MODELING AND MEASURING FERRY BOAT WAKE PROPAGATION

Brad Perfect[1]
Dr. Jim Thomson [2]
Dr. James Riley [3]

ABSTRACT

Surface waves generated by vessels were studied for two classes of ferries in the Washington State Department of Transportation (WSDOT) system. The vessel wakes were measured using a suite of free-drifting GPS buoys. The wakes were modeled using an analytical potential flow model, which has a strong dependence on vessel Froude number and vessel aspect ratio. Both the measurements and the model are phase-resolving, and thus direct comparisons are made, in addition to comparing bulk parameters. The model has high fidelity for the parameters describing the operating conditions for WSDOT vessels, successfully reproducing the measured difference between the two vessel classes and the dependence on vessel speed for each vessel class.

Keywords: Free Surface, Ship Wake, Kelvin Wake.

---

[1] PhD Student, Dept. of Mechanical Engineering. Univ. of Washington. 3900 Stevens Way NE, Seattle, WA 98195. E-mail: bperfect@uw.edu.

[2] Principal Oceanographer, Applied Physics Laboratory. Univ. of Washington. 1013 NE 40th St, Box 355640, Seattle, WA 98105. E-mail: jthomson@apl.washington.edu

[3] PACCAR Prof. of Engr., Dept. of Mechanical Engineering. Univ. of Washington. 3900 Stevens Way NE, Seattle, WA 98195. E-mail: rileyj@u.washington.edu

INTRODUCTION

Washington State Ferries (WSF) operates a fleet of 22 vessels in the Puget Sound region of Washington State, USA, and British Columbia, Canada. It is the largest ferry system in the United States and services the highest volume of automobiles of any ferry service in the world. The Seattle-Bremerton route, servicing 2.33 million passengers and 642,000 vehicles per year (Smith 2013), is the primary subject of the current study. In a section of this route named Rich Passage, denoted by the box in Fig. **??**, the channel narrows to 600m (**?**). Due to the narrow channel and steep bathymetry of the region, vessel wake-induced erosion is a concern for the surrounding shoreline.

A vessel operating protocol (power limit) of 8.2 m/s (16 knots) is presently in place to reduce wake-induced morphological effects within the channel (Klann 2002). The WSF vessels that serve the Seattle-Bremerton route are known as Issaquah Class and Super Class vessels. While of qualitatively similar size, the Super Class vessels are longer, narrower, and heavier than the Issaquah Class vessels. The protocol is applied to the Issaquah Class only, because the Issaquah Class wakes have been classified, visually, as much larger than the Super Class wakes. This paper seeks to quantify the differences between the wakes of the two vessel classes through the use of field data and an analytical model.

Rich Passage has been the subject of a comprehensive wake and shoreline study conducted upon the introduction of the Rich Passage I, a passenger-only fast ferry that served the Seattle-Bremerton route for Kitsap Transit. The study, conducted by Golder and Associates, concluded that the Rich Passage I meets wake height guidelines specified by

$$H = \begin{cases} H < 0.2 & T \leq 3.5 \\ H < 1.16T^{-1.4} & T > 3.5 \end{cases} \tag{1}$$

where H is the the wake height in meters measured 300m from the sailing line of the vessel, and T is the wake period in seconds. This criterion is based on the observation that low-frequency waves contain more erosive potential because of their higher energy flux, and therefore have a more restrictive associated height limit. The morphological shoreline response was largely inconclusive (Cote et al. 2013).

WSF commissioned a similar study of the car ferry fleet in 1985 to measure vessel wakes (Nece et al. 1985). This study involved Issaquah, Superclass, and Evergreen class models in open water in Puget Sound. No modeling efforts accompanied this study, and no curve fitting or statistical analysis were performed. The results establish qualitatively that the Super class vessel wakes grow at a much slower rate as vessel speed increases when compared to the Issaquah class. However, noise in wake data and difficulties in establishing the buoy-boat distance make the conclusions difficult to quantify. Despite these shortcomings, it was found that the data from this study could be used to supplement the field data from the current study. Uncertainty in wake height due to natural variation in time-dependent surface wave conditions dominated uncertainty due to the buoy-boat distances.

