

Radiation Detection and Measurement

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Types of radiation relevant to Nuclear Medicine

Particle	Symbol	Mass (MeV/c ²)	Charge
Electron	e ⁻ , β ⁻	0.511	-1
Positron	e ⁺ , β ⁺	0.511	+1
Alpha	α	3700	+2
Photon	γ	no rest mass	none

α Particle Range in Matter

mono-energetic

- Loses energy in a more or less continuous slowing down process as it travels through matter.
- The distance it travels (range) depend only upon its initial energy and its average energy loss rate in the medium.
- The range for an α particle emitted in tissue is on the order of μm's.

β Particle Range in Matter

continuous energy spectrum

- β particle ranges vary from one electron to the next, even for βs of the same energy in the same material.
- This is due to different types of scattering events the β encounters (i.e., scattering events, bremsstrahlung-producing collisions, etc.).
- The **β range** is often given as the **maximum distance** the **most energetic** β can travel in the medium.
- The range for β particles emitted in tissue is on the order of mm's.

Interactions of Photons with Matter

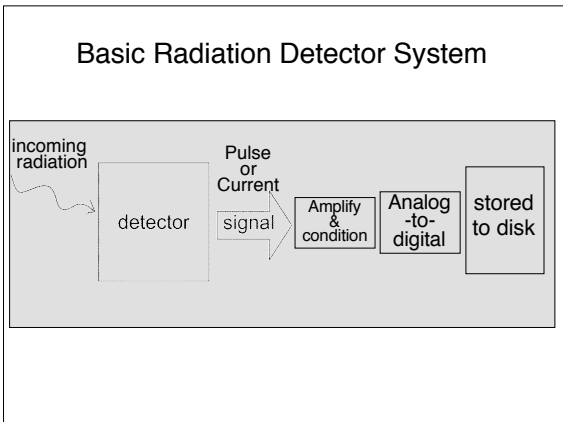
Exponential Penetration: $N=N_0e^{-\lambda x}$

Photoelectric effect
photon is absorbed

Compton scattering
part of the energy of the photon is absorbed
scattered photon continues on with lower energy

Pair production
positron-electron pair is created
requires photons above 1.022 MeV

Coherent (Rayleigh) scattering
photon deflected with very little energy loss
only significant at low photon energies (<50 keV)



Basic Radiation Detector Systems

What do you want to know about the radiation?

Energy?

Position (where did it come from)?

How many / how much?

Important properties of radiation detectors

(depends on application)

Energy resolution

Spatial resolution

Sensitivity

Counting Speed

Pulse Mode versus Current Mode

- Pulse mode
 - Detect individual photons
 - Required for NM imaging applications
- Current mode
 - Measures average rates of photon flux
 - Avoids dead-time losses

Types of Radiation Detectors

detection modes / functionality

- Counters
 - Number of interactions
 - Pulse mode
- Spectrometers
 - Number and energy of interactions
 - Pulse mode
- Dosimeters
 - Net amount of energy deposited
 - Current mode
- Imaging Systems
 - CT = current mode
 - NM = pulse mode

Types of Radiation Detectors

physical composition

- Gas-filled detectors
- Solid-state (semiconductor) detectors
- Organic scintillators (liquid & plastic)
- Inorganic scintillators

scintillators operate with a
photo-sensor
(i.e. another detector)

Gas-filled Detectors

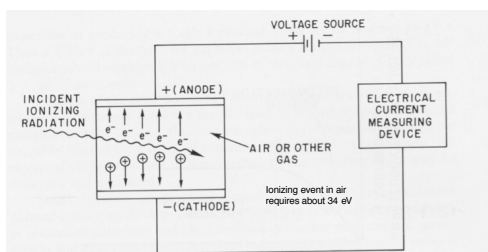
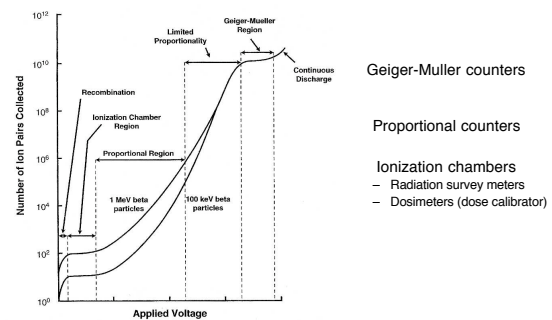


Fig. 4-1. Basic principles of a gas-filled detector. Electrical charge liberated by ionizing radiation is collected by positive and negative electrodes.

