure that the Euclidean norm of the out-of-balance force vector is within prescribed tolerances. Output consists of both the forces and displacements of the initial static response and time histories of the dynamic response. These time histories may consist of nodal displacements, nodal accelerations, member forces, and, for nonlinear elements, member nonlinear (i. e., plastic) displacements.

Discrete Foundation Element

As previously discussed, the foundation models for soil-structure interaction may range in complexity from simple linear spring supports to those employing a number of internal nodes, masses, dampers, and nonlinear springs. Accordingly, the Discrete Foundation (DF) element was formulated as a general purpose element to enhance the capabilities of NEABS. The element connects two nodes, which may actually occupy the same location, as in a simple foundation model. The DF element is a parallel combination of a spring and viscous damper. Thus, to model the more complex systems shown in Fig. 1, several DF elements and internal foundation nodes would be required. For example, model (d) in Fig. 1 would require five DF elements and two internal nodes. Note that the DF element used to model c5 would include damping and zero stiffness.

The model built with DF elements connects the base of the structure element (e. g., a column) and a fixed support. Separate properties are used for each of six local degrees-of-freedom, and there is no stiffness or damping coupling. Mass and mass moments of inertia may be lumped at each end node

(including internal foundation nodes) and each degree-of-freedom independently.

The DF element spring stiffness is bilinear to allow elasto-plastic behavior and hysteretic material damping. Kinematic strain hardening is incorporated as the default, but isotropic hardening or a combination of the two may be specified. A gap and stiffness degradation, as a function of deformation, may also be included.

The damping coefficients for each DF element may be specified separately for all degrees-of-freedom, allowing discrete dampers to be included in a foundation model. This damping is independent of the Rayleigh viscous damping in that the contribution of the DF element to the global mass and stiffness matrix is not considered when determining the Rayleigh contribution to the global damping matrix. Thus, the Rayleigh damping concept may be used for the bridge structure without affecting the concentrated dampers present in the foundation models. A complete description of the DF element may be found elsewhere (22).

PARAMETRIC STUDIES

A set of parametric studies was undertaken to investigate the effects of incorporating foundation models of varying complexity into bridge seismic analysis. The purpose was to compare various foundation models with each other and with a fixed support to evaluate their effect on the structural response of two typical bridge bents. One should note that, since the study results were not correlated with experimental response data, the study does not constitute a verification test of these models' accuracy. Rather, it is an exploration of the structural response effects of incorporating these models in seismic bridge

analysis. The foundation models are consistent with reasonable engineering assumptions, as previously discussed. Establishing consistency between these assumptions and actual behavior is beyond the scope of this report.

Spread Footing Foundation Study

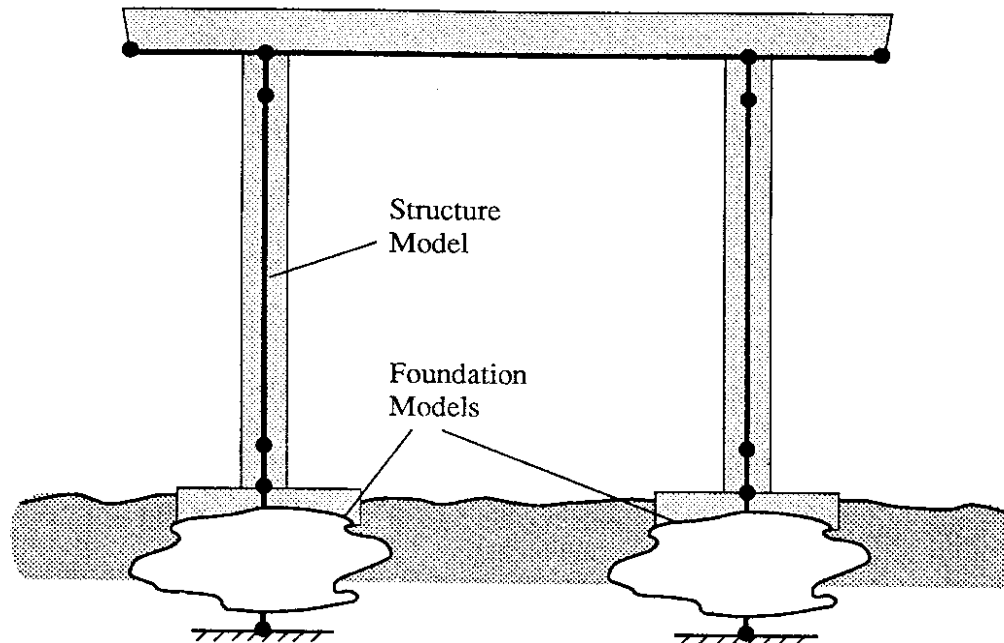
Description of the Model

An existing highway bridge was chosen to provide guidance for the development of the structural analysis model. A solitary bridge bent was modeled so that only the effects of the spread footing foundation, and not that of abutments, would be included.

The bent consisted of two 25 foot long, 36 inch diameter reinforced concrete columns on spread footings, supporting a cross beam, which supported the bridge superstructure. The 42 inch wide, 36 inch deep cross beam was cast monolithically with the diaphragm and deck and, because the resulting composite assembly was quite stiff in comparison with the columns, the cross beam was assumed to be rigid. The bent was assumed to support a dead load of approximately 472 kips. The centerlines of the two columns were 24 feet apart. Longitudinal reinforcing bars were spaced evenly around the cross-section perimeter and they extended into the crossbeam with no splice. The spread footing dimensions were 9.5 feet square in plan and 24 inches deep. A schematic of the model that was analyzed is shown in Fig. 2. Specific details of the bent are given elsewhere ([22](#), [23](#)).

The bent was modeled with nine beam elements and it was supported on the various foundation models, composed of DF elements. The foundation properties were independently assigned to the three planar degrees-of-

(a) Structure Model



(b) Support Models

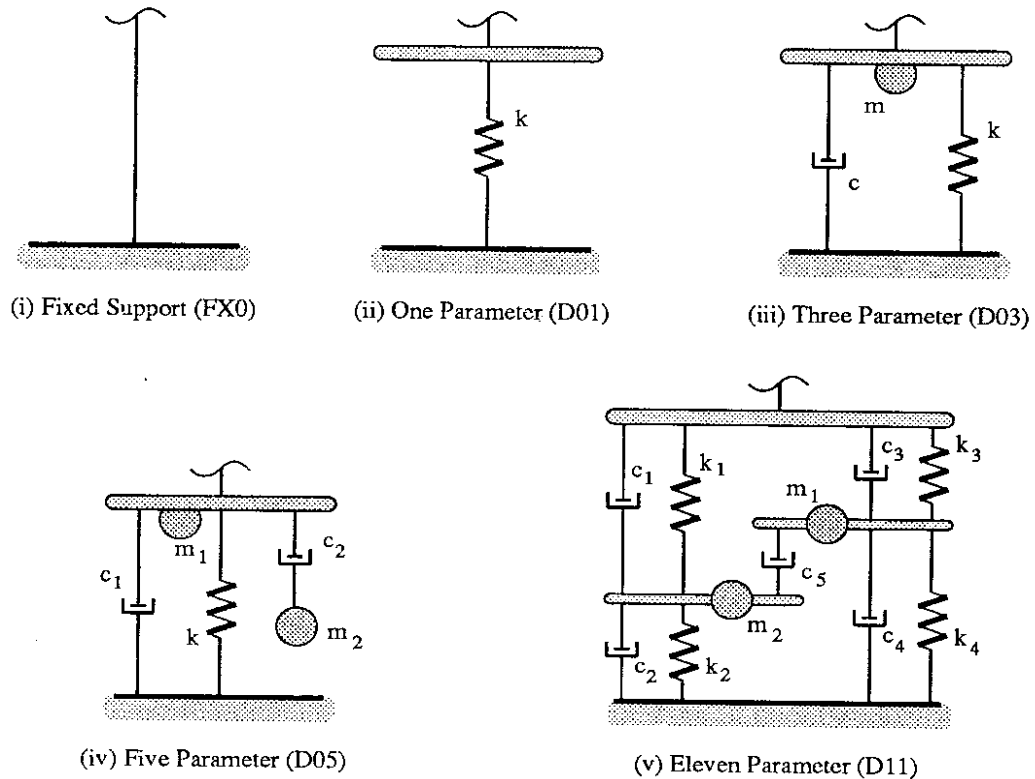


Figure 2. Schematic of NEABS models for the spread footing foundation study; (a) bent structure; (b) foundation models.

freedom: horizontal translation, vertical translation, and rocking. All other degrees-of-freedom were constrained. The modulus of elasticity that was used for the columns was $E = 4700$ ksi. The moment of inertia that was used was half that of the gross column cross-section to account for the effect of initial concrete cracking. The yield surface for the elasto-plastic beam elements was based on the axial force-bending moment strength interaction curve. Rayleigh damping, corresponding to five percent of critical for the fundamental period of the fixed-base bent, was added to the structure.

Five foundation models were considered, as shown in Fig. 2. One model consisted of fixed supports, one consisted of elastic supports, and three had damped elastic supports that required three, five, and eleven parameters per degree-of-freedom, respectively. All but the fixed support are discrete approximations of the elastic half-space continuum model, but with increasing levels of complexity. The footings were not assumed to be embedded. Because the half-space is elastic, the damping that is present in the foundation models corresponds to radiation damping only. Energy dissipation from material damping has not been quantified and, therefore, it is not included.

Three soil stiffness values were used in testing each model. The stiffnesses were selected to span a range of values commonly encountered. The unit weight of the soil was taken to be 110 pcf. Three shear wave velocities, of 300, 700, and 1300 fps, were chosen to produce the three soil stiffnesses. For the given soil density and the assumption of small strain, these corresponded to soil shear moduli, G , of 2140, 11,650, and 40,170 psi, respectively. Poisson's ratio for the soil was taken to be $\nu = 0.33$. The stiffness, mass, and damping values that were assigned to each foundation model are given in Table 1. Formulas for obtaining these values may be found elsewhere ([19](#), [22](#), [24](#), [25](#)). The fundamental periods for the bent ranged from 0.53

Table 1: Spread Footing Foundation Models: Parameter Values*

		Soft Soil				Intermediate Soil				Stiff Soil			
		D01	D03	D05	D11	D01	D03	D05	D11	D01	D03	D05	D11
Lateral Translation	k1	866.5	866.5	866.5	591.1	4718	4718	4718	3218	16271	16271	16271	11100
	k2				1700				9254				31919
	k3				571.6				3112				10733
	k4				1798				9791				33770
	c1		11.78	10.29	13.11		27.49	24.01	30.6		51.06	44.59	56.83
	c2			0.0	36.02			0.0	84.04			0.0	156.1
	c3				1.983				4.627				8.593
	c4				24.75				57.77				107.3
	c5				30.39				70.90				131.7
	m1		0.0452	0.0	0.0527		0.0452	0.0	0.0527		0.452	0.0	0.0527
m2			0.0	0.2975			0.0	0.2975			0.0	0.2975	
Vertical Translation	k1	983.1	983.1	983.1	1091	5353	5353	5353	5943	18461	18461	18461	20499
	k2				1393				7582				26149
	k3				450.2				2451				8453
	k4				1778				9680				33385
	c1		17.78	14.01	13.67		41.49	32.69	31.89		77.05	60.71	59.23
	c2			4.700	31.43			10.90	73.34			20.24	136.2
	c3				7.306				17.04				31.66
	c4				24.48				57.12				106.1
	c5				27.96				65.23				121.1
	m1		0.1202	0.0	0.3931		0.1202	0.0	0.3931		0.1202	0.0	0.3931
m2			0.0345	0.2537			0.0345	0.2537			0.0345	0.2537	
Rocking	k1	3.00E6	3.00E6	3.00E6	1.31E6	16.34E6	16.34E6	16.34E6	7.113E6	56.37E6	56.37E6	56.37E6	24.53E6
	k2				5.07E6				27.61E6				95.24E6
	k3				4.55E6				24.79E6				85.49E6
	k4				3.26E6				17.77E6				61.28E6
	c1		16553	0.0	-16789		38624	0.0	-39175		71732	0.0	-72754
	c2			21389	64412			49909	150.3E6			92688	279.1E3
	c3				39968				93259				173.2E6
	c4				3164				7383				13712
	c5				33788				78839				146.4E3
	m1		276.6	0.0	1228		276.6	0.0	1228		276.6	0.0	1228
m2			255.9	392.4			255.9	392.4			255.9	392.4	

* unit of force = kip; unit of length = inch; unit of rotation = radian; unit of time = second

seconds for the fixed-base foundation to 0.68 seconds for the most flexible foundation.

