Introduction to Nuclear Physics and Nuclear Decay

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Course website (Nuclear Medicine Imaging)

http://depts.washington.edu/uwmip/

Atoms

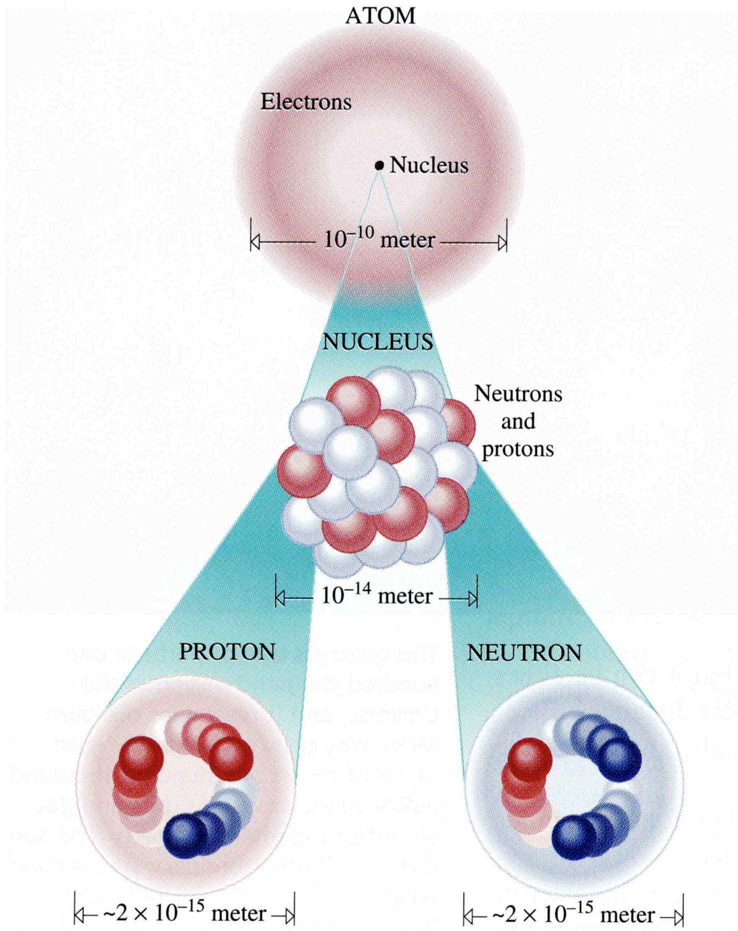
Nucleus: ~10^{-14} m diameter
~10^{17} kg/m^3

Electron clouds: ~10^{-10} m diameter
(= size of atom)

water molecule: ~10^{-10} m diameter
~10^3 kg/m^3

Nucleons (protons and neutrons) are ~10,000 times smaller than the atom, and ~1800 times more massive than electrons.
(electron size < 10^{-22} m (only an upper limit can be estimated))

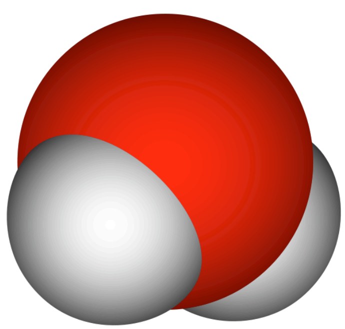
Nuclear and atomic units of length
10^{-15} = femtometer (fm)
10^{-10} = angstrom (Å)



Hecht, *Physics*, 1994

Molecules

mostly empty space



Water
(wikipedia)

Mass and Energy Units and Mass-Energy Equivalence

Mass

atomic mass unit, u (or amu): mass of $^{12}C \equiv 12.000000$ u

1 u = 1.660540×10^{-27} kg = 931.494×10^{6} eV/c^2

Energy

Electron volt, eV $\equiv$ kinetic energy attained by an electron accelerated through 1.0 volt

1 eV = 1.6×10^{-19} J

$$E = mc^2$$

$c = 3 \times 10^{8}$ m/s
speed of light

mass of proton, m_p	= 1.6724×10^{-27} kg	= 1.007276 u	= 938.3 MeV/c^2
mass of neutron, m_n	= 1.6747×10^{-27} kg	= 1.008655 u	= 939.6 MeV/c^2
mass of electron, m_e	= 9.108×10^{-31} kg	= 0.000548 u	= 0.511 MeV/c^2

