Introduction to Nuclear Physics

Structure of Matter

Basic Nuclear Phenomenology

Nuclear Stability and Decay

Ruth E. Schmitz, PhD - rschmitz@u.washington.edu, 543-3316

Course Website with slides, practice questions/answers:
http://depts.washington.edu/uwmip/
login: imaging password: physics

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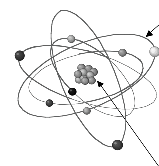
Structure of Matter

- Molecules: grouping of atoms
- Atoms:

Large sparse outer cloud: electron shells - chemistry



Atomic MODEL

Small dense core: nucleus - nuclear physics

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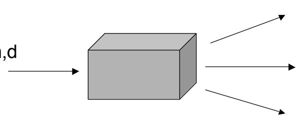
Nuclear Model

We can't see the nucleus!

Treat as black box: typically probe it with particles and/or gamma rays - see what comes out and build a model



Model: mathematical description that allows to calculate observed phenomena and to visualize the underlying processes.

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Basic Constituents of Matter

- Nucleons
  - » Constituents of the nucleus:
    - Protons (charge +1)
    - Neutrons (charge 0)
  - » Held together by the strong nuclear force
- Photons
  - » Transmit the electromagnetic force
  - » Massless
  - » A Gamma-ray is a photon produced in a nuclear reaction or decay
- Electrons, Positrons (Antielectrons)
  - » Charge -1, +1
  - » Interact via the electromagnetic and weak forces but not the strong nuclear force
  - » **Nuclear β radiation consists of electrons (β^-) or positrons (β^+)**
- ν, $\bar{\nu}$ (Neutrinos, Antineutrinos)
  - » approx. massless
  - » weak interaction only

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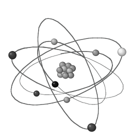
Classification of Atoms

Atom =

Z electrons orbiting a nucleus
with Z protons and N neutrons



- » Nucleus: A nucleons (A = N + Z)
  - Z protons -- 10^{-13} cm, 1.7 x 10^{-24} g (938 MeV/c^2)
  - N neutrons -- 10^{-13} cm, 1.7 x 10^{-24} g (940 MeV/c^2)

  Z = Atomic Number = Number of Protons
  N = Neutron Number
  A = N + Z = Mass Number
- » Atomic Size: 10^{-8}-10^{-7} cm = 10^{-10}-10^{-9} m
- » Nuclear Size: ~ 10^{-13} cm = 10^{-15} m = 1 fm (Fermi)
- » Atomic Mass: A atomic mass units (amu)

  1 amu ~ 1.66 x 10^{-24} g (~ 931.5 MeV/c^2) ≡ 1/12 of the mass of C-12
- » Electron Mass: $M_e = m_0 = 9.1 \times 10^{-28}$ g (0.511 MeV/c^2)

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Examples of Atom/Nucleus Classification

- Notation:

  Element (symbol X) with Z protons, N neutrons, A mass number:
  ⇒ ${}^{A}_{Z}X_{N}$, often only ${}^{A}_{Z}X$ or ${}^{A}X$ (equivalent to X-A)

  Example: Fluorine: symbol F, atomic number 9,
  isotope with 18 nucleons (-- neutron number?)
  ⇒ ${}^{18}_{9}F_{9}$, ${}^{18}_{9}F$, ${}^{18}F$, or F-18
- Nuclides:

  Nuclear species of atoms uniquely identified by number of protons, number of neutrons, and energy content of the nucleus.

  Groups that share properties:

  Isotopes - nuclides with the same proton (atomic) number, Z
  Isotones - nuclides with the same neutron number, N
  Isobars - nuclides with the same mass number, A
  Isomers - nuclides with the same A and Z, but different energy

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Chart of the Nuclides

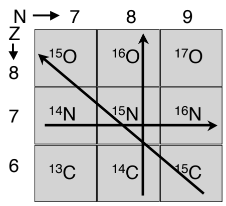
- Analogous to the Periodic Table of the elements
- Rows of constant Z (proton number): Isotopes (same chemical properties)