From: Physics in Nuclear Medicine (Sorenson and Phelps)

Gas-filled detectors

(operates in three ranges)



Geiger-Muller counters

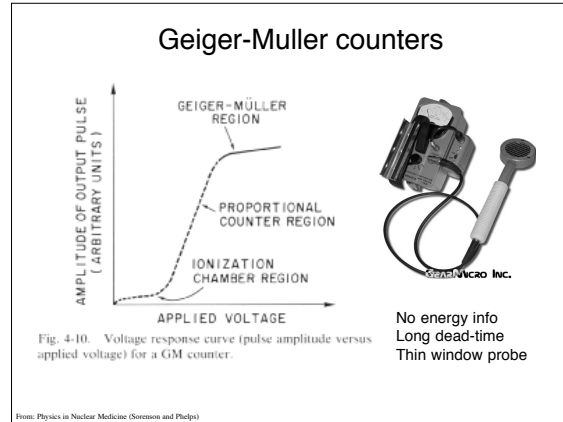
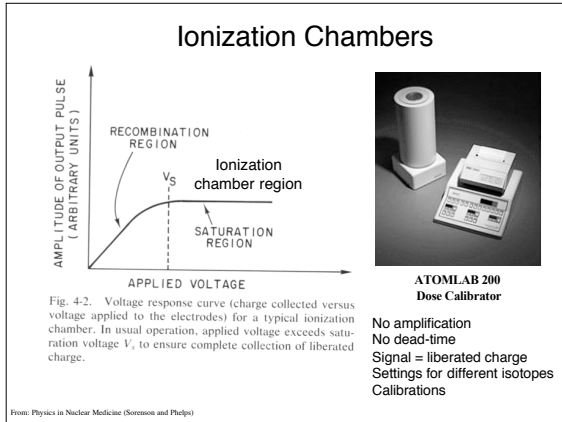
Proportional counters

Ionization chambers

– Radiation survey meters

– Dosimeters (dose calibrator)

From: Radiation Detection and Measurement (Knoll, GP)



Semiconductor Detectors

- Works on same principle as gas-filled detectors (i.e., production of electron-hole pairs in semiconductor material)
- Only ~3 eV required for ionization (~34 eV, air)
- Usually needs to be cooled (thermal noise)
- Usually requires very high purity materials or introduction of "compensating" impurities that donate electrons to fill electron traps caused by other impurities

Semiconductor Detectors

- CdZnTe detectors - can operate at room temperature - starting to show up in some dedicated cardiac gamma cameras

Position Sensitive CZT Detectors

SIEMENS
Advanced Research
Siemens Medical Systems, Inc.
Nuclear Medicine Group
2501 North Harrington Road
Hoffman Estates, Illinois 60135

Organic Liquid Scintillators

(liquid scintillator cocktail)

- Organic solvent - must dissolve scintillator material and radioactive sample
- Primary scintillator (p-terphenyl and PPO)
- Secondary solute (wave-shifter)
- Additives (e.g., solubilizers)
- **Effective for measuring beta particles** (e.g., H-3, C-14).