Elements

Named for their number of *protons*

X = element symbol

Z (*atomic number*) = number of protons in nucleus

N = number of neutrons in nucleus

A (*atomic mass number*) = *Z* + *N*

[*A* is different than, but approximately equal to the *atomic weight* of an atom]

$$ {}^{A}_{Z}X_{N} \qquad {}^{A}_{Z}X \qquad {}^{A}X $$

Examples; oxygen, lead

Electrically neural atom, ${}^{A}_{Z}X_{N}$ has *Z* electrons in its atomic orbit. Otherwise it is *ionized*, and holds net electric charge.

$$ {}^{16}_{8}O_{8} \qquad {}^{208}_{82}Pb_{126} $$

Standard periodic table

Z

Group → / ↓ Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Uub	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo
			* Lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
			** Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Nuclide Groups/Families

A nuclide is a nucleus with a specific *Z* and *A*

~1500 nuclides exist

(Periodic Table typically lists distinct *Z*)

Nuclides with the same

Z (#protons) are ***Isotopes***

N (#neutrons) are ***Isotones***

A (#nucleons) are ***Isobars***

A nuclide with the same *Z* and *A* (& thus also *N*) can also exist in different (excited & ground) states; these are ***Isomers***

N vs. *Z* Chart of Nuclides

$N > Z$ for the majority
($N = Z$ for low Z elements)

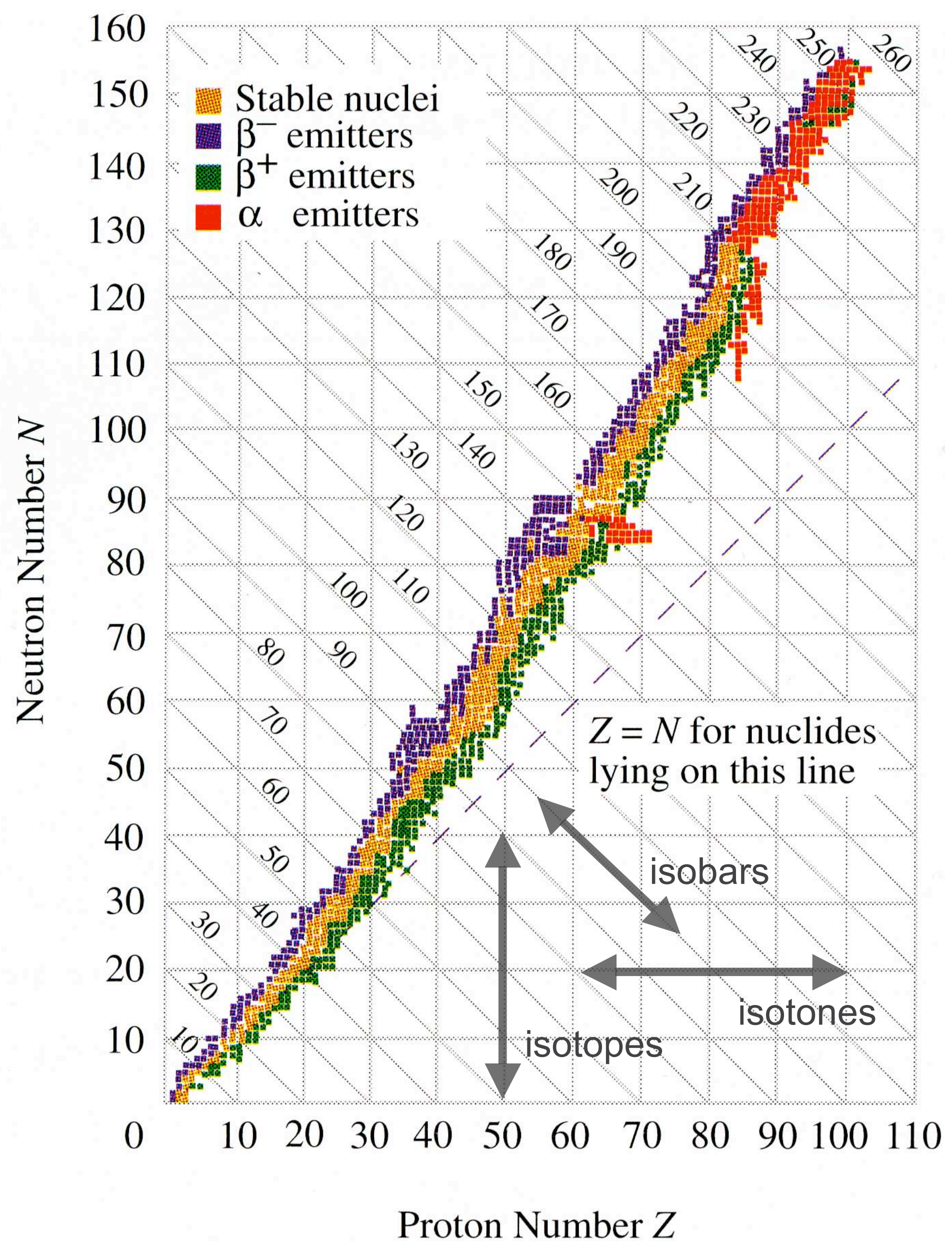
The *line of stability* (gold band) represents the stable nuclei.