The more general objective of predicting boat wakes has generally been approached as either a computational fluids problem, or as an empirical quantification of wave height. Previous modeling efforts of phase resolved vessel wakes have been confined to domains of only a few boat lengths due to computational requirements (Raven 1998), which provide limited information about potential shoreline impact. Sophisticated near-vessel models are primarily used to model wave breaking and wave making resistance for naval architectural applications (Landrini et al. 1999; Huang and Yang 2016). Phase-averaged models have been successfully implemented (Cote et al. 2013), but the resultant wake structure is lost. Other phase-resolved models have focused on vessels with transcritical Froude numbers in shallow water (Kofoed-Hansen et al. 1999). Empirical models have been used to great success within the bounds of the space sampled (Sorensen 1997; Kriebel et al. 2003; Ng and Bires)

The analytic study of ship wakes was first considered mathematically in 1887, when Lord Kelvin showed that the disturbance caused by a point pressure source moving across a fluid surface forms a predictable structure which now bears his name (Thomson 1887). Subsequent work focused on the geometry and structure of wakes, with extensions for arbitrary motion of the pressure point (Stoker 1957). Significant gains have been made more recently, with results detailing surface velocity fields (Yun-gang and Ming-de 2006), the inclusion of viscous forces (Lu and Chwang 2007), and finite vessel dimensions (Darmon et al. 2014; Benzaquen et al. 2014).

The impact of boat wakes on shoreline, and mitigation thereof, has been a continuous concern for caretakers of waterways, as well as a subject of many previous studies (Glamore 2008; Aage et al. 2003; Verhey and Bogaerts 1989). In particular, work has been done on the sediment transport associated with vessel-generated waves (Velegrakis et al. 2007).

The effects of bow wakes, which are known to have complex hull-dependent forms (Noblesse et al. 2013) are not considered in this study, though results suggest that they may dominate the vessel-generate wave field for low Froude number.

METHODS

Analytical Model

The first analytic study of wake patterns was published in 1887, when Lord Kelvin showed that the disturbance caused by a point pressure source moving across a fluid surface forms a predictable structure which now bears his name (Thomson 1887). Subsequent work focused on the geometry and structure of wakes, with extensions for arbitrary motion of the pressure point (Stoker 1957). A closed form expression for the sea surface height caused by a finite pressure distribution moving across a fluid surface was recently developed (Benzaquen et al. 2014; Darmon et al. 2014). The equations developed by Darmon, Benzaquen, et al. are the basis for the analytical modeling presented below.

The analytical model must be confined to potential flow theory in order to yield tenable solutions for the sea surface height. Therefore, the restrictions

$$\mu = 0 \qquad \text{and} \qquad \vec{\omega} = \nabla \times \vec{u} = 0, \tag{2}$$

are required. Here, μ is the viscosity of the flow, $\vec{u}$ is the flow field in Euclidean coordinate space, and $\vec{\omega}$ is the vorticity.

The sea surface height generated by a moving pressure distribution $p(x, y)$ (Raphaël and deGennes 1996) is given by

$$\zeta(x, y) = -\lim_{\epsilon \to 0} \int_0^\infty \int_0^\infty \frac{dk_x dk_y}{4\pi^2 \rho} \frac{k\hat{p}(k_x, k_y) e^{-i(k_x x + k_y y)}}{\omega(k)^2 - U^2 k_x^2 + 2i\epsilon U k_x} \tag{3}$$

where $\hat{p}(k_x, k_y)$ is the Fourier transform of the pressure distribution $p(x, y)$, ρ is the constant fluid density, $\omega(k)$ is the surface wave dispersion relation, $k = \sqrt{k_x^2 + k_y^2}$, and U is the pressure distribution's constant forward velocity. The parameter ϵ is a variable with units s^{-1} that approaches zero, ensuring that the radiation boundary condition is satisfied. For a derivation of Eq. (3), the reader is referred to Raphaël and deGennes (1996). Define a coordinate system such that the vessel is located at the origin, and is moving in the $-x$-direction, with the sea surface displacement in the z-direction. The y-axis defines the neutral sea surface perpendicular to the sailing line of the vessel. Following the method developed by Darmon et al., the nondimensionalization conditions for a boat of length b are:

$$\begin{gathered} X = \frac{x}{b} \qquad Y = \frac{y}{b} \qquad K_X = k_x b \\ K_Y = k_y b \qquad Z = \frac{4\pi^2 \zeta}{b} \qquad \hat{P} = \frac{\hat{p}}{\rho g b^3} \qquad \tilde{\epsilon} = \epsilon \sqrt{\frac{b}{g}}, \end{gathered} \tag{4}$$