Liquid scintillator

Inorganic Scintillators

(physical characteristics)

Absorption of radiation lifts electrons from valence to conduction band

Impurities (activators) create energy levels within the band gap permitting visible light scintillations

Inorganic Scintillators

(physical characteristics)

	NaI(Tl)	BGO	LSO(Ce)	GSO(Ce)
Density (gm/cm³)	3.67	7.13	7.4	6.71
Effective Atomic Number	51	75	66	59
Attenuation Coefficient (@ 511 keV, cm⁻¹)	0.34	0.955	0.833	0.674
Light Output (photons/MeV)	40K	~8K	~30K	~20K
Decay Time	230 ns	300 ns	12 ns 40 ns	60 ns
Wavelength	410 nm	480 nm	420 nm	430 nm
Index of Refraction	1.85	2.15	1.82	1.85
Hygroscopy	yes	no	no	no
Rugged	no	yes	yes	no

relevant detector property

sensitivity

energy & spatial resol.

counting speed

photo-sensor matching manufacturing / cost

}
}

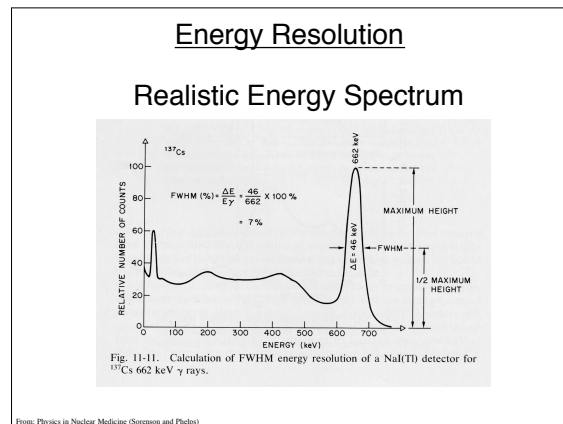
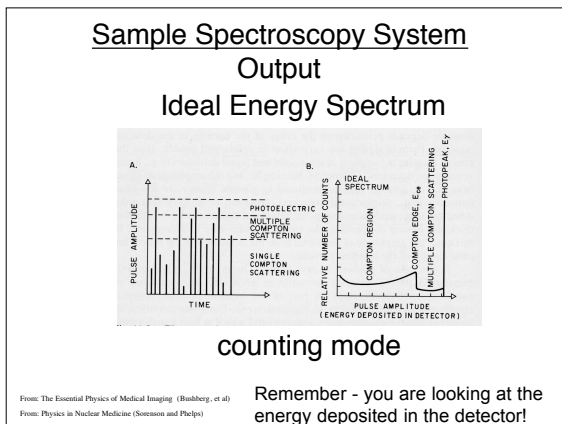
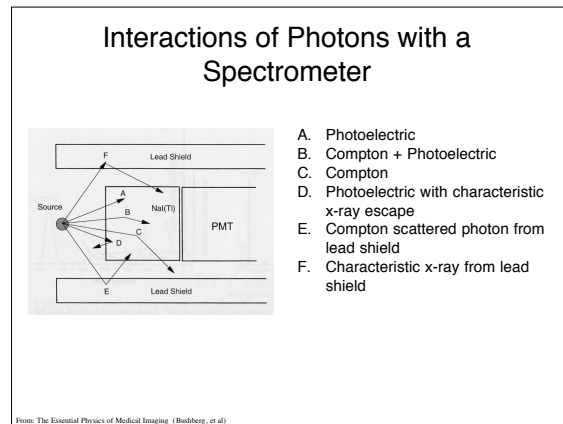
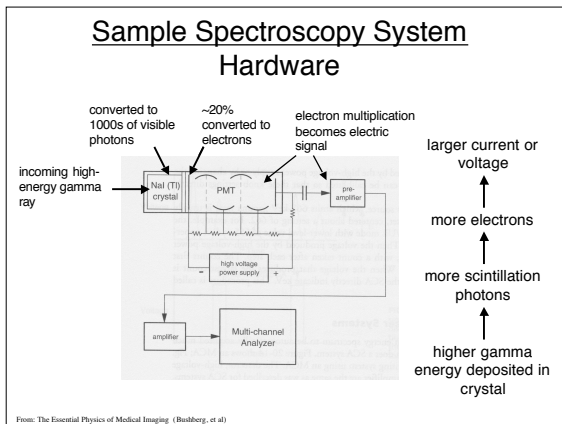
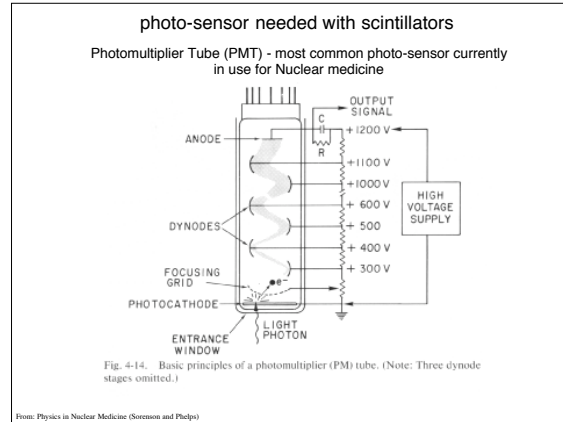
relevant detector property