Distribution of stable nuclei:

Z	N	#stable nuclei
even	even	165
even	odd	57
odd	even	53
odd	odd	4

279 stable nuclei exist
(all have $Z < 84$)

~1200 unstable (radioactive)
(65 natural, remaining are human-made)



Hecht, *Physics*, 1994

Nuclear Shell Structure

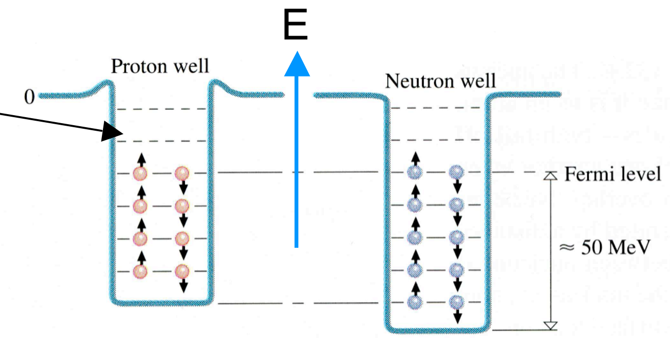
- Similar to atomic structure, the nucleus can be modeled as having quantized allowed energy states (shells) that the nucleons occupy.
- The lowest energy state is the *ground state*.
- Nuclei can exist in *excited states* with energy greater than the ground state.
- Excited nuclear states that exist for > 10^{-12} sec. are *metastable* states (*isomeric*).
- Nucleons held together by the 'strong force'; short range, but strong.
- This overcomes the repulsive electrostatic force of similar charged protons
- Also similar to atomic theory:
  - → Electrons swirl around in clouds about the nucleus; likewise, the nucleus is a dynamic swirl of nucleons.
  - → Nucleons, like electrons, are paired in energy states - each with opposite spin.
  - → Closed electron shells lead to chemically inert atoms. Magic numbers of nucleons (analogous to closed shells) form particularly stable nuclei.

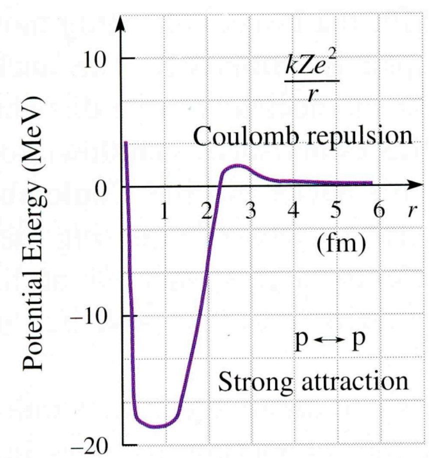
Schematic energy diagrams

E=0: particle is unbound (free)

E<0: particle is bound (e.g. in nucleus, in an atom)

E>0: free & has excess energy (can be potential or kinetic)