  Example: C-12 and C-14: Z=6
- Columns of constant N: Isotones

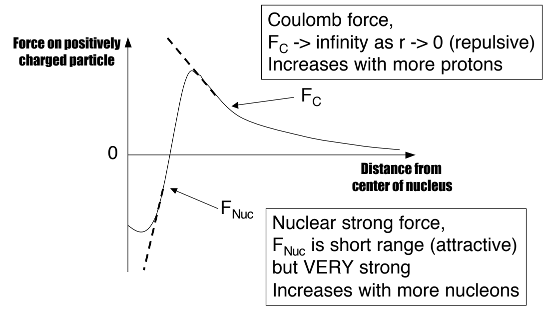
  Example: $^{131}_{53}$I (N=78) and $^{132}_{54}$Xe (N=78)
- Isobars lie on diagonals of constant mass number A

  Example: ^{99}Tc and ^{99}Mo
- Isomers are the same entry with different energy levels

  Example: ^{99m}Tc and ^{99}Tc



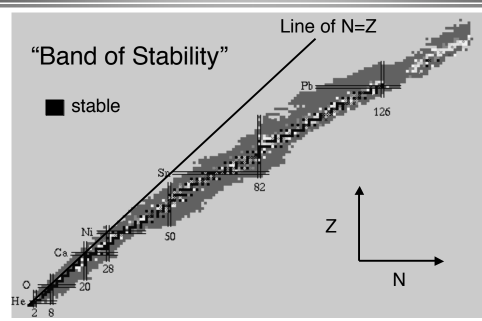
Forces in the nucleus



Factors in Nuclear Stability

- Nuclear stability represents a balance between:
  - Nuclear "strong force" (basically attractive)
  - Electrostatic interaction (Coulomb force) between protons (repulsive)
  - Pauli exclusion principle
  - Residual interactions ("pairing force", etc.)
- Stability strongly favors N approximately equal to (but slightly larger than) Z. This results in a "band of stability" in the Chart of the Nuclides.

Full Chart of the Nuclides



Phenomenology of Stability

- Stability strongly favors nuclides with even numbers of protons and/or neutrons
  - ~50% are Even-Even
  - ~25% are Odd-even
  - ~25% are Even-Odd
  - **Only 4 out of 266 stable nuclides are Odd-Odd!** The heaviest stable Odd-Odd nuclide is ^{14}N.
- "Magic Numbers" -- analogous to closed atomic shells
  - Result in many stable isotopes or isotones
  - Magic nuclei are particularly stable and more "inert"
  - Magic #'s: 2,8,20,28,50,82,126

Nuclear Binding and Stability

- Protons and neutrons are more stable in a nucleus than free. The binding energy is the amount by which the nucleus' energy (i.e. mass) is reduced w.r.t. the combined energy (i.e. mass) of the nucleons.
- Example: N-14 atom - Measured mass of N-14 = 14.0037

  mass of 7 protons = 7 * (1.00727 amu) = 7.05089 amu

  mass of 7 neutrons = 7 * (1.00866 amu) = 7.06062 amu

  mass of 7 electrons = 7 * (0.00055 amu) = 0.00385 amu

  mass of component particles of N-14 = 14.11536 amu

Binding energy is mass difference: E_{bind} = 0.11229 amu = 104.5 MeV

Ruth E. Schmitz <rschmitz@u.washington.edu>

Fundamental Concepts

- Total energy = $E = mc^2$
- Rest energy = $E_o = m_o c^2$
- Classic kinetic energy = $1/2(mv^2)$
- Classic momentum = $P = mv$
- Binding energy per nucleon = E_b (total Binding E)/A

Raphex Questions

- **Raphex 2003, G 16.** In heavy nuclei such as ^{235}U:
  - A. There are more protons than neutrons.
  - B. Protons and neutrons are equal in number.
  - C. There are more neutrons than protons.
  - D. Cannot tell from information given.

⇒ **C.** With higher mass number, more neutrons needed to balance the attraction of all masses (nucleons) with the repulsion between positively charged protons.

- **Raphex 2003, G12.** A 10MeV _____ travels at the greatest speed in a vacuum.
  - A. Alpha particle
  - B. Neutron
  - C. Proton
  - D. Electron

⇒ **D.** 10MeV is the kinetic energy of the particle. The lightest one travels fastest.

More Raphex Questions

- **Raphex 2001, G 15.** The number of neutrons in a U-238 atom (Z=92) is:
  - A. 330
  - B. 238
  - C. 146
  - D. 92
  - E. Cannot tell from information given.

