WXML Final Report: Prime Spacing

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Autumn 2016

1 Introduction

1.1 The initial problem

In Mathematics, prime gaps are widely studied due to the potential insights they could tell about the distribution of the primes, and methods they could reveal about obtaining large prime numbers.

Our research of prime spacing focused on studying the prime intervals, which is defined as $p_n - p_{n-1} - 1$, where p_n is the n^{th} prime. Specifically, we looked at what we defined as prime-prime intervals, that is primes having $p_n - p_{n-1} - 1 = r$ where r is prime. Also, we looked at $p_n - p_{n-k} - 1 = r$ where k is an integer greater than 1 in addition to having r be prime.

1.2 New directions

The idea of studying the primeness of prime intervals had not been previously researched, so there were many directions to take from the initial proposed problem. We saw multiple areas that needed to be studied; the prevalence of prime-prime intervals in the set of all prime intervals, the distribution of the values of prime intervals, and how the statistics on prime-prime intervals changed when we expanded from looking at $p_n - p_{n-1} - 1$ to looking at $p_n - p_{n-k} - 1$.

2 Progress

2.1 Computational

The first step taken was to see how many prime intervals up to the first n^{th} prime were prime. Our original thoughts were that the amount of prime-prime intervals would grow at a rate of $\pi(\pi(x))$, since the function π tells us how many primes there are up to some number x. However, our findings suggest that the ratio of prime-intervals that are prime is close to $\frac{2}{3}\pi(x)$ up to $\approx$ 100,000,000.

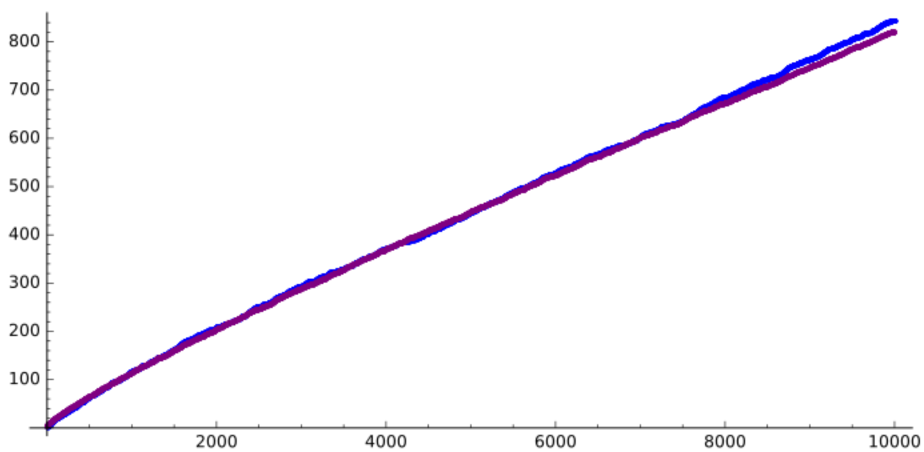

Figure 1: In purple we have $\pi(x)$, in blue we have $I(x)$

Likely, this strange finding is due to the bias that primes have towards small numbers, as 62.5% of the first 16 odd numbers are prime.

We have also looked into the number of prime-prime intervals if we take $p_n - p_{n-k} - 1$ for different values of k. We found that if we take the ratio of these intervals for all k that the ratio of prime-prime intervals to all the intervals gets smaller and smaller.