Recorded acceleration histories from actual earthquakes formed the basis of the seismic excitation applied to the bent-foundation system. The two earthquake records chosen were the S00E component of the El Centro record of the 1940 Imperial Valley Earthquake (referred to as the "El Centro" record) and the N86E component of the Olympia record of the 1949 Western Washington earthquake (or "Olympia" record). Acceleration history plots are given in Fig. 3.

To incorporate variations in record intensity in the study, both records were scaled to an intensity of 0.25g effective peak acceleration ("lower" intensity) and to an intensity of 0.40g effective peak acceleration ("higher" intensity). The definition of effective peak acceleration is outlined in the recommendations of the National Earthquake Hazards Reduction Program (26).

Results

The performance of the various foundation models was assessed in terms of its effects on the response of the bent structure. Specifically, three aspects of the bent's response were selected to be studied: column displacement (i. e., the displacement of the column top relative to the bottom), the moment at the top of the column, and the plastic hinge rotation at the column top. This information was provided by the program in the form of time histories. The results were then interpreted in terms of their implications for column ductility demand and energy dissipation demands. One should note that the column moment values reported by NEABS include a dynamic component from damping in addition to the usual moment that results from stiffness.

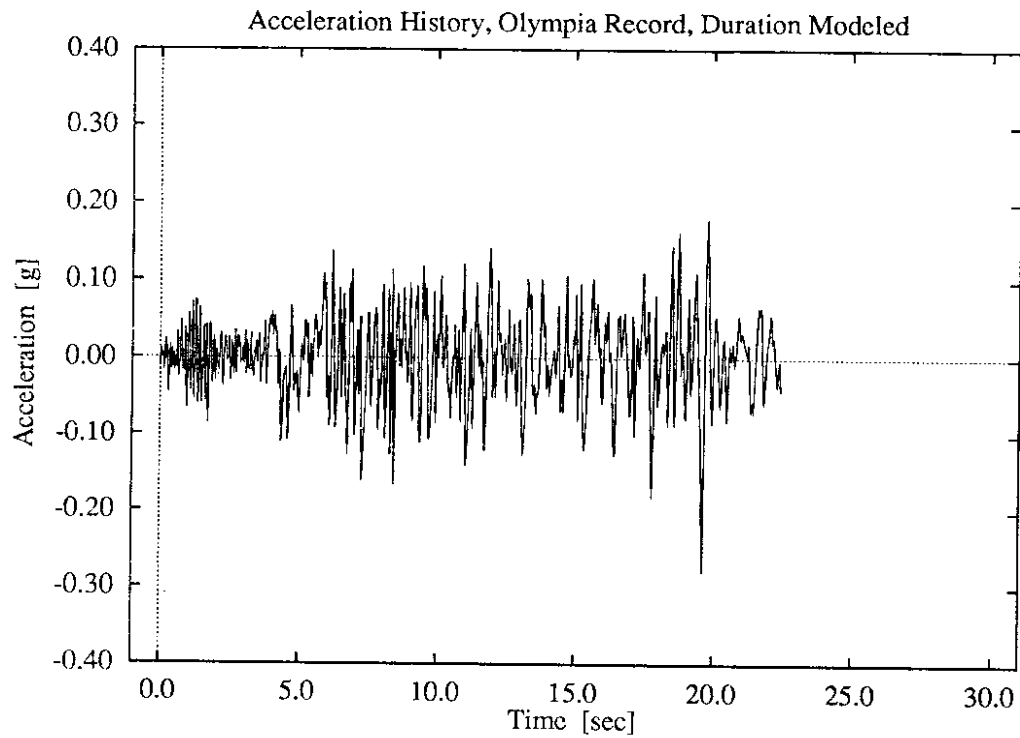
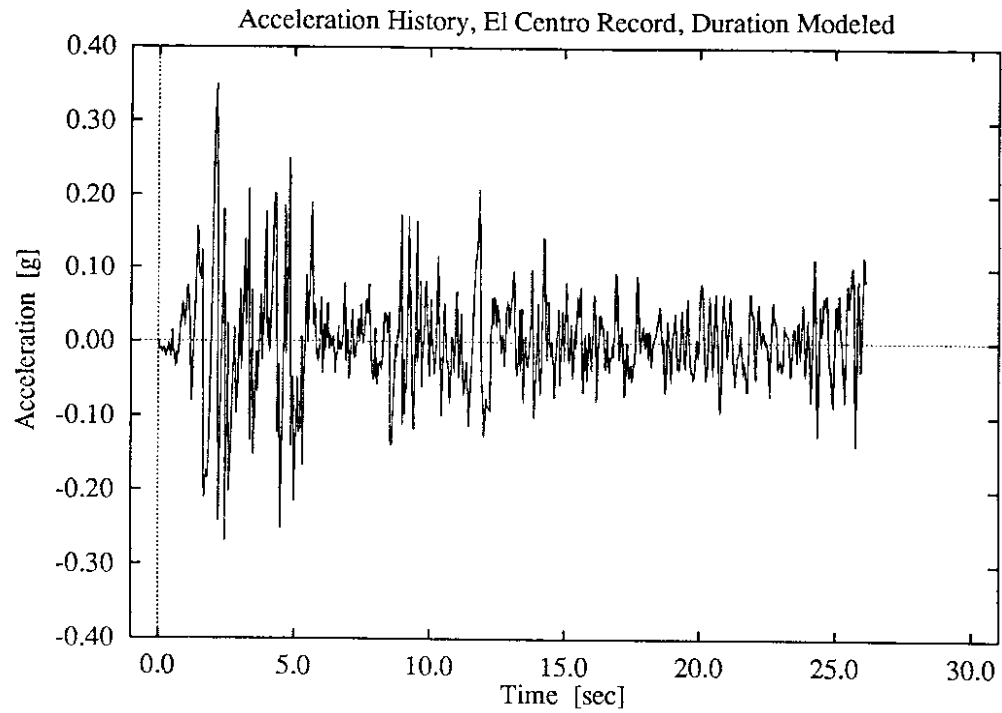


Figure 3. Earthquake acceleration history plots.

A number of analyses were performed, consisting of five foundation models, three soil stiffness values, and four seismic input records. Four graphs of the data from each NEABS analysis were used, examples of which are shown in Figs. 4 and 5. In Fig. 4, the time histories of the column displacement and column moment for the higher intensity El Centro earthquake record, soft soil, and eleven parameter foundation model are given. The third graph, shown in Fig. 5 for the same analysis, depicts the column moment-displacement hysteresis, which may be used as an indicator of energy dissipation demand. The fourth graph, also shown in Fig. 5, is a time history of the plastic hinge rotation at the top of the column. While the column remains elastic, the moment in the column does not produce plastic rotation; this condition results in a horizontal line in this graph. A vertical line indicates that a plastic hinge has formed at the column top, and it is being rotated by the moment. The magnitude of these plastic rotations are indicative of instantaneous ductility demand at the top of the column. Also, if the axial force on the columns is assumed to be constant, or nearly so, over the duration of the excitation, then the moment required to yield this column will also be constant. If this is the case, then work done on the plastic hinge over the excitation duration will be the yield moment multiplied by the sum of the absolute values of plastic rotation, represented by the vertical lengths on the graph. As the assumption of nearly constant axial force is reasonable, this graph can also provide an indication of the cumulative energy dissipation demand of the top of the column.

To summarize and compare these results, the maximum plastic rotation (measured from the undeformed state) and the sum of all plastic rotation was calculated for each run. As mentioned, these quantities are related to ductility and energy dissipation demands. This data is given in Figs. 6 - 9. Each figure shows a set of bar charts of both the rotation maxima and rotation sums for the