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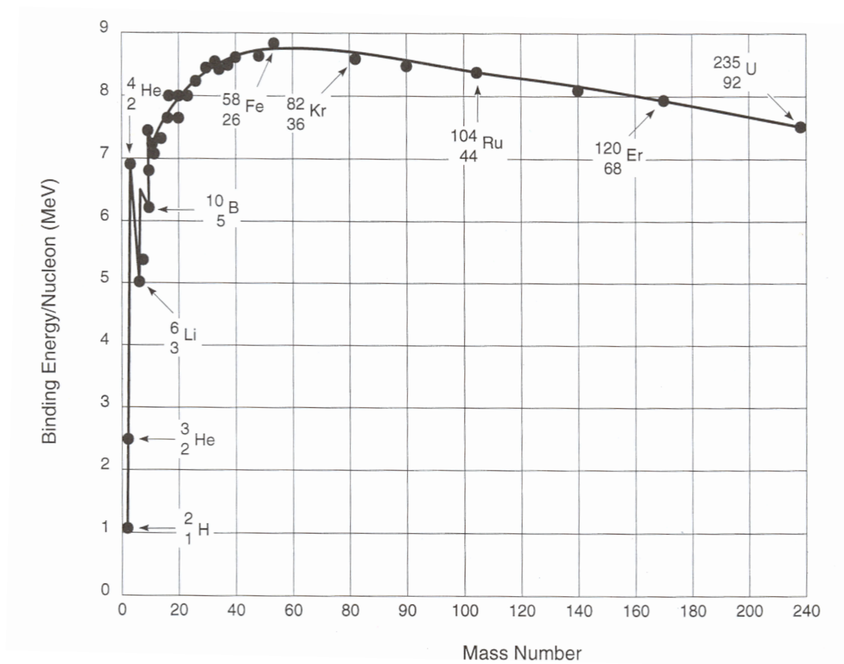
Binding Energy

The mass of a nuclide *is less than* the mass of the sum of the constituents. The difference in energy is the *binding energy*.

The consequence is that energy is liberated when nucleons join to form a nuclide.

The binding energy per nucleon dictates results when nuclides break apart (fission) or fuse together (fusion)

(keep in mind that binding energies are thought of as negative, as in energy level diagrams on previous slide)



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Radioactive Decay

Unstable nuclei change (decay) towards stable states

The transformation involves emission of secondary particles:

➔ Radiation

$$^{A}_{Z}X \rightarrow {}^{A'}_{Z'}Y^{[*]} + W + Q$$

X = parent nucleus, *Y* = daughter nucleus [possibly excited *],
W = radiation particle(s), *Q* = additional energy liberated in the decay; *Q* is shared between the *X*, *Y*, and *W* particles. *Y* is frequently unstable itself.

Conservation principles:

- Energy (equivalently, mass)
- linear momentum
- angular momentum (including intrinsic *spin*)
- charge

are all conserved in radioactive transitions

Radioactive Decay Processes

The decay processes are named for the (primary) radiation particle emitted in the transition:

- **alpha**

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha + Q $$

- **beta**

  isobaric

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z\mp 1}Y + \beta^{\pm} + \nu + Q $$

  alternative mechanism to β^+ decay is *electron capture*

- **gamma**

  isomeric

$$ {}^{A[m]}_{Z}X^{[*]} \rightarrow {}^{A}_{Z}X + \gamma $$

  alternative mechanism is *internal conversion*

The ionization (net charge) on particles can also be specified (upper-right)

Decay Time

The rate at which radionuclides decay is governed by a characteristic *decay time constant,* λ
(units of λ are inverse-time, i.e. frequency or rate)

$$N(t) = N_0 e^{-\lambda t}$$

$N(t)$ = number of radionuclides at time t
N_0 = number at time t = 0
λ = characteristic decay time constant

The *half-life*, $T_{1/2}$, is the time it takes for a sample to decay to one-half of its original number, or half of its original *activity*.

$$T_{1/2} = \frac{ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

$$N(t) = N_0 2^{-\left(\frac{t}{T_{1/2}}\right)}$$

Alpha Decay

An alpha particle is the same as a helium nucleus;

(two protons and two neutrons)

$$\alpha = {}^{4}_{2}\mathrm{He}^{+2}$$

General form of alpha decay process

$${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}^{+2} + \mathrm{Q}$$

- Alpha particle always carries Q energy as kinetic energy (monoenergetic)
- Alpha decay occurs with heavy nuclides (A > 150)
- Commonly followed by isomeric emission of photons,
- which can also result in electron emission (see internal conversion slide)

Beta Decay

A beta(minus, β^-) particle is an electron (or, it is indistinguishable from an electron).