Nondimensionalization yields

$$\begin{aligned} \frac{Z(X,Y)b}{4\pi^2} &= -\lim_{\tilde{\epsilon}\to 0} \int_{-\pi/2}^{\pi/2} \int_0^\infty \frac{dK_X dK_Y}{4\pi^2 b^2 \rho} \frac{\frac{K}{b}\rho g b^3 \hat{P}(K_x, K_y) e^{-i(K_X X + K_Y Y)}}{\frac{gK}{b} - U^2 \frac{K_X^2}{b^2} + 2i\tilde{\epsilon}\sqrt{\frac{g}{b}} U \frac{K_X}{b}} \\ Z(X,Y) &= -\lim_{\tilde{\epsilon}\to 0} \int_{-\pi/2}^{\pi/2} \int_0^\infty \frac{dK_X dK_Y}{b^4 \rho} \frac{K \rho g b^3 \hat{P}(K_x, K_y) e^{-i(K_X X + K_Y Y)}}{\frac{gK}{b} - U^2 \frac{K_X^2}{b^2} + 2i\tilde{\epsilon}\sqrt{\frac{g}{b^3}} U K_X} \\ &= -\lim_{\tilde{\epsilon}\to 0} \int_{-\pi/2}^{\pi/2} \int_0^\infty \frac{dK_X dK_Y K \hat{P}(K_x, K_y) e^{-i(K_X X + K_Y Y)}}{K - Fr^2 K_X^2 + 2i\tilde{\epsilon} Fr K_X} \end{aligned} \tag{5}$$

An elliptic transform

$$X = \breve{R}\cos\breve{\varphi} \qquad Y = W\breve{R}\sin\breve{\varphi} \qquad K_X = \breve{K}\cos\breve{\theta} \qquad K_Y = W^{-1}\breve{K}\sin\breve{\theta}, \tag{6}$$

permits solutions for an elliptic Gaussian pressure distribution. Define an aspect ratio $W > 0$ to be the ratio of the y to x characteristic dimensions of $P(x, y)$. Applying this transformation,

$$\begin{aligned} & Z(\breve{R}, \breve{\varphi}) = \\ & -\lim_{\tilde{\epsilon}\to 0} \int_{-\pi/2}^{\pi/2} \int_0^\infty \frac{\breve{K}}{W} d\breve{K} d\breve{\theta} \frac{\breve{K}\sqrt{\cos^2\breve{\theta} + W^{-2}\sin^2\breve{\theta}}\,\breve{\hat{P}}(\breve{K}) W e^{-i\breve{K}\breve{R}(\cos\breve{\varphi}\cos\breve{\theta} + \sin\breve{\varphi}\sin\breve{\theta})}}{\sqrt{\cos^2\breve{\theta} + W^{-2}\sin^2\breve{\theta}} - Fr^2 \breve{K}^2 \cos^2\breve{\theta} + 2i\tilde{\epsilon} Fr \breve{K}\cos\breve{\theta}}, \end{aligned} \tag{7}$$

where the Jacobian of the coordinate transformation in Eq. (6) is $\frac{\breve{K}}{W}$, and $\breve{\hat{P}}(\breve{K}) = \frac{\hat{P}(K_X, K_Y)}{W}$ to account for the reduced dimension. Simplifying,

$$
\begin{aligned}
&Z(\breve{R},\breve{\varphi}) = \\
&-\lim_{\tilde{\epsilon}\to 0}\int_{-\pi/2}^{\pi/2}\int_{0}^{\infty}\frac{\sqrt{W^2\cos^2\breve{\theta}+\sin^2\breve{\theta}}\,\breve{\hat{P}}(\breve{K})e^{-i\breve{K}\breve{R}\cos(\breve{\theta}-\breve{\varphi})}\breve{K}d\breve{K}d\breve{\theta}}{\sqrt{W^2\cos^2\breve{\theta}+\sin^2\breve{\theta}}-WFr^2\breve{K}\cos^2\breve{\theta}+2i\tilde{\epsilon}FrW\cos\breve{\theta}}.
\end{aligned}
\tag{8}
$$

The Sokhotski-Plemelj formula (Appel 2007) states that for a function $f(x)$ with Cauchy principal value Q,

$$
\lim_{\tilde{\epsilon}\to 0}\int_a^b \frac{f(x)}{x\pm i\tilde{\epsilon}}dx = \mp i\pi f(0) + Q\int_a^b \frac{f(x)}{x}dx. \tag{9}
$$

Eq. (8) is manipulated into this form using

$$
\begin{aligned}
x &= \frac{\sqrt{W^2\cos^2\breve{\theta}+\sin^2\breve{\theta}}-WFr^2\breve{K}\cos^2\breve{\theta}}{2FrW\cos\breve{\theta}} \qquad dx = -\frac{Fr\cos\breve{\theta}}{2}d\breve{K} \\
f(x) &= \breve{K}\frac{\sqrt{W^2\cos^2\breve{\theta}+\sin^2\breve{\theta}}\,\breve{\hat{P}}(\breve{K})e^{-i\breve{K}\breve{R}\cos(\breve{\theta}-\breve{\varphi})}}{Fr^2W\cos^2\breve{\theta}}.
\end{aligned}
\tag{10}
$$