  ⇒ C. Neutron Number $N = A - Z = 146$

- **Raphex 2000, G15.** Elements which have the same Z but different A are called:
  - A. Isotopes
  - B. Isomers
  - C. Isotones
  - D. Isobars

  ⇒ Isotopes have the same number of protons (atomic number, Z)

Nuclear Decay Occurs…

…when a nucleus is unstable

- An unstable nucleus metamorphoses ("decays") into a more stable nucleus
- Difference in energy levels ==> mass and kinetic energy of the decay products
- Mass is converted into energy ==> radiation

$$E = mc^2$$

Nuclear (Radioactive) Decays

- Fission -- only very heavy (high Z) elements (U, Pu, etc.) spontaneously fission. Nucleus splits into two smaller nuclei.
- Alpha decay -- like very asymmetric fission, usually occurs in heavy elements "above" the valley of stability. Nucleus emits an alpha particle: the same as a He nucleus, (2p 2n).
- Beta decay -- element X transforms into neighbor element X'. Nucleus converts a neutron to a proton or vice versa and emits a beta particle (electron): $n \rightarrow p + e^- + \nu$. - Can also occur as Electron Capture
- Gamma decay -- "excited" nucleus reduces its excitation energy without changing nuclear species (N, Z). Nucleus emits a gamma ray (electromagnetic quantum: the photon). - Can also occur as Internal Conversion Electron.

Alpha Decay

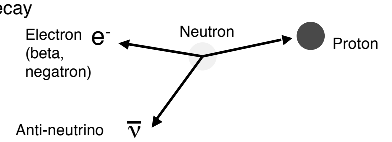
- Spontaneous emission of an α particle (2p 2n = He-4 nucleus)
- Only occurs with heavy nuclides (A>150)
- [often followed by gamma and characteristic x-ray emission]
- Emitted with discrete energy (nuclide-dependent, 2-10 MeV)
- Not used in medical imaging

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He^{+2} + \text{transition energy}$$

Example: $^{220}_{86}Rn \rightarrow {}^{216}_{84}Po + {}^{4}_{2}He^{+2}$ + 6.4 MeV transition energy

Beta (β) Decay

Basis:
- A free neutron decays:
  neutron ==> proton + electron + antineutrino
- Half-life ($T_{1/2}$) = 10.5 minutes (for a free (unbound) neutron)
- The released energy is split between 3 decay products, so each has a spectrum of possible energies up to the max
- This basic process (and its inverse) forms the basis of all β decay



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Beta (β) Decay - II

- Free neutron decay: $n \Rightarrow p + e^- + \bar{\nu}$
- Beta (β^-) emission: ${}^{A}_{Z}X_{N} \Rightarrow {}^{A}_{Z+1}Y_{N-1}$
- Positron (β^+) emission: $p \Rightarrow n + e^+ + \nu$

$${}^{A}_{Z}X_{N} \Rightarrow {}^{A}_{Z-1}Y_{N+1}$$

- Electron (e^-) capture: $p + e^- \Rightarrow n + \nu$
  Orbital electron captured, characteristic x-ray emission follows

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Gamma Decay (Isomeric Transition)

- Nucleus in excited state with lower-lying nuclear energy levels open (usually formed as daughter product of other decay)
  - Excited state marked by * (e.g. ^{99*}Tc)
- Gamma ray (high-energy photon) emitted during transition to stable state
- Usually occurs instantaneously
- Some excited states persist longer (10^{-12} sec - 600 years!)
  - Metastable or isomeric state (e.g. ^{99m}Tc,)
- Can also emit internal conversion electron - all energy is transferred to inner shell electron, which is ejected, characteristic x-rays follow to fill the opening

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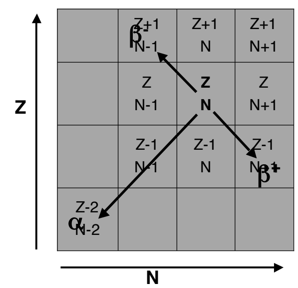
Decay in the Chart of the Nuclides

Nuclear Decay Modes



Beta (β^-) decay: ${}^{A}_{Z}X_{N} \Rightarrow {}^{A}_{Z+1}Y_{N-1}$

Positron (β^+) decay: ${}^{A}_{Z}X_{N} \Rightarrow {}^{A}_{Z-1}Y_{N+1}$

Alpha (α) decay: ${}^{A}_{Z}X_{N} \Rightarrow {}^{A-4}_{Z-2}Y_{N-2}$

Gamma (γ) decay: ${}^{A(m)}_{Z}X_{N} \Rightarrow {}^{A}_{Z}X_{N}$

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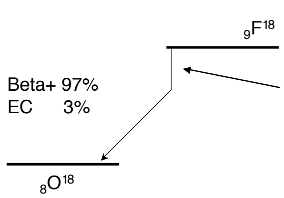
Energy Level Diagram: Positron decay