There are also beta(plus, β^+) particles. These are indistinguishable from electrons, *except with positive charge* (of the same magnitude).

The general form of β^-, β^+ decay:

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + \beta^- + \bar{\nu} + \mathrm{Q} \qquad\qquad {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + \beta^+ + \nu + \mathrm{Q} $$

e.g. $${}^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + \beta^+ + \nu + 0.635\,\mathrm{MeV}$$

In β^- decay, a nuclear ***neutron is converted into a proton*** (Z→Z+1)
In β^+ decay, a nuclear ***proton is converted into a neutron*** (Z→Z-1)

In each case, the decay products include a neutrino (ν) or an anti-neutrino ($\bar{\nu}$)
Neutrinos have no charge, spin 1/2, and mass ~ 0.1 - 1 eV (?)

Beta decay is mediated by the 'weak force'.

Electron Capture

An alternative (and competing mechanism) to β^+ decay is electron capture.

In electron capture, a proton is converted to a neutron, as in β^+ decay, however, rather than emitting a β^+, an orbital electron (usually from inner electron shells) is captured by the nucleus, conversion of a proton to a neutron occurs, and a neutrino (and additional energy, Q) are emitted from the decay process:

$$^{A}_{Z}X + e^- \rightarrow {}^{A}_{Z-1}Y + \nu + Q$$

Capture of an electron creates a vacancy in an inner electron shell, which is filled by another electron from a higher shell. This results in characteristic x-rays, or Auger electrons.

An example of e.c. relevant to nuclear medicine is the following decay:

$$^{201}_{81}Tl + e^- \rightarrow {}^{201}_{80}Hg + \nu + Q$$

None of the products of this decay are used in imaging, rather, characteristic x-rays filling the vacancy are detected by gamma cameras.

Gamma Emissions

Gamma decay is an isomeric transition that follows the occurrence of alpha or beta decay.

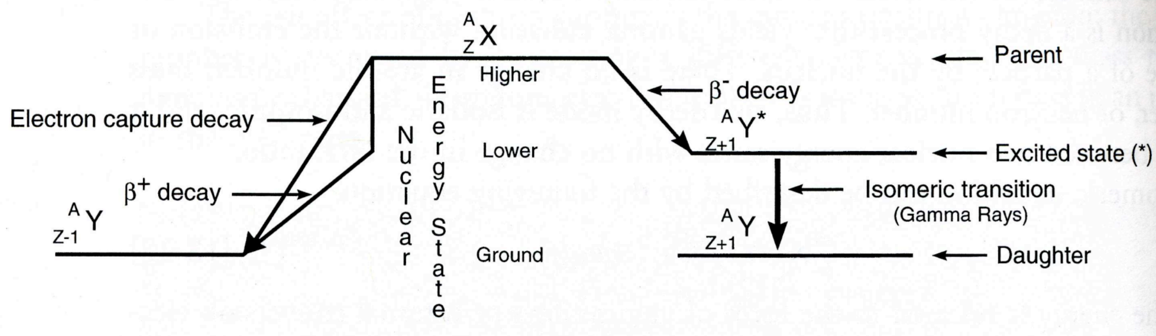
$$^{A[m]}_{Z}X^{[*]} \longrightarrow {}^{A}_{Z}X + \gamma$$

The parent in this case (which is the daughter of the preceding α or β decay, or electron capture) can be in an excited state, * ,that (essentially) immediately transitions to a lower state via emission of a gamma, or it can be in a *metastable* state *m*, which can have a life-time of between 10^{-12} sec. and ~600 years. Decay of metastable states also follow the exponential decay law, and thus have characteristic decay times.