The zero point in f is determined uniquely by $\breve{K}$ since it is the only independent variable. Therefore, designate

$$
\breve{K}_0 \equiv \frac{\sqrt{W^2\cos^2\breve{\theta}+\sin^2\breve{\theta}}}{WFr^2\cos^2\breve{\theta}} \tag{11}
$$

by solving with $x = 0$ in $\breve{K}$. It is argued in Benzaquen et al. (2014) that $Q\int_a^b f(x)dx/x$ is a rapidly decreasing function that can be ignored. Therefore, the sea surface height is given by

$$
Z(\breve{R},\breve{\varphi}) \approx i\pi\int_{-\pi/2}^{\pi/2}\frac{\breve{\hat{P}}(\breve{K}_0)\left(W^2\cos^2\breve{\theta}+\sin^2\breve{\theta}\right)e^{-iK_0R\cos\left(\breve{\theta}-\breve{\varphi}\right)}}{W^2\mathrm{Fr}^4\cos^4\breve{\theta}}d\breve{\theta}. \tag{12}
$$

The pressure distribution is assumed to be of the form

$$p(x,y) = \frac{\pi m g}{W b^2} e^{-\left(\frac{x\pi}{b}\right)^2 - \left(\frac{y\pi}{Wb}\right)^2}. \tag{13}$$

The leading coefficient is chosen such that the integral of the pressure function Eq. (13) over the plane of the sea surface is equal to the total force of the vessel on the water, mg (i.e., $p(x,y)$ has a net 'weight' equal to that of the vessel.)

The b and W parameters can be tuned to approximate the geometry of the physical hulls. To that end, $p(x,y)$ may be converted into an equivalent $\zeta(x,y)$ to represent the sea surface elevation at which that pressure is experienced:

$$\zeta(x,y) = \frac{p(x,y)}{\rho g}. \tag{14}$$

The volume between $\zeta(x,y)$ and the xy axis defines a 3-dimensional figure, S that can be geometrically compared to a CAD hull model, C. In subsequent analysis, the W and b parameters are chosen such that the absolute norm $\int_{\mathbb{R}^3} |S - C| \, dV$ is minimized. The pressure distribution then matches the weight of the vessel and the shape is tuned to match that of the physical hull, within the constraints of the analytical model.

The physical pressure distribution is then converted into a form compatible with Eq. (12). In the nondimensional form specified by Eq. (4),

$$P(X,Y) = \frac{\pi m}{W \rho b^3} e^{-(X\pi)^2 - \left(\frac{Y\pi}{W}\right)^2}. \tag{15}$$

After a Fourier transform,

$$\hat{P}(K_X, K_Y) = \frac{m}{2\pi \rho b^3} e^{-\frac{1}{4\pi^2}\left(K_X^2 + W^2 K_Y^2\right)}. \tag{16}$$

And finally, applying the elliptic transform Eq. (6),

$$\breve{\hat{P}}(\breve{K}) = \frac{m}{2\pi\rho b^3} e^{-\frac{\breve{K}^2}{4\pi^2}}. \tag{17}$$

Measurements

On 17 September 2014 and 18 November 2014, the wakes of the ferry boats servicing the Seattle-Bremerton route were measured via data buoys deployed from R/V Jack Robertson. An approximate sailing line for an approaching ferry was obtained from Automatic Identification System (AIS) vessel tracking data. Buoys were then deployed in a line running approximately perpendicular to the predicted sailing line of the ferry. The ferry crossing time was recorded as the time when the AIS coordinates of the ferry intersected the time-dependent best fit line of the array of buoys.

The data buoys in use consisted of four SWIFT and eight μSWIFT buoys (Thomson 2012). These buoys are free-floating spar-type buoys equipped with GPS loggers. The μSWIFT uses a QStarz BT-Q1000eX and the SWIFT uses a Microstrain 3dm-35. Data were sampled at rates varying from 4 Hz to 10 Hz. The GPS logs both positional velocimitry data, but the horizontal positional accuracy of commercial GPS devices such as this one are only on the order of 10m–significantly larger than is required to phase-resolve individual waves. The relative horizontal velocity resolution, however, is approximately 0.05 m/s through Doppler phase processing of the raw signals (Herbers et al. 2012; Thomson 2012). The vertical elevation and velocity do not have sufficient accuracy to be considered for data processing. Therefore, the positional GPS data are used to obtain approximate locations and trajectories of the buoys, and the horizontal velocity data are used to capture the motion of the sea surface. Typically, buoy motion due to ocean current was less than 1 m/s.