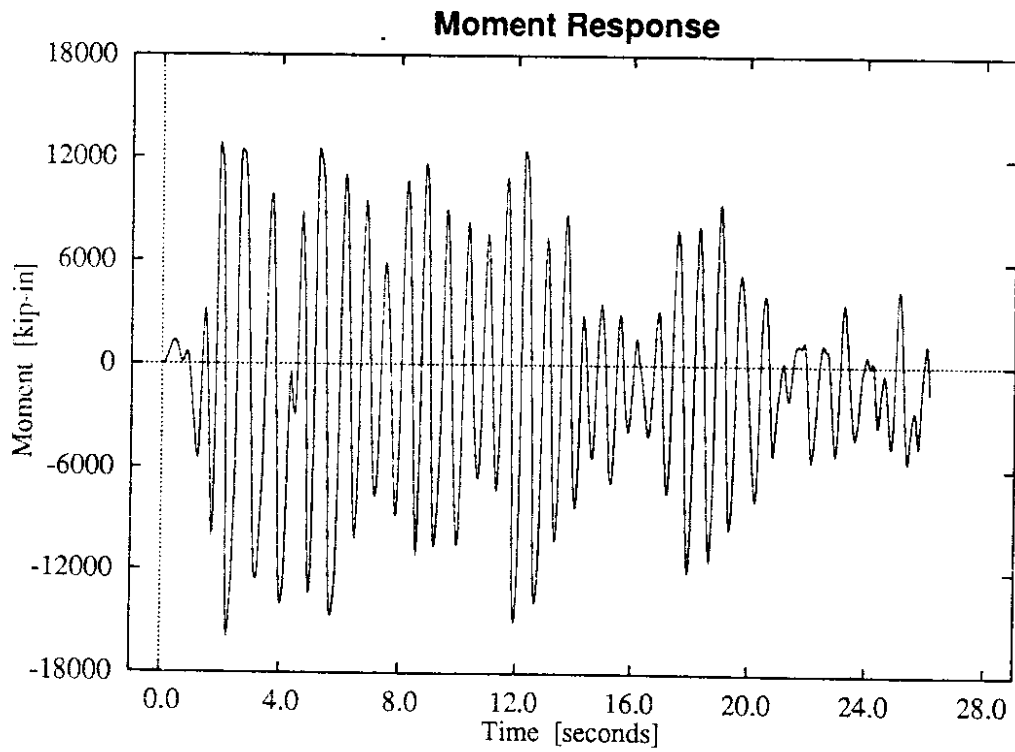
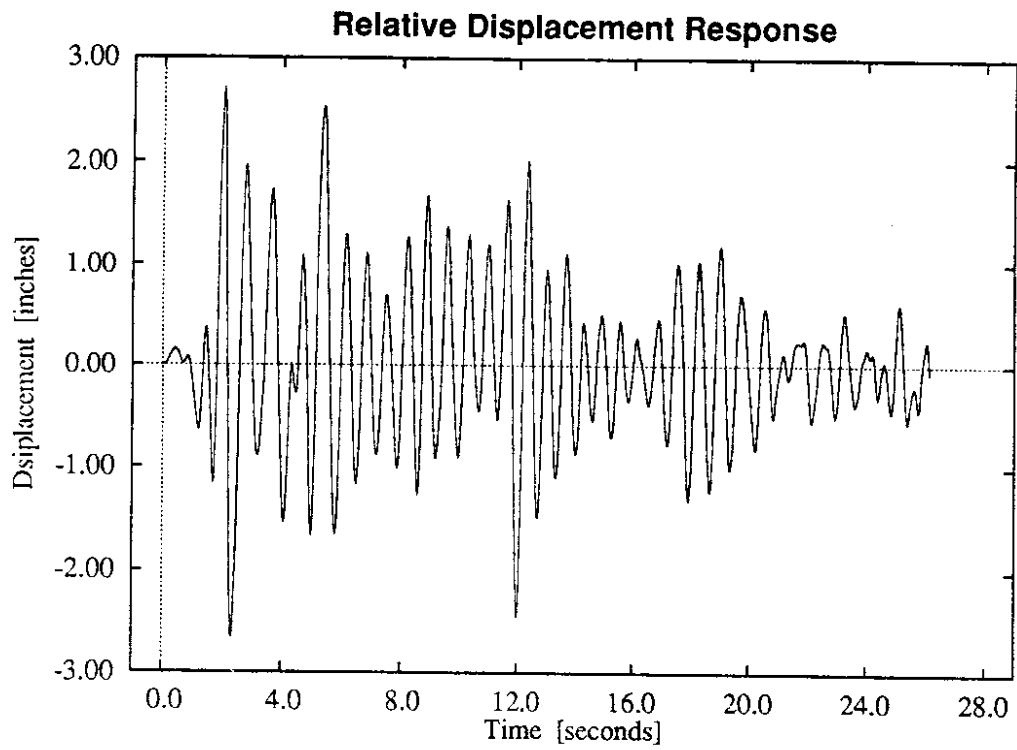


Figure 4. Typical time history results.

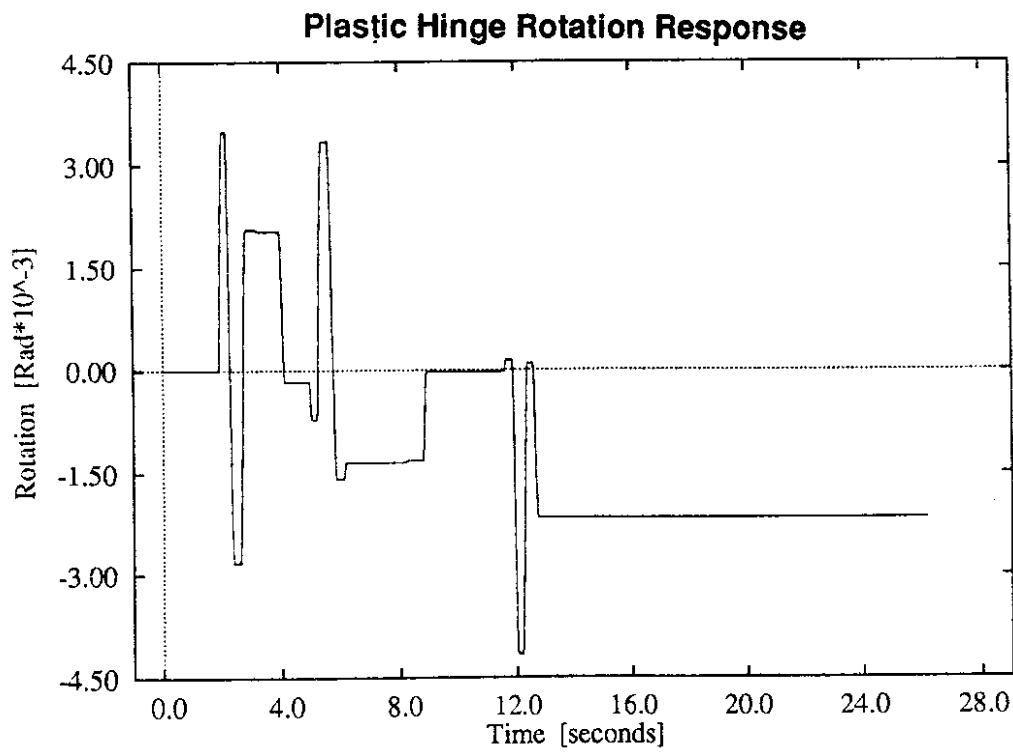
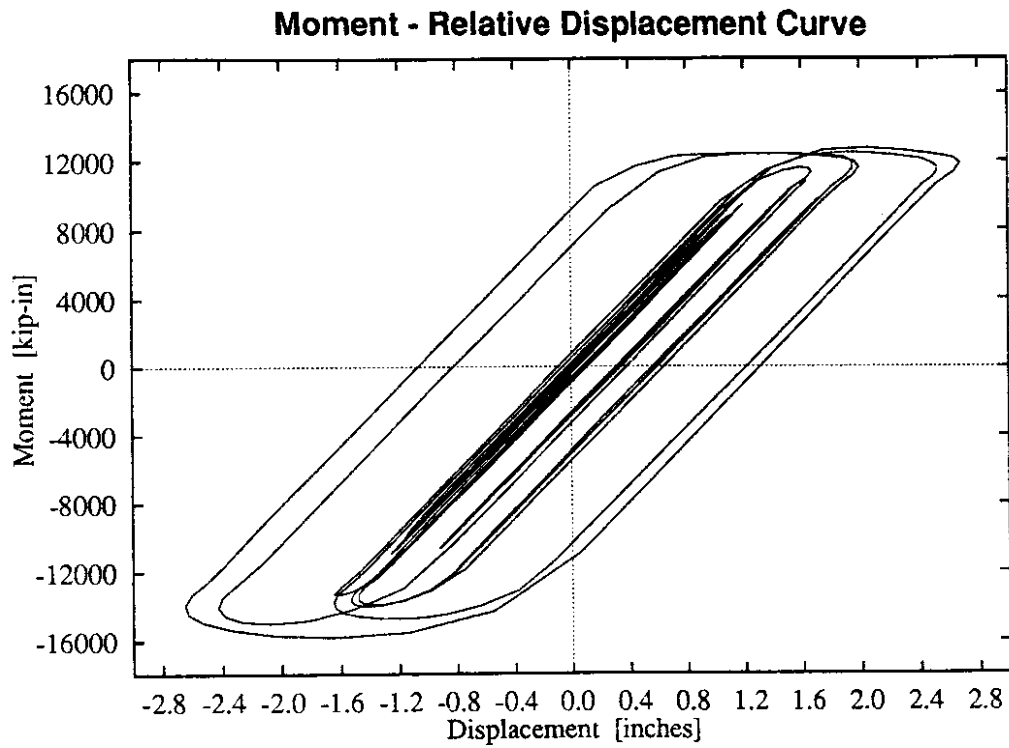


Figure 5. Typical hysteresis and plastic hinge rotation results.

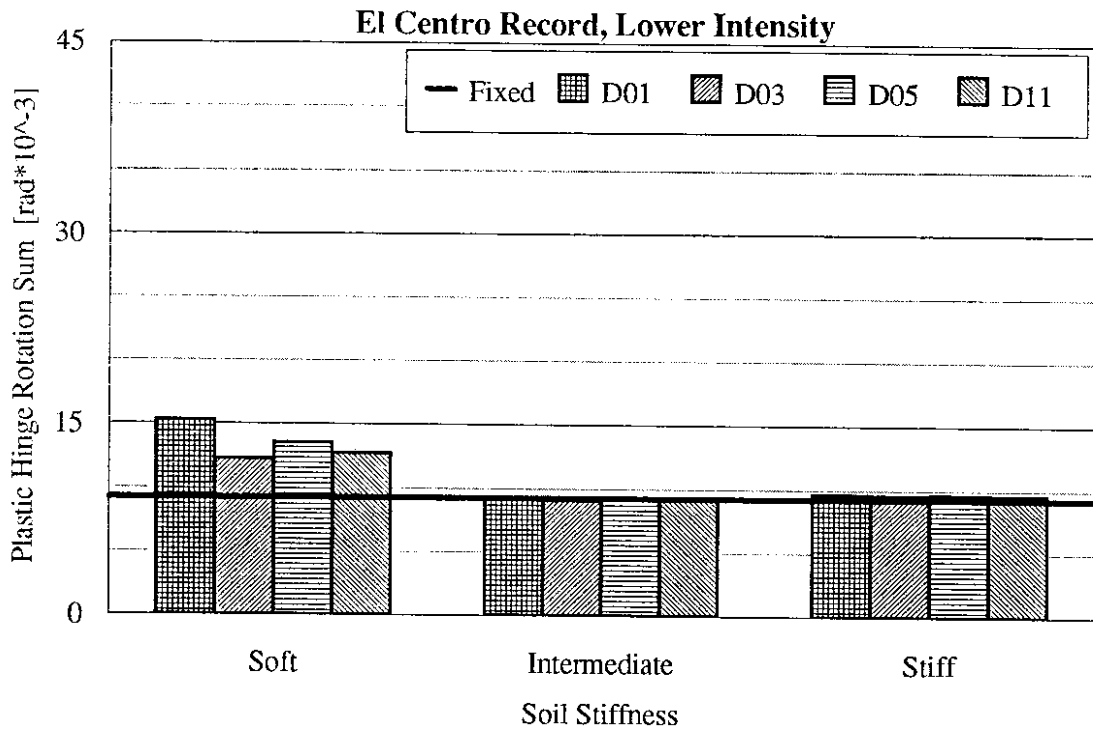
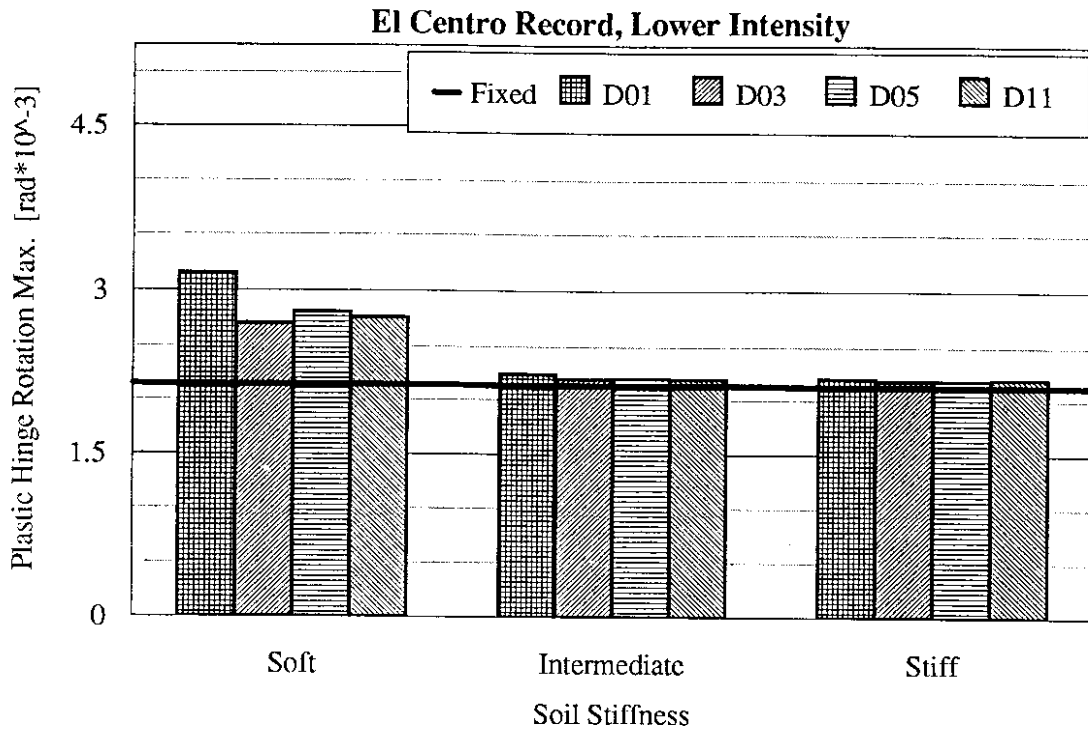