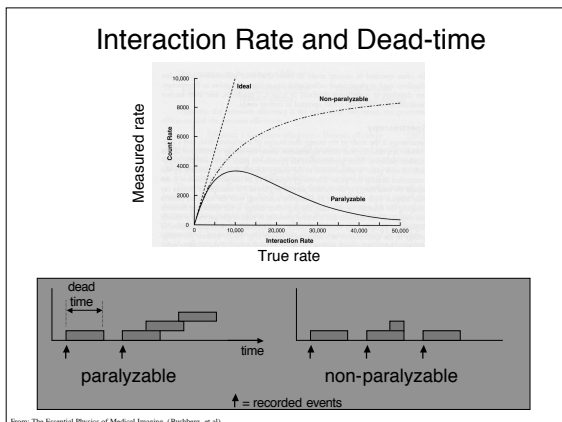
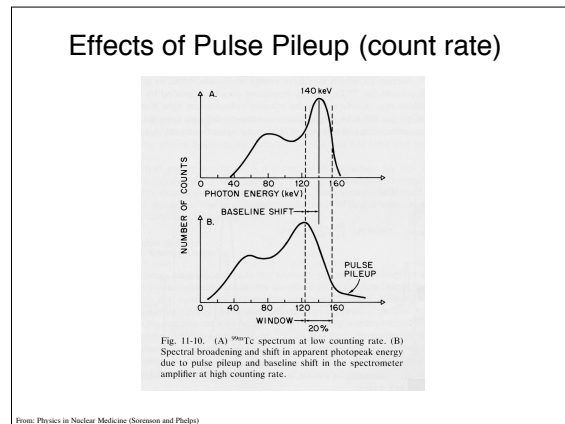
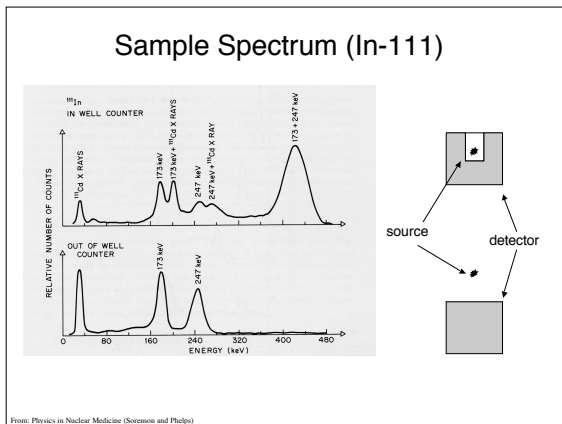
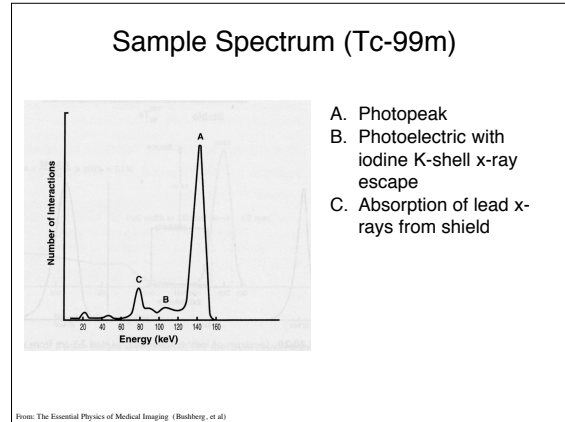
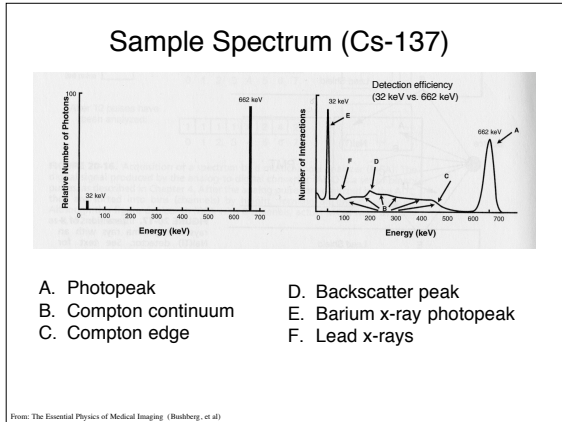
sensitivity