In linear wave theory, the motion of inertial particles influenced by sinusoidal wave motion is given by

$$v(t) = a\omega e^{kz} \cos\theta(t) \tag{18}$$

for wave amplitude a, angular frequency ω, wavenumber k, and distance below the surface z. Similarly, the surface elevation is given by

$$\zeta(t) = a\cos\theta(t). \tag{19}$$

For a particle on the surface, the sea surface height may be obtained from the velocity via the relation

$$\zeta(t) = \frac{v(t)}{\omega(t)}. \tag{20}$$

The natural frequency of both types of drifter buoy is well above the frequencies of the observed wake pattern. As such, the drifters are treated as inertial surface particles. The goal of the time domain analysis is, therefore, to obtain accurate and well-behaved values for ω and $v(t)$ in order to reconstruct the sea surface height. Here, well-behaved refers to avoiding low-frequency velocity drift and artificially small values of ω.

The surface reconstruction is performed by a process of

- Calculating and subtracting low-pass filtered velocity vectors, in order to remove the effects of buoy drift
- Obtaining a scalar velocity by calculating the principal axes for buoy kinetic energy in the plane of the ocean and taking only the velocity of the major axis
- Calculating a local wave frequency ω by calculating the zeros in the scalar velocity
- Applying Eq. (20) to obtain the sea surface height. The conversion from

scalar velocity to sea surface height is shown in Fig. **??**

In a rough sense, the surface disturbance caused by a passing ferry boat can be expressed as a near-Dirac disturbance, or equivalently, a broadband wave generation. Denote such a function

$$\zeta(x,0) = \int_{-\infty}^{\infty} a(k) e^{ikx} dk \tag{21}$$

where $a(k)$ specifies a magnitude associated with a given wavenumber k. Then, for the outward spreading of this disturbance, the dispersion relation $\omega(k) = \sqrt{gk}$ is used. This is the deep water limit, which for the shallowest in-situ depth encountered of 20 meters, is permissible for wavelengths of less than 40 meters. The time-dependent sea surface height is then

$$\zeta(x,t) = \int_{-\infty}^{\infty} a(k) e^{i\left(kx-\sqrt{gk}t\right)} dk. \tag{22}$$

In the limit of large x and t (as in the case of the field data), a stationary phase approximation (Stoker 1957) may be invoked in order to evaluate Eq. (22), yielding

$$\zeta(x,t) \approx a(k_0)\sqrt{\frac{gt^2}{\pi x^3}} \cos\left(k_0(x) - \omega(k_0)t \pm \frac{\pi}{4}\right) \tag{23}$$

where $k_0 = gt^2/(4x^2)$ solves the equation $\partial/\partial k \left[kx - \sqrt{gk}t\right] = 0$.

Therefore, for a given (x,t) displacement from the initial disturbance (i.e., a time t after a vessel's closest approach to a point x of observation), the observed frequency of waves should be given by

$$\omega(x,t) = \frac{gt}{2x}. \tag{24}$$

Note that in Eq. (24) the expression for velocity is the group velocity, $g/(2\omega)$, rather than the phase velocity. For constant $x = d$ (i.e. a buoy that is stationary in the reference frame of the ocean surface) the observed frequency should increase linearly in time with slope $g/(2d)$. Comparisons of predicted and observed wave frequencies (Fig. **??** being a typical case) show that treating the vessels as generating broadband wave pulses serves as a good predictor for the arrival of wave frequencies observed by the buoys. This is a useful check in cases where the distance from sailing line is difficult to determine.

RESULTS

Analytical Model

The analytical model described by Eq. (12) and Eq. (13) produces a steady state sea surface height as a function of boat length b, boat aspect ratio W, boat mass m, water density ρ, and the boat velocity. If Eq. (12) is expressed with the leading coefficients in dimensional form, the expression becomes

$$\zeta(x,y) = \frac{m}{4\pi^2 \rho b^2} \mathrm{Im} \left[\int_{-\pi/2}^{\pi/2} \frac{e^{-\frac{\breve{K}_0}{4\pi^2}} \left(W^2 \cos^2 \breve{\theta} + \sin^2 \breve{\theta} \right) e^{-iK_0 R \cos\left(\breve{\theta} - \breve{\varphi}\right)}}{W^2 \mathrm{Fr}^4 \cos^4 \breve{\theta}} d\breve{\theta} \right] \tag{25}$$