P mass = 1.67252 x 10^{-27} kg
N mass = 1.67482 x 10^{-27} kg
E mass = 0.0009 x 10^{-27} kg
Neutrino mass = 0

So, part of Energy -> mass

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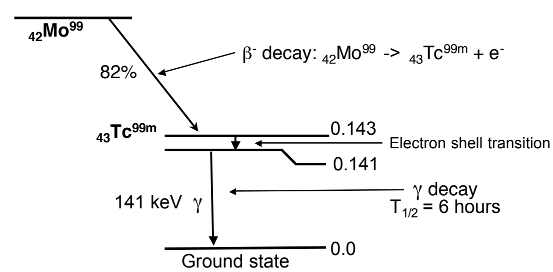
Energy Level Diagram

Nuclear Medicine Example: $_{43}Tc^{99m}$



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Decay Terms

- **Activity, A**
  - Number of radioactive decays per unit time (t) - or
  - Change in number of radioactive nuclei present: A = -dN/dt
  - Depends on number of nuclei present. During decay of a fixed initial number of nuclei, A will decrease.
  - Measured in Becquerel (Bq):
    1 Bq = 1 disintegration per second (dps)
    traditionally in Curies (Ci):
    1 Ci = 3.7×10^{10} Bq (1mCi = 37 MBq)
- **Decay Constant, λ**
  - Fraction of nuclei that will decay per unit time: $\lambda = (-dN/dt) / N = A / N$
  - Related to activity: $A = \lambda N$
  - Constant in time, characteristic of each nuclide
  - Example: Tc-99m has $\lambda = 0.1151\ hr^{-1}$, i.e. 11.5% decay per hour
    Mo-99 has $\lambda = 0.252\ day^{-1}$, i.e. 25.2% decay per day

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Decay Terms - II

- **Half-life, $T_{1/2}$**
  - » Time after which half of the initially present nuclei (N_0) will have decayed
  - » After n half-lives, $N = N_0 \times (1/2)^n$ nuclei will be left
  - » Also characteristic of nuclide, constant in time
  - » Related to decay constant, λ, by natural log of 2:
    $\lambda = ln\,2\ /\ T_{1/2} = 0.693 / T_{1/2}$

Examples:

Radionuclide	$T_{1/2}$	λ
Fluorine 18	110 min	$0.0063\ min^{-1}$
Technetium 99m	6.2 hr	$0.1152\ hr^{-1}$
Iodine 123	13.3 hr	$0.0522\ hr^{-1}$
Molybdenum 99	2.75 d	$0.2522\ d^{-1}$
Iodine 131	8.02 d	$0.0864\ d^{-1}$

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Fundamental Decay Equation

$$N_t = N_0\, e^{-\lambda t} = N_0\, e^{-t\, \log_e(2)/T_{1/2}}$$

$$A_t = A_0\, e^{-\lambda t} = A_0\, e^{-t\, \log_e(2)/T_{1/2}}$$

- N_t/A_t: Number of nuclei / activity present after time t
- λ: decay constant
- $T_{1/2}$: half-life

Example: Patient injected with 10 mCi F-18 FDG, scan started 60 min later. How much activity is present in the scan?

⇒ $A(60min) = A_0 \times e^{-\lambda t} = 10mCi \times e^{-(60min*0.0063/min)}$
$= 10\ mCi \times 0.685 = 6.85\ mCi$

Nuclear decay is statistical process => can only predict averages!

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Raphex Questions

Raphex 2003 G 28. The following radioactive transformation represents ____.

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \gamma + \nu$$

A. Alpha Decay
B. Beta minus Decay
C. Beta plus Decay
D. Electron capture
E. Isomeric transition

Answer: D -- As Z decreases by 1, it must be either beta plus or electron capture. However, no positron is created, so beta plus is ruled out.

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More Raphex Questions

Raphex 2002 G 23-30. Match the mode of decay to the description below:

A. Beta minus
B. Beta plus
C. Alpha
D. Isomeric

G23. Ra-226 to Rn-222
G24. Z increases by 1
G25. Z decreases by 1
G27. A and Z remain constant
G28. Tritium (H-3) to Helium (He-3)
G29. Tc-99m to Tc-99
G30. Electron capture can be a competing mode of decay to this.