Figure 6. Comparison of instantaneous and cumulative column demands, lower intensity El Centro earthquake record, spread footing foundation.

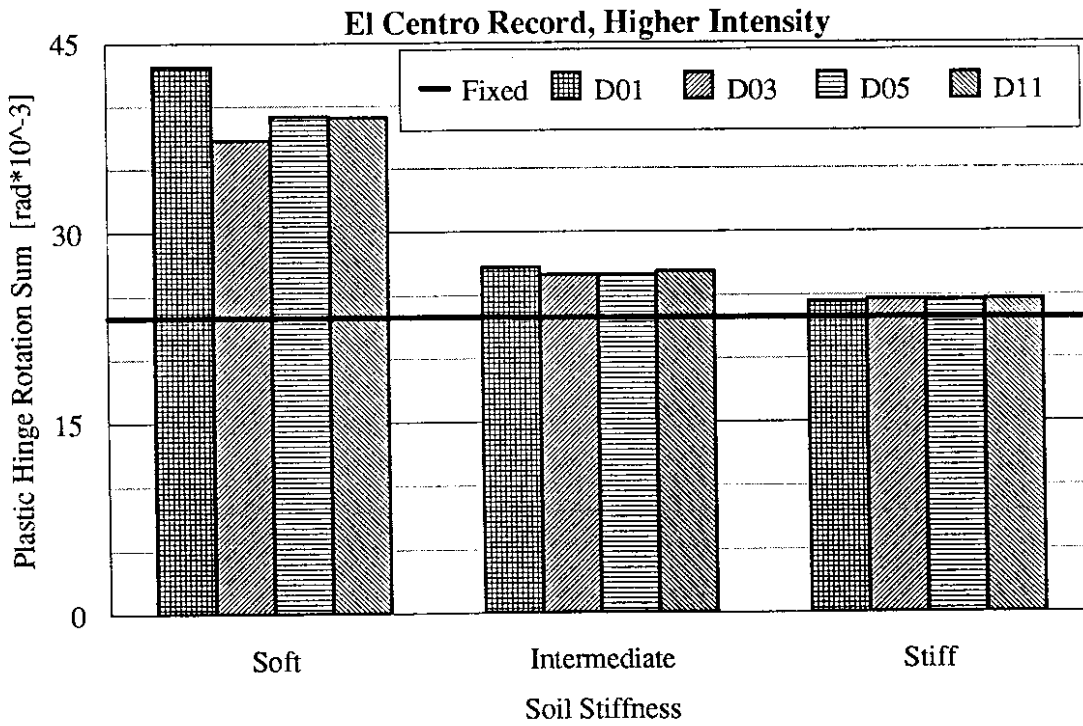
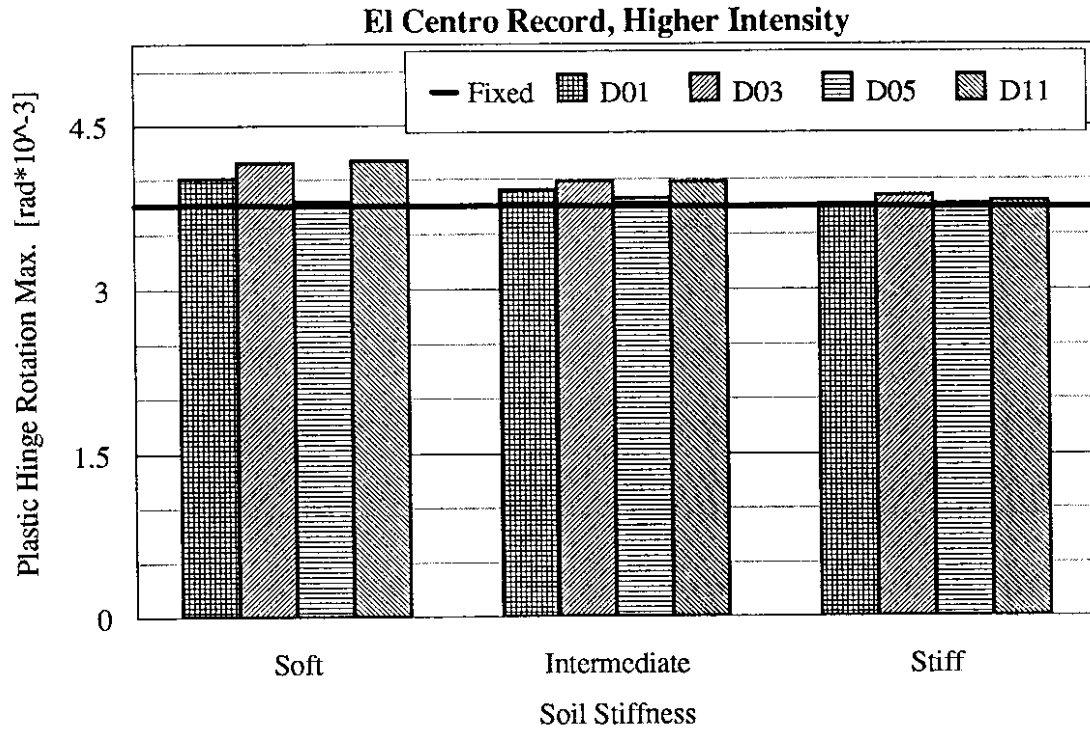


Figure 7. Comparison of instantaneous and cumulative column demands, higher intensity El Centro earthquake record, spread footing foundation.

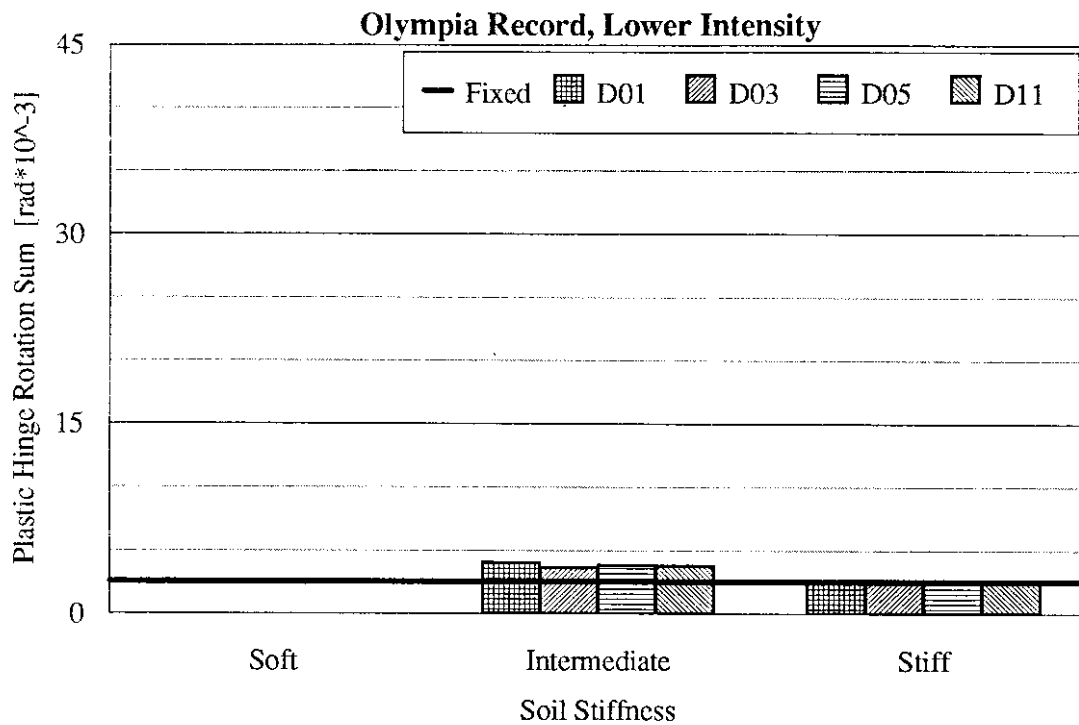
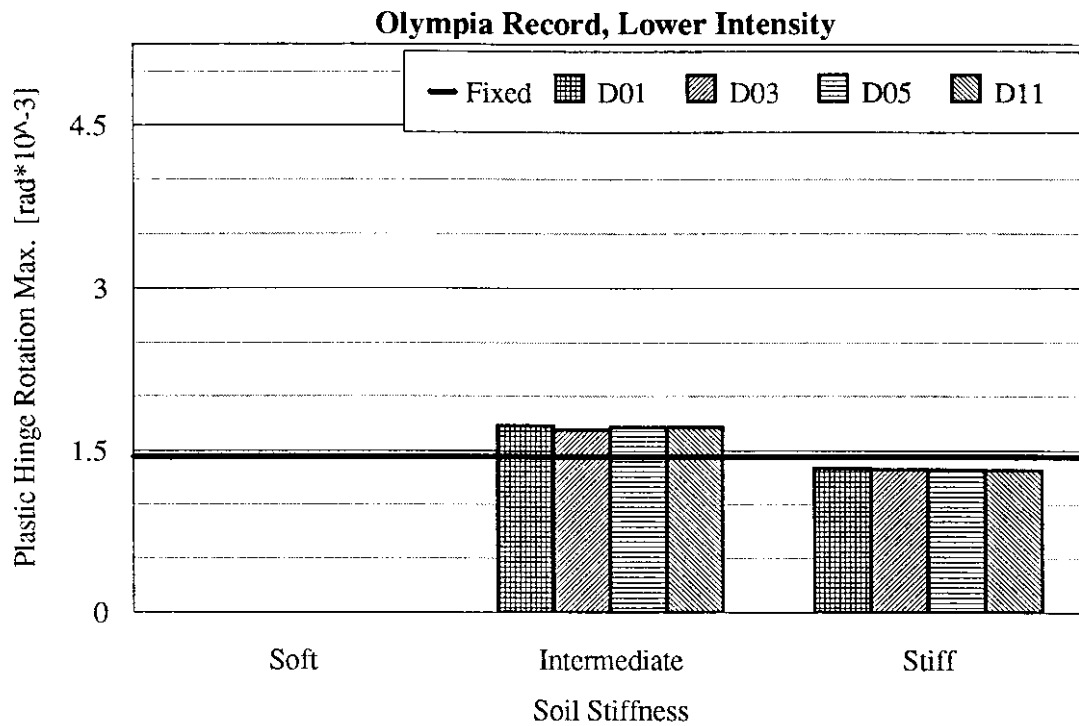


Figure 8. Comparison of instantaneous and cumulative column demands, lower intensity Olympia earthquake record, spread footing foundation.