One may conclude, then, that the sea surface height scales linearly as the boat mass, and inversely as the water density. Eq. (25) contains a complex dependence on the remaining variables. These remaining dependencies are shown graphically in Fig. **??**. The sea surface forms a distinctive Kelvin wake pattern. In Fig. **??**, two such solutions are shown, for different values of W. At low W, the largest waves are located at the intersection between the angled diverging wake and the approximately horizontal transverse wake. At $W = 1$, indicating a circular pressure distribution, the greatest perturbation in the sea surface is located directly behind the vessel, in the centers of the transverse wake waves. The growth of the diverging wake relative to the transverse wake is typically associated with a larger vessel Froude number (higher velocity, or smaller vessel length when all else is held constant). The effects of this transition are further explored on page 16.

The plots in Fig. **??** explore the maximum wake height, as this is the relevant parameter under study for the ferry boats. Maximum wake height is defined to be the absolute value of the greatest deformation in the free surface, at a given perpendicular distance d from the axis of vessel motion. Of the variables other than m and ρ, the maximum wake height is determined by the Froude number and the aspect ratio W. However, because the Froude number contains two physically relevant variables, velocity and characteristic length, their independent effects are explored. In each case, in Fig. **??**, the vessel mass and water density are unchanged.

In order to address the physical wakes, the model parameters were chosen to represent an Issaquah class and a Super class vessel. Using the available vessel dimensions, mass, and CAD models, Gaussian pressure distributions of the form in Eq. (13) were created. The resulting wake profiles, shown in Fig. **??** and generated from the values in Table **??**, were highly dependent on the boat length b, but in all cases, the Issaquah class model produced much larger wakes than the Super class model. The presented models match velocity rather than Froude number, for dual reasons. The ferry operating conditions are determined by the time schedule, and so the same velocity must be attained when comparing performance. In addition, the Froude number of the Issaquah class is larger by a small enough amount that slowing the vessel to attain Froude number parity would still not eliminate the difference in wake height. The wake profiles, presented at the same scale, indicate significantly smaller wake for the Super Class, as well as a weaker diverging wake relative to the transverse wake.

Measurements

A total of 19 ferry crossings were recorded, yielding 79 useful wake histories from the drifter buoy data. These wake histories represent 65 measurements of the Issaquah class at a variety of vessel speeds, and 14 measurements of the Super class, all at approximately its standard operating speed of 18 knots.

For one wake event, a μSWIFT and a SWIFT buoy were placed within 10m of each other in order to compare buoy performance. In general, the SWIFT buoy provides less noisy data, but the comparison in Fig. **??** suggests that both varieties of buoy have similar spectra at the frequencies of interest.

The raw buoy velocities also indicate close agreement. However, the sea surface height reconstruction is highly sensitive to the exact recorded velocities. Despite qualitatively similar buoy velocities, the final sea surface height differs considerably between the two measurements. This reflects the highly localized nature of sea surface velocities. The spread in maximum sea surface height at consistent experimental conditions (see Fig. **??**) is qualitatively similar for both buoy types when the aggregate data are taken. For this reason, data from both buoy types are used interchangeably.

The angular frequency associated with the maximum wake amplitude for a wake event was recorded. In general, the observed frequency was lower at larger distances from the sailing line, as the dispersion relation for deep water waves would suggest. However, the relationship between angular frequency and maximum wake height, Fig. **??**, demonstrated a trend that appears to depend on the vessel speed. The most energetic waves are the ones with low angular frequency and large maximum height. Therefore, it is particularly concerning that the largest wakes associated with higher vessel speeds are also the lowest frequency wakes.

Comparisons

Because one of the primary concerns for WSDOT ferry wakes is the overtopping of sea walls, and the shoreline morphology is quite steep, the maximum sea surface height is considered as a metric for comparison. Maximum sea surface height versus distance from sailing line was compared across all measurements and models. The analytical model's parameters were tuned to match the shape and mass of the Issaquah and Super Class hulls, as described on page 4. Fig. **??** displays the overall comparison between the various wake measurements studied. The analytical model does a good job of capturing the decay in wake height with increased distance from the sailing line that is observed in the field data. At low vessel speed, however, the analytical model tends to predict a smaller disturbance than is physically observed.

Another method of visualizing the model's predictive ability is to treat each buoy as an inertial particle moving on the surface of the ocean, in the reference frame of the vessel. The buoy's location is converted to a time varying displacement from the vessel, and the analytic solution is calculated along the transect the buoy takes, parameterized by time. The parameterized analytical solution $\zeta(t)$ then mimics the history of the sea surface height experienced by a numerical buoy taking the same trajectory as the buoy in the field. This permits direct comparisons with reconstructed sea surfaces from the wave buoys.