energy & spatial resol.

counting speed

photo-sensor matching manufacturing / cost





Calibrations

- Energy calibration (imaging systems/spectroscopy)
 - Adjust energy windows around a known photopeak
 - Often done with long-lived isotopes for convenience
 - Cs-137: $E_\gamma = 662 \text{ keV}$ (close to PET 511 keV), $T_{1/2} = 30 \text{ yr}$
 - Co-57: $E_\gamma = 122 \text{ keV}$ (close to Tc99m 140 keV), $T_{1/2} = 272 \text{ d}$
- Dose calibration (dose calibrator)
 - Measure activity of known reference samples (e.g., Cs-137 and Co-57)
 - Linearity measured by repeated measurements of a decaying source (e.g., Tc-99m)

Raphex Question

D58. The window setting used for Tc-99m is set with the center at 140 keV with a width of +/-14 keV i.e., 20%. The reason for this is:

- A. The energy spread is a consequence of the statistical broadening when amplifying the initial energy deposition event in the NaI(Tl) crystal.
- B. The 140 keV gamma ray emission of Tc-99m is not truly monoenergetic but the center of a spectrum of emissions.
- C. The higher and lower Gaussian tails are a consequence of Compton scattering within the patient.
- D. The result of additional scattered photons generated in the collimator.
- E. A consequence of patient motion during scanning.

Raphex Answer

D58. The window setting used for Tc-99m is set with the center at 140 keV with a width of +/-14 keV i.e., 20%. The reason for this is:

- A. Photons, which impinge upon the crystal, lose energy by Compton scattering and the photoelectric effect. Both processes convert the gamma ray energy into electron energy. On average approximately one electron hole pair is produced per 30 eV of gamma ray energy deposited in the crystal. These electrons result in the release of visible light when trapped in the crystal. These light quanta are collected and amplified by photomultiplier tubes. The statistical fluctuation in the number of light quanta collected and their amplification is what causes the spread in the detected energy peak, even when most of the Tc-99m photons deposit exactly 140 keV in the NaI(Tl) crystal.

Counting Statistics

Sources of Error

- Systematic errors
 - Consistently get the same error
- Random errors
 - Radiation emission and detection are random processes
- Blunder
 - operator error

Definition of terms

Series - a number of test results which possess common properties that identify them uniquely.

Range - the difference in magnitude between the highest and lowest value in the series.

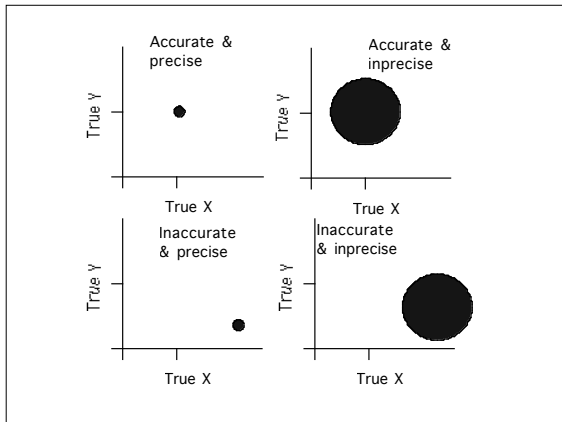
Mean Value - the hypothetical "true" value calculated by the sum of the events divided by the number of events.