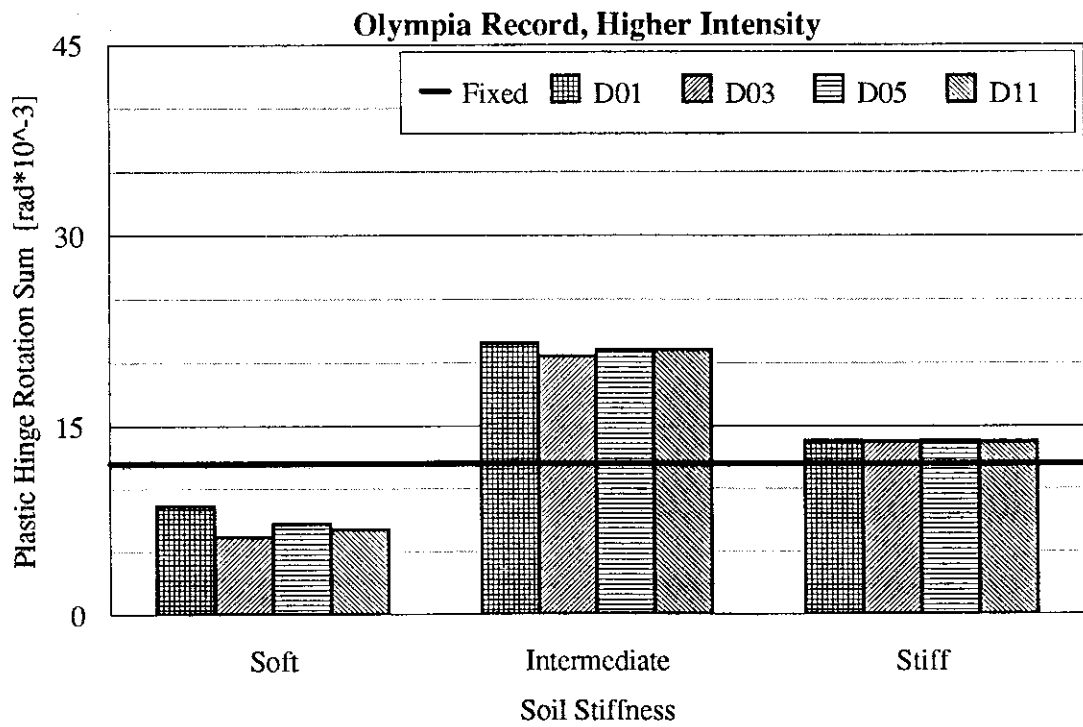
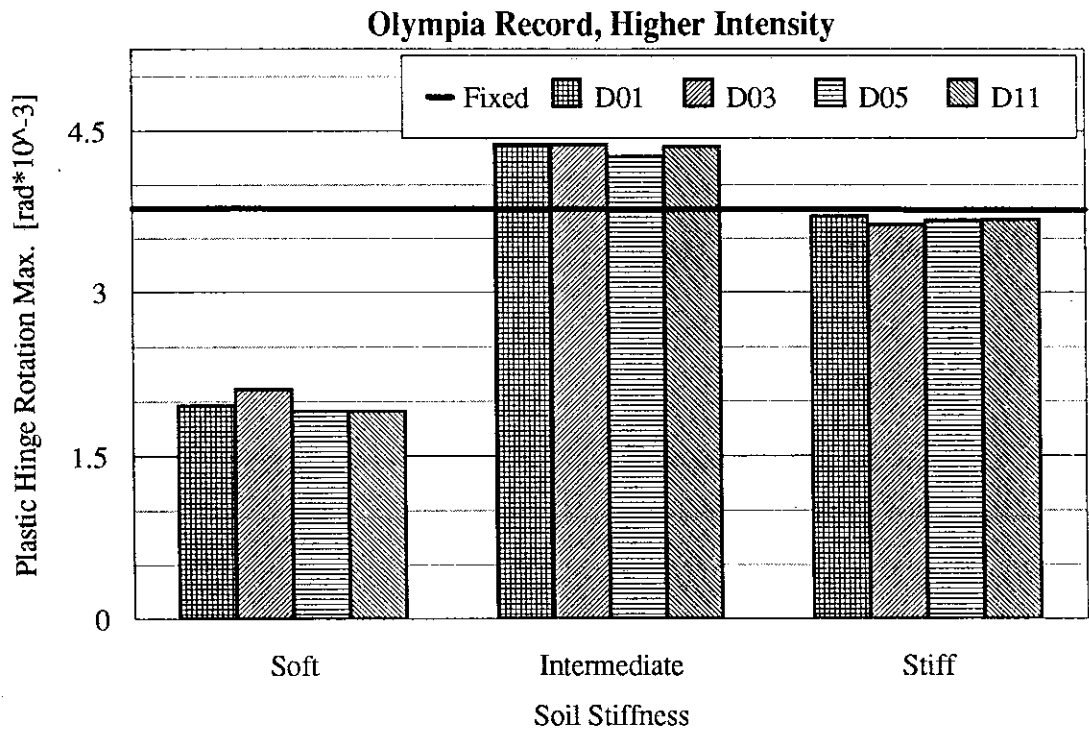


Figure 9. Comparison of instantaneous and cumulative column demands, higher intensity Olympia earthquake record, spread footing foundation.

given excitation record. Each bar chart shows the results of the four discrete foundation models for each soil stiffness, and allows a comparison with the fixed-support results. In Fig. 2, a schematic of each foundation model is shown.

Discussion

The response of the bridge bent to the two earthquake records is somewhat different, although the intensity of each earthquake resulted in plastic hinge formation for almost all analyses. For both El Centro records, the stiff and intermediate foundation models led to nearly the same instantaneous and cumulative demands as those of the fixed-base model. The soft foundation model resulted in a significant increase in cumulative demand for both intensities, and it led to increased instantaneous demand for the lower intensity record. The instantaneous demand for the higher intensity El Centro record was approximately the same for all of the foundation models.

The flexible foundation caused an increase rather than a reduction in column demand. By comparing the earthquake record of Fig. 3 with the example plastic hinge rotation history of Fig. 5, one may observe that much of the damage results from peak accelerations at approximately 2 seconds, 5 seconds, and 12 seconds. The pulse at 12 seconds seems to be the major source of the increase in demand over the other foundations because its period of application is close to the fundamental period of the structure with the flexible foundation.

Damping in the discrete foundation model had a negligible effect on the column demands for the intermediate and stiff foundations. However, the damped foundations (i. e., the three, five, and eleven parameter models) caused a reduction in demand in the order of 15 to 20 percent, when compared to the spring foundation alone, for the soft soil. Also, little change was observed

between the simple and more complex damped models. This is likely to be due to the fact that the damping and mass values for the three parameter model are relatively insensitive to the loading frequency for translational motion, which seemed to dominate the response.

For the Olympia earthquake records, the column instantaneous demands were of the same order as those of the El Centro records for the intermediate and stiff foundation models, but much less for the soft foundation model. Indeed, no column yielding was indicated for the lower intensity Olympia record and the soft foundation. This appears to be the result of the frequency content of the earthquake versus the natural frequencies of the structure-foundation system.

The cumulative demand, however, was significantly less for all foundations when compared to that of the El Centro earthquake. The two earthquake records were scaled to the same effective peak accelerations, but, from Fig. 3, it is apparent that the Olympia record is dominated by a single peak at approximately 20 seconds. Because the majority of the column damage is caused by this peak, as opposed to several different peaks in the El Centro record, the total amount of plastic hinge rotation is reduced.

As with the El Centro earthquake, radiation damping was only significant for the column cumulative demand and the soft foundation. The reduction in demand from damping ranged from approximately 25 to 35 percent.

Pile Foundation Study

Description of the Model

Interstate 90 crosses Mercer Slough in south Bellevue, Washington by means of several bridges. The structure carrying the westbound lanes was

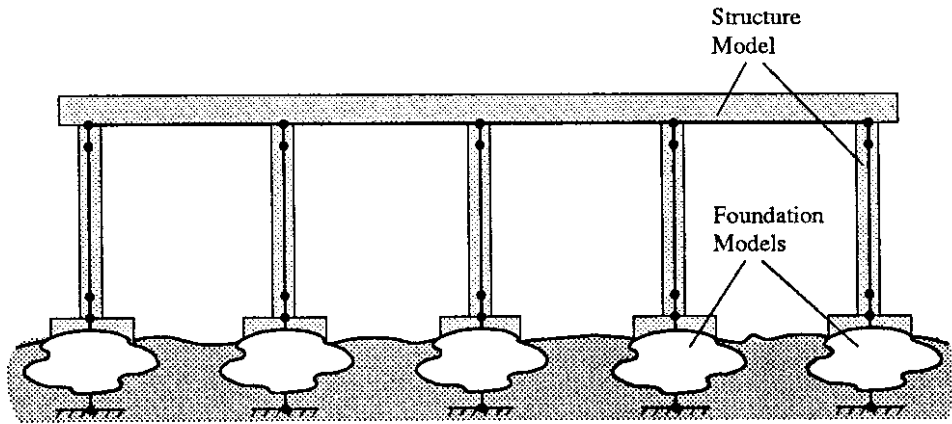
recently the subject of a WSDOT research project (27, 28). Because it was constructed on pile foundations in marshy soil, this bridge was chosen for the second parametric study. In their research, McLean and Cannon (27) observed that, away from the ends of the bridge, each bridge section between expansion joints responded nearly independently to lateral seismic excitation. Therefore, as with the spread footing study, the pile foundation study was confined to analyzing an isolated bridge bent.

The spans consisted of reinforced concrete T-girders, supported on pile-founded, five column, reinforced concrete bents. Each span was 60 feet wide, incorporating a 6-inch thick slab and nine 16-inch wide T-girder webs at 7.5-foot centers. The section depth was typically 33 inches from the bottom of the web to the top of the slab. The bent caps formed diaphragms between the T-girder webs. A schematic of the model that was used is shown in Figure 10.

An approximate subsurface soil profile of Mercer Slough along the Interstate 90 alignment is shown in Figure 11. In a WSDOT study of the slough by Kramer (29), the soil was found to be very weak and soft. Because the seismic response of soft soils can change, and often amplify, ground free-field motions, the free-field motions likely to occur at the Mercer Slough site were determined (29). The soil profile was divided into zones and filtered earthquake records were obtained by applying recorded earthquake acceleration time histories to the base of the soil models. Two earthquake records were considered, entitled "El Centro" and "Lake Hughes" (29).