Several such comparisons are presented in Fig. **??**. This method is quite sensitive to clock synchronization between the vessel and buoy, as well as vessel speed. For each comparison, the vessel class and speed are matched between the analytical model and the observations for the measured vessel crossing. The initial wake structure is well-captured, though the timing may have an offset due to ocean currents and the exact location of the AIS receiver on the ferry. The wave packeting that is often seen by the physical buoys was not reliably reproduced by the model, and the reasons for this are explored on 16.

DISCUSSION

Models and data indicate that at similar operating speeds, the Issaquah Class vessels make significantly larger wakes than the Super Class vessels. This result is somewhat unintuitive, as the Super Class is a longer and heavier vessel. Results from the analytical model point to a dual justification for the wake height disparity. The increased vessel length for the Super Class has the effect of reducing the vessel Froude number, and the decreased aspect ratio W also works to decrease the wake. These two effects counteract the increased weight to yield an overall lower wake.

Furthermore, wake height reduction is achieved by reducing the vessel speed for the Issaquah Class. The Super Class data do not include significant speed variation, but the data displayed in Fig. **??** demonstrate that the full-speed Superclass produces much smaller peak wake height than the full-speed Issaquah, and qualitatively comparable wake heights to the speed-limited Issaquah class. The field data support the conclusions obtained in the 1985 study (Nece et al. 1985). Analytic data indicate a roughly linear relationship between peak wake height at a given distance from the sailing line and vessel speed in the regime of interest. Therefore, in assessing a given distance and peak height objective, the field data plots should be used as two data points, with a linear interpolation to determine appropriate speed.

At low Froude numbers, the transverse wake dominates the wake field of a vessel. However, field observations and CFD modeling have shown that, particularly at low Froude numbers, the vessels produce a significant bow wake. The bow wake is a consequence of the physical displacement of the water in front of the vessel and is not captured by the model. Particularly at the lowest operational speeds of the ferries, the bow wake dominates the waves produced by the vessel. This is the proposed mechanism for the model failure at low Froude number (below 0.35).

In addition, the packeting observed in Fig. **??** may be a beat frequency produced by the interaction of the bow wake and the Kelvin wake. At $Fr > 0.35$, the bow wake was not observed to affect the maximum wake height measurements.

CONCLUSIONS

The wakes produced by Issaquah and Super Class WSF vessels were studied using an array of GPS buoys and with an analytic potential flow wake model. Each of these methods produced results that indicate a maximum wake height for a given distance from the sailing line of the vessel and a given vessel speed. These results are summarized in Fig. **??**. The in-situ buoy measurements are regarded to be the most reliable, but also the contain the most sparse data, as well as significant noise. A higher-fidelity sea surface height reconstruction may be possible using vertical acceleration, given a high enough sampling rate. The 4-10 Hz sampling rate used presently was insufficient to accurately represent buoy heave.

Despite being an older and larger vessel, the Super Class ferry produces smaller wakes than the Issaquah class at a given speed. The relatively simple analytic representation of these vessels is able to attribute difference in wake height to the geometric qualities of aspect ratio and vessel length, as well as vessel mass. The measurements and subsequent model comparison constitute a novel approach the question of vessel wake management.

NOTATION

Variable Name	Description
ρ	Seawater density. Assigned a value of $1025 kg/m^3$ for Puget Sound.
ω	Angular frequency of a wave.
$\{x, y, z\}$	Cartesian coordinates in the frame of the ferry boat. x denotes the streamwise direction, y is the crossstream direction, and z is the vertical direction. The origin of the grid is located at the mass centroid in x and y, and at the flat sea surface in z.
k_x, k_y	Wavenumbers associated with the analytic wake model.
W	Aspect ratio of a ferry boat in the analytic wake model. This is the ratio of the major to minor axis of the ellipse representing the boat. A smaller W implies a more streamlined vessel.
b	Characteristic length of the ferry.
Fr	Length Froude number, defined by $U/(\sqrt{gb})$.
U	Free stream velocity of flow past the ferry boat.
ζ	Value of the sea surface elevation, measured as a signed perturbation from a flat surface of value 0.
Z	Nondimensional sea surface, defined by $4\pi^2\zeta/b$.
g	Gravitational acceleration, given a value of $9.8 m/s^2$.
h	Maximum wake height. This is the maximum of the absolute value of the perturbation in the free surface.
a	Wave amplitude, for the purpose of linear wave analysis.

REFERENCES

Aage, C., Bell, A., Bergdahl, L., Blume, A., Bolt, E., Eusterbarkey, H., Hiraishi, T., Kofoed-Hansen, H., Maly, D., Single, M., Rytkonen, J., and Whittaker, T. (2003). *Guidelines for Managing Wake Wash From High-Speed Vessels.*

Appel, W. (2007). *Mathematics for Phyiscs and Physicists.* Princeton University Press.

Benzaquen, M., Darmon, A., and Raphael, E. (2014). "Wake pattern and wave resistance for anisotropic moving disturbances." *Physics of Fluids.*

Cote, J., Curtiss, G., and Osbourne, P. (2013). "Rich passage I wake validation and shore response study." *Ports.*

Darmon, A., Benzaquen, M., and Raphael, E. (2014). "Kelvin wake pattern at large froude numbers." *J. Fluid Mechl.*

Glamore, W. C. (2008). "A Decision Support Tool for Assessing the Impact of Boat Wake Waves on Inland Waterways." *International Conference on Coastal and Port Engineering in Developing Countries*, (1), 20.

Herbers, T., Jessen, P., Janssen, T., Colbert, D., and MacMahan, J. (2012). "Observing ocean surface waves with gps-tracked buoys." *J. Atmos. Oceanic Technol.*, 29, 944–959.

Huang, F. and Yang, C. (2016). "Hull form optimization of a cargo ship for reduced drag." *Journal of Hydrodynamics, Ser. B*, 28(2), 173–183.

Klann, A. (2002). "Property owners, state, settle fast ferry lawsuit." *Hagens Berman Sobol Shapiro LLP*, <https://www.hbsslaw.com/cases/closed-case/pressrelease/closed-case-property-owners-state-settle-fast-ferry-lawsuit>.

Kofoed-Hansen, H., Jensen, T., Kirkegaard, J., and Fuchs, J. (1999). "Prediction of wake wash from high-speed craft in coastal areas." *Proc. Conf. Hydrodynamics of High Speed Craft*, 24–25.

Kriebel, D., Seelig, W., and Judge, C. (2003). "Development of a unified description of ship-generated waves (presentation).” 1–37.

Landrini, M., Colagrossi, A., and Tulin, M. (1999). “Breaking bow and stern waves: Numerical simulations.” *15th Int. Workshop on Water Waves and Floating Bodies*.

Lu, D. and Chwang, A. (2007). “Interfacial viscous ship waves near the cusp lines.” *Wave Motion*, 44(7-8), 563–572.

Nece, R., McCaslin, M., and Christensen, D. (1985). “Ferry wake study.” *Report No. WA-RD-70.1*, Washington State Dept. of Transportation.

Ng, M. and Bires, R. “Vessel wake study.

Noblesse, F., Delhommeau, G., Liu, H., Wan, D., and Yang, C. (2013). “Ship bow waves.” *Journal of Hydrodynamics*, 25(4), 491–501.

Raphaël, E. and deGennes, P. (1996). “Capillary gravity waves caused by a moving disturbance.” *Phys. Rev. E*.

Raven, H. (1998). “Inviscid calculations of ship wave making — capabilities, limitations and prospects.” *Proc. 22nd Symposium on Naval Hydrodynamics*.

Smith, B. (2013). “It’s a boat, its a train it’s super-transit!.” *Multimodal Operations Planning Workshop*.

Sorensen, R. (1997). “Prediction of Vessel-Generated Waves with Reference to Vessels Common to the Upper Mississippi River System.” (December 1997), 50.

Stoker, J. (1957). *Water Waves; the Mathematical Theory with Applications*. Interscience, New York.

Thomson, J. (2012). “Wave breaking dissipation observed with swift drifters.” *J. Atmos. Oceanic Technol.*, 29.

Thomson, W. (1887). “On the waves produced by a single impulse in water of any depth.” *Proc. R. Soc. London Ser. A42*, 80–83.

Velegrakis, A. F., Vousdoukas, M. I., Vagenas, A. M., Karambas, T., Dimou, K., and Zarkadas, T. (2007). “Field observations of waves generated by passing ships: A note.” *Coastal Engineering*, 54(4), 369–375.

Verhey, H. and Bogaerts, M. (1989). “Ship waves and the stability of armour layers protecting slopes.” *9th International Harbour Congress, Antwerp, Belgium*, Delft Hydraulics.

Yun-gang, W. U. and Ming-de, T. A. O. (2006). “An Asymptotic Solution of Velocity Field in Ship Waves.” *Journal of Hydrodynamics*, 18(4), 387–392.