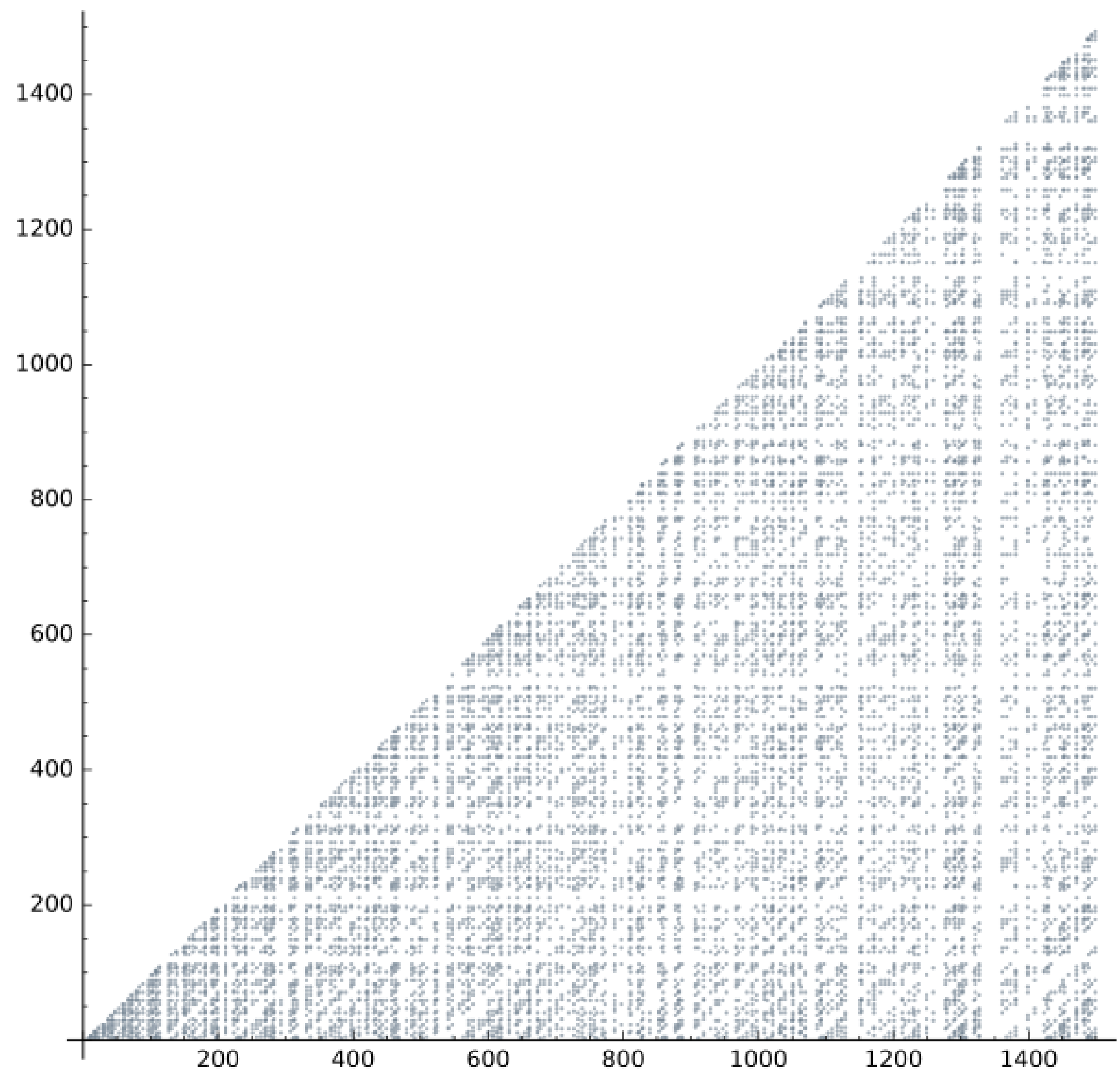

Figure 2: View of all prime intervals $p_n - p_{n-k} - 1$, with p_n on the x-axis, p_{n-k} on the y-axis

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AllPrimeIntervals(k)
AllPrimeIntervals(100)
0.518400000000000
AllPrimeIntervals(1000)
0.403486394557823
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AllPrimeIntervals(10000)
0.301329214447966
AllPrimeIntervals(100000)
0.232280092842172
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The percentages above show that the ratio of prime-prime intervals to all prime intervals up to some number, k, is slowly decreasing.

To further examine the list of prime-prime intervals, we sorted them into lists of their remainder mod 3 and found for primes up to 430,000,000 that there were 1609019 prime-prime intervals divisible by 3, 4022318 prime-prime intervals congruent to 1 mod 3, and 9480830 prime-prime intervals congruent to 2 mod 3. We did the same with mod 5 and found 2843566 were divisible by 5, 2851839 were congruent to 1 mod 5, 3284512 were congruent to 2 mod 5, 4176292 were congruent to 3 mod 5, and 1955958 were congruent to 4 mod 5.

2.2 Theoretical

We developed no mathematical proofs that related to prime-prime intervals. However, we have hypothesized that there are an infinite number of prime intervals that are prime. We also have evidence to believe that every prime shows up as a prime prime interval at least once. Since primes are infinite, there is a strong possibiliy that the prime intervals will continue to increase infinitey as well.

Extending off of our computational findings, we believe that the ratio of all prime-prime intervals $p_n - p_{n-k} - 1 = r$, to those r that are not prime, is going to zero. This is what is being displayed in the $AllPrimeIntervals()$ function.

3 Future directions

For the future, we would like to explore more into the $\frac{2}{3}$ ratio and see how it is changing. It is possible that this is really a function of the number x, so that in $I(x) = c * \pi(x)$, $c = c(x)$ is changing as x increases. We would also like to look more into the gaussian primes, which have problems when being used in research of *intervals*, as the complex plane is not an ordered set.