Answers:
G 23: C
G 24: A
G 25: B
G 27: D
G 28: A
G 29: D
G 30: B

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Another Question

C-11 has a half-life of 20 minutes. Initial sample has 1000 nuclei.

Q: How many are left after 40 minutes?
After 80 minutes?
When is less than 1 left?

A: After 2 half-lives (40 min), $1/2^2$=1/4 of the initial activity is left (25%).
After 4 half-lives (80 min), one 16th is left (6.25%).
Less than one left happens after 10 half-lives, because $(1/2)^{10} = 1/1024$, so after 200 minutes (3 hrs 20 mins).

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Extra: Models of the Nucleus

- Liquid Drop model
- Shell model
- Optical model
- Collective model (includes ‘modern’ notions of string vibration states, etc).

⇒ The one of interest to Nuclear Medicine is the Shell model

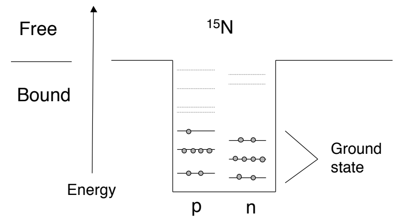
⇒ It needs to explain nuclear stability and decay

Shell model

- Similar to the electron shell model in atoms

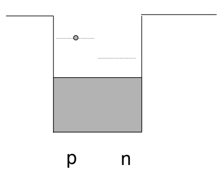
=> “Magic numbers”

- Complicated by two kinds of nucleons (proton, neutron)



^{18}F to ^{18}O

- Decay occurs because there is a neutron level open at a lower energy than an occupied proton level

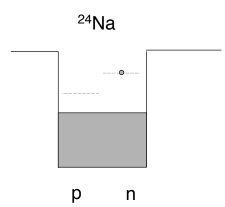


$p \rightarrow n + e^+ + \nu$

Positron decay

^{24}Na to ^{24}Mg

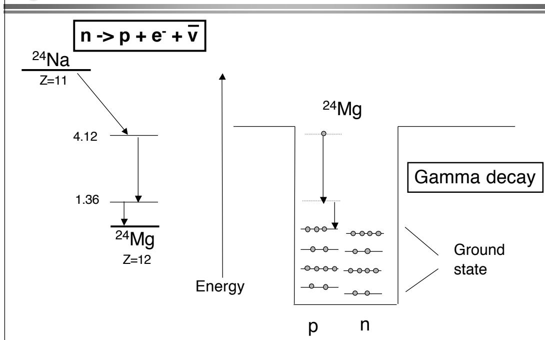
- Decay occurs because there is a proton level open at a lower energy than an occupied neutron level



$n \rightarrow p + e^- + \bar{\nu}$

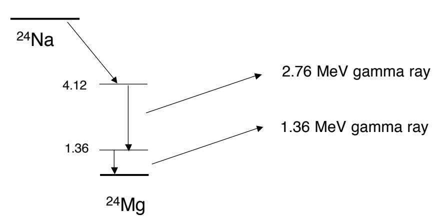
Beta decay

Consider ^{24}Mg



Where does the energy go?

- When the nucleon changes levels (but not species), the energy is usually emitted as a gamma ray (or internal conversion electron).



Ruth E. Schmitz <rschmitz@u.washington.edu>

Lets recap a few points

Nuclear Decay Occurs...

…when a nucleus is unstable (lower open energy levels)

- An unstable nucleus metamorphoses ("decays") into a more stable (more tightly bound) nucleus
- Difference in binding energy ==>
  mass and kinetic energy of the decay products
- Mass is converted into energy ==> radiation

$E = mc^2$

Nuclear Decay Characteristics

- Type of decay (fission, alpha, beta, electron capture, etc.)
- Decay constant (transformation rate)
  - $N = N_0 e^{-\lambda t}$ Half-life, $T_{1/2} = 0.693 / \lambda$
- Radiation type (β^+, β^-, α, fission fragments, etc.)
- Emission energy -- if continuum, then express as maximum energy or mean (average) energy
- Associated gamma (γ) or x rays
- "Daughter nucleus"
  - is it stable?
  - Produced in "ground state" or "excited state"?
  - With what probabilities ("branching ratios")?

What's next

Next week we will take a look at
Radiation detection and measurements
Dr. Thomas K. Lewellen