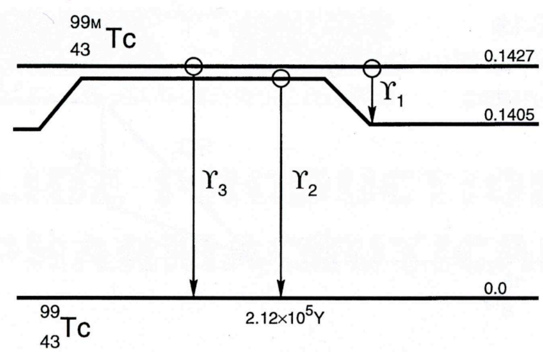
Internal Conversion

- Alternatively, the energy liberated from the isomeric transition can be delivered to an electron ejected from the atom (like Auger electrons vs. char. x-rays).
- Again, electrons rearrange to fill the vacancy left by the i.c. electron, resulting in characteristic x-rays and/or Auger electrons.
- Gamma emission and i.c. electron compete in the same nuclide decay.

Decay Schemes



Example: ^{99m}Tc



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Decay Data Table

Radiation		Mean Number per Disintegration	Mean Energ per Particl (MeV)
Gamma	1	0.0000	0.0021
M Int Con Elect		0.9860	0.0016
Gamma	2	0.8787	0.1405
K Int Con Elect		0.0913	0.1194
L Int Con Elect		0.0118	0.1377
M Int Con Elect		0.0039	0.1400
Gamma	3	0.0003	0.1426
K Int Con Elect		0.0088	0.1215
L Int Con Elect		0.0035	0.1398
M Int Con Elect		0.0011	0.1422
K Alpha-1 X-Ray		0.0441	0.0183
K Alpha-2 X-Ray		0.0221	0.0182
K Beta-1 X-Ray		0.0105	0.0206
KLL Auger Elect		0.0152	0.0154
KLX Auger Elect		0.0055	0.0178
LMM Auger Elect		0.1093	0.0019
MXY Auger Elect		1.2359	0.0004

- **Raphex 2001, G 15.** The number of neutrons in a U-238 atom (Z=92) is:
  - A. 330
  - B. 238
  - C. 146
  - D. 92
  - E. Cannot tell from information given.

  ⇒ C. Neutron Number $N = A - Z = 146$

- **Raphex 2000, G15.** Elements which have the same Z but different A are called:
  - A. Isotopes
  - B. Isomers
  - C. Isotones
  - D. Isobars

  ⇒ Isotopes have the same number of protons (atomic number, Z)

- **Raphex 2003, G 16.** In heavy nuclei such as ^{235}U:
  - A. There are more protons than neutrons.
  - B. Protons and neutrons are equal in number.
  - C. There are more neutrons than protons.
  - D. Cannot tell from information given.

⇒ **C.** With higher mass number, more neutrons needed to balance the attraction of all masses (nucleons) with the repulsion between positively charged protons.

- **Raphex 2003, G12.** A 10MeV _____ travels at the greatest speed in a vacuum.
  - A. Alpha particle
  - B. Neutron
  - C. Proton
  - D. Electron

⇒ **D.** 10MeV is the kinetic energy of the particle. The lightest one travels fastest.

Raphex 2003 G 28. The following radioactive transformation represents ____.

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \gamma + \nu$$

- A. Alpha Decay
- B. Beta minus Decay
- C. Beta plus Decay
- D. Electron capture
- E. Isomeric transition

Answer: D -- As Z decreases by 1, it must be either beta plus or electron capture. However, no positron is created, so beta plus is ruled out.

C-11 has a half-life of 20 minutes. Initial sample has 1000 nuclei.

Q: How many are left after 40 minutes?
After 80 minutes?
When is less than 1 left?

A: After 2 half-lives (40 min), $1/2^2=1/4$ of the initial activity is left (25%).
After 4 half-lives (80 min), one 16th is left (6.25%).
Less than one left happens after 10 half-lives, because $(1/2)^{10} = 1/1024$, so after 200 minutes (3 hrs 20 mins).

Raphex 2002 G 23-30. Match the mode of decay to the description below:

A. Beta minus
B. Beta plus
C. Alpha
D. Isomeric

G23. Ra-226 to Rn-222
G24. Z increases by 1
G25. Z decreases by 1
G27. A and Z remain constant
G28. Tritium (H-3) to Helium (He-3)
G29. Tc-99m to Tc-99
G30. Electron capture can be a competing mode of decay to this.

Answers:
G 23: C
G 24: A
G 25: B
G 27: D
G 28: A
G 29: D
G 30: B