Definition of terms

Accuracy - the degree of correctness of an answer - ie, how closely does it approximate the true answer.

Error - the amount that a value deviates from the true value. $E = X - \bar{X}$

Precision - the value as related to the spread of values of similar measurements. The degree of reproducibility of a series of measurements



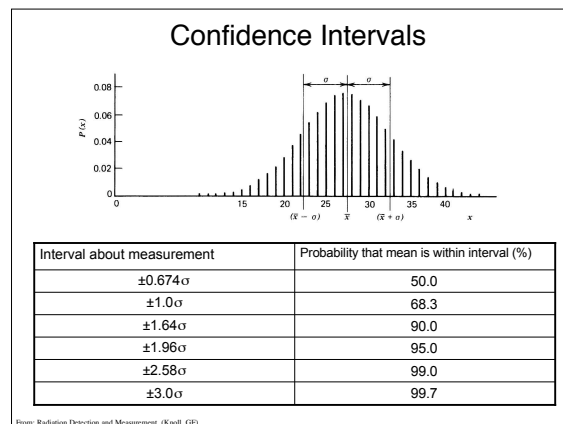
Measures of Central Tendency

- Mean
 - Average value
- Median
 - Middlemost measurement (or value)
 - Less affected by outliers

Example: 8, 14, 5, 9, 12
Mean = 9.6
Median = 9

Measures of Variability

- Variance
 - Measure of variability:
$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_N - \bar{x})^2}{N - 1}$$
- Standard deviation
 - Square root of variance
$$\sigma = \sqrt{\sigma^2}$$



Two Basic Things to Keep in Mind

Radioactive Decay is a Random Phenomenon

The distribution of random events is described mathematically by the Poisson distribution.

Poisson Probability Distribution Function

- Radioactive decay and detection are Poisson random processes
 - Observation time is short compared to the half-life of the source
 - probability of radioactive decays (i.e., p) remains constant
 - probability of a given nucleus undergoing decay is small
- Variance
 - Variance = mean = $pN = x$
 - where p = probability of an event and N = total number of events.
- Standard deviation
 - Standard deviation = $\sqrt{\text{variance}} = \sqrt{pN} = \sqrt{x}$
- Can estimate standard deviation from a single measurement
- If we make a single measurement (x) then there is a 68.3% probability that the real mean values is within $\pm \sqrt{x}$ of x

Raphex Question

D70. How many counts must be collected in an instrument with zero background to obtain an error limit of 1% with a confidence interval of 95%?

- A. 1000
- B. 3162
- C. 10,000
- D. 40,000
- E. 100,000

Raphex Answer

D70. How many counts must be collected in an instrument with zero background to obtain an error limit of 1% with a confidence interval of 95%?

- D. A 95% confidence interval means the counts must fall within two standard deviations (SD) of the mean (N). Error limit = 1% = 2 SD/N, but SD = $N^{1/2}$. Thus $0.01 = 2(N^{1/2})/N = 2/N^{1/2}$. Where $[0.01]^2 = 4/N$ and $N = 40,000$.

Propagation of Error

Description	Operation	Standard Deviation
Multiplication of a number with random error (x) by a number without random error (const., c)	cx	c σ
Division of a number with random error (x) by a number without random error (c)	x/c	σ/c
Addition of two numbers containing random errors	$x_1 + x_2$	$\sqrt{\sigma_1^2 + \sigma_2^2}$
Subtraction of two numbers containing random errors	$x_1 - x_2$	$\sqrt{\sigma_1^2 + \sigma_2^2}$

Note the PLUS sign: combination of two measurements with error leads to result with greater overall error - errors add "in quadrature". Errors compound, they can't 'cancel' each other.