Four foundation models were compared, as well as fixed-base supports. Three of the four foundation models were discrete springs or spring/damper systems which modeled the pile cap behavior "seen" by the column bases. The fourth foundation model was a Winkler-type pile foundation. These five support conditions are shown in Figure 10.

(a) Structure Model



(b) Support Models

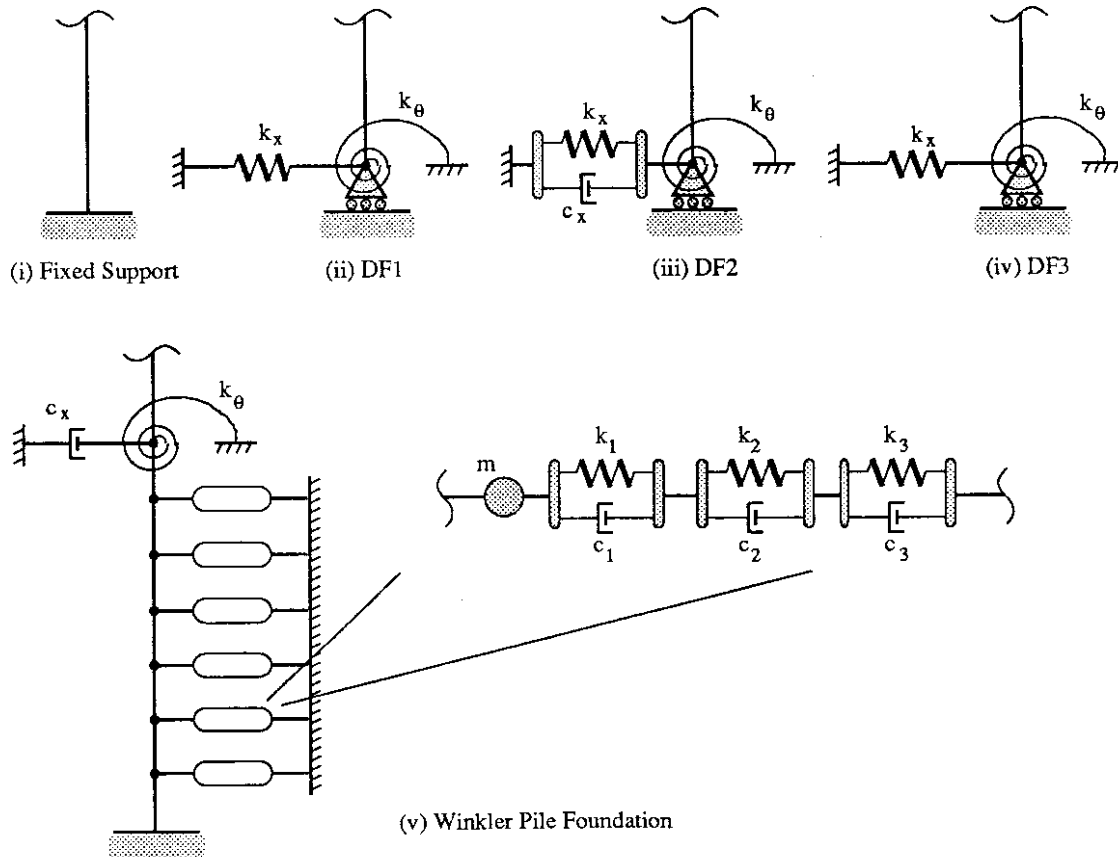


Figure 10. Schematic of NEABS models for the pile foundation study: (a) bent structure; (b) foundation models.

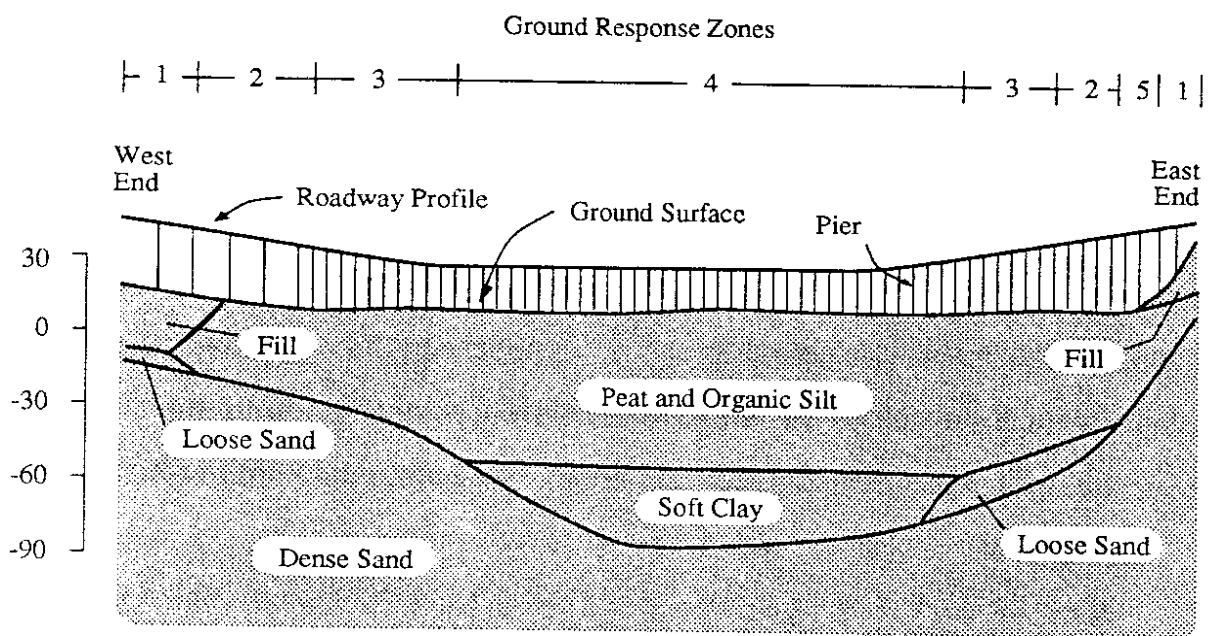


Figure 11. Approximate soil profile of Mercer Slough along alignment of Interstate 90.

The first discrete pile cap model, referred to as the "DF1" model, used a lateral spring with a linear stiffness of $k = 8.2$ kips/inch. This pile head stiffness was used by McLean and Cannon (27) in their modeling of the Mercer Slough bridge. In addition, a rotational pile head spring was applied. The stiffness of this spring was determined from the resistance to rotation of the center of the pile cap due to the eccentric axial reaction of the piles. The piles were assumed to be elastic and end-bearing for this calculation. This rotational spring was used for all four foundation models.

The second discrete pile cap model, or "DF2" model, utilized a hysteretic bilinear lateral spring in conjunction with a viscous lateral damper. The DF2 model's stiffness values were obtained by modeling Kramer's 8-inch test pile (29) with Winkler-type soil reaction springs. The soil spring stiffness values were adjusted to match the experimentally determined pile stiffness. To appropriately model the actual four pile foundations, the soil spring stiffness values were then scaled to reflect the difference in pile diameters, and an equivalent pile head stiffness was obtained. This procedure was followed for both portions of the bilinear load-deflection curve of the test pile.

A lateral viscous damper was also used in the DF2 model. The damping coefficient was based on the lateral pile head damping coefficients obtained by Kramer for the 8-inch test pile (29). These damping coefficients were obtained from a dynamic test conducted at low soil-strain amplitudes, so they account primarily for radiation damping. The damping was observed to be relatively independent of the frequency of applied vibration. Because radiation damping is a far-field phenomenon, and because it was deemed unlikely that the far-field effects would vary significantly between an 8-inch and a 12-inch pile, the damping coefficient that was used was obtained by multiplying the average coefficient reported by Kramer by four piles.

The third discrete pile head model, or "DF3" model, was not strictly a unique model, because it contained a simple lateral spring like the DF1 model. However, a linear secant stiffness was used, based on the results of the seismic analysis of the bridge bent utilizing the DF2 model. In this respect, it represents a model of intermediate sophistication. It was employed to test the use of an elastic secant stiffness to approximate nonlinear pile head stiffness, and by comparison with the DF1 model results, to test the sensitivity of the bridge bent response to the elastic stiffness of the lateral pile head foundation spring. The secant stiffness was determined by recording the maximum pile head lateral deflection of the DF2 model under each of the applied seismic excitation records. The internal force of the bilinear spring used in the DF2 model at these recorded maximum pile head deflections, divided by these maximum deflections, defined the secant stiffness used in the DF3 model. However, a practical secant stiffness value could also be obtained by iterating to a maximum pile head deflection on a standard pile "p-y" curve. As with the DF1 model, no foundation damping was included with the DF3 model.

The fourth foundation model was a Winkler-type pile foundation, and it is referred to as the "WPF" model. The pile groups were each modeled as a single "pile" using beam elements coupled to the column bases. Six lateral soil reaction models were used with each of the pile group models. The soil reaction models that were used were based on the soil far-field sub-model proposed by Nogami and Konagai ([30](#)), illustrated in Figure 10. This model is derived from the representation of the soil as a vertically-stratified series of plane-strain elastic layers. The procedure for determining the stiffness and damping parameters is described elsewhere ([22](#)) and the stiffness, damping, and mass coefficients that were used are given in Table 2.

Six different acceleration time histories were applied to the bent models

Table 2: Pile Foundation Models: Parameter Values*

All Models:

Elastic Rotational Pile Head Spring: $k = 516,000$
 Lateral Pile Head Damper (when used): $c = 0.1625$

Pile Head Models:

	DF1	DF2	DF3
Elastic Stiffness	8.2	11.6	9.0
Yield Force	-	26.8	-
Plastic Stiffness	-	2.1	-

WPF Model:

depth	m	k1	c1	k2	c2	k3	c3
43	0.01060	68.28	43.90	69.50	9.756	107.3	3.634
105	0.01164	74.96	48.19	76.30	10.71	117.8	3.989
179	0.01306	84.08	54.06	85.59	12.01	132.1	4.475
266	0.01517	97.67	62.80	99.42	13.96	153.5	5.198
370	0.01884	121.3	77.98	123.5	17.33	190.6	6.455
515	0.02835	182.5	117.4	185.8	26.08	286.9	9.715

* unit of force = kip; unit of length = inch; unit of rotation = radians; unit of time = second

as seismic excitation. The four most severe ground motions that were determined by Kramer (29) were used, consisting of the El Centro record applied to zones three and four and the Lake Hughes record applied to zones one and two (i. e., El Centro Z3, El Centro Z4, Lake Hughes Z1, and Lake Hughes Z2). These were obtained by applying recorded earthquake acceleration time histories to the base of the soil models of Mercer Slough. As the soft soil models responded to the seismic excitation, the surface ground motion changed, and thus these ground motions are referred to as "filtered" records. The two "unfiltered" records are the lower and higher intensity El Centro records that were used in the spread footing study, applied directly to the structure/foundation system.

Discussion of Results

As with the spread footing study, the performance of the various foundations was assessed in terms of their effects on the response of the bent structure. The same three aspects of the bent response were selected to be studied: column drift (the displacement of the column top relative to the bottom), the internal moment at the column top, and the rotation of the plastic hinge at the column top. The maxima of the three parameters were again recorded for each analysis. This data is presented in Figures 12 and 13. One should note that, with one exception, the columns remained elastic for the fixed base bent model and for the three discrete pile head foundation models when subjected to the four filtered records. Because the plastic rotation of the one exception was very small, only bar charts for drift and column top moment are provided for the filtered records. For the two unfiltered records, bar charts are given for the plastic hinge rotation maxima and sums.

Of the filtered records, the Lake Hughes Z1 record produced the highest

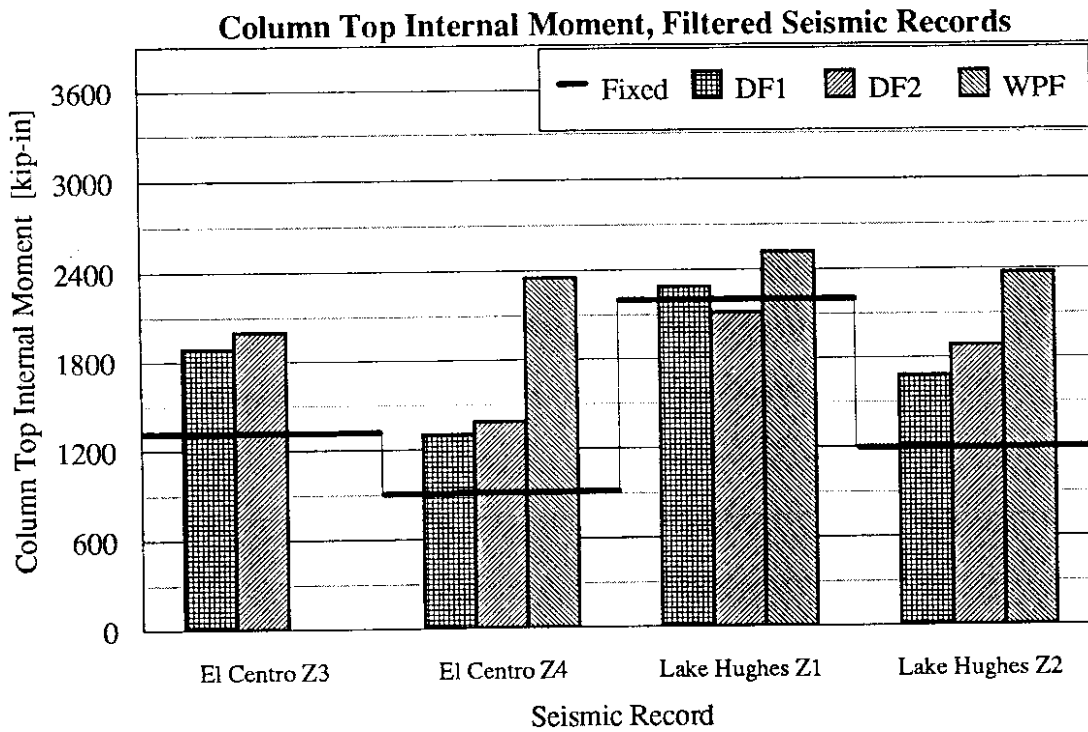
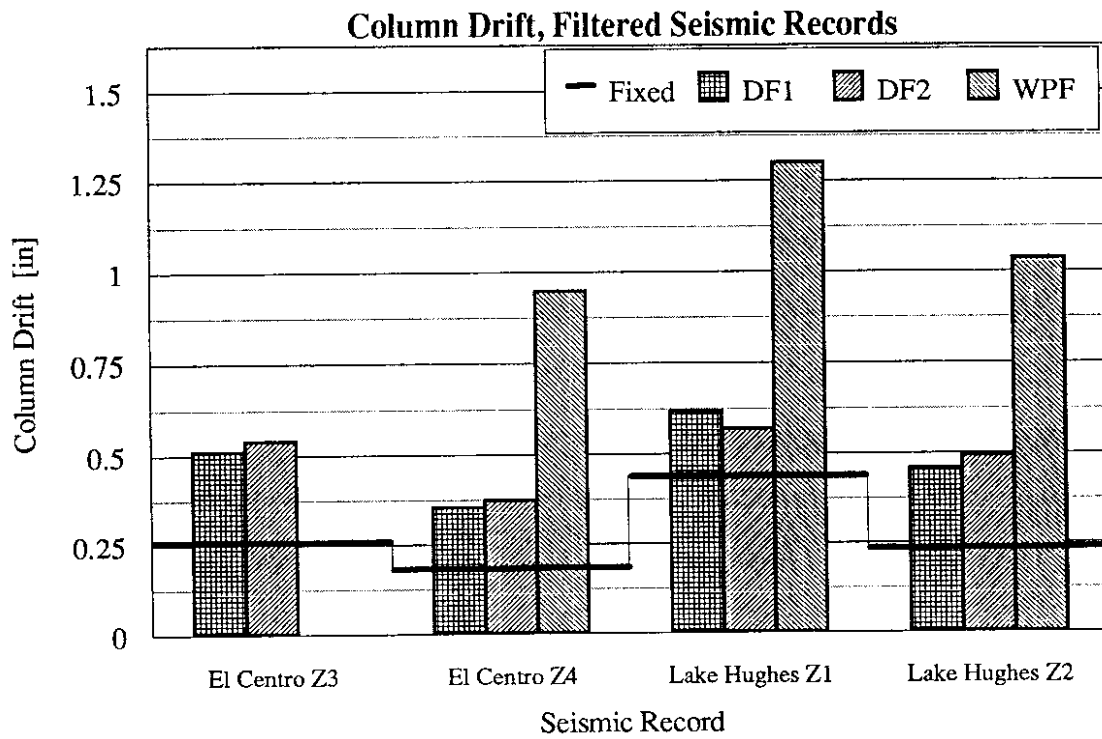


Figure 12. Comparison of column demands, filtered seismic records: (top) column drift maxima; (bottom) column top internal moment maxima.

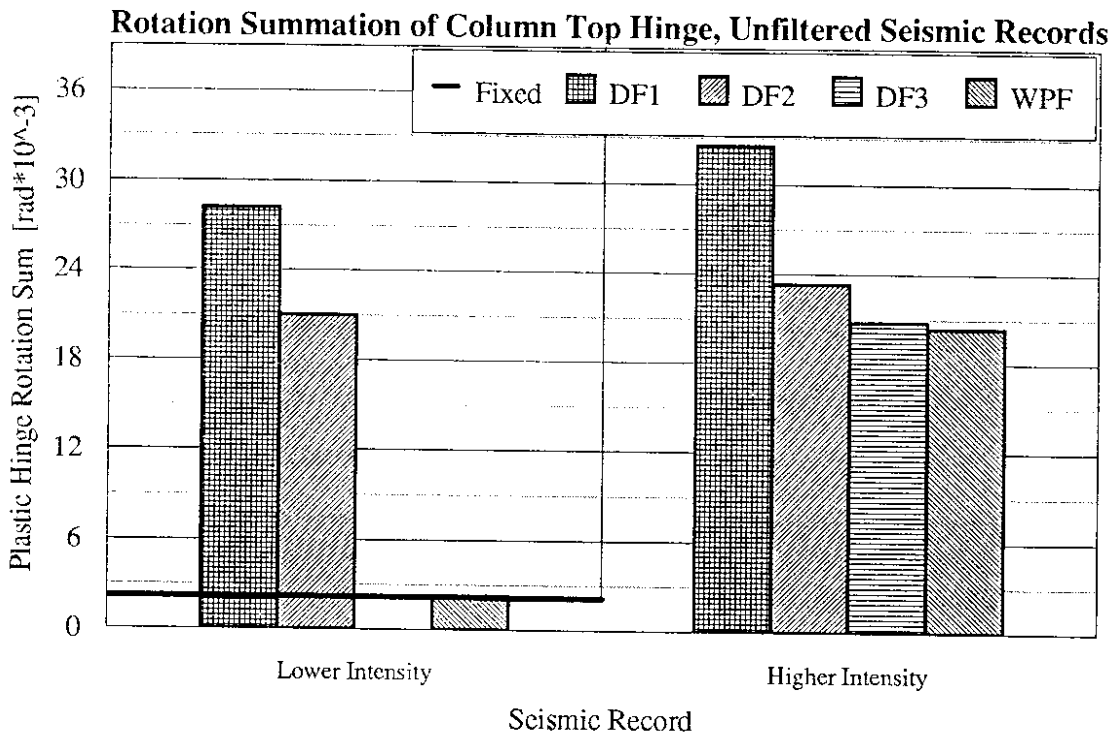
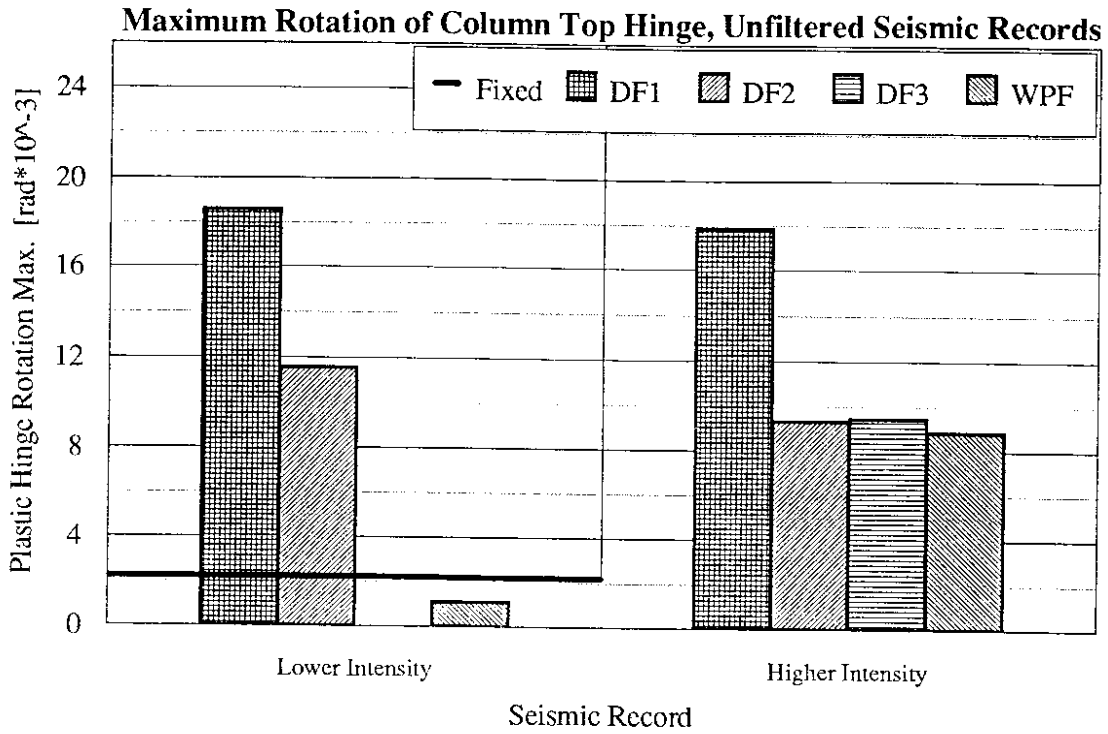


Figure 13. Comparison of column demands, unfiltered seismic records: (top) maximum rotation attained (related to ductility demand); (bottom) total rotation sustained (related to energy demand).

values of all the measured response parameters for the fixed base model, followed by the El Centro Z3 and Lake Hughes Z2 records (which produced very close results) and then by the El Centro Z4 record. This same ordering of response severity was observed for the bents using the DF1 and DF2 foundation models. The column drift and column top moment response for these two models, as shown in Figure 12, were in close agreement with each other, and they were both significantly more severe than the fixed base response to each of these records. The DF1 model's response was slightly greater than the DF2 model's response for the Lake Hughes Z1 record, while the opposite was true for the other three records.

In no case was the response great enough to produce a maximum lateral pile head displacement in excess of 2.3 inches for the DF2 model, the yield displacement of the lateral spring. Therefore, the differences between the DF1 and DF2 models resulted solely from the higher elastic stiffness of the DF2 model (approximately 40 percent stiffer) and the additional laterally-acting damper. The close correlation of the results of these two models appears to indicate that the column top moment is not especially sensitive to the stiffness of the pile head equivalent elastic springs. Further, the column top moment is not appreciably affected by radiation damping at the structure/frequency combinations analyzed, even though the Mercer Slough soil is very soft. Also, because the DF2 model's lateral spring did not yield during any of the analyses using the filtered records, the DF3 model's secant stiffness is the same as that of the DF2 model. Therefore, no unique DF3 model was created for these analyses.

The column top response is significantly more severe for the DF1 and DF2 models than for the fixed base model, contrary to the typical assumption that including the effects of soil-structure interaction in seismic analysis tends to

reduce the predicted response. However, one should note that this phenomenon could be influenced by the moment redistribution that occurs when the fixed-base rotation constraint is removed.

The most flexible foundation model, in terms of lateral pile head stiffness, is the WPF model. The response it produced was significantly more severe than that of the DF1 or DF2 models, and yielding of the column top was observed for all seismic records. This increased severity was most noticeable for the El Centro Z4 record. No results are reported for the WPF model subjected to the El Centro Z3 record due to an apparent numerical instability that developed during the analysis and caused the response to grow unreasonably.

The observed response of the bent models subjected to the two unfiltered records was more severe than that produced by the filtered records, and yielding of the column top was observed in all analyses. The higher intensity El Centro record was the only record to produce yielding of the lateral pile head spring used in the DF2 model. Thus, only one DF3 secant stiffness model was created, with a stiffness value of 9.0 kips/inch. One should note that, because the unfiltered records are quite different from expected ground motions at the Mercer Slough site, using these records serves as an exercise to evaluate the effect of the foundation models.

The responses of the column top plastic hinge for the DF1 and DF2 models, as shown in Figure 13, were quite different from each other. However, they were similar for each model between the lower and higher intensity records. For the lower intensity record, both the DF1 and DF2 models produced a much more severe response than did the fixed base model. An apparent numerical instability developed during the analysis of the fixed base model subjected to the higher intensity record, and caused the response to grow

unreasonably.

The response of the DF1 model was significantly higher than that of the DF2 model for both records. The response of the one DF3 secant stiffness model was much closer to the response of the DF2 model (from which the DF3 model's stiffness was derived) than to the DF1 model, even though the DF3 model has an elastic spring stiffness only approximately ten percent greater than that of the DF1 model. The DF3 model is apparently able to adequately reproduce the response of the bilinear DF2 model. The results of the analyses using the unfiltered records appear to indicate that the response is sensitive to the stiffness of lateral elastic spring models of the pile cap, contrary to the trend observed for the filtered records. However, this may again indicate that the effect of incorporating foundation models on the predicted response is very dependent on the natural frequencies of the structure and the frequency content of the earthquake.

Also unlike the trend observed for the filtered record analyses, the response of the WPF model was the lowest of all the foundation models for the unfiltered analyses. From the response histories, the fundamental period of transverse vibration of the bent using the WPF model appears to be approximately 1.5 seconds. The lower predicted response is likely due to the lack of significant excitation of this period inherent in the El Centro record.

APPLICATION AND IMPLEMENTATION

From the results of this study, it is recommended that the interaction of bridge foundations with the founding material be rationally included in seismic analysis. For spread footings, this can usually be accomplished (at least to account for soil far-field effects) by employing simple spring-only foundation models, with spring stiffness values based on the static stiffness of an elastic

half-space. For foundations that are soft in comparison with the bridge, however, neglecting radiation damping may cause an over-estimation of the response. The three-parameter spring-mass-damper model adequately includes the effects of radiation damping, and it may be used for soft soils. Procedures for determining the three-parameter model properties are presented by Veletsos and Verbic (31) and Wolf (19).

For pile foundations, either the Winkler pile or pile head models may be used, but the pile head models are recommended because far fewer elements are required for the analysis. The foundation properties may be determined from either field testing, finite element analyses, or pile software, such as COM624. If the data is available, the optimal arrangement of the pile head model is a near field, hysteretic spring in series with a far field spring/damper system to include radiation damping. However, the spring defined with a secant stiffness alone was shown to result in a reasonable solution for the case that was considered.

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REFERENCES

1. J. Penzien, R. Imbsen, and W. D. Liu. NEABS, Nonlinear Earthquake Analysis of Bridge Systems. National Information Service for Earthquake Engineering, Earthquake Engineering Research Center, University of California, Berkeley, 1981.
2. S. D. Werner, J. L. Beck, and M. B. Levine. Seismic Response Evaluation of Meloland Road Overpass Using 1979 Imperial Valley Earthquake Records. Earthquake Engineering and Structural Dynamics, Vol. 15, 1987, pp. 249-274.
3. W. D. Liu, F. S. Nobari, and R. A. Imbsen. Dynamic Response Prediction for Earthquake Resistance Design of Bridge Structures. Proceedings, ASCE Structures Congress, Seismic Engineering: Research and Practice, 1989, pp. 1-10.
4. J. C. Wilson and B. S. Tan. Bridge Abutments: Formulation of a Simple Model for Earthquake Response Analysis. Journal of Engineering Mechanics, ASCE, Vol. 116, No. 8, 1990, pp. 1828-1837.
5. I. G. Buckle, R. L. Mayes, and M. R. Button. Seismic Design and Retrofit Manual for Highway Bridges. Final Report, FHWA, U. S. Department of Transportation, 1987.
6. I. Lam and G. R. Martin. Seismic Design for Highway Bridge Foundations. Proceedings, Lifeline Earthquake Engineering: Performance Design and Construction, ASCE, 1984, pp. 7-21.
7. J. Penzien. Soil-Pile Foundation Interaction. Earthquake Engineering, R. L. Wiegel, ed., Prentice-Hall, 1970, pp. 349-381.
8. C. B. Crouse, B. Hushmand, and G. B. Martin. Dynamic Soil-Structure Interaction of a Single Span Bridge. Earthquake Engineering and Structural Dynamics, Vol. 15, 1987, pp. 711-729.
9. C. C. Spyrakos. Assessment of SSI on the Longitudinal Seismic Response of Short Span Bridges. Engineering Structures, Vol. 12, No. 1, 1990, pp. 60-66.
10. J. B. Mander. ERBS - Earthquake Resistant Bridge Systems, A Coordinated Research Initiative. Proceedings, Second Workshop on Bridge Engineering Research in Progress, Reno, Nevada, 1990, pp. 197-200.
11. J. C. Wilson. Stiffness of Non-Skew Monolithic Bridge Abutments for Seismic Analysis. Earthquake Engineering and Structural Dynamics, Vol. 16, 1988, pp. 867-883.

12. E. Maragakis, B. Douglas, and S. Vrontinos. Analysis of the Effects of the Impact Energy Losses Occurring Between the Bridge Deck and Abutments. Proceedings, Second Workshop on Bridge Engineering Research in Progress, Reno, Nevada, 1990, pp. 201-204.
13. M. E. Barenberg and D. A. Foutch. Evaluation of Seismic Design Procedures for Highway Bridges. Journal of Structural Engineering, ASCE, Vol. 114, No. 7, 1988, pp. 1588-1605.
14. G. Norris. Lateral and Rotational Stiffness of Pile Foundations. Proceedings, Ninth Structures Congress, ASCE, Indianapolis, Indiana, 1991, pp. 749-752.
15. A. Ghobarah and H. M. Ali. Seismic Performance of Highway Bridges. Engineering Structures, Vol. 10, No. 3, 1988, pp. 157-166.
16. A. Ghobarah. Seismic Behavior of Highway Bridges with Base Isolation. Canadian Journal of Civil Engineering, Vol. 15, No. 1, 1988, pp. 72-78.
17. I. G. Buckle and R. L. Mayes. The Application of Seismic Isolation to Bridges. Proceedings, ASCE Structures Congress, Seismic Engineering: Research and Practice, 1989, pp. 633-642.
18. J. P. Wolf. Dynamic Soil-Structure Interaction. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1985.
19. J. P. Wolf. Soil-Structure Interaction Analysis in Time Domain. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1988.
20. F. E. Richart, Jr., J. R. Hall, Jr., and R. D. Woods. Vibrations of Soils and Foundations. Prentice-Hall, International, Inc., New Jersey, 1970.
21. W. S. Tseng and J. Penzien. Analytical Investigations of the Seismic Response of Long Multiple-Span Highway Bridges. College of Engineering, U. C. Berkeley, Earthquake Engineering Research Center, Report No. EERC 73-12, June, 1973.
22. J. W. McGuire, W. F. Cofer, and D. I. McLean. Analytical Modeling of Foundations for Seismic Analysis of Bridges. Washington State Department of Transportation Final Report, October, 1993.
23. M. O. Eberhard, M. L. Marsh, T. O'Donovan, and G. Hjartarson. Lateral-Load Tests of Reinforced Concrete Bridge. In Transportation Research Record 1371, TRB, National Research Council, Washington, D. C., 1992, pp. 92-100.

24. A. S. Veletsos and B. Verbic. Vibration of Viscoelastic Foundations. Earthquake Engineering and Structural Dynamics, Vol. 2, 1973, pp. 87-102.
25. W. Y. Jean, T. W. Lin, and J. Penzien. System Parameters of Soil Foundations for Time Domain Dynamic Analysis. Earthquake Engineering and Structural Dynamics, Vol. 19, 1990, pp. 541-553.
26. NEHRP. Recommended Provisions for the Development of Seismic Regulations for New Buildings. Earthquake Hazards Reduction Series No. 65, FEMA, 1991.
27. D. I. McLean and I. B. S. Cannon. Seismic Analysis of the I-90 Mercer Slough Westbound Lanes, Washington State Department of Transportation Final Report, 1992.
28. I. B. S. Cannon. Seismic Analysis of the Interstate 90 Mercer Slough Westbound Lanes, Thesis submitted in partial fulfillment of the degree of Master of Science, Washington State University, August, 1992.
29. S. L. Kramer. Seismic Response - Foundations in Soft Soils, Washington State Department of Transportation Draft Technical Report, July, 1992.
30. T. Nogami and K. Konagai. Time Domain Flexural Response of Dynamically Loaded Single Piles. Journal of Engineering Mechanics, ASCE, Vol. 114, No. 9, September, 1988, pp. 1512-1525.
31. A. S. Veletsos and B. Verbic. Vibration of Viscoelastic Foundations. International Journal of Earthquake Engineering and Structural Dynamics, Vol. 2, No. 1, 1973, pp. 87-102.