Propagation of errors:

Multiplication and division

For N_1, N_2, \dots, N_n the variance, V, is: $V_i = \frac{\sigma_{N_i}}{N_i}$

$$V(A * / B) = \sqrt{V_A^2 + V_B^2}$$

Count Rate

$$\text{Rate} = R = \frac{N}{T}$$

$$\sigma_R = \frac{\sigma_N}{T} = \frac{\sqrt{N}}{T} = \sqrt{\frac{R}{T}}$$

Raphex question

G74. A radioactive sample is counted for 1 minute and produces 900 counts. The background is counted for 10 minutes and produces 100 counts. The net count rate and net standard deviation are about ____ and ____ counts.

- A. 800, 28
- B. 800, 30
- C. 890, 28
- D. 890, 30
- E. 899, 30

Raphex answer

G74. A radioactive sample is counted for 1 minute and produces 900 counts. The background is counted for 10 minutes and produces 100 counts. The net count rate and net standard deviation are about ____ and ____ counts/min.

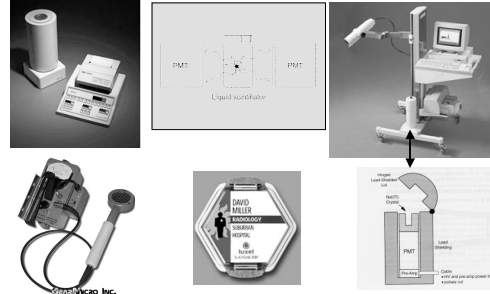
D. The net count rate is:

$$[(N_s/t_s) - (N_b/t_b)] = [(900/1) - (100/10)] = 890.$$

The net standard deviation, σ is:

$$[(N_s/t_s) + (N_b/t_b)]^{1/2} = [(900) + (1)] = 30.$$

What piece of equipment would you use to measure the activity of a pure beta emitter?



From: The Essential Physics of Medical Imaging (Bushberg, et al)

Radiation detectors used in Nuclear Medicine



Statistical Models for Random Trials

- Binomial Distribution
- Poisson Distribution
 - Simplification of binomial distribution with certain constraints
- Gaussian or Normal Distribution
 - Further simplification if average number of successes is large (e.g., >20)

Binomial process

- Trial can have only two outcomes

Trial	Definition of a success	Probability of a success
Toss of a coin	"Heads"	1/2
Toss of a die	"A four"	1/6
Observation of a radioactive nucleus for a time "t"	It decays	$1 - e^{-\lambda t}$
Observation of a detector of efficiency E placed near a radioactive nucleus for a time "t"	A count	$E(1 - e^{-\lambda t})$

Source: Adapted from Knoll, GF. *Radiation detection and measurement*, 3rd ed. New York: John Wiley, 2000.

From: The Essential Physics of Medical Imaging (Bushberg, et al)

Binomial probability density function (PDF)

$$P(x) = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

- N is total number of trials
- p is probability of success
- x is number of successes

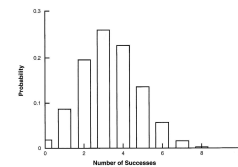


FIGURE 20-28. Binomial probability distribution function when the probability of a success in a single trial (p) is 0.5 and the number of trials (N) is 10.

From: The Essential Physics of Medical Imaging (Bushberg, et al)

**Binomial probability density function
mean and variance**

$$\bar{x} = pN \quad \text{and} \quad \sigma = \sqrt{pN(1-p)}$$

- N is total number of trials
- p is probability of success
- \bar{x} is mean, σ is standard deviation

If p is very small and a constant then:

$$\sigma = \sqrt{pN(1-p)} \approx \sqrt{pN} = \sqrt{\bar{x}}$$

Same as Poisson random process.

An example for a flood imaging from a gamma camera

64 x 64 image array (OK for cardiac, small for other studies)

Rectangular field-of-view covering 3400 pixels

Total image counts	counts/pixel	sigma	% sigma
100,000	29	5.4	18.60%
500,000	147	12.1	8.20%
1,000,000	294	17.1	5.80%
2,000,000	588	24.2	4.10%
10,000,000	2941	54.2	1.80%
50,000,000	14706	121.3	0.80%
100,000,000	29412	171.5